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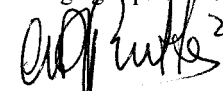
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## Compositional Specification of a Multi-Agent System by Stochastically and Dynamically Coloured Petri Nets

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**Summary.** For safety-critical operations in the nuclear and chemical industries, Petri nets have proven to be useful for the compositional specification of appropriate accident risk assessment models. For air traffic operations, the development of such model is more challenging due to the high distribution of and complex interactions between the multiple agents involved. The specific problems are: A) Need for a hierarchy from low level Petri nets to the complete Petri net; B) Duplication of arcs and transitions within a low level Petri net; C) Cluttering of interconnections. The chapter develops adequate solutions for each of these problems. The solution approaches are first explained graphically, and next formally. The approach developed is illustrated for an air traffic operation example.

### 1 Introduction

The aim of this chapter is to extend the compositional specification power of Petri nets for application to a multi-agent hybrid system. The motivating type of application is accident risk assessment of safety-critical operations in general, and of air traffic operations in particular. For safety-critical operations in e.g. nuclear and chemical industries, it is common practice that accident risk assessment models are being developed to provide valuable feedback to the process of design and certification of a change (e.g. [16], [18]). Accident risk assessment could play a similar valuable role in the design of novel air traffic operations.

By the very nature of air traffic management, the various decision-makers are highly distributed: per aircraft there is a crew of pilots, and per air traffic control centre there are many human operators. In addition, the safety related decision-making process involves interactions of these humans with each other and with:

- a random and often unpredictable environment, e.g. varying wind, thunderstorms, etc.,



- a large set of procedural rules and guidelines,
- many technical and automation support systems,
- decision-makers at airline operation centres.

These aspects make accident risk assessment for air traffic operations a very challenging application area, the decision making process of which is significantly more complex than it is of operations in other safety-critical industries as is illustrated in Figure 1 of the Introduction of this book. This makes the specification of an unambiguous mathematical model of air traffic operations a very challenging task.

The most advanced approaches that have been developed in literature to model accident risk of safety-critical operations in nuclear and chemical industries make use of the compositional specification power of Petri nets to instantiate a model, and subsequently use stochastic analysis and Monte Carlo simulation (e.g. [16]) to evaluate the model. Since their introduction in the 1960s, Petri nets have shown their usefulness for many practical applications in different industries (e.g. [4]). Various Petri net extensions and generalisations, new analysis techniques, and numerous supporting computer tools have been developed, which further increased their modelling opportunities, though falling short for air traffic operations. In order to capture the characteristics of air traffic operations through a Petri net, [7] extended Dynamically Coloured Petri Net (DCPN) of [6], [5], [8] to Stochastically and Dynamically Coloured Petri Net (SDCPN) and proved that there exists a close relationship with the larger class of Generalised Stochastic Hybrid Processes (GSHP) needed to model air traffic operations. Basically, a DCPN is an extension of Coloured Stochastic Petri Net (e.g. [12]), in the sense that in DCPN the token colours evolve in time (dynamically) as solutions of differential equations while the tokens reside in their places. In an SDCPN, differential equations are replaced by stochastic differential equations. The DCPN formalism has been successfully used in practical air traffic applications, (e.g. [2], [3]). However, it was found that when being used for modelling more and more complex multi-agent hybrid systems, the compositional specification power of Petri nets reaches its limitations. More specifically, the following problems were identified:

- A. Need for a hierarchy from low level Petri nets to the complete Petri net. For the modelling of a complete Petri net for complex systems, a hierarchical approach is necessary in order to be able to separate local modelling issues from global or interaction modelling issues.
- B. Duplication of arcs and transitions within a low level Petri net. Often the addition of an interconnection between two low-level Petri nets leads to a duplication of transition and arcs in the receiving Petri net.
- C. Cluttering of interconnections. The number of interconnections between the different low level Petri nets tends to grow quadratically with the size of the Petri net.



## 2 Compositional Specification Challenge

In literature, approaches have been developed to address problem A. These approaches are outlined below.

Ref. [14] introduced Hierarchical Coloured Petri Nets. These Hierarchical CPNs allow a set of subnets, called pages, to be related to each other, in such a way that together they constitute a single model. The pages interact with each other in a well-defined way. A page can also be substituted by a place or a transition, in order to show its role in the larger model, or to postpone its detailed modelling until later. In addition to these substitution transitions and places, Hierarchical CPN allow invocation transitions (CPN is temporarily extended with a new instance of an invocation subpage), place fusion (a set of places is folded into a single place) and transition fusion (a set of transitions is folded into a single transition). The pages that interact solve problem A.

More recent approaches also address problem A; they consider elementary Petri nets that have input (or entry) and output (or exit) places through which these Petri nets are coupled with other Petri nets. One example approach is  $B(PN)^2$  (Basic Petri Net Programming Notation), introduced by Best and Hopkins (see e.g. [10]). The compositional denotational semantics of  $B(PN)^2$  programs can be given in terms of M-nets (modular multilabelled nets), which form an algebra of composable high-level Petri nets. These Petri net components have at least one entry place and at least one exit place. Several composition operations (e.g. parallel composition, sequential composition) are defined to couple the Petri nets. Communication is performed by transition synchronisation. Another example approach is by [15], who introduced the concept of Petri net Components and showed how systems can be composed from components. These components have input and output places and components can be connected at these input and output places. Ref. [15] also provides the compositional semantics.

Also addressing problem A are [9] and [11], who consider sub-Petri nets that model parallel systems, and draw these sub-Petri nets in separate boxes. Places and transitions in different sub-Petri nets are coupled by arcs to model interactions. Ref. [9] uses Synchronous Interpreted Petri Nets (SIPN) as basis and shows how the interactions can be used to model synchronisation or priority of the parallel systems. Ref. [9] also allows hierarchy: a macroplace can be exploded (or imploded) to form (or hide) a complete sub-Petri net. Ref. [11] uses Generalised Stochastic Petri nets (GSPNs), refers to the sub-Petri nets as modules, and adopts the requirement that there should be exactly one token in each module; transitions in a module are not allowed to consume a token from another module without returning one immediately. Therefore, [11] introduced three module coupling mechanisms: 1) marking tests; 2) common transitions; 3) interconnection blocks. In addition, in order to improve the compactness of the module, [11] recommends two rules, called optimisa-

tion rules: 1) avoidance of immediate internal transitions; 2) module folding using memories.

For addressing problem B, some ideas from literature are useful. In order to avoid the duplication of transitions, one might apply transition fusion as proposed by [14], or module folding of [11].

The aim of this chapter is to combine and adopt the approaches from literature that solve problem A and to develop approaches to solve problems B and C, and to organise these new developments into a compositional specification approach for SDCPN. In addition, also the effectiveness of the approach is illustrated for the modelling of an air traffic example. The relations between our approach and those found in literature are explained in Sections 4 and 5, and are summarised below.

To solve problem A, the compositional specification of a SDCPN for a complex process or operation starts with developing a Local Petri Net (LPN) for each agent that exists in the process or operation (e.g. air traffic controller, pilot, navigation and surveillance equipment). Counterparts of LPNs in literature are the modules of [11], the pages of [14] and the components of [15]. An essential difference is that our LPNs (and [11]'s modules) are connected with each other such that the number of tokens residing in an LPN is not influenced by these interconnections, while [14] and [15] do not pose this restriction.

We use two types of interconnections between nodes and arcs in different LPNs:

- Enabling arc (or inhibitor arc) from one place in one LPN to one transition in another LPN. These types of arcs have been used widely in Petri net literature, including [11] for inhibitor arcs and [9] for both types.
- Interaction Petri Net (IPN) from one (or more) transition(s) in one LPN to one (or more) transition(s) in another LPN. These IPNs are similar to the interconnection blocks of [11]. If an IPN consists of one place only, then the connection of two LPNs through an IPN also has some similarity with place fusion, see e.g. [14] or [15], except that our IPN will not change the number of tokens in its connecting LPNs.

Each LPN is surrounded by a box. This boxing idea has also been used by e.g. [9] or [15]. Next, to solve problems B and C, we identify additional interconnections between LPNs that allow, with well-defined meanings, arcs to initiate and/or to end on the edge of the box surrounding an LPN. To the authors knowledge this element has no counterpart in Petri net literature; however, it is based on how [13] composes statecharts. The meaning of these interconnections from or to an edge of a box allows several arcs or transitions to be represented by only one arc or transition. In that sense, there is a relation with transition fusion used by [14] and with module folding used by [11].

This chapter is organised as follows: Section 3 briefly outlines SDCPN; for a more complete definition we refer to [7]. Section 4 outlines how a SDCPN can be specified in a logical sequence for each entity of an agent, and explains how the entities of agents are connected without changing the structure of low



level entities. This solves problem A above. Section 5 defines some new Petri net clustering types which avoid the internal duplication problem (problem B) and the problem of cluttering interconnections (problem C). It is noted that these clustering types can also be applied to other Petri net extensions than SDCPN. Section 6 extends the SDCPN definition of Section 4 to include these new clustering types. In Section 7 the approach developed is illustrated for an air traffic operation example. Finally, Section 8 gives concluding remarks.

### 3 Stochastically and Dynamically Coloured Petri Net

This section gives a definition of SDCPN. Subsection 3.1 describes the SDCPN elements, while Subsection 3.2 describes the SDCPN evolution rules. These elements and evolution rules together form the SDCPN definition. For a more formal SDCPN definition and a simple SDCPN example we refer to [5] and to [7].

#### 3.1 SDCPN Elements

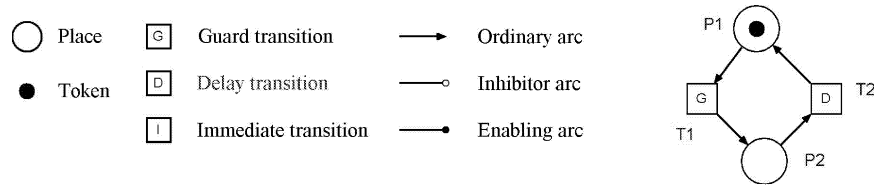
A Stochastically and Dynamically Coloured Petri Net [7] is given by the following tuple:  $\text{SDCPN} = (\mathcal{P}, \mathcal{T}, \mathcal{A}, \mathcal{N}, \mathcal{S}, \mathcal{C}, \mathcal{V}, \mathcal{W}, \mathcal{G}, \mathcal{D}, \mathcal{F}, \mathcal{J})$ , where:

- $\mathcal{P}$  is a set of places
- $\mathcal{T}$  is a set of transitions which consists of a set of guard transitions ( $\mathcal{T}_G$ ), a set of delay transitions ( $\mathcal{T}_D$ ) and a set of immediate transitions ( $\mathcal{T}_I$ ).
- $\mathcal{A}$  is a set of arcs which consists of a set of ordinary arcs ( $\mathcal{A}_o$ ), a set of enabling arcs ( $\mathcal{A}_e$ ) and a set of inhibitor arcs ( $\mathcal{A}_i$ ).
- $\mathcal{N}$  is a node function which maps each arc to an ordered pair of one transition and one place; multiple arcs between the same place and transition are allowed.
- $\mathcal{S}$  is a set of colour types for the tokens occurring in the net (a colour is the value of an object or process in Petri net terminology). Each colour type is to be of the form  $\mathbb{R}^n$ .
- $\mathcal{C}$  is a colour function which maps each place to a colour type in  $\mathcal{S}$ .
- $\mathcal{V}$  and  $\mathcal{W}$  are sets of place-specific colour functions which together describe the behaviour of the colour of a token while it resides in its place. For each place, these elements determine a stochastic differential equation, which is locally Lipschitz continuous.
- $\mathcal{G}$  is a set of Boolean-valued transition guards associating each transition in  $\mathcal{T}_G$  with a guard function. This guard function is continuously evaluated when the transition has a token in each of its input places, i.e., when there is at least one token per input arc of the transition present. The guard function must evaluate to True before the transition is allowed to fire (i.e. remove and produce tokens). This happens when the colours of the input tokens of the transition (which can change value through time) reach particular transition-specific value combinations.



- $\mathcal{D}$  is a set of transition delays associating each transition in  $\mathcal{T}_D$  with a delay function. This delay function is continuously evaluated when the transition has a token in each of its input places. The delay function determines for how long the transition must wait before it is allowed to fire (i.e. remove and produce tokens). The firing rate depends on the colours of the input tokens of the transition (which can change value through time) and is determined by a Poisson point process.
- $\mathcal{F}$  is a set of (probabilistic) firing functions. For each transition it describes the quantity and colours of the tokens produced by the transition at its firing. A transition produces 0 or 1 token per outgoing arc; this quantity and the colour of the produced tokens is according to a transition-specific probabilistic mapping rule that may depend on the colours of the input tokens.
- $\mathcal{J}$  is an initial marking which defines the set of tokens initially present, i.e., it specifies in which places they initially reside, and the colours they initially have.

The set of places  $\mathcal{P}$ , the set of transitions  $\mathcal{T}$ , the set of arcs  $\mathcal{A}$  and the node function  $\mathcal{N}$  are defined in a Petri net graph. Figure 1 shows the graphical representation of the elements in  $\mathcal{P}$ ,  $\mathcal{T}$  and  $\mathcal{A}$ . The node function  $\mathcal{N}$  describes how these components are connected into a Petri net graph.



**Fig. 1.** Notation for places, tokens, transitions and arcs in Petri net graphs. On the right-hand- side there is a simple example Petri net graph, with two places and two transitions and a token in place P1

### 3.2 SDCPN Evolution

Tokens and the associated colour values in a SDCPN evolve over time quite similar as in a Coloured Stochastic Petri Nets (e.g. [12]). The main additions are that the colour of a token while it is residing in a place is an element of  $\mathbb{R}^n$  (where  $n$  is place-specific) and may evolve according to a differential equation that is governed by the colour functions  $\mathcal{V}$  and  $\mathcal{W}$  of the specific place where the token resides, and that guard and delay transitions take the evolving colour values into account. For DCPN, this differential equation is an ordinary differential equation, completely described by  $\mathcal{V}$  only. For SDCPN,





this differential equation is a stochastic differential equation, described by  $\mathcal{V}$  (for the flow part) and  $\mathcal{W}$  (for the Brownian motion term).

Tokens can be removed from places by transitions that are connected to these places by incoming ordinary arcs. A transition can only remove tokens if two conditions are both satisfied. If this is the case, the transition is said to be enabled. The first condition is that the transition must have at least one token per ordinary arc and one token per enabling arc in each of its input places and have no token in the input places to which it is connected by an inhibitor arc. When the first condition holds, the transition is said to be pre-enabled. The second condition differs per type of transition. For immediate transitions the second condition is automatically satisfied if the transition is pre-enabled. For guard transitions the second condition is specified by the set of transition guards  $\mathcal{G}$  and for delay transitions it is specified by the set of transition delays  $\mathcal{D}$ , see their description in Subsection 3.1.

When these two conditions are satisfied, the transition removes the tokens from the input places by which it is connected through an ordinary arc. It does not remove the tokens from places by which it is connected through an enabling arc. Subsequently, the transition produces a token for some or all of its output places, specified by the firing function  $\mathcal{F}$ . The colour of a produced token (which must be of the correct type, indicated by what  $\mathcal{C}$  defines for the output place), and the place for which it is produced is also specified by the firing function  $\mathcal{F}$ . The evaluation of  $\mathcal{G}$ ,  $\mathcal{D}$  and  $\mathcal{F}$  may be dependent on the colours of the input tokens of the corresponding transition.

In order to avoid ambiguity, for a DCPN the following rules apply when two transitions are enabled simultaneously:

- $R_0$  The firing of an immediate transition has priority over the firing of a guard or a delay transition.
- $R_1$  If one transition becomes enabled by two or more disjoint sets of input tokens at exactly the same time, then it will fire these sets of tokens independently, at the same time.
- $R_2$  If one transition becomes enabled by two or more non-disjoint sets of input tokens at exactly the same time, and the firing of one set disables the other, then the set that is fired is selected randomly.
- $R_3$  If two or more transitions become enabled at exactly the same moment by disjoint sets of input tokens, then they will fire at the same time.
- $R_4$  If two or more transitions become enabled at exactly the same moment by non-disjoint sets of input tokens, then the transition that will fire is selected randomly, with the same probability for each transition.

Note that in Rule  $R_2$  there is a conflict between tokens fighting for the same transition and in Rule  $R_4$  there is a conflict between transitions fighting for the same set of tokens; these rules settle these fights by appointing a random winner. In Rules  $R_1$  and  $R_3$  there is no conflict between tokens fighting for the same transition or for transitions fighting for the same tokens; these firings can occur independently and these rules only take care of the timing of these



individual firings. For a motivation of why these rules are chosen like this, see [5].

## 4 Local Petri Nets-Based Specification of an SDCPN

The compositional specification of a Stochastically and Dynamically Coloured Petri Net for a complex process with many different interacting agents such as exist in air traffic operations (e.g. air traffic controllers, pilots, navigation and surveillance equipment), is a bottom-up process (note that for DCPN this process is the same, except that no Brownian motion terms need to be defined). Prior to starting this compositional process, per agent the relevant low level functional entities have to be identified based on expert domain knowledge of that agent. For this chapter we assume that these low-level functional entities are given per agent. The compositional specification idea is then first to specify one small Petri net per functional entity of an agent, and refer to this as a Local Petri Net (LPN). Next, the interactions between these LPNs are specified. Note that our LPN definition has similar counterparts in Petri net literature. For example, [11] considers Generalised Stochastic Petri Nets to be composed of Modules in a similar way as we propose for SDCPN to be composed of LPNs. Ref. [14] proposes Hierarchical Coloured Petri Nets to be composed of Pages, while [15] considers the composition of Petri Net Components. An essential difference is that our LPNs (and Fotas modules) are connected with each other such that the number of tokens residing in an LPN (or module) is not influenced by these interconnections, while [14] and [15] do not pose this restriction.

The specification of the various elements of one LPN is explained in Subsection 4.1; this has to be accomplished for all LPNs. Subsection 4.2 describes how the interconnections between these LPNs are established.

### 4.1 Specification of Local Petri Net

#### Specification of elements $\mathcal{P}$ , $\mathcal{T}$ , $\mathcal{A}$ , $\mathcal{N}$

First, places (drawn as circles) are identified for the LPN. These places may represent operational or physical conditions (nominal modes and non-nominal modes). Next, the transitions are identified: If between two places, say  $P_1$  and  $P_2$ , a switch might occur, one transition (rectangle) is drawn, with two arcs (arrows) connecting the places with the transition. The places are gathered in the set of places  $\mathcal{P}$ , the transitions are gathered in the set of transitions  $\mathcal{T}$ , and the arcs are gathered in the set of arcs  $\mathcal{A}$ . The node function  $\mathcal{N}$  describes for each arc which place and transition it connects.



### Specification of elements $\mathcal{S}$ , $\mathcal{C}$ , $\mathcal{V}$ and $\mathcal{W}$

A complex stochastic dynamic process such as in air traffic operations cannot be described by places and transitions alone. Usually, some (piecewise) continuous valued timed processes are identified, which can be influenced by and which can influence the LPN places and transitions. In SDCPN, continuous valued processes are associated with tokens that reside in the places. In general, if a token resides in a particular place, its value changes according to a differential equation that is associated with that place. In this step, all places are checked on whether a continuous valued process can be associated with it. Note that on the other hand, it may happen that identified continuous valued processes lead to a necessary introduction of new places. For example: one continuous-valued process appears to alternately follow two different differential equations; in that case two places need to be introduced, each associated with one of these differential equations. All continuous valued process types are collected in the set  $\mathcal{S}$ . The mapping of each place of the Petri net to one of these types is described by  $\mathcal{C}$ . The sets of place-specific colour functions  $\mathcal{V}$  and  $\mathcal{W}$  describe how the colour of a token changes while it is staying in a place. For each place,  $\mathcal{V}$  and  $\mathcal{W}$  specify the coefficients of a differential equation which describes the rate of change of the token colour: if  $\mathcal{V}_P$  and  $\mathcal{W}_P$  define the token colour functions for place  $P$  that has colour type  $\mathcal{C}_P$ , then the colour  $c_t^P \in \mathcal{C}_P$  of a token in place  $P$  at time  $t$  satisfies:  $dc_t^P = \mathcal{V}_P(c_t^P)dt + \mathcal{W}_P(c_t^P)dw_t$ , where  $\{w_t\}$  is Brownian motion.

### Specification of elements $\mathcal{G}$ , $\mathcal{D}$ , $\mathcal{F}$ and $\mathcal{J}$ in Local PN terms

Next, for each transition, one should determine whether it is a guard transition, a delay transition or an immediate transition. A guard transition fires based on the combined colours of its input tokens reaching some value. A delay transition models a duration, e.g. of an action. An immediate transition fires without delay. The guard transitions are collected in the set  $\mathcal{T}_G$ , the delay transitions are collected in  $\mathcal{T}_D$  and the immediate transitions are collected in the set  $\mathcal{T}_I$ . Subsequently, the guards  $\mathcal{G}$  and the delays  $\mathcal{D}$  are specified in detail. The firing function  $\mathcal{F}$  describes the colours of the tokens fired by a transition into its output places, given the colours of the tokens in the input places. Finally, the initial marking  $\mathcal{J}$  describes which place(s) of the LPN initially contain one or more token(s) and describes the initial colour values of these tokens, hence it describes the initial state of the process modelled by the SDCPN.

## 4.2 Interconnections Between LPNs

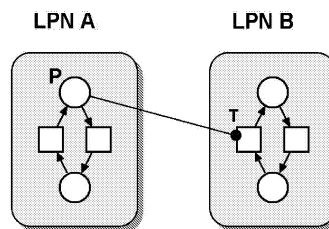
The interconnections between the LPNs have to be specified in a way that allows to start at the lowest level and then step by step going up to the highest level, and such that an interconnection at a higher level does not

imply a significant change at a lower level. The typical exception on this is caused by non-local influences on  $\mathcal{G}$ ,  $\mathcal{D}$  and  $\mathcal{F}$ . In order to improve into this desired direction, in this subsection some specific types of interconnections are identified.

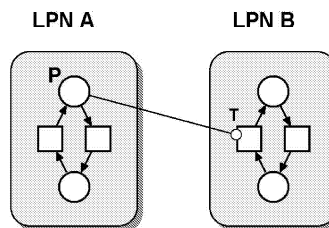
Following [11] one step in enabling a systematic bottom-up specification of a Petri net is to ensure that each LPN always contains exactly one token. For air traffic types of applications it often is useful to allow multiple tokens to be within one LPN, e.g. one for each aircraft. Hence we relaxed the Fota-principle to the following requirement: all interconnections between LPNs shall be such that the number of tokens in an LPN is not directly influenced by these interconnections. Subsequently we identified two types of interconnections that satisfied our above requirement:

- Enabling arc (or inhibitor arc) from one place in one LPN to one transition in another LPN.
- Interaction Petri Net (IPN) from one (or more) transition(s) in one LPN to one (or more) transition(s) in another LPN.

Enabling and inhibitor interconnections are illustrated in Figures 2 and 3, respectively. Note that in these figures, each LPN is surrounded by a box. This boxing idea has also been used by e.g. [9] or [15].



**Fig. 2.** Illustration of an enabling arc from one place in LPN A to one transition in LPN B

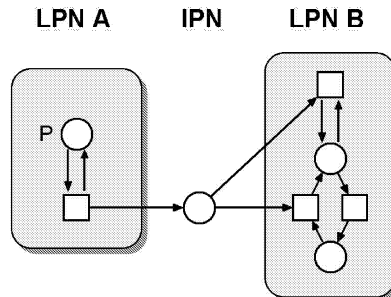


**Fig. 3.** Illustration of an inhibitor arc from one place in LPN A to one transition in LPN B



Enabling and inhibitor arcs are used to describe how agents modelled by individual LPNs influence each other. The transition at the tip of the arc (i.e. transition  $T$  in LPN B in Figures 2 and 3) can only fire if the process modelled by LPN A is in a particular state or marking, and when it fires, it may use the information existing in this marking of LPN A. For example, it may appear that the guard or delay of transition  $T$  is dependent of the colour of the token in place  $P$ . In those cases, the Petri net graph needs to be extended with (enabling) arcs from place  $P$  to transition  $T$  in order to get access to this information (and the guard or delay of the transition, which in the previous subsection has only been defined locally, needs to be adapted). Since tokens are not consumed through enabling arcs at a transition firing, the state of LPN A is not changed through this firing. Ref. [9] uses enabling arcs like this to model synchronisation. [11] uses GSPN which do not support enabling arcs although they do support inhibitor arcs; however, [11] does allow tokens of other modules to be consumed and immediately placed back, which is similar to using an enabling arc.

An Interaction Petri Net (IPN) consists of at least one place, and zero or more transitions. It connects, by means of ordinary arcs, one or more transition(s) in one LPN with one or more transition(s) in another LPN. If there are transitions in the IPN, and if these transitions are connected with other LPNs, then only enabling or inhibitor arcs can be used for the connections of these transitions with other LPNs. An example of an IPN is illustrated in Figure 4. It can be easily verified that an IPN does not influence the number of tokens in the LPNs it connects.



**Fig. 4.** Illustration of an Interaction Petri Net from one transition in LPN A to two transitions in LPN B

Interaction Petri Nets are used when enabling or inhibitor arcs are insufficient to model the interconnection between two agents. For example, it can hold on to state information from its input LPN (i.e. LPN A in Figure 4) while the state of LPN A itself evolves further. Also, IPNs can be used to connect two transitions, while enabling or inhibitor arcs always connect a place with a transition. Note that our IPNs are similar to the Interconnection blocks

of [11]. The connection of two LPNs through an Interaction Petri Net also has some similarity with Place Fusion, see e.g. [14] or [15], except that our Interaction Petri Net will not change the number of tokens in its connecting LPNs.

## 5 Extension with Interconnection Mapping Types

Interconnections between LPNs through enabling (or inhibitor) arcs and IPNs might lead to a combinatorial growth of the number of interconnections with the size of the Petri net. To avoid this combinatorial growth as much as possible, in this section hierarchical clustering and interconnection mapping approaches are graphically developed, based on how [13] composes statecharts:

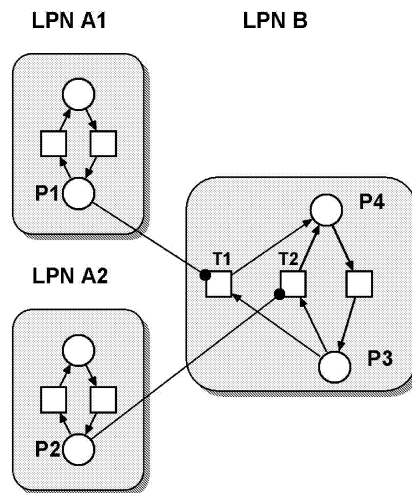
1. Interconnection mapping types I and II are defined to avoid possible duplication of transitions and arcs within LPNs caused by specifying interconnections between LPNs.
2. Interconnection mapping types III, IV and V are defined to avoid cluttering of interconnections between places and transitions of different LPNs.
3. Interconnection mapping types VI and VII define interconnections from or to hierarchical clusters of LPNs, which reduce the cluttering of interconnections.
4. Combinations of interconnection mapping types, and an additional interconnection mapping type VIII that avoids a duplication of transitions and arcs within an LPN and duplication of arcs between LPNs.

In Section 6, the SDCPN definition is extended to include these interconnection mapping types.

### 5.1 Avoid Duplication of Transitions and arcs within an LPN

Figure 5 shows an example where interconnections between LPNs lead to duplication of transitions and arcs within one of these LPNs. A transition from place P3 to place P4 occurs if either P1 or P2 contains a token. To model this, it is necessary to use two transitions T1 and T2 between P3 and P4. The use of only one transition between P3 and P4 would model an and relation (i.e. both P1 *and* P2 contain a token) instead of an or relation.

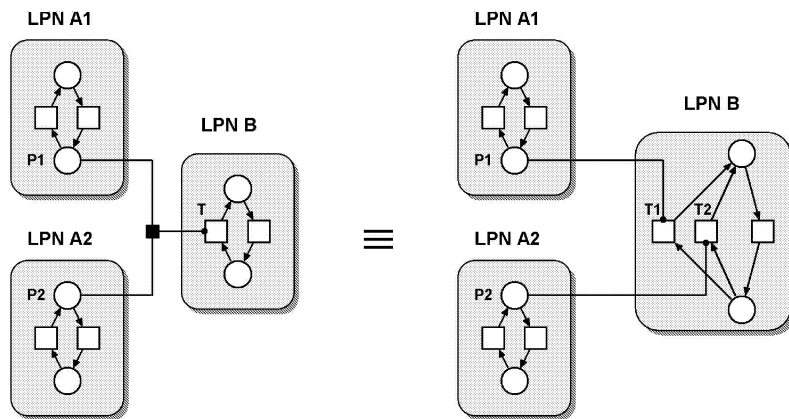
In most cases, the duplicated transitions and arcs do not have an essentially different meaning, and they are mostly introduced to be able to make use of colours of tokens residing in other LPNs. In particular, these duplicated transitions have the same guard or delay and the same firing function. This makes that duplication leads to reduced readability. This subsection presents some interconnection mapping types to avoid such duplication.



**Fig. 5.** Illustration of duplication of transitions within an LPN

*LPN interconnection mapping type I*

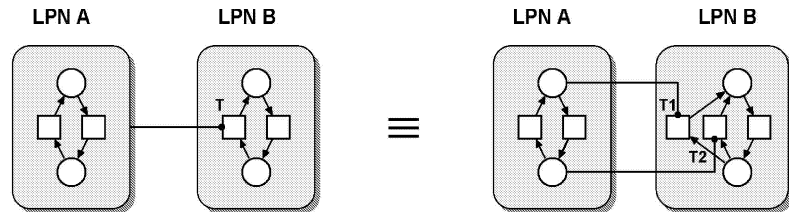
A set of  $s$  enabling arcs initiating on  $s$  places, merging into one arc and ending on one transition, means that this transition is duplicated  $s$  times, and that  $s$  enabling arcs are drawn between the  $s$  places and the  $s$  resulting transitions. This type of arc is called merging arc. The transition at the end of the merging arc should be in a different LPN than the  $s$  places that are at the beginning of the arc. Figure 6 shows an example of this interconnection mapping type. Note that in order to avoid confusion when using this interconnection mapping type, the  $s$  duplicated transitions should have the same guard or delay function and the same firing function and their input places should have the same colour type. Interconnection mapping type I is not defined with inhibitor or ordinary arcs instead of enabling arcs.



**Fig. 6.** LPN interconnection mapping type I. The point where several arcs merge into one arc is represented by a small black square

*LPN interconnection mapping type II*

An enabling arc initiating on the edge of an LPN box and ending on a transition in another LPN box, means that enabling arcs initiate from all places in the first LPN and end on duplications of this transition in the second LPN. Figure 7 shows an example of this interconnection mapping type. The duplicated transitions should have the same guard or delay function and the same firing function and their input places should have the same colour type. Interconnection mapping type II is not defined with inhibitor or ordinary arcs instead of enabling arcs.



**Fig. 7.** LPN interconnection mapping type II

**5.2 Avoid Cluttering of Interconnections between LPNs**

The interconnection mapping types in the previous subsection avoid the duplication problem, but not the cluttering due to the many enabling arcs and IPNs between places and transitions of different LPNs. If several LPNs are

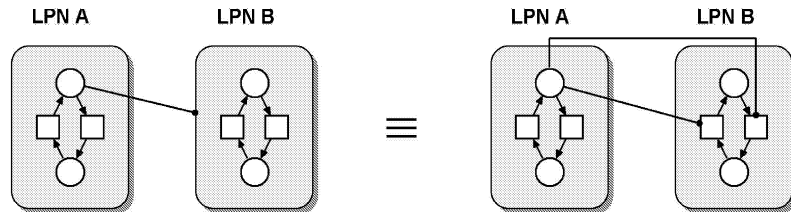


interconnected in one graph the result becomes unreadable. This subsection presents some interconnection mapping types to avoid this:

- Interconnection mapping type III can be applied to avoid enabling arcs cluttering.
- Interconnection mapping types IV and V can be applied to avoid IPNs cluttering.

*LPN interconnection mapping type III*

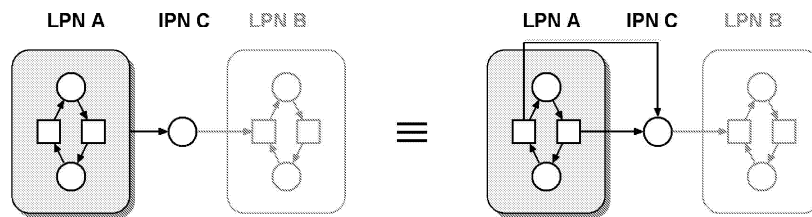
An enabling arc ending on the edge of an LPN box, means that enabling arcs end on each transition in this LPN. Figure 8 shows an example of this interconnection mapping type. Interconnection mapping type III can also be used with inhibitor arcs instead of enabling arcs. It cannot be used with ordinary arcs, due to the restriction that the number of tokens in an LPN should remain the same.



**Fig. 8.** LPN interconnection mapping type III

*LPN interconnection mapping type IV*

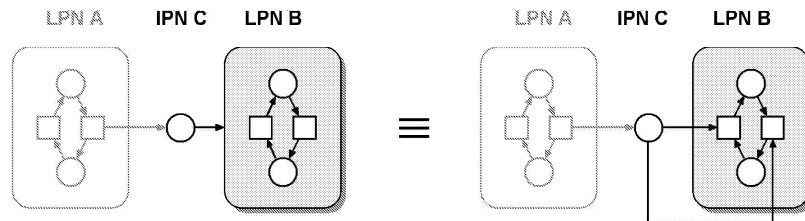
An ordinary arc initiating on the edge of an LPN box and ending on a place within an IPN means that ordinary arcs initiate from all transitions in this LPN. Figure 9 shows an example of this interconnection mapping type. Interconnection mapping type IV is not defined with enabling or inhibitor arcs instead of ordinary arcs.



**Fig. 9.** LPN interconnection mapping type IV

*LPN interconnection mapping type V*

An ordinary arc ending on the edge of an LPN box and starting from a place within an IPN means that ordinary arcs end on each transition in this LPN. Figure 10 shows an example of this interconnection mapping type. Interconnection mapping type IV is not defined with enabling or inhibitor arcs instead of ordinary arcs.



**Fig. 10.** LPN interconnection mapping type V

### 5.3 Clustering of LPNs

In this subsection, we define enabling arcs that go from or to a cluster of LPNs. This is done following the next two interconnection mapping types. Figures 11 and 12 show examples of these interconnection mapping types.

*LPN interconnection mapping type VI*

Suppose there is one LPN A and a set of  $n$  LPNs  $B_i$  ( $i=1,2, \dots, n$ ) which is enclosed by a large box. An enabling arc initiating on the edge of LPN A and ending on the edge of the large box with the set of LPNs  $B_i$ , means that the enabling arc represents  $n$  actual enabling arcs, initiating on the edge of LPN A and ending on the edge of each LPN  $B_i$ . Interconnection mapping type VI can also be defined from a place to a large box of LPNs (by means of an enabling arc), or from a place within an IPN to a large box of LPNs (by means of an ordinary arc). It is not defined with inhibitor arcs instead of enabling arcs. Note that the right hand side of Figure 11 makes use of a combination of interconnection mapping types II and III. For more examples of such combinations, see Subsection 5.4.

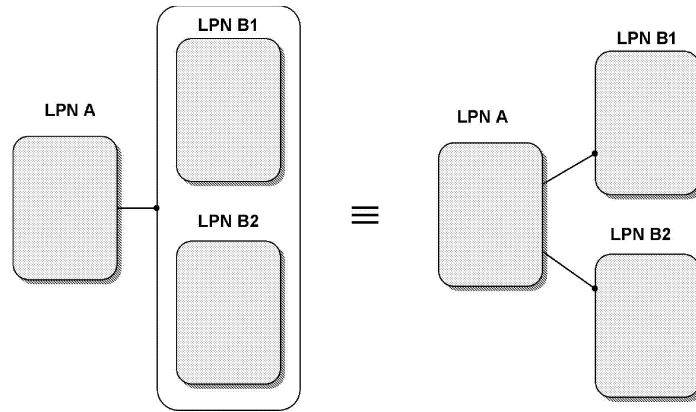


Fig. 11. LPN interconnection mapping type VI

*LPN interconnection mapping type VII*

Suppose there is a set of  $n$  LPNs  $A_i$  ( $i=1,2, \dots, n$ ) which is enclosed by a large box and one LPN B. An enabling arc initiating on the edge of the large box with the set of LPNs  $A_i$  ( $i=1,2, \dots, n$ ) and ending on the edge of LPN B, means that the enabling arc represents  $n$  actual enabling arcs, initiating on the edge of each LPN  $A_i$  and ending on the edge of LPN B. Interconnection mapping type VII can also be defined from a large box to a transition. It is not defined with ordinary or inhibitor arcs instead of enabling arcs.

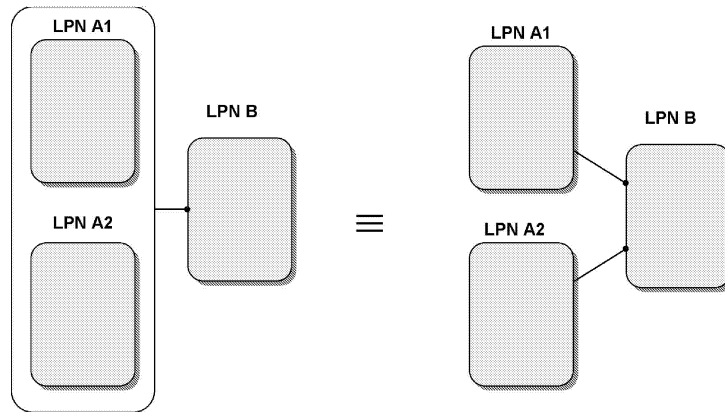


Fig. 12. LPN interconnection mapping type VII

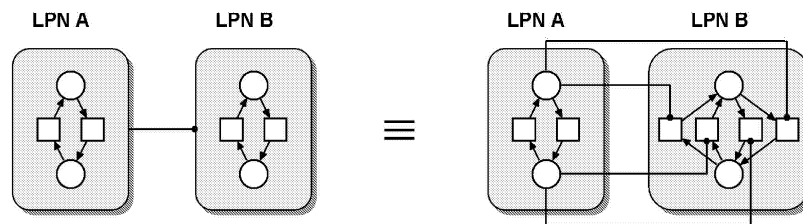


### 5.4 Combinations of Interconnection Mapping Types and Additional Type

Interconnection mapping types can also be combined, such as interconnection mapping type I with II, type II with III, type IV with V, or type VI with VII. An illustration of combination of II with III is given below.

*LPN interconnection mapping types II and III combined*

An enabling arc initiating on the edge of an LPN box and ending on the edge of another LPN box, means that enabling arcs initiate from all places in the first LPN and end on duplications of all transitions in the second LPN. Figure 16-fig:LPN IMP II and III shows an example of this combination of interconnection mapping types II and III.

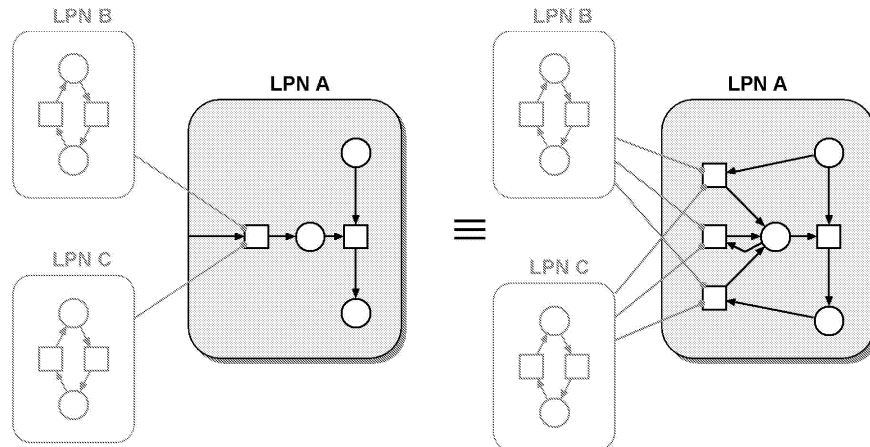


**Fig. 13.** LPN interconnection mapping types II and III combined

Finally, we introduce an additional interconnection mapping type which avoids duplication of transitions and arcs within an LPN, and consequently cluttering of arcs between LPNs:

*LPN interconnection mapping type VIII*

An ordinary arc initiating on the edge of an LPN box and ending on a transition inside the same box, means that ordinary arcs initiate from all places in the LPN box to duplications of this transition. The duplicated transitions should have the same guard or delay function and the same firing function and their set of input places should have the same set of colour types. Figure 14 illustrates how this avoids both the duplication of transitions and arcs within an LPN, and the duplication of arcs between LPNs.



**Fig. 14.** LPN interconnection mapping type VIII, which avoids duplication of arcs and transitions within an LPN and duplication of arcs between LPNs

**Remark:** The interconnection mapping types introduced in this section could also be used for other types of Petri nets than SDCPN, provided that these other types of Petri nets support the same graphical elements as SDCPN, such as enabling arcs. If this is not the case, the interconnection mapping types might still be used, but then the restriction that the number of tokens in an LPN cannot be changed by the interconnections must be removed.

## 6 Extension of SDCPN with Interconnection Mapping Types I through VIII

This subsection extends the SDCPN definition of [7] to include the interconnection mapping types identified in Section 5. The extension is referred to as  $\text{SDCPN}^{imt}$ .

$\text{SDCPN}^{imt}$  is a tuple  $(\mathcal{P}, \mathcal{T}, \mathcal{B}, \mathcal{A}^{imt}, \mathcal{L}, \mathcal{N}^{imt}, \mathcal{S}, \mathcal{C}, \mathcal{V}, \mathcal{W}, \mathcal{G}, \mathcal{D}, \mathcal{F}, \mathcal{J})$ , where  $\mathcal{P}, \mathcal{T}, \mathcal{S}, \mathcal{C}, \mathcal{V}, \mathcal{W}, \mathcal{G}, \mathcal{D}, \mathcal{F}, \mathcal{J}$  are as in the definition of SDCPN (Section 3 or Ref. [7]), and the other elements are outlined below:

$\mathcal{A}^{imt}$  is the set of arcs in the  $\text{SDCPN}^{imt}$ . It equals the set of arcs  $\mathcal{A} = \mathcal{A}_o \cup \mathcal{A}_e \cup \mathcal{A}_i$  as defined in [7], but extended with a set of merging arcs ( $\mathcal{A}_m$ ). In other words,  $\mathcal{A}^{imt} = \mathcal{A} \cup \mathcal{A}_m$ .

A merging arc is a set of  $s \geq 2$  enabling arcs merging into one arc, where  $s$  can be different for each merging arc. The merging point is denoted by a small black square.

$\mathcal{B}$  is a set of boxes which consists of a set  $\mathcal{B}_L$  of LPN-boxes and a set  $\mathcal{B}_C$  of Cluster-boxes.



Each box in  $\mathcal{B}$  is drawn as a rectangle with rounded corners. Note that at this definition level, each element of  $\mathcal{B}$  is just an empty box. The box function  $\mathcal{L}$  (see definition next) will specify the actual contents (i.e. the places and transitions or other boxes) of each box in  $\mathcal{B}$ . There, each LPN-box in  $\mathcal{B}_L$  will be associated with an LPN, and each cluster-box in  $\mathcal{B}_C$  will be associated with a cluster of LPNs.

$\mathcal{L}$  is a box function which specifies the contents of each box in  $\mathcal{B}$ :  $\mathcal{L}$  maps each place in  $\mathcal{P}$  to zero or one box in  $\mathcal{B}$ , each transition in  $\mathcal{T}$  to zero or one box in  $\mathcal{B}$ , and each box in  $\mathcal{B}_L$  to zero or one box in  $\mathcal{B}_C$ . Places (and transitions) that form IPNs are not mapped to an LPN-box but can be mapped to a cluster-box, and at least two LPN-boxes should be mapped to each cluster-box.

For each LPN-box in  $\mathcal{B}_L$ , the box function specifies which places in  $\mathcal{P}$  and which transitions in  $\mathcal{T}$  are drawn in it to form an LPN; for each cluster-box in  $\mathcal{B}_C$  it specifies which (at least two) LPN-boxes in  $\mathcal{B}_L$  are drawn in it to form a cluster of LPNs. Some places (and transitions) are not inside any LPN-box; these form the IPNs. It is however possible that IPNs are part of a cluster-box (although they are not part of an LPN-box). Similarly, not all LPN-boxes need to be inside a cluster-box.

$\mathcal{N}^{imt}$  is a node function which maps each arc in  $\mathcal{A}_m$  to an ordered pair of which the first component is a set of places (but not in IPNs) or boxes, and the second component is a transition. Furthermore,  $\mathcal{N}^{imt}$  maps each arc in  $\mathcal{A} = \mathcal{A}_o \cup \mathcal{A}_e \cup \mathcal{A}_i$ , to an ordered pair of nodes, where a node is a place, a transition, an LPN-box or a cluster-box. Multiple arcs between the same pair of nodes are allowed (but not both an inhibitor arc and another type of arc). There are the following restrictions:

- Ordinary arcs can only be drawn from a place to a transition within the same LPN- box, from a transition to a place within the same LPN- box, from a place in an IPN to a transition, from a transition to a place in an IPN, from a place in an IPN to an LPN-box, from an LPN-box to a place in an IPN, from a place in an IPN to a cluster-box, or from an LPN box to a transition in the same LPN box.
- Enabling arcs can only be drawn from a place to a transition within the same LPN- box, from a place to a transition in a different LPN-box or in an IPN, from a place (but not in an IPN) to an LPN- or cluster-box, from an LPN- or cluster-box to a transition, or between two boxes (i.e. LPN-LPN, LPN-cluster, cluster-LPN or cluster-cluster).
- Inhibitor arcs can only be drawn from a place to a transition within the same LPN- box, from a place to a transition in a different LPN-box or in an IPN, or from a place (but not in an IPN) to an LPN box.
- Merging arcs can only be drawn from a set of places (but not in IPNs) or boxes, to a transition that is in another LPN than these places or boxes. The input places of a merging arc should be of the same type.



Note that the guards  $\mathcal{G}$ , delays  $\mathcal{D}$ , and firing functions  $\mathcal{F}$  defined for  $\text{DCPN}^{imt}$  are equal to those defined for SDCPN. However, since the elements  $\mathcal{G}$ ,  $\mathcal{D}$ , and  $\mathcal{F}$  use the colours of the transition input tokens as input, their evaluation is a little more complicated in the sense that from the  $\text{SDCPN}^{imt}$  graph it is not immediately obvious which places are the input and output places of the transitions. These input and output places become clear if the  $\text{SDCPN}^{imt}$  graph is extended to a SDCPN graph, i.e. the cluttered one without the interconnection mapping types. Some rules that avoid this for the most-often used interconnection mapping types are given below. Here, only the between-LPN interconnections are considered. The pre-enabling, enabling or firing of each transition is also dependent on the colours of the input tokens along the within-LPN connections, but to keep the description brief, these are not considered here.

- If a transition has an incoming merging arc (see e.g. interconnection mapping type I), it is pre-enabled if it has a token in at least one of the places also connected to this merging arc. The transition is enabled if it is enabled by this input token as described in [7] (e.g. its guard evaluates to true, or its delay has passed). If there are tokens in several of these input places, the transition guard or delay function uses their colours in parallel for its evaluation.
- If a transition has an (enabling) incoming arc connected with an LPN-box (see e.g. interconnection mapping type II), then it is pre-enabled if there is at least one token somewhere in this input LPN-box (and this is usually the case). It is enabled if it is enabled by this input token as described in [7].
- If the LPN-box in which a transition resides has an input place (see e.g. interconnection mapping type III), then the transition is pre-enabled if there is a token in this input place, and its guard or delay uses the colour of this token for its evaluation as described in [7].
- If the LPN-box in which a transition resides has an input LPN-box (see e.g. interconnection mapping type II combined with III), then the transition is pre-enabled if there is at least one token somewhere in the input LPN-box (and this is usually the case) and its guard or delay uses the colour of this token for its evaluation as described in [7].

## 7 Free Flight Air Traffic Example

The compositional specification approach described has been used to specify an initial  $\text{SDCPN}^{imt}$  for a risk assessment of the Free Flight based air traffic operation adopted in [17]. Free Flight -sometimes referred to as Self Separation Assurance- is a concept where pilots are allowed to select their trajectory freely at real time, at the cost of acquiring responsibility for conflict prevention. It changes ATM in a fundamental way: the centralised control becomes a



distributed one, responsibilities and tasks transfer from ground to air, ATC sectorization and routes are removed and new technologies are brought in. Before such concept can be implemented, it is necessary to determine its level of safety. The aim of this section is to illustrate the SDCPN<sup>imt</sup> specification developed in [1] for a collision risk model for Free Flight.

### 7.1 LPNs of the Free Flight Air Traffic Example

In the Free Flight air traffic example, the airspace is an En-Route Airspace without fixed routes or an active ATC specifying routes. All aircraft flying in this airspace are assumed to be properly equipped and enabled for Free Flight: the pilots can try to optimise their trajectory, due to the enlarged freedom to choose path and flight level. The pilots are only limited by their responsibility to maintain airborne separation, in which they are assisted by a system called ASAS (Airborne Separation Assistance System). This can be considered as a system processing the information flows from the data-communication links between aircraft, the navigation systems and the aircraft guidance and control systems. ASAS detects conflicts, determines conflict resolution manoeuvres and presents the relevant information to the aircrew.

The number of agents involved in the Free Flight operation is huge and ranges from the Control Flow Management Unit to flight attendants. In the setting chosen for an initial risk assessment, the following agents are taken into account:

- A Pilot-Flying in each aircraft,
- A Pilot-Non-Flying in each aircraft,
- A number of systems and entities per aircraft, like the aircrafts position evolution and the Conflict Management Support systems,
- A number of global systems and entities, like the communication frequencies and the satellite system.

As explained in the beginning of Section 4, LPNs are specified for each relevant functional entity of each agent. It was judged sufficient to specify the following number of LPNs for the agents:

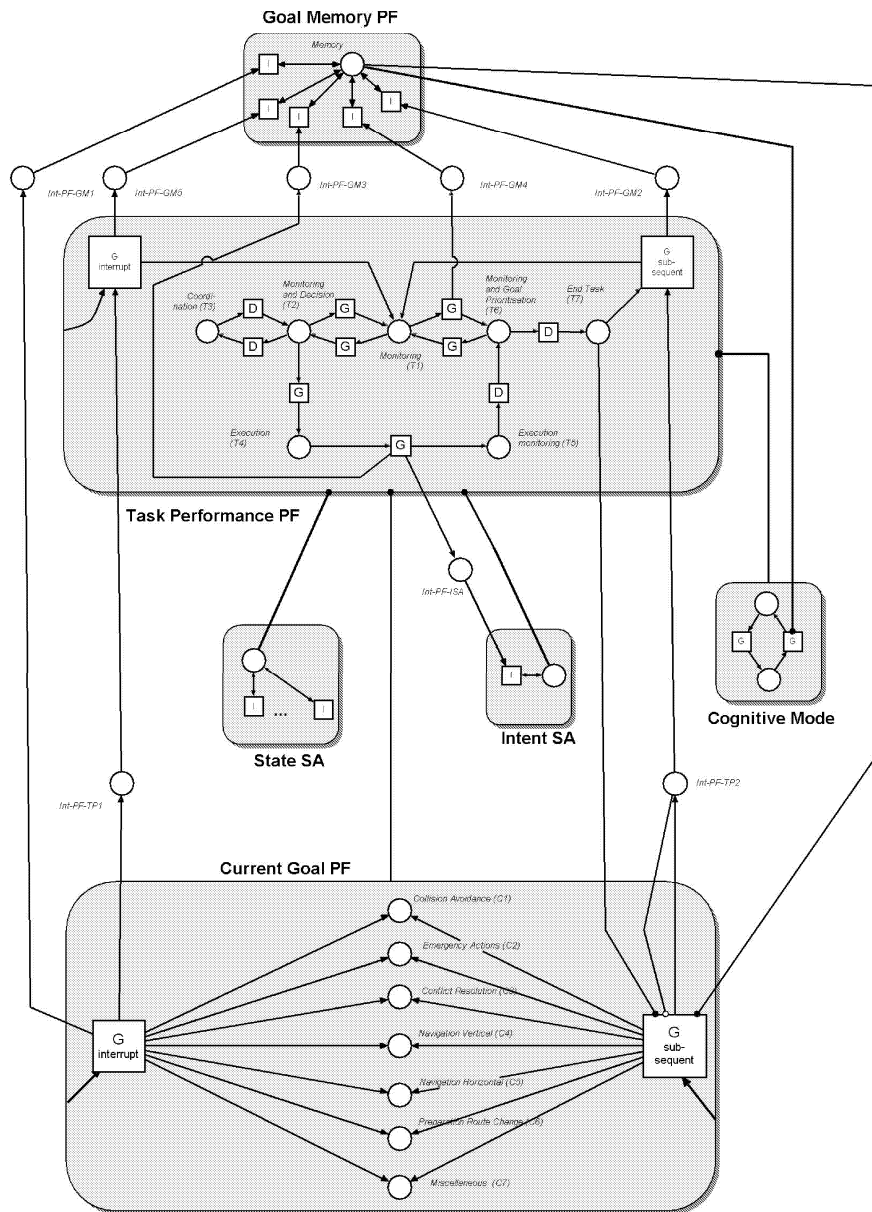
- 6 LPNs for each Pilot-Flying,
- 2 LPNs for each Pilot-Non-Flying,
- 36 LPNs for the systems and entities of each aircraft,
- 7 LPNs for the environment.

The actual number of LPNs in the whole model then depends on the number  $N$  of aircraft involved, and equals  $7 + N \times (6 + 2 + 36)$ .

### 7.2 Interconnected LPNs of Pilot Flying

This subsection illustrates, for the specific Free Flight air traffic example, a Petri Net model for the Pilot Flying as agent. A graphical representation of all LPNs the Pilot-Flying consists of, is given in Figure 15.





**Fig. 15.** The agent Pilot-Flying in Free Flight is modelled by 6 different LPNs, and a number of ordinary and enabling arcs and some IPNs, consisting of one place and input and output arcs



The Human-Machine-Interface where sound or visual clues might indicate that attention should be paid to a particular issue, is represented by a LPN that does not belong to the Pilot-Flying as agent and is therefore not depicted in the Figure. Similarly, the arcs to or from any other agent are not shown in Figure 15. Because of the very nature of Petri Nets, these arcs can easily be added during the follow-up specification cycle. To get an understanding of the different LPNs, a good starting point might be the LPN Current Goal (at the bottom of the figure) as it represents the objective the Pilot-Flying is currently working on. Examples of such goals are Collision Avoidance, Conflict Resolution and Horizontal Navigation. For each of these goals, the pilot executes a number of tasks in a prescribed or conditional order, represented in the LPN Task Performance. Examples of such tasks are Monitoring and Decision, Execution and Execution Monitoring. If all relevant tasks for the current goal are considered executed, the pilot chooses another goal, thereby using his memory (where goals deserving attention might be stored, represented by the LPN Goal Memory) and the Human-Machine-Interface. His memory where goals deserving attention might be stored is represented as the LPN Goal Memory in Figure 15.

So, the LPNs Current Goal, Task Performance, and Goal Memory are important in the modelling of which task the Pilot-Flying is executing. The other three LPNs are important in the modelling on how the Pilot-Flying is executing the tasks. The LPN State SA, where SA stands for Situation Awareness, represents the relevant perception of the pilot about the states of elements in his environment, e.g. whether he is aware of an engine failure. The LPN Intent SA represents the intent, e.g. whether he needs to leave the Free Flight Airspace. The LPN Cognitive mode represents whether the pilot is in an opportunistic mode, leading to a high but error-prone throughput, or in a tactical mode, leading to a moderate throughput with a low error probability.

**Table 1.** Numbers of interconnection mapping types and Petri net elements before and after application of interconnection mapping types. The number of places (i.e. 19 places within LPNs and 8 places between LPNs) does not change due to the interconnection mapping types

|              | Number of elements In Figure 15 | Without interconnection mapping types |
|--------------|---------------------------------|---------------------------------------|
| Within LPNs  | 27 transitions                  | 279 transitions                       |
|              | 66 arcs                         | 642 arcs                              |
| Between LPNs | 16 ordinary arcs                | 293 ordinary arcs                     |
|              | 7 enabling arcs                 | 1023 enabling arcs                    |
|              | 1 inhibitor arc                 | 7 inhibitor arcs                      |
| Total        | 117                             | 2244                                  |

There are many interactions (which, in some cases, are complex) between these individual LPNs, which are depicted as enabling arcs and IPNs with one



place only. The use of the new interconnection mapping types makes that the figure is still readable. Interconnection mapping types I, IV, V, VI and VII have not been used, while type II has been used 2×, type III 4×, and type VIII 3×. Table 1 shows that without the use of these interconnection mapping types the figure really would be cluttered with duplicated transitions and arcs within LPNs, and with connections drawn between LPNs.

## 8 Concluding Remarks

For the compositional specification of a multi-agent hybrid system this chapter has introduced a hierarchical extension of the compositional specification power of Petri nets, which avoids the need for all kinds of low level changes once making connections at a higher model level. Moreover the problem of combinatorial growth of the number of interconnections with the size of the Petri net is remedied. The effectiveness of the SDCPN based compositional specification is illustrated for an air traffic example of a taxiing aircraft crossing an active runway.

After a  $\text{SDCPN}^{imt}$  has been specified for a particular application, the next step is to analyse it, and to investigate and assess particular characteristics of the application. This can be done in various ways; for example, the  $\text{DCPN}^{imt}$  can be directly used as basis of a computer (e.g. Monte Carlo) simulation. Instead of this, or in addition to this, a particular SDCPN property can be used: In [7] it has been proven that, under a few conditions, a SDCPN is equivalent to a particular powerful subclass of hybrid state Markov process, named Generalised Stochastic Hybrid Process (GSHP), see 16-BujorianuLygerosGloverPola2003. Due to this equivalence, typical GSHP properties can be used to analyse the  $\text{SDCPN}^{imt}$ , even without elaborating the particular transformation from  $\text{SDCPN}^{imt}$  to GSHP for the application considered, and Monte Carlo simulations can be run which make use of GSHP mathematical properties. By those means, a collision risk model for Free Flight is instantiated using Petri-nets and the new interconnection mapping types. The usage of the new interconnection mapping types improves simplicity, readability and resilience against modelling errors.

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