A ray acoustics model for the propagation of aircraft noise through the atmosphere

Customer

National Aerospace Laboratory NLR

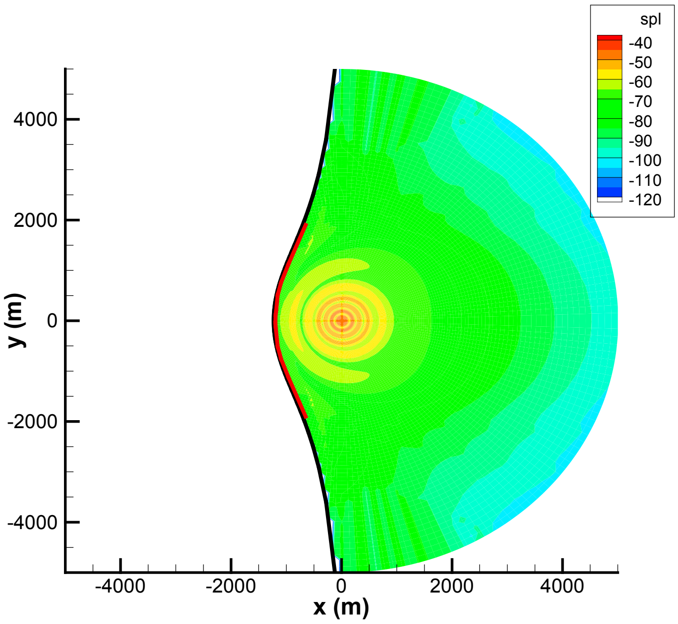
NLR-TP-2014-520 - December 2014

__NLR__ – Dedicated to innovation in aerospace

National Aerospace Laboratory NLR
Anthony Fokkerweg 2
1059 CM Amsterdam
The Netherlands
Tel +31 (0)88 511 3113
www.nlr.nl

EXECUTIVE SUMMARY

A ray acoustics model for the propagation of aircraft noise through the atmosphere



Report no.
NLR-TP-2014-520

Author(s)
H.H. Brouwer

Report classification
UNCLASSIFIED

Date
December 2014

Knowledge area(s)
Aëro-akoestisch en experimenteel aërodynamisch onderzoek

Descriptor(s)
Sound Propagation
Aircraft Noise

Problem area

The reliable prediction of the noise of aircraft as heard on the ground requires high quality tools for modeling long-range sound propagation in the atmosphere. These tools have to be capable of incorporating atmospheric conditions that vary along the altitude. Especially the gradients of temperature and wind can have a significant effect on how sound propagates, and on the sound level at the observer. Today, most state-of-the-art propagation modelling tools in practical use are based on acoustic ray tracing. These methods are capable of computing the sound wave propagation from source to observer, with incorporation of locally varying (but time-averaged) atmospheric conditions. Furthermore, atmospheric absorption can be included, as well

This report is based on a paper published in the International Journal of Aeroacoustics, volume 13, number 5 & 6, 2014.

as reflections by a ground surface that is characterized by an acoustic impedance. Ray tracing in general describes the trajectory of a sound ray, but not its amplitude. The present paper addresses the formulation of a ray tracing method, RAYTRAC, that extends to the computation of the amplitude.

Description of work

The method presented in this paper is based on the expansion of the acoustic pressure into orders of the inverse frequency. By solving the convective wave equation in leading order, the eikonal equation is obtained. Application of the method of characteristics
leads to equations which describe the path of the sound ray. In the next order a differential equation for the ray amplitude is obtained. This equation is shown to be one of a larger set of coupled differential equations, which has to be solved simultaneously.

Results and conclusions

The main results provide an expression for the acoustic amplitude that correctly incorporates spherical spreading and the variation in density and temperature. Furthermore, an algorithm has been included that finds the ray connecting the source and a specific observer point. The program has been extended with the possibility to deal with multiple rays, such as a direct and a reflected ray.

Applicability

The RAYTRAC program can be applied to predict the noise of aircraft as heard on the ground, for various atmospheric conditions, if the source is specified. For example, it is used to assess the so-called en-route noise of aircraft driven by prospective propulsion systems, such as contra-rotating open rotors.

National Aerospace Laboratory NLR
Anthony Fokkerweg 2, 1059 CM Amsterdam,
P.O. Box 90502, 1006 BM Amsterdam, The Netherlands
Telephone +31 (0)88 511 31 13, Fax +31 (0)88 511 32 10, www.nlr.nl

A ray acoustics model for the propagation of aircraft noise through the atmosphere

H.H. Brouwer

Customer
National Aerospace Laboratory NLR
December 2014

This report is based on a paper published in the International Journal of Aeroacoustics, volume 13, number 5 & 6, 2014.

This publication has been refereed by the Advisory Committee AEROSPACE VEHICLES.

Customer	National Aerospace Laboratory NLR
Contract number	-----
Owner	NLR
Division NLR	Aerospace Vehicles
Distribution	Unlimited
Classification of title	Unclassified
Date	December 2014

Approved by:

Author	Reviewer	Managing department
H. Brouwer	M. Tuinstra	J. Hakkaart
Date 2/12/2014	Date 3/12/2014	Date 8/12/14.

A ray acoustics model for the propagation of aircraft noise through the atmosphere

by

Harry H. Brouwer

reprinted from

international journal of aeroacoustics

volume 13 · number 5 & 6 · 2014

published by MULTI-SCIENCE PUBLISHING CO. LTD.,
5 Wates Way, Brentwood, Essex, CM15 9TB UK
E-MAIL: mscience@globalnet.co.uk
WEBSITE: www.multi-science.co.uk

Content

A ray acoustics model for the propagation of aircraft noise through the atmosphere

Harry H. Brouwer*

National Aerospace Laboratory NLR, P.O. Box 90502, 1006 BM Amsterdam, The Netherlands

Submitted: Oct 8, 2013; Revised May 28, 2014; Accepted Aug 19, 2014

ABSTRACT

A ray-tracing tool, called RAYTRAC, has been developed to compute the propagation of aircraft noise through a stratified atmosphere. The mathematical model on which RAYTRAC is based is derived from first principles, i.e. the linearized Euler equations for a non-uniform atmosphere. The main result of the model consists of two equations:one for the location of the rays and one for the change in amplitude along each ray. An analysis of these equations is given with respect to their possible solutions under various atmospheric circumstances. Also an efficient iteration procedure is presented that finds the ray connecting the source and a specific observer point. It is shown how multiple rays can be added, with incorporation of a reflecting surface. Finally, the tool is demonstrated by application to two examples that are representative for an aircraft emitting sound to an observer on the ground.

1. INTRODUCTION

The reliable prediction of the noise of aircraft as heard on the ground requires high quality tools for modelling long-range sound propagation in the atmosphere. These tools have to be capable of incorporating atmospheric conditions that vary along the altitude. Especially the gradients of temperature and wind can have a significant effect on how sound propagates, and on the sound level at the observer. Today, most state-of-the-art propagation modelling tools in practical use are based on acoustic ray tracing , e.g. [1] to [3]. These methods are capable of computing the sound wave propagation from source to observer, with incorporation of locally varying (but time-averaged) atmospheric conditions. Furthermore, atmospheric absorption can be included, as well as reflections by a ground surface that is characterized by an acoustic impedance. Because of the basic assumptions of ray acoustics, the validity of the methods is restricted to high frequencies, i.e. the acoustic wavelength is assumed to be much

*Corresponding author: E-mail: Harry.Brouwer@nlr.nl

smaller than the other dimensions in the problem, such as the characteristic lengths of temperature- and wind gradients.

The basic equation of ray tracing is the so-called eikonal equation, which describes the trajectory of a point of constant phase, i.e. the location of a ray. This equation is common to all ray acoustics models. Apart from the location of the ray, one is usually also interested in its amplitude, or rather the change in amplitude from a specified initial value. A widely used method is to compute the trajectories of a few adjacent rays, at least three, and to calculate the area enclosed. Together with an expression for the acoustic intensity, the amplitude follows from the conservation of acoustic energy. Although this method works quite well in practice, it has the disadvantage that at least three rays have to be computed. Furthermore, the definition of acoustic intensity in a non-uniform flow is different from that in a quiescent medium, see [4] and [5].

The method presented in this paper is based on the expansion of the acoustic pressure into orders of the inverse frequency. By solving the convective wave equation in leading order, the eikonal equation is obtained. Application of the method of characteristics leads to equations which describe the path of the sound ray. In the next order a differential equation for the ray amplitude is obtained. This equation is shown to be one of a larger set of coupled differential equations, which has to be solved simultaneously.

An initial version of RAYTRAC, derived along the same lines, was published by Schulten [2]. However, his model contained some approximations and errors which have been removed in the present study. In particular the differential equation for the amplitude, and some additional equations required to solve it, are significantly different.

Section 2 of this paper contains the mathematical model. In section 3 an analysis of these equations is given with respect to their possible solutions under various atmospheric circumstances. Section 4 is devoted to the numerical solution of the equations, whereas in section 5 the results for two relevant examples are discussed.

2. MATHEMATICAL MODEL

2.1. Basic equations

We start with the linearized Euler equations:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \left(\rho_0 \vec{u}\right) + \nabla \cdot \left(\rho \vec{w}\right) = 0 \tag{1}$$

$$\begin{aligned} &\frac{\partial \rho_0 \vec{u}}{\partial t} + \frac{\partial \rho \vec{w}}{\partial t} + \nabla \cdot \left(\vec{w} \otimes \rho_0 \vec{u}\right) + \nabla \cdot \left(\vec{w} \otimes \rho \vec{w}\right) + \\ &\nabla \cdot \left(\vec{u} \otimes \rho_0 \vec{w}\right) + \nabla p + \rho \nabla \Phi = 0 \end{aligned} \tag{2}$$

with p the acoustic pressure fluctuation, ρ the acoustic density fluctuation, $\vec{u}$ the acoustic particle velocity, and Φ the gravitational potential. $\otimes$ denotes the dyadic product, i.e.: $\left[\nabla\cdot\left(\vec{w}\otimes\vec{u}\right)\right]_j = \sum_i \frac{\partial w_i u_j}{\partial x_i}$. The mean density ρ_0, pressure p_0, temperature T, and the wind vector $\vec{w}$ are considered to be time-independent. The gravity term has to be included because the method will be applied to cases with a large vertical distance between source and observer.

Multiplying eq. (1) with $\vec{w}$ and subtracting it from eq. (2) yields:

$$\frac{\partial \rho_0 \vec{u}}{\partial t} + \rho_0 \vec{u}\cdot\nabla\vec{w} + \rho\vec{w}\cdot\nabla\vec{w} + \nabla\cdot\left(\vec{w}\otimes\rho_0\vec{u}\right) + \nabla p + \rho\nabla\Phi = 0 \tag{3}$$

Throughout this paper we assume a stratified medium, i.e. p_0, ρ_0, $\vec{w}$, and the temperature T vary only in vertical direction z, and $w_z = 0$. Then eqs. (1) and (3) become:

$$\frac{\mathrm{D}\rho}{\mathrm{D}t} + \nabla\cdot\left(\rho_0\vec{u}\right) = 0 \tag{4}$$

$$\frac{\mathrm{D}\vec{u}}{\mathrm{D}t} + \vec{u}\cdot\nabla\vec{w} + \frac{1}{\rho_0}\nabla p + \frac{p}{\gamma\, p_0}\nabla\Phi = 0 \tag{5}$$

with $\frac{\mathrm{D}}{\mathrm{D}t} = \frac{\partial}{\partial t} + \vec{w}\cdot\nabla$. The isentropic relation between the acoustic density and pressure fluctuations is (section 2.4 of reference [4]):

$$c^2\left(\frac{\mathrm{D}\rho}{\mathrm{D}t} + \vec{u}\cdot\nabla\rho_0\right) = \frac{\mathrm{D}p}{\mathrm{D}t} + \vec{u}\cdot\nabla p_0 \tag{6}$$

Using eq. (4) and the ideal gas law, $\rho_0 = \gamma p_0/c^2$, we thus have:

$$\frac{\mathrm{D}p}{\mathrm{D}t} + \vec{u}\cdot\nabla p_0 + \gamma\, p_0 \nabla\cdot\vec{u} = 0 \tag{7}$$

Left multiplication of eq. (7) with $\frac{\mathrm{D}}{\mathrm{D}t}$ and eq. (5) with $\gamma p_0\nabla$ and subtraction of the resulting equations yields:

$$\left[\left(\frac{\mathrm{D}}{\mathrm{D}t}\right)^2 - \gamma\, p_0\nabla\cdot\frac{1}{\rho_0}\nabla\right]p - \left(\nabla p_0 + \gamma\, p_0\nabla\right)\cdot\left(\frac{p}{\gamma\, p_0}\nabla\Phi\right) -$$

$$\frac{\nabla p_0}{\rho_0}\cdot\nabla p-\left(\nabla p_0+2\gamma\, p_0\nabla\right)\cdot\left(\vec{u}\cdot\nabla\right)\vec{w}=0 \tag{8}$$

This is still an equation in two unknowns, p and $\vec{u}$. The fourth term can be eliminated by using eq. (5) once more:

$$\begin{aligned}&\frac{\mathrm{D}}{\mathrm{D}t}\left[\left(\frac{\mathrm{D}}{\mathrm{D}t}\right)^2-\gamma\, p_0\nabla\cdot\frac{1}{\rho_0}\nabla\right]p-\\&\frac{\mathrm{D}}{\mathrm{D}t}\left[\left(\nabla p_0+\gamma\, p_0\nabla\right)\cdot\left(\frac{p}{p_0}\nabla\Phi\right)+\frac{\nabla p_0}{\rho_0}\cdot\nabla p\right]+\\&\left(\nabla p_0+2\gamma\, p_0\nabla\right)\cdot\left[\left(\frac{1}{\rho_0}\nabla p+\frac{p}{\gamma\, p_0}\nabla\Phi\right)\cdot\nabla\right]\vec{w}=0\end{aligned} \tag{9}$$

This is the Pridmore-Brown equation [6], extended with gravity terms.
Next we assume a single tone of frequency ω and write:

$$p=Pe^{i\omega(t-\tau)} \tag{10}$$

and expand in powers of ω^{-1}. In lowest order, i.e. ω^3, we find, as in reference [2], the eikonal equation:

$$q^2=\left[\frac{1-\vec{w}\cdot\vec{q}}{c}\right]^2 \tag{11}$$

with $\vec{q}=\nabla\tau$ is the slowness vector, and $q=\left|\vec{q}\right|$.
In the order ω^2 we find:

$$\begin{aligned}&2\left[\left(1-\vec{w}\cdot\vec{q}\right)\vec{w}+c^2\vec{q}\right]\cdot\nabla P+\\&\left[c^2\nabla\cdot\vec{q}-\vec{w}\cdot\left(\vec{w}\cdot\nabla\right)\vec{q}+\nabla\Phi\cdot\vec{q}+\frac{\nabla p_0}{\rho_0}\cdot\vec{q}-\frac{c^2}{\rho_0}\vec{q}\cdot\nabla\rho_0+\right.\\&\qquad\left.\frac{2cq_z}{q}\vec{q}\cdot\frac{\partial\vec{w}}{\partial z}\right]P=0\end{aligned} \tag{12}$$

For a stratified atmosphere and with $\nabla p_0 = -\rho_0 \nabla \Phi$ this can be written as:

$$\nabla \cdot \left\{ \frac{P^2}{q \rho_0 c^3} \left[\frac{\vec{q}}{q} c + \vec{w} \right] \right\} = 0 \tag{13}$$

This is the expression for the conservation of acoustic energy: $\nabla \bar{I} = 0$, with the time-averaged acoustic intensity $\bar{I}$ given for an atmosphere with wind.

2.2. Ray tracing

In general, a nonlinear partial differential equation given by:

$$H(\vec{q}, f, \vec{x}) = 0 \tag{14}$$

with $\vec{q} = \nabla f$, can be solved by the method of characteristics, see e.g. section 8.6 of reference [4].

Eq. (14) can be replaced by 3 ordinary differential equations:

$$\begin{aligned} \frac{\mathrm{d}\vec{x}}{\mathrm{d}t} &= \nabla_q H \\ \frac{\mathrm{d}\vec{q}}{\mathrm{d}t} &= -\left(\frac{\partial H}{\partial f} \vec{q} + \nabla H \right) \\ \frac{\mathrm{d}f}{\mathrm{d}t} &= \vec{q} \cdot \nabla_q H \end{aligned} \tag{15}$$

with the introduction of a new parameter t.

It is assumed that the wind speed is well below the speed of sound, which allows us to rewrite the eikonal equation eq. (11) as:

$$H(\vec{q}, \tau, \vec{x}) = cq - 1 + \vec{w} \cdot \vec{q} = 0 \tag{16}$$

and find:

$$\frac{\mathrm{d}\vec{x}}{\mathrm{d}t} = \nabla_q H = c\frac{\vec{q}}{q} + \vec{w} \tag{17}$$

$$\frac{\mathrm{d}\vec{q}}{\mathrm{d}t} = -\frac{\partial H}{\partial \tau} \vec{q} - \nabla H = -q \nabla c - \vec{q} \times (\nabla \times \vec{w}) - (\vec{q} \cdot \nabla) \vec{w} \tag{18}$$

$$\frac{\mathrm{d}\tau}{\mathrm{d}t} = 1 \tag{19}$$

Note that the specific choice of H in eq. (16) leads to the fact that the parameter t is consistent with time. If we recall eq. (10), eq. (19) means that a ray describes the propagation of a point of constant phase. Eq. (18) applied to a stratified atmosphere yields:

$$\begin{aligned} \frac{\mathrm{d}q_x}{\mathrm{d}t} &= \frac{\mathrm{d}q_y}{\mathrm{d}t} = 0 \\ \frac{\mathrm{d}q_z}{\mathrm{d}t} &= -q\frac{\partial c}{\partial z} - \vec{q}\cdot\frac{\partial \vec{w}}{\partial z} \end{aligned} \tag{20}$$

Eqs. (17) and (20) form the ray tracing equations to be solved numerically, and yield the position of the ray in space for a given set of initial conditions.

A ray tracing equation for the amplitude is obtained from eq. (12):

$$\begin{aligned} &\frac{\mathrm{d}P}{\mathrm{d}t} = \nabla P\cdot\frac{\mathrm{d}\vec{x}}{\mathrm{d}t} = \\ &\frac{P}{2q}\left[\frac{\vec{w}\cdot(\vec{w}\cdot\nabla)\vec{q}}{c} - c\nabla\cdot\vec{q} + \frac{c}{\rho_0}\vec{q}\cdot\nabla\rho_0 - \frac{2q_z\vec{q}}{q}\cdot\frac{\partial\vec{w}}{\partial z}\right] \end{aligned} \tag{21}$$

The first term at the right-hand side represents the offset by the wind. The second term is responsible for the spherical spreading. The third and fourth term represent corrections for the density and wind gradients. With use of the ideal gas law and $\nabla\Phi = g\vec{i}_z$ the third term can also be expressed as the sum of a gravity term and the gradient of the speed of sound:

$$\frac{\mathrm{d}P}{\mathrm{d}t} = \frac{P}{2q}\left[\frac{\vec{w}\cdot(\vec{w}\cdot\nabla)\vec{q}}{c} - c\nabla\cdot\vec{q} - 2q_z\left(\frac{\gamma g}{2c} + \frac{\partial c}{\partial z} + \frac{\vec{q}}{q}\cdot\frac{\partial\vec{w}}{\partial z}\right)\right] \tag{22}$$

The amplitude is determined by integrating eq. (22) for a given ray. The right hand side contains spatial derivatives of $\vec{q}$. To obtain these, we have to integrate their derivatives, for example $\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial q_z}{\partial z}$, along a ray. For any field $f(\vec{x})$ that does not explicitly depend on time, we have:

$$\frac{\mathrm{d}f}{\mathrm{d}t} = \frac{\mathrm{d}\vec{x}}{\mathrm{d}t}\cdot\nabla f \tag{23}$$

If this is applied to q_i and $\frac{\partial q_i}{\partial x_j}$, we find:

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial q_i}{\partial x_j} = \frac{\partial}{\partial x_j}\frac{\mathrm{d}q_i}{\mathrm{d}t} - \nabla q_i \cdot \frac{\partial}{\partial x_j}\frac{\mathrm{d}\vec{x}}{\mathrm{d}t} \tag{24}$$

Using eqs. (17) and (20) we find for example for $\frac{\partial q_z}{\partial z}$:

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial q_z}{\partial z} = &-\nabla q_z \cdot \left[\frac{c}{q}\left(\frac{\partial \vec{q}}{\partial z} - \frac{\vec{q}}{q^2}\vec{q}\cdot\frac{\partial \vec{q}}{\partial z}\right) + \frac{\vec{q}}{q}\frac{\partial c}{\partial z} + \frac{\partial \vec{w}}{\partial z}\right] \\ &- \frac{\partial \vec{q}}{\partial z}\cdot\left[\frac{\vec{q}}{q}\frac{\partial c}{\partial z} + \frac{\partial \vec{w}}{\partial z}\right] - \left[q\frac{\partial^2 c}{\partial z^2} + \vec{q}\cdot\frac{\partial^2 \vec{w}}{\partial z^2}\right]\end{aligned} \tag{25}$$

Similar expressions can be found for the other components, although it is mentioned here for future reference that the expression for $\frac{\partial q_z}{\partial z}$ is the only one that contains the second derivatives of c and $\vec{w}$. These 9 expressions, together with (the components of) eqs. (17), (20), and (22) form a system of 16 coupled ordinary differential equations that have to be integrated numerically. Note that for the computation of the location of the ray only 6 equations have to be solved.

2.3. Initial conditions

In the original version of RAYTRAC the choice for the initial conditions corresponded to the sound field of a monopole, at a short distance from its position, without incorporation of the wind. In some cases this caused errors in the computation of the amplitude. In this section it is shown how more accurate expressions for the initial values of $\nabla \otimes \vec{q}$ are derived.

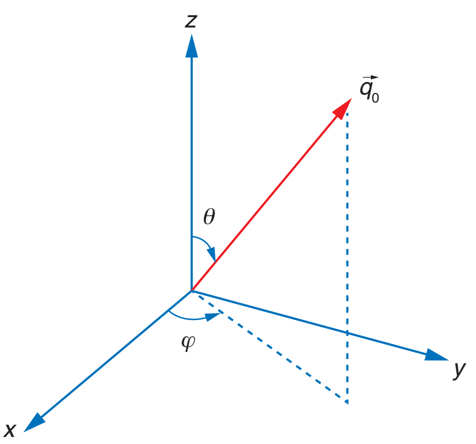


Figure 1: Coordinate system.

Consider a ray with the emission angles θ and φ, i.e. at the source we have, see Figure 1:

$$q_x = q_0 \cos\varphi \sin\theta, q_y = q_0 \sin\varphi \sin\theta, q_z = q_0 \cos\theta \tag{26}$$

This vector will be denoted by $\vec{q}_0$. For convenience the source is placed at the origin. From the eikonal equation (11) an explicit expression for q_0 can be derived:

$$q_0 = \frac{1}{c_s + \left(w_{s,x} \cos\varphi + w_{s,y} \sin\varphi\right) \sin\theta} \tag{27}$$

Here, c_s is the speed of sound and $\vec{w}_s$ the wind velocity at the source. After an infinitesimal short time t, the position of the ray is:

$$\vec{x} = \left(c_s \frac{\vec{q}_0}{q_0} + \vec{w}_s \right) t \tag{28}$$

Next we want to find an expression for e.g. $\partial q_x / \partial x$, which is $\mathrm{d}q_x/\mathrm{d}x$ with $\mathrm{d}y$ = $\mathrm{d}z$ = 0. A second ray is considered that leaves at slightly different angles $\vartheta + \mathrm{d}\vartheta$ and $\varphi + \mathrm{d}\varphi$ arriving at $x + \mathrm{d}x$ at time $t + \mathrm{d}t$. From $\mathrm{d}z = 0$ we find:

$$\mathrm{d}t = \tan\theta t \mathrm{d}\theta \tag{29}$$

and from $\mathrm{d}y = 0$:

$$\mathrm{d}\varphi = -\mathrm{d}\theta \left[\frac{\tan\varphi}{\sin\theta \cos\theta} + \frac{w_{s,y}}{c_s} \frac{1}{\cos\theta \cos\varphi} \right] \tag{30}$$

and we find:

$$\begin{aligned} \mathrm{d}x = \left[c_s \frac{1}{\sin\theta \cos\varphi} + w_{s,x} + w_{s,y} \tan\varphi \right] \mathrm{d}t = \\ \frac{\mathrm{d}t}{q_0 \sin\theta \cos\varphi} \end{aligned} \tag{31}$$

From eqs. (27) and (29) to (31) we then find:

$$\begin{aligned} \frac{\partial q_x}{\partial x} = q_0^3 c_s \frac{1}{t} \Big\{ \cos^2\theta + \sin^2\theta \sin^2\varphi + \\ 2M_{s,y} \sin\varphi \sin\theta + M_{s,y}^2 \sin^2\theta \Big\} \end{aligned} \tag{32}$$

with $\vec{M}_s = \vec{w}_s / c_s$. .

Similar expressions can be derived for the other components of the initial values of $\nabla \otimes \vec{q}$.

3. LIMITING SOLUTIONS

3.1. Limiting angles

RAYTRAC has been developed with the aim to analyze the propagation of aircraft noise, i.e. with a source that emits sound rays downward to an observer. However, not for all values of $\theta > \pi/2$ the ray reaches the height of the observer, and in many cases a ray that connects the source and the observer does not exist. It is very useful to know in advance whether a solution exists, and, if so, in which range of θ the solution has to be sought. The conditions which determine this range for given source and observer altitudes, are the atmospheric profile and the azimuthal emission angle φ. The limiting values of θ are those at which the emitted ray travels horizontally at some altitude, i.e. at this altitude $q_z = 0$. Reminding that q_x and q_y are constant along a ray and are given by eq. (26) we then find from the eikonal equation, eq. (11):

$$c_e(z) = \frac{1}{q_0 \sin\theta} \tag{33}$$

where $c_e(z)$ is the effective speed of sound for a horizontal ray, given by $c_e = c + w_1$, with w_1 the component of the wind vector in the initial azimuthal direction of the ray: $w_1 = w_x\cos\varphi + w_y\sin\varphi$. Using eq. (27) this can also be written as:

$$\sin\theta = \frac{c_{e,s} - w_{l,s}}{c_e(z) - w_{l,s}} \tag{34}$$

where the subscript s refers to source altitude.

This equation, relating the altitude where a ray becomes horizontal and the polar emission angle θ, can be used to characterize some specific situations.

3.1.1. c_e has its maximum value at observer level.

This is the common situation if there is no wind (or in the upwind direction), as the temperature usually decreases with height. In this case the rays are bent upward, and a shadow zone exists. The minimum value of θ, denoted by θ_{min}, corresponds to the ray that just touches the observer position:

$$\sin\theta_{\min} = \frac{c_{e,s} - w_{l,s}}{c_{e,m} - w_{l,s}} \tag{35}$$

where the subscript m refers to observer (microphone) altitude. The profile of c_e and the limiting sound ray are depicted in Figure 2.

If the horizontal distance of the observer to the source is larger than that of the turning point of the limiting sound ray, then the observer is in the shadow zone. If not, as depicted in Figure 2, a sound ray connecting the source and the observer exists, and its polar emission angle is in the range $[\theta_{min}, \pi]$.

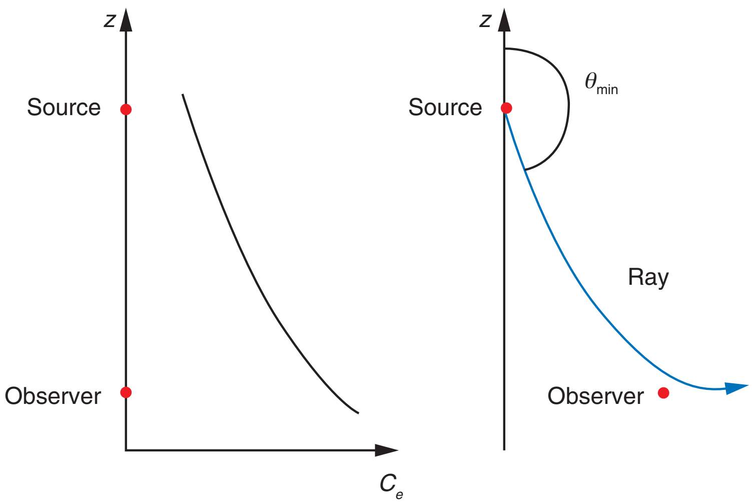


Figure 2: Profile of effective speed of sound c_e (left) for case I, and limiting sound ray (right).

3.1.2. c_e has its maximum value at source level

In some atmospheric conditions the temperature increases with altitude (temperature inversion). If there is no wind (or in the downwind direction), the effective speed of sound then increases with altitude. In this case the rays are bent downward. Any ray will eventually reach the ground and no shadow zone exists. In some cases the ray that reaches the observer first goes in the upward direction until it reaches a turning point. The altitude of the turning point is given by the solution of eq. (34), with $\theta < \pi/2$. A solution exists if c_e continues to increase above source level, see Figure 3.

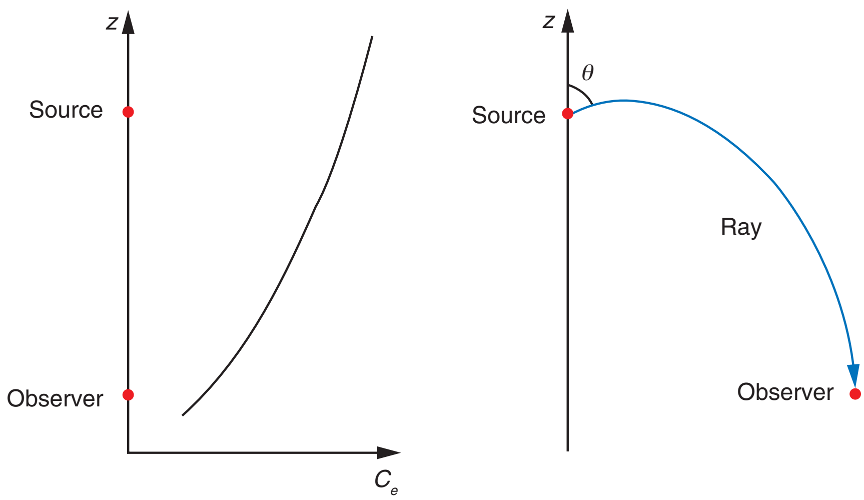


Figure 3: Profile of effective speed of sound c_e (left) for case II, and example of a sound ray (right).

3.1.3. c_e has its maximum value somewhere between observer and source level.
The altitude at which c_e has its maximum value is denoted by z^*, see Figure 4.

If a ray is emitted at an angle of:

$$\sin\theta_{\min} = \frac{c_{e,s} - w_{l,s}}{c_e\left(z^*\right) - w_{l,s}}, \quad \theta_{\min} > \frac{\pi}{2} \tag{36}$$

then at $z = z^*$ not only θ_z vanishes, but also dq_z/dt (see eq. (20)). This means that a ray emitted at θ_{min} will asymptotically approach the horizontal at $z = z^*$. At an emission angle larger than θ_{min} the ray will pass the level $z = z^*$ and then bend downward to the ground. Any distance can be reached and there is no shadow zone. The polar emission angle of the ray reaching the observer is to be found in the interval $(\theta_{min}, \pi]$.

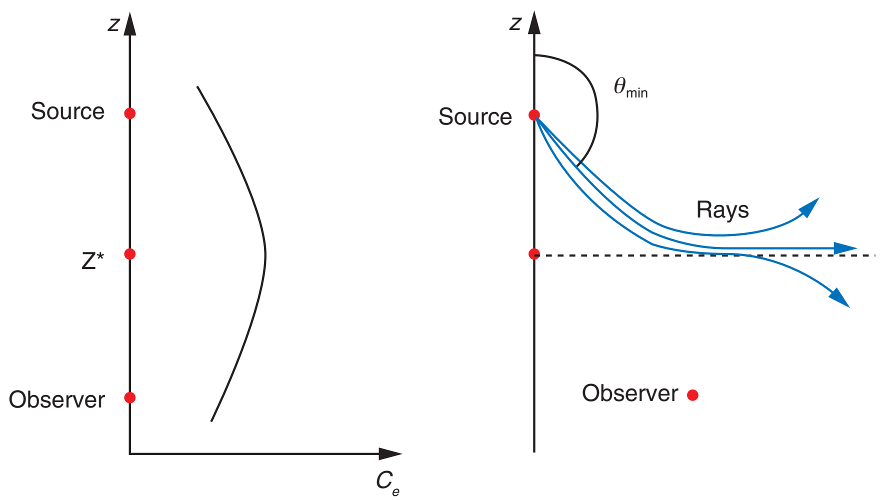


Figure 4: Profile of effective speed of sound c_e (left) for case III, and some rays (right).

3.2. Caustics

It sometimes happens that adjacent rays cross each other, which means that the cross section of the ray tube enclosed vanishes. At that point the amplitude is singular. The curve connecting such singular points is called a caustic. In this section we investigate how such a singularity can be explained from the mathematical formulation of section 2.

In a uniform atmosphere the amplitude decreases along a ray as $1/t$. The same can be shown for the spatial derivatives of the slowness vector, i.e. for the 9 components of $\nabla \otimes \vec{q}$. In the case of nonzero gradients of temperature and/or wind, it might be assumed that this behavior is approximately the same. This is not consistent however with eq. (25), where the last term, containing the second derivatives of c and $\vec{w}$, does not contain any spatial derivatives of $\vec{q}$ and will remain finite for large t. It is stated again that the expression for $\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial q_z}{\partial z}$ is the only one that contains these second derivatives.

In the following analysis we still assume that all derivatives of $\vec{q}$ go to zero for large t, except for $\frac{\partial q_z}{\partial z}$, which is assumed to become large. Then, retaining only the leading terms at large t, eq. (25) becomes:

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial q_z}{\partial z} = -\frac{c}{q^3}\left(q_x^2 + q_y^2\right)\left(\frac{\partial q_z}{\partial z}\right)^2 - \left[q\frac{\partial^2 c}{\partial z^2} + \vec{q}\cdot\frac{\partial^2 \vec{w}}{\partial z^2}\right] \tag{37}$$

Next we approximate the factor $\frac{c}{q^3}\left(q_x^2 + q_y^2\right)$ by a (positive) constant A, which is not a bad approximation assuming that $w \ll c$, and replace the last term by a constant ε, which may be justified if in the region of interest the ray is propagating mainly in the horizontal direction. We then have:

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial q_z}{\partial z} = -A\left(\frac{\partial q_z}{\partial z}\right)^2 - \varepsilon \tag{38}$$

For $\varepsilon = 0$ the solution is:

$$\frac{\partial q_z}{\partial z} = \frac{1}{A\left(t - t_0\right)} \tag{39}$$

which is consistent with the remarks above. The solution for $\varepsilon > 0$ is:

$$\frac{\partial q_z}{\partial z} = \sqrt{\frac{\varepsilon}{A}}\frac{1}{\tan\left(\sqrt{A\varepsilon}\left(t - t_0\right)\right)} \tag{40}$$

Initially, i.e. for $t - t_0 \ll \frac{1}{\sqrt{A\varepsilon}}$, this solution behaves as that of eq. (39), but at $t - t_0 = \frac{\pi}{\sqrt{A\varepsilon}}$ it has a (second) singularity.

For $\varepsilon < 0$ the solution is:

$$\frac{\partial q_z}{\partial z} = \sqrt{\frac{-\varepsilon}{A}}\coth\sqrt{-A\varepsilon}\left(t - t_0\right) \tag{41}$$

This solution has no other singularity than the one at $t = t_0$.

The singularity in the right hand side of eq. (40) corresponds to a caustic. If the amplitude of a ray that hits a caustic is computed by numerical integration of eq. (22), the integration will stall at that point. In the analysis above the approximations made are too crude to use it for an accurate computation of the value of t where the caustic occurs, but it can be used for an estimate if and in which region it may occur. It is clear from eq. (37) that caustics can be expected where the vertical profiles of c and $\vec{w}$ are strongly curved. This is usually only the case for $\vec{w}$ near the ground, if a no-slip condition is imposed. A commonly used shape of the wind profile near the ground is $w \propto \ln(1+z/z_0)$. In the downwind direction $\vec{q} \cdot \dfrac{\partial^2 \vec{w}}{\partial z^2}$ is negative ($\varepsilon < 0$), in the upwind direction it is positive ($\varepsilon > 0$). In conclusion, caustics may be expected near the ground in the upwind direction.

If a downward travelling ray ($q_z < 0$) is approaching a caustic, the value of $\partial q_z / \partial z$ goes to $-\infty$ which means that q_z has to increase until it equals zero. For $q_z \approx 0$ it follows from eq. (20) that q_z is proportional to $t - t_c$, with t_c the time the ray passes the caustic. This implies that if a ray passes a caustic at altitude z, its location is on the border of the shadow zone at that altitude. Although the trajectory of the ray can be computed after passing the caustic, its amplitude has a singularity and is not defined for $t > t_c$. As an example 3 rays are plotted in Figure 5 for a source at z = 300 m, and a wind in the positive x direction. The ray at the right just touches the ground, and marks the border of the shadow zone at ground level.

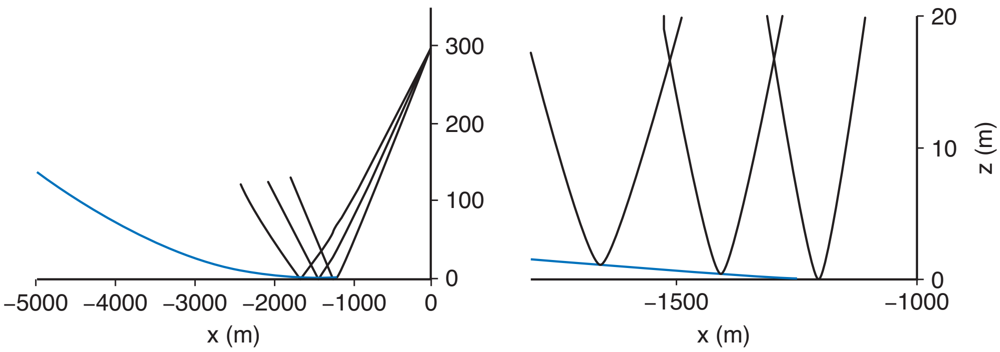


Figure 5: Black curves are rays, blue curve is the caustic. The plot on the right is an enlargement.

4. NUMERICAL SOLUTION

The computation of the sound level at a specific observer location consists of two steps:

1. Firstly, the relations of section 3.1 are used to assess whether a ray exists that connects the source and the observer. If not, the observer is in the shadow zone. If it does exist, there may be more than one because of reflection at the ground. An iteration procedure is used to find these rays.
2. Integration of the differential equation for the amplitude along the rays found.

4.1. Iteration procedure

Each ray can be specified by its emission angles θ and φ, see Figure 1, and finding the correct ray amounts to finding the correct emission angles. In a uniform atmosphere these would be the angles corresponding to the straight line connecting the source and the observer. The most straightforward procedure to find the correct angles is to start with a suitable set of emission angles, integrate eqs. (17) and (20) with some time-stepping method until the smallest distance between the ray and the observer location is reached, apply a correction step to the emission angles and reiterate this procedure until a convergence criterion is reached. In the present version of RAYTRAC the time-stepping is replaced by a definite integration, as described in section 11.1 of reference [7]. It is assumed here that the source is at a higher altitude than the observer. If a ray propagates from an altitude z_s to an altitude z_m, $z_m < z_s$, the total distance travelled in x-direction is:

$$s_x = \int_{z_s}^{z_m} \frac{\mathrm{d}x}{\mathrm{d}t}\left(\frac{\mathrm{d}z}{\mathrm{d}t}\right)^{-1} \mathrm{d}z = \int_{z_s}^{z_m} \frac{1}{q_z}\left(q_x + \frac{w_x}{c} q\right) \mathrm{d}z \tag{42}$$

and similar for s_y. Note that q_x and q_y are constant along the ray, so q_z and q can be evaluated for every z by using the eikonal equation (eq. (11)). If the ray passes a turning point or is reflected, i.e. q_z changes sign, the integrand becomes multi-valued, and the integration has to be split into parts where the integrand is single-valued. In these and other cases q_z becomes zero at one of the integration limits. It can be proven that if q_z equals zero at z_s or z_m, it behaves as $\sqrt{|z_{s,m} - z|}$. To remove the singularities, the integration variable is changed to $\zeta = \sqrt{\sqrt{z_s - z_m} - \sqrt{z - z_m}}$ in the numerical implementation.

Next it is described how the 'shooting' direction, i.e. θ and φ, is determined such that a ray actually hits the specified observer position. First a ray is shot downward, i.e. $\theta = \pi$. The point where this ray reaches the observer height, denoted by $\vec{x}_0 = (x_0, y_0)$, is taken as the origin of a new coordinate system. The ξ-axis is chosen along the line connecting the new origin $\vec{x}_0$ with the observer point $\vec{x}_m$, and the η-axis is perpendicular to that, see Figure 6. The starting value of the azimuthal angle in the iteration procedure is set at $\varphi = \mathrm{atan}\left(\frac{y_m - y_o}{x_m - x_o}\right)$. Next a root finding procedure is applied to the function $s_\xi(\vartheta) - s_\mathrm{m}$,

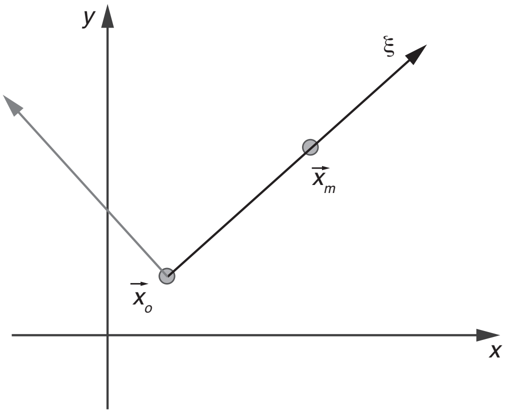


Figure 6: Coordinate system used in iteration procedure.

with:

$$s_m = \left|\vec{x}_m - \vec{x}_0\right| \tag{43}$$

s_ξ is the equivalent of s_x, defined in eq. (42), in the new coordinate system. Next the root finding procedure is applied to the function $s_\eta\,(\varphi)$, which yields a new value for φ. This procedure is reiterated until convergence is reached. The change in the azimuthal angle φ is caused by the crosswind component of the wind, denoted by $w_\perp$, and will be small, in the order of the average value of $w_\perp/c$, unless the observer is close to the point directly under the source. Therefore, convergence is usually reached after a few (~ 3) iterations.

Note that if instead of $\vec{x}_0$ the point vertically below the source would be taken as origin, i.e. $(x_s, y_s, 0)$, this iteration procedure would not work well for points close to the origin in the case of any wind.

If a ray is sought that is first reflected at the ground ($z = 0$), the integrals as in eq. (42) are split into two parts according to:

$$\int_{z_s}^{0} + \int_{0}^{z_m} \tag{44}$$

4.2. Computation of the amplitude

Once the emission angles are known, a direct integration of the system of the 16 coupled ordinary differential equations derived in section 2.2 is still necessary for the computation of the amplitude P. The evaluation of a definite integral expression like in eq. (42) is not possible for P, as the right hand side of eq. (22) contains variables that are not known beforehand at each z.

In RAYTRAC the equations are integrated in time by using a 4th-order Runge-Kutta method with adaptive time step size, e.g. Press et al. [8]. To include the atmospheric absorption, the term:

$$-\frac{\ln(10)}{20}\alpha(z)c(z)P \tag{45}$$

is added to the right hand side of eq. (22), with $\alpha(z)$ the coefficient of atmospheric absorption in dB per unit distance.

As explained in section 3.2 the numerical integration will stall near a caustic. Based on the analysis of section 3.2 the presence of a caustic is detected, which will terminate the integration.

4.3. Reflection at the ground

If a microphone is placed at some height above the ground, it can be reached by a direct ray and by a ray that is reflected at the ground. The emission angles of the reflected ray are determined as explained at the end of section 4.1. If the acoustic property of the ground is given by a normalized impedance Z, the reflection coefficient is given by:

$$R = \frac{Z\cos\theta_g + 1}{Z\cos\theta_g - 1} \tag{46}$$

with θ_g the value of the polar direction angle at the ground:

$$\cos\theta_g = \left.\frac{q_z}{q}\right|_{z=0} \tag{47}$$

the value of which follows directly from the emission angles and the atmospheric conditions at the ground. Here q_z has to be evaluated just before reflection and thus has a negative value. (Note that $\theta_g = \pi\text{-}\theta_i$, with θ_i the angle of incidence.) We assume for simplicity that the rays arrive at the observer at a time that the phase *at the source* is zero. If the travel time of the direct ray, with amplitude P_1, is t_1, its phase equals $-\omega t_1$. The same for the ray that is reflected at the ground, with P_2 the amplitude that is obtained by the integration of eq. (22). The total amplitude at the microphone is thus given by:

$$P = \left|P_1 e^{-\omega t_1} + RP_2 e^{-\omega t_2}\right| \tag{48}$$

Once the emission angles are known, the travel time t_t of a ray can be computed by integration over z of:

$$\left(\frac{\mathrm{d}z}{\mathrm{d}t}\right)^{-1} = \frac{q}{cq_z} \tag{49}$$

where again the integral has to be split up for the reflected ray.

5. NUMERICAL EXAMPLES

5.1. Source at low altitude

As a first example we analyze the following configuration:

- Microphone at $z_m = 1.2$ m.
- Source at $\vec{x}_s = (0, 0, 300\,\text{m})$.
- Frequency: 1000 Hz.
- Linear temperature profile: 15 °C on the ground, decreasing with 6.5°C/km.
- Approximately logarithmic wind speed profile in x-direction, with $w_x(z{=}10\text{m}) = 7$ m/s.
- Constant relative humidity: 50%.
- Normalized ground impedance 2.646 – i 2.235.

The wind profile is shown in Figure 7.

The normalized impedance has been derived with the expression of Delany & Bazley [9] from a flow resistivity of $\sigma = 100{,}000$ Pa s/m^2, which is a typical value for grass. In Figure 8 the profiles of the effective speed of sound c_e in downwind and upwind direction are plotted.

The profile in downwind direction has (barely visible) a maximum at $z = 255$ m, which means that there are no multiple reflections and there is no shadow zone (type III in section 3.1). The profile in the upwind direction is steadily increasing with z, and a shadow zone exists (type I in section 3.1). In Figure 9 the results for the transmission loss are plotted for a grid of observer positions at a height of 1.2 m. The transmission loss is defined here as the difference of the sound level at the observer and that at 1 m from the source. The pattern caused by the interference of the direct and indirect rays is clearly visible. The black curve indicates the border of the shadow zone at ground level, and the red line the location of the caustic.

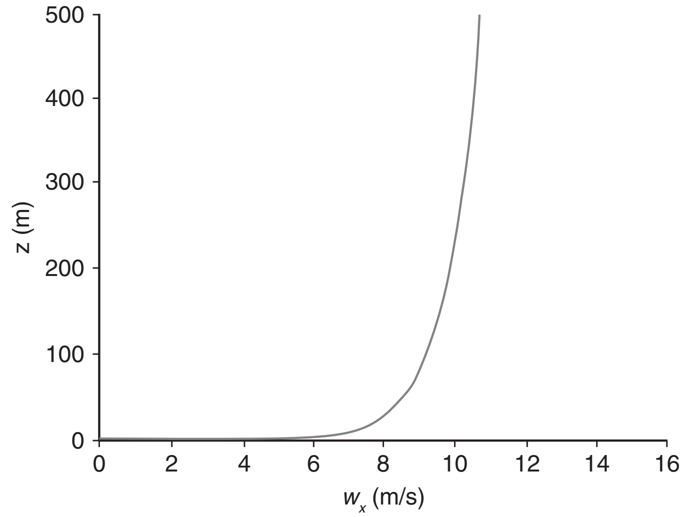


Figure 7: Wind profile of example.

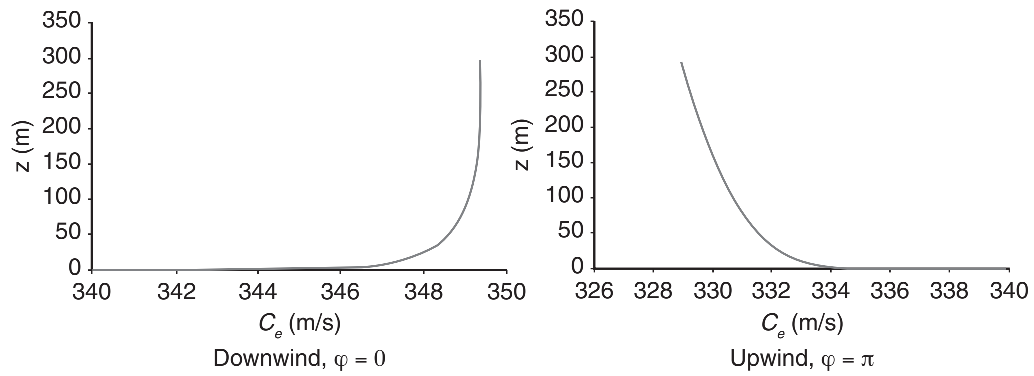


Figure 8: Profiles of effective speed of sound c_e in downwind (left) and upwind direction (right).

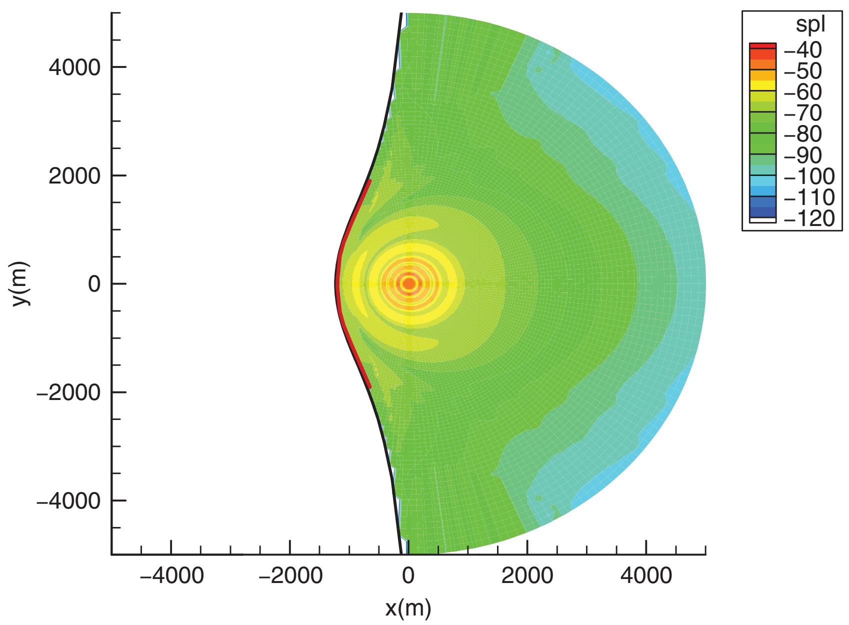


Figure 9: Transmission loss for example 1, at observer height 1.2 m.

5.2. Source at high altitude

As a second example a configuration is defined that corresponds to an aircraft at cruise altitude:

- Microphone at z_m = 1.2 m.
- Source at $\vec{x}_s$ = (0,0,1000 m).
- Frequency: 334 Hz.
- Constant relative humidity: 70%.

· Normalized ground impedance 3.2272 – i 2.9982.
The wind and temperature profile are the same as above, extended to z = 10000 m.
The results for the transition loss in this case are shown in Figure 10.
Again, the shadow zone is clearly visible at the left hand side.

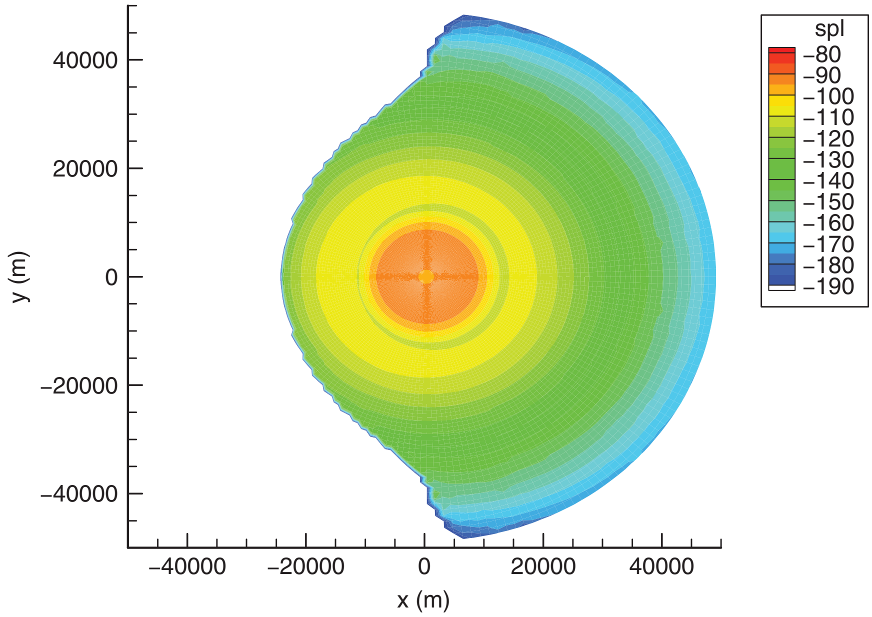


Figure 10: Transmission loss for example 2, at observer height 1.2 m.

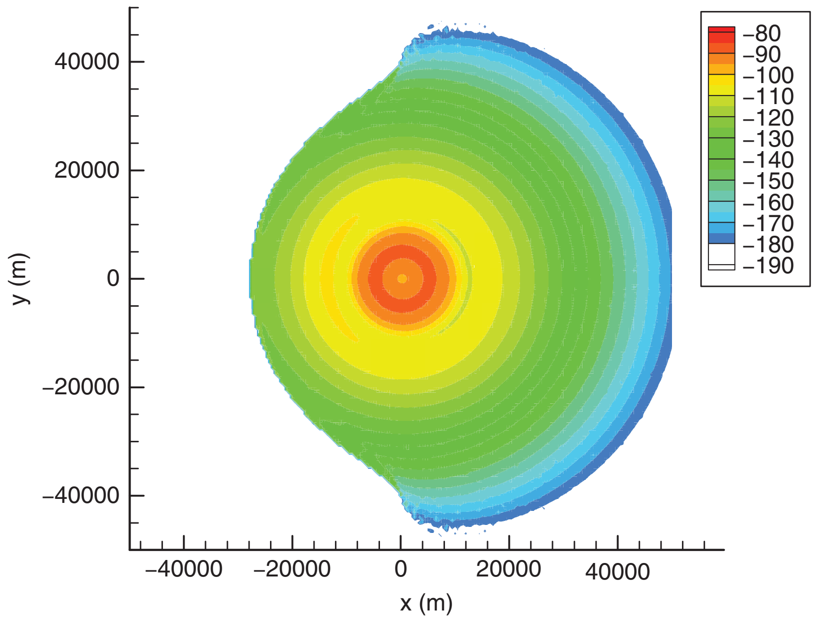


Figure 11: Results of APHRODITE for example 2.

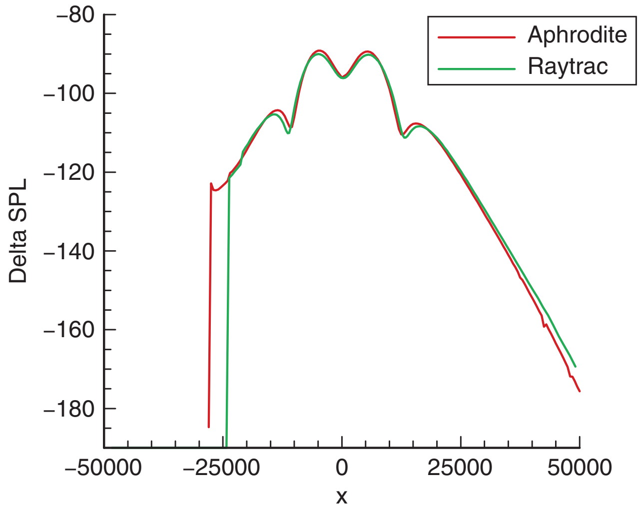


Figure 12: Comparison of transmission loss results from RAYTRAC and APHRODITE.

This case has also been computed with the APHRODITE program. APHRODITE is a ray tracing program based on a different approach: the stratified atmosphere is divided in segments with a linearly varying speed of sound and wind velocity. In each segment the ray follows then a circular path. By matching these paths at the interfaces, the complete path is obtained. The amplitude is obtained by computing a number of adjacent rays, and determining the area enclosed by the ray tube. Details are presented in another paper in this issue, reference [10].

The results are shown in Figure 11.

Some differences occur near the origin. To have a better view of the magnitude of these differences, the transmission loss for $y = 0$ is plotted as function of x, for both sets of results, in Figure 12.

This plot shows that the differences are quite small, except for the location of the shadow zone. This is caused by the choice made in RAYTRAC that, for simplicity, the boundary of the shadow zone is determined for an observer at ground level. For an observer level of 1.2 m the shadow zone is shifted a little in upwind direction. As the noise level near the boundary is not adequately captured by ray theory anyhow, this is of little practical importance.

Overall, the agreement is very satisfactory.

6. CONCLUSIONS

This paper describes the mathematical model on which the ray tracing program RAYTRAC is based. The main result of the mathematical analysis consists of a number of differential equations, describing:

1. the ray paths, i.e. the propagation of a point of constant phase,
and
2. the evolution of the amplitude along a ray path.

Especially the latter poses a difference with most of the existing ray tracing methods, where the amplitude is determined by computing the cross section of a ray tube.

An analysis of these equations is given with respect to their possible solutions under various atmospheric circumstances. It is shown under which conditions a shadow zone occurs, and how the limiting ray, connecting the source and the border of the shadow zone, can be determined. Also an analysis of caustics is presented, showing how the amplitude may become singular in some regions. An efficient iteration procedure is presented that finds the ray connecting the source and a specific observer point. It is shown how multiple rays can be added, with incorporation of a reflecting surface. Finally, the tool is demonstrated by application to two examples that are representative for an aircraft radiating sound to an observer on the ground. The results of the second example are compared to those of an alternative ray tracing method. Despite a completely different approach, both sets of results show only minor differences.

It is concluded that RAYTRAC is an efficient and accurate tool for the computation of sound propagation through a stratified atmosphere.

REFERENCES

[1] Pierce, A.D., Acoustics - An Introduction to Its Physical Principles and Applications, McGraw-Hill, New York, 1981.

[2] Schulten, J.B.H.M., Computation of aircraft noise propagation through the atmospheric boundary layer, *Proceedings of the Fifth International Congress on Sound and Vibration*, University of Adelaide, Australia, 1997.

[3] Mcaninch, G.L. and Heath, S.L., On The Determination of Shadow Boundaries and Relevant Eigenrays For Sound Propagation In Stratified Moving Media, *AIAA Paper* 2006–0412, 2006.

[4] Rienstra, S.W. and Hirschberg, A., An Introduction to Acoustics, *Report IWDE 01 – 03*, Eindhoven University of Technology, 2001 (available on *http://www.win.tue.nl/÷sjoerdr/papers/boek.pdf)*.

[5] Myers, M.K., An exact energy corollary for homentropic flow, *Journal of Sound and Vibration* 109, pp. 277–284, 1986.

[6] Pridmore-Brown, D. C., Sound Propagation in a Fluid Flowing through an attenuating Duct, *Journal of Fluid Mechanics* 4, pp. 393–406, 1958.

[7] Attenborough, K., Li., K.M., and Horoshenkov, K., Predicting outdoor sound, Taylor & Francis, 2007.

[8] Press, W.H., Flannery, B.P., Teukolsky, S.A., Vetterling, W.T., Numerical Recipes, Cambridge University Press, Cambridge, 1989.

[9] Delany, M.E. and Bazley, E.N., Acoustical properties of fibrous absorbent materials, *Appl. Acoust.*, Vol. 3, pp. 105–116, 1970.

[10] Tuinstra, M., Aphrodite: Atmospheric refraction prediction method, this issue.

This page is intentionally left blank.

WHAT IS NLR?

The NLR is a Dutch organisation that identifies, develops and applies high-tech knowledge in the aerospace sector. The NLR's activities are socially relevant, market-orientated, and conducted not-for-profit. In this, the NLR serves to bolster the government's innovative capabilities, while also promoting the innovative and competitive capacities of its partner companies.

The NLR, renowned for its leading expertise, professional approach and independent consultancy, is staffed by client-orientated personnel who are not only highly skilled and educated, but also continuously strive to develop and improve their competencies. The NLR moreover possesses an impressive array of high quality research facilities.

***NLR** – Dedicated to innovation in aerospace* www.nlr.nl