Nationaal Lucht- en Ruimtevaartlaboratorium

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NLR-TP-2013-299

Accuracy analysis of propellant gauging systems

R.C. van Benthem

Co-authors

P. van Put

R. Matthijsen

Nationaal Lucht- en Ruimtevaartlaboratorium
National Aerospace Laboratory NLR

Anthony Fokkerweg 2
P.O. Box 90502
1006 BM Amsterdam
The Netherlands
Telephone +31 (0)88 511 31 13
Fax +31 (0)88 511 32 10
Web site: http://www.nlr.nl

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Executive summary

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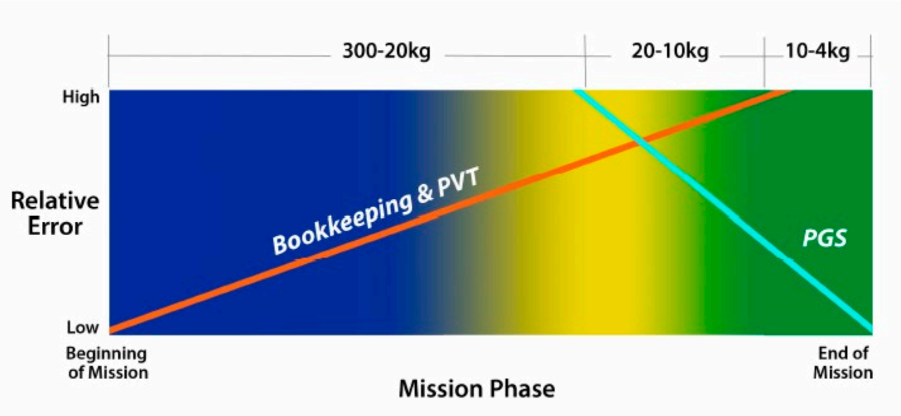

Schematic representation of propellant gauging systems error built-up (ref 1.)

Report no.
NLR-TP-2013-299

Author(s)
R.C. van Benthem

Report classification
UNCLASSIFIED

Date
March 2014

Knowledge area(s)
Ruimtevaarttoepassingen

Descriptor(s)
Propellant Systems
Gauging Systems
Error Analysis

Problem area
Gauging systems currently in use have an end-of-life prediction accuracy of typically a few months up to a year for geostationary satellites. Replacement of a scientific or commercial satellite after 10-15 years in service is very expensive, putting pressure on improvement of end-of-life gauging accuracy for life-time extension.

Description of work
For comparison of gauging methods in use, a system analysis is performed on typical mono- and bi-propellant systems to evaluate the factors influencing gauging over life related to the propellant load from ground filling towards end-of-life operations in orbit. In general the gauging accuracy reduces over time. Three common gauging methods have been investigated: PVT (gas law), Bookkeeping (propellant flow integration) and (T)PGS (“Thermal Knocking”).

Results and conclusions
The accuracy analysis shows that the propagation of uncertainties is related to the propellant load, system design, ground filling, orbital operations and applied sensors. Using state-of-the-art data it was found that Bookkeeping currently provides the highest gauging accuracy between ±3 months for 10 years up to ±12 months for 15 years. A significant improvement of the system analysis, ground operations and in-flight sensor technology is required to improve the gauging accuracy up to ±1 months for new satellites.

Applicability
Satellite gauging systems.

This report is based on a presentation held at the AIAA/ICES Conference, Vail, Colorado, USA, July 15-18, 2013.

Nationaal Lucht- en Ruimtevaartlaboratorium, National Aerospace Laboratory NLR

Anthony Fokkerweg 2, 1059 CM Amsterdam,
P.O. Box 90502, 1006 BM Amsterdam, The Netherlands
Telephone +31 88 511 31 13, Fax +31 88 511 32 10, Web site: www.nlr.nl

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This publication has been refereed by the Advisory Committee AEROSPACE SYSTEMS.

Customer	Netherlands Space Office
Contract number	1020301
Owner	NLR
Division NLR	Aerospace Systems
Distribution	Unlimited
Classification of title	Unclassified
	March 2014

Approved by:

Author	Reviewer	Managing department
R. van Benthem	J. van Es	M. Keuning
Date: 28/3/14	Date: 31/03/14	Date: 31/03/2014

Contents

Accuracy analysis of propellant gauging systems

R.C. van Benthem [1] and J. van Es [2]
National Aerospace Laboratory, NLR, Amsterdam, The Netherlands

P van Put [3] and R Matthijsen [4]
Moog Bradford Engineering, Heerle, The Netherlands

Gauging systems currently in use have an end-of-life prediction accuracy of typically a few months up to a year for geostationary satellites. Replacement of a scientific or commercial satellite after 10-15 years in service is very expensive, putting pressure on improvement of end-of-life gauging accuracy for life-time extension. For comparison of gauging methods in use, a system analysis is performed on typical mono- and bi-propellant systems to evaluate the factors influencing gauging over life related to the propellant load from ground filling towards end-of-life operations in orbit. In general the gauging accuracy reduces over time. Three common gauging methods have been investigated: PVT (gas law), Bookkeeping (propellant flow integration) and (T)PGS ("Thermal Knocking"). The accuracy analysis shows that the propagation of uncertainties is related the propellant load, system design, ground filling, orbital operations and applied sensors. Using state-of-the-art data it was found that Bookkeeping currently provides the highest gauging accuracy between ±3 months for 10 years up to ±12 months for 15 years. A significant improvement of system analysis, ground operations and in-flight sensor technology is recommended to improve the gauging accuracy up to ±1 months for new satellites.

Abbreviations

AIT	=	Assembly Integration & Testing on ground
BK	=	Book-keeping
BOL	=	Begin-of-Life orbital operations
EOL	=	End-of-life orbital operations
GTO	=	Geostationary Transfer Orbit
HAPT	=	High Accuary Pressure Transducer (Moog Bradford)
MID	=	Mid-Life orbital operations
OMS	=	Orbital Maneuver System (Space shuttle)
PVT	=	Pressure, Volume, Temperature
RCS	=	Reaction Control System (Space shuttle)
RSS	=	Root Sum Square
TPGS	=	Thermal Propellant Gauging System "Thermal Knocking"
UFM	=	Ultrasonic Flow Meter (Moog Bradford)

[1] R&D Engineer Thermal Control, Space Department, tel: +31 88 511 4231, fax: +31 88 511 4210, email: roel.van.benthem@nlr.nl
[2] R&D Manager Thermal Control, Space Department, tel: +31 88 511 4231, fax: +31 88 511 4210, email: johannes.van.es.@nlr.nl
[3] Technical Director, tel: +31-165 305 161, fax. +31 165 304 422, email:p.vanput@bradford-space.com.
[4] Senior Project Lead, Chemical Propulsion Components, tel: +31 165 305 168, fax : +31 165 304 422, email: r.matthijssen @bradford-space.com

Nomenclature

β_M	=	Propellant load/mass ratio (=M_L/M_{AIT}) related to a full tank [-]
C	=	Effective conduction tank supports [W/K]
C_p	=	Specific heat capacity (tank, propellant) [J/Kg/K]
D_T	=	Tank diameter (cylinder) [m]
D_m	=	Mission duration [days, months, years]
ε	=	Absolute measurement error [unit]
h	=	Tank height (cylinder) [m]
H_L	=	Heat capacity propellant [J/K]
H_s	=	Heat capacity system (= tank + propellant) [J/K]
H_T	=	Heat capacity tank [J/K]
M_{AIT}	=	Propellant mass fill on ground [kg]
m_{He}	=	Helium mass pressurant tank [kg]
M_L	=	Propellant mass in tank [kg]
n	=	Burn number [-]
N	=	Total number of burn until end-of orbital operations [-]
P_e	=	Electrical (heater) power [Watt]
P_P	=	Pressurant (Helium) pressure [bar]
P_U	=	Ullage pressure [bar]
$P_{U,0}$	=	Ullage pressure on ground during AIT [bar]
V_L	=	Propellant volume [liter]
$V_{L,0}$	=	Propellant volume on ground during AIT [liter]
V_P	=	Helium tank volume [liter]
V_T	=	Propellant tank volume [liter]
V_U	=	Ullage volume [liter]
$V_{U,0}$	=	Ullage volume on ground during AIT [liter]
R_{He}	=	Specific Gas constant Helium [2077 J/kg/K]
T	=	Temperature [^{o}C]
ΔT	=	Temperature difference [^{o}C]
t	=	Time [sec]
Δt	=	Time period (burn time or heater on) [sec]
μ	=	Averaged system property at a point [-]
z_0	=	System property at a point [-]
Φ	=	Propellant mass flow rate during burn [Kg/sec]
'Gas law'	=	PV=mRT

I. Introduction

For a cost effective decommissioning it is crucial to have accurate propellant gauging systems on board of commercial and scientific satellites. However, by a combination of sensor accuracies, design and cost limitations, the best gauging systems in use typically have an End-of- life (EOL) prediction of ± 4-6 months for a geostationary satellite. In order to guarantee a 10-15 year of continuous operation satellite builders usually take appropriate margins in the amount of propellant on board. A more accurate monitoring of the remaining propellant could extend operational life and delays replacement. Also in case of emergencies, having an accurate gauging, orbital life can be extended by the operators by reducing station-keeping operations to a bare minimum. Since improving gauging accuracy is linked with the system design, sensor accuracies and costs, a general comparison of all gauging methods is hard to perform. This paper focuses on three most common methods that are analyzed in more detail how to improve gauging accuracy.

- PVT ("Gas law"),
- Book-Keeping ("Propellant Flow Measurement"),
- (T)PGS ("Thermal Knocking")

From **Figure 1** it is concluded that whereas the uncertainty of PVT and Bookkeeping increases over time the accuracy of TPGS is higher when the propellant load becomes below 20kg, indicating that a combination of methods could be beneficial to improve gauging.

A system function of these common propellant systems is constructed for a comparison of the gauging errors built-up over life related to propellant load. The error analysis investigates how design margins and sensor uncertainties propagate through the system during its lifetime. Since not all errors contribute in the same manner, the analysis considers what margins to apply for each error contribution. From a system design point of view all error contributions should be balanced as much as possible in a cost effective way. This paper proposes a linear method to investigate the propagation of errors e.g. a set of system and sensors requirements to investigate the accuracy currently achievable using state-of the-art technology and how to improve the gauging accuracy up to ±1 month for new satellite systems.

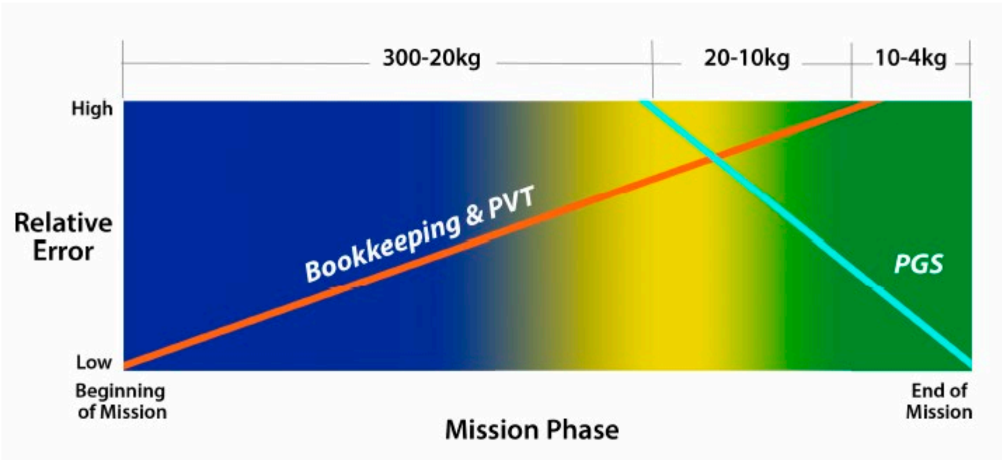

Figure 1 Schematic representation of propellant gauging errors (ref 1.)

II. Gauging requirements

Typical parameters for propellant systems for satellites are hard to obtain but propellant tank sizes are usually optimized for life time operations including margins. The uncertainty of gauging is related to the system design for instance a mono or bi-propellant system with or without pressuring tanks, the measurements method(s) applied, the sensor accuracy and the propellant fill ratio. For a general analysis of the system performance over life and system function is defined. A way to investigate how errors build-up is to consider the propellant mass ratio: the *relative* amount of propellant with respect to the propellant mass fill during ground testing. On ground (AIT) propellant tanks are typically loaded for about 90% on volume base. From the literature [ref 3] most propellant on board, between 75-90%, is used for transfer the satellite to its geostationary orbit (GTO), starting Begin-of-Life (BOL) orbital operations with about 10-25% propellant on board. At End-of-Life (EOL), before transfer to its graveyard orbit, a minimum of 1-2% propellant is required to complete its mission. See **Table 1** for the relative propellant mass depletion over life for a typical geostationary satellite and for example a system having a 500 kg tank.

Acronym	Life phase	Propellant load factor, β_M	Propellant mass [Kg] (example for a 500 Kg tank)
AIT	Assembly, Integration & Testing, propellant tank filled	1	500
GTO/BOL	End of transfer to GTO/Start of orbit operations	0.1-0.25	50-130
MID	Mid-life orbit operations	0.04-0.12	20-60
EOL	End of operations before transfer into graveyard orbit	0.01-0.02	5-10

Table 1: Propellant load during the subsequent life phases of a typical geostationary satellite

In order to estimate the amount of propellant expelled per month for a 10-15 years missions duration the above values are graphically displayed between an optimistic and pessimistic scenario in **Figure 2**.

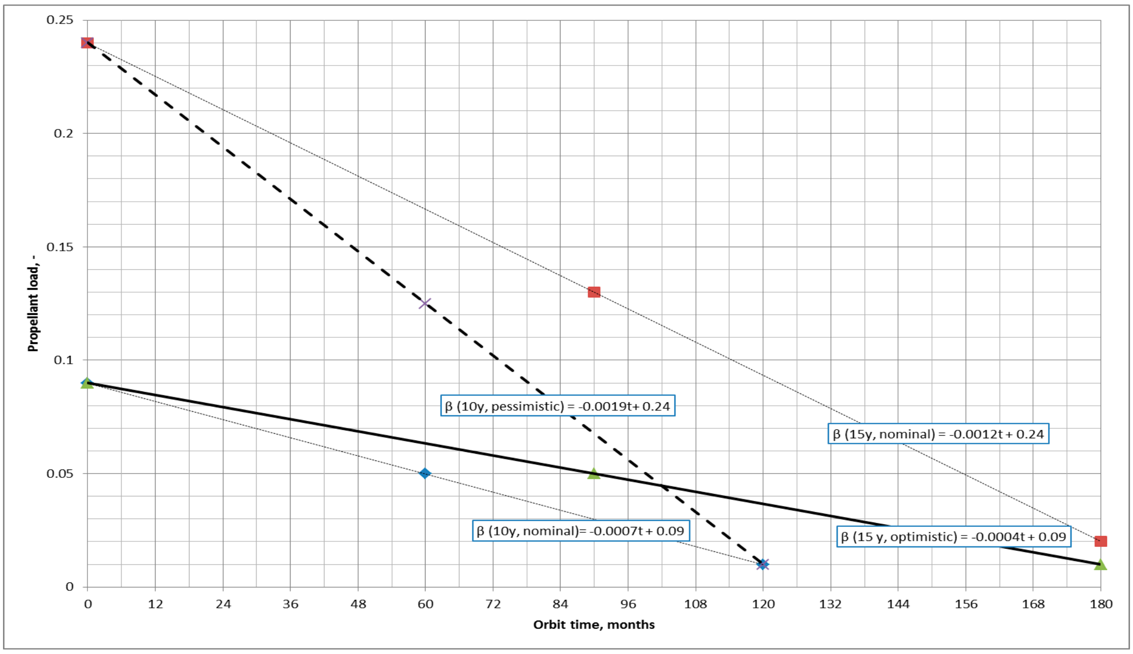

Figure 2: Estimation of optimistic or pessimistic propellant load for a 10 to 15 years mission

From the slope of the black solid and dotted lines in **Figure 2** the required relative propellant load accuracy is between ±0.19-0.04% for 10-15 years to obtain a ±1 months EOL gauging error. For ±6 months EOL gauging error this is multiplied by 6, resulting in ±1.2-0.24% for 10-15 years relative propellant load accuracy. For very optimistic cases (future missions, very low monthly propellant consumption) the target relative accuracy could be set on ±0.04%. See also **Table 2** for the corresponding mass range for a 500 kg propellant tank. That this is an ambitious target is clear from the fact that the relative accuracy for the loaded propellant mass during AIT is currently about ±0.1% [ref 3] which indicates that a factor of 5 to 10 improvement is needed for ground filling operations to achieve a ±1 months gauging accuracy at all.

Mission scenario	Expelled propellant (mass) per month	Required relative propellant (mass) accuracy for an EOL gauging error of	
	Δβ/dt	**± 1 months**	**± 6 months**
15 years (optimistic)	-0.04% *[-0.2 kg]**	±0.04% *{±0.2 kg]**	±0.24% *[±1.2 kg]**
10 years (pessimistic)	-0.19% *[-0.95 kg]**	±0.19% *[±0.95 kg]**	±1.2% *[±6 kg]**

Table 2: Expelled propellant per month and the corresponding 1-6 months EOL gauging accuracy requirements. As an example the propellant mass is established for a 500 kg tank

** Example calculation for a 500 kg tank*

For an EOL gauging prediction target within ± 1 month relative load accuracy during AIT between ±0.04% and ±0.2% for a 15years optimistic or a 10 years pessimistic scenario respectively is required. Since the gauging accuracy relates to the propellant system design and relative tank fill ratio (see section III), the gauging method (see section IV) and the sensor errors this is investigated in the next sections.

III. Typical Propellant Systems

To model how design uncertainties and measurement errors affect the gauging accuracy over life information about typical mono and bi-propellant systems is collected. To find relative scaling factors related to the tank sizes, design data of two typical systems is collected: (I) Mono-propellant (Hydrazine or Green Propellants), (II) Bi-propellant (Hydrazine & oxidizer). Since the vapor pressure of most propellants and Oxidizers such as hydrazine and Nitrogen tetroxide is quite low at room temperature the tank pressure is increased to typically 15-20 Bar using a pressurant such as Helium. A mono-propellant system has a single tank (**Figure 3**a) in which the propellant is contained that is pressurized by gaseous Helium. The system pressure is related to amount of Helium, the tank size and propellant volume. No pumps are needed but a disadvantage is that the system pressure significantly drops (which is bad for gauging) when the propellant is expelled. For a bi-propellant system (**Figure 3**b) both the fuel and oxidizer tanks are pressurized via a latch-valve with gaseous Helium. The advantage of this arrangement is that the tanks are pressure controlled until blown down. Mechanical fuel pumps are not needed to produce a flow which increasing the systems reliability. Auxiliary high pressure tanks (typically at 200-350 Bar, **Table 3**) provide for the Helium. Typical propellant systems design parameters are hard to assess but for the Space Shuttle's Orbital Maneuver System (OMS) and Reaction Control System (RCS) some design data is given in **Table 3**. Note that the ratios for the tank volume, pressure and Helium for the OMS and RCS are (nearly) the same indicating similar design considerations. For the gauging analysis these design ratios as found for the space shuttle are generally assumed to apply for comparable propellant systems.

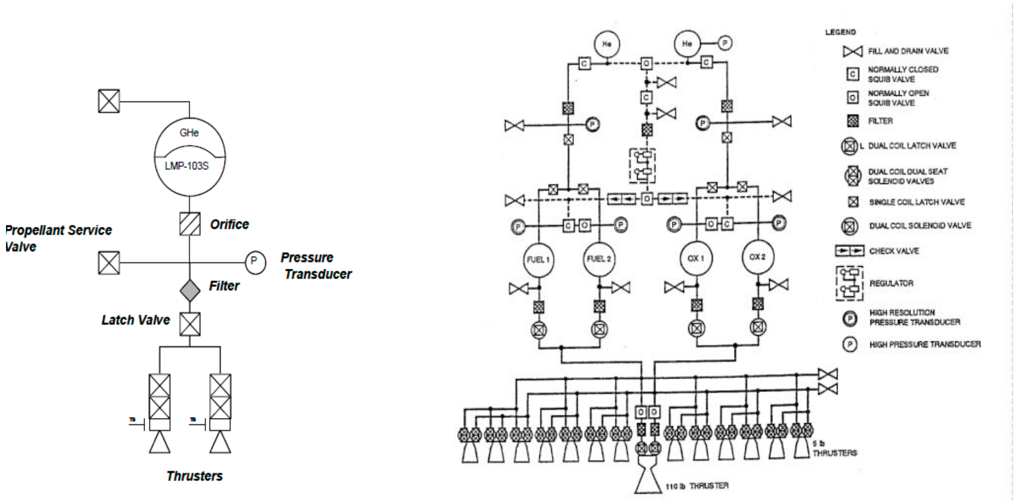

Figure 3: Typical mono- (left) and bi-propellant (right) system using Helium as pressurant

Symbol	Item	OMS	RCS	Unit
m_O	Oxidizer (N_2O_4) weight	6800	664	Kg
m_F	Fuel (MMH) weight	4080	418	Kg
V_T	Propellant tank volume	2550	506	Liter
V_U	Ullage volume (full tank)	220	34-42	Liter
V_P	Helium tank volume	481	100	Liter
P_U	Tank pressure (nominal)	17	19	Bar
$P_{P,\ AIT}$	Helium storage pressure BOL/AIT	324	248	Bar
$P_{P,\ EOL}$	Helium storage pressure EOL*	233	156	Bar
$m_{He,AIT}$	Helium mass auxiliary tank AIT**	25	4	Kg
$m_{He,EOL}$	Helium mass auxiliary tank EOL***	7	1.5	Kg

Design Ratios	OMS	RCS
m_F/m_O	0.6	0.63
V_L/V_T	0.91	0.92
V_P/V_T	0.19	0.2
P_U/P_P	0.052	0.076
$Pp_{,EOL}/Pp_{,BOL}$	0.71	0.63
$m_{He,AIT}/m_{He,BOL}$	0.28	0.38

Table 3: Typical design data of the Orbital Maneuver (OMS) and Reaction Control System (RCS) of the space shuttle

Remarks:

** The pressure drop is related to the work the Helium gas provides to blown down the propellant tank*
*** Estimated at T=300K using gas law R_{He}=2077 J/Kg/K => m=PV/RT*
**** The residual Helium in the storage tanks after blown down could be used for cold gas propulsion.*

IV. Gauging Systems Investigated

Three most common gauging methods are investigated in detail for the analysis which is either based on:

(I) PVT (e.g. gas law),
(II) Bookkeeping (integration of propellant flow)
(III) TPGS (thermal knocking).

The above methods are investigated with respect to their accuracy for life-time gauging. Typical gauging sensors applied are: pressure transducers, temperature sensors and flow-meters.

Method	Acronym	Description	Sensors
I	**PVT**	Pressure, Volume, Temperature (gas law): based on estimation of system properties by measurements of absolute pressure and temperature differences after re-pressurization	Pressure Transducers, Temperature sensors
II	**BK**	Book-Keeping: based on counting burns and summing propellant flow measurements. Also thrust feed-back can be applied (not investigated in this paper).	Flow-meter Temperature, pressure sensors, accelerometers (thrust)
III	**TPGS**	Thermal Propellant Gauging System: based on the measured of the temperature response (thermal mass) of the insulated fuel tank after heating with an electrical heater.	Temperature sensors, Power measurements

Table 4: Typical parameters and sensors for three gauging systems

For a first order calculation of an error function for gauging the following physical influences are neglected: gas compressibility, propellant vapor pressure, pressurant solvability, tank stretch, thermal expansion of tank and the propellant. For TPGS most of the propellant is assumed to be located at the tank walls using capillary material rather than floating around in low gravity. The thermal response of the tank and propellant is assumed to be optimal having a uniform temperature distribution. Detailed analysis of the propellant distribution inside the tank is however required for using TPGS optimally.

V. Error Analysis

A. First order system function

Any system function **f** to calculate a property (**z**) using the measured parameters x_i is given as:

$$\mathbf{z = f\ (x_1, x_2, x_3, ...)} \tag{1}$$

To estimate the error on the outcome $\mathbf{z_0}$ at a point a usual approach is a linearization around that point, taking all partial derivatives of the system function f.

$$\frac{dz}{z_0} = f'(x_1, x_2, ...) = \frac{1}{z_0}\left\{\frac{\partial f}{\partial x_1}dx_1 + \frac{\partial f}{\partial x_2}dx_2 + ..\right\} \tag{2}$$

For small variations or absolute measurement errors this can be approximated to $\varepsilon_i = \Delta x_i$ for each sensor:

$$\Delta z \approx \frac{\partial f}{\partial x_1}\Delta x_1 + \frac{\partial f}{\partial x_2}\Delta x_2 + ... \le \left|\frac{\partial f}{\partial x_1}\right|\varepsilon_1 + \left|\frac{\partial f}{\partial x_2}\right|\varepsilon_2 + ... = \varepsilon_z \tag{3}$$

For small relative variations $\Delta z/\mathbf{z_0}$ the error function f' becomes:

$$f'(x_1, x_2, ..) = \frac{\Delta z}{z_0} = \frac{1}{z_0}\left\{\frac{\partial f}{\partial x_1}\Delta x_1 + \frac{\partial f}{\partial x_2}\Delta x_2 + ...\right\} \le \frac{1}{z_0}\left\{\left|\frac{\partial f}{\partial x_1}\right|\varepsilon_1 + \left|\frac{\partial f}{\partial x_2}\right|\varepsilon_2 + ...\right\} = \frac{\varepsilon_z}{z_0} \tag{4}$$

The above approach is usually worst case for the estimation of the total error in z assuming that all random and systematic errors add up.

$$\varepsilon_i = \varepsilon_{sys,i} + \varepsilon_{rnd,i} \tag{5}$$

For industrial processes, for instance the error in the total thickness tolerance for of a stack of layers, systematic errors are usually neglected by using statistical analysis. The statistical approach involves estimation of the sigma value (σ_i) taking a number of samples (N) of a single parameter xi during a time period with μ_i is the averaged value of parameter x_i. The systematic error is included in the averaged.

$$\sigma_i = \sqrt{\frac{1}{N}\sum_{j=1}^{N}(x_{i,j} - \mu_i)^2} \tag{6}$$

The Root Sum Square or RSS is an approximation of the statistical error by neglecting systemic errors and assuming pure random variations of fully independent variables (cross terms are neglected) The RSS is calculated by taking the quadratic partial derivatives of f and the quadratic parameter i sigma's σi^2 using:

$$RSS = \sigma_z{}^2 \approx \left(\frac{\partial f}{\partial x_1}\right)^2 \sigma_1^2 + \left(\frac{\partial f}{\partial x_2}\right)^2 \sigma_2^2 + ... \tag{7}$$

Normally, 3σ equals a 99.9% certainty which is normally sufficient to cover the random measurement errors ε_{rnd} in z.

$$\varepsilon_{rnd,z} = 3\sigma_z \tag{8}$$

Below (Table 5) both the linear and statistical error approaches are compared for their suitability for gauging analysis

<table>
<tr><td rowspan="2"></td><td colspan="2">System function
z = f (xx_1, x_2, x_3, ...)</td><td rowspan="2">Relative error function
$\Delta z/z_0 = f'(x_1, x_2, x_3, \ldots)$</td></tr>
<tr><td>Measurement</td><td>Measurement error parameter i</td></tr>
<tr><td>Method</td><td></td><td></td><td></td></tr>
<tr><td>Linear (systematic errors included)</td><td>System measurement point z_0=f(x_1, x_2,..)</td><td>$\varepsilon_i = \varepsilon_{sys,i} + \varepsilon_{rnd,i}$</td><td>$\frac{\varepsilon_z}{z_o} \leq \frac{1}{z_o}\left\{\left|\frac{\partial f}{\partial x_1}\right|\varepsilon_1 + \left|\frac{\partial f}{\partial x_2}\right|\varepsilon_2 + ..\right\}$</td></tr>
<tr><td>Statistic (fully random errors assumed, systematic errors neglected)</td><td>Statistical measurement μ_z=f(μ_1, μ_2,..)</td><td>$\mu_i = \frac{1}{N}\sum_{i=1}^{N} x_i$
$\sigma_i = \sqrt{\frac{1}{N}\sum_{j=1}^{N}(x_{i,j} - \mu_i)^2}$
(sample rate)</td><td>$\frac{\sigma_z^{\ 2}}{\mu_z^{\ 2}} \approx \frac{1}{\mu_z^{\ 2}}\left\{\left(\frac{\partial f}{\partial x_1}\right)^2 \sigma_1^2 + \left(\frac{\partial f}{\partial x_2}\right)^2 \sigma_2^2 + ...\right\}$
(ε_{rnd}=3σ => 99.9%, z_o= μ_z)</td></tr>
</table>

Table 5: Comparison of linear and statistic error methods based on a system function

Note that both methods require calculation of the partial derivatives from parameter x_i of the system function z = f (x_1, x_2, x_3, ...). For the uncertainty analyses the system function f must be known around the system point z_0 based on the measurements and know system values. For the calculation of the error function f' the partial derivatives around that point are estimated. Conclusion is that since the statistical approach neglects systematic errors and requires frequent sampling of parameters this method is less suitable for gauging analysis. The linear error analysis is better equipped because it includes systemic errors and can be directly applied to the system function. In the sections following a first order system function is defined for the three common gauging methods to estimate the propellant mass based on in orbit measurement of system parameters. Systematic error propagation is derived from the partial derivatives of the system function. In the summary the results of the error functions for the different gauging methods are compared.

B. Linear error analysis method I: Gas law (PVT)

This method [ref 7] is based on the system properties derived from the gas law (PV=mRT) valid for low pressure gases using measurements of both pressure and temperature. The propellant volume V_L is estimated from a subtraction of the known Tank volume V_T and the Ullage volume V_U using:

$$V_L = V_T - V_U \tag{9}$$

The propellant mass M_L is calculated from propellant density as function of temperature:

$$M_L = V_L\rho(T_L) = (V_T - V_U)\rho(T_L) \tag{10}$$

Let us introduce the propellant load factor β_M (or relative system function) which is the ratio between the propellant mass and the tank fill mass during AIT that varies over life between β=1 (AIT) and β~0 (EOL). Calculation of this ratio delivers the system function f as function of all relevant system parameters x_i.

$$\beta_M = M_L/M_{AIT} = f(X_1, X_2, \ldots, X_i) \tag{11}$$

The momentary propellant mass M_L is defined as:

$$M_L = M_{AIT}\beta_M \tag{12}$$

The Ullage volume is estimated from the pressure drop using the gas law assuming no leakages of the Pressurant gas using:

$$V_U = \frac{P_{U,0}}{P_U}\frac{T_U}{T_{U,0}}V_{U,0} \tag{13}$$

Note that the propellant volume is the tank volume *minus* the Ullage volume: $V_{L,0} = V_T - V_{U,0}$ at the start (t=0). The system function f to estimate the momentary propellant mass ratio β_M of the momentary propellant mass and the AIT fill mass using PVT becomes:

$$\beta_M = f_{PVT}(V_T, M_{AIT}, V_{U,0}, P_U, P_{U,0}, T_U, T_{U,0}, T_L) = \frac{V_T\rho_L(T_L)}{M_{AIT}}\cdot\left(1 - \frac{P_{U,0}}{P_U}\frac{T_U}{T_{U,0}}\frac{V_{U,0}}{V_T}\right) \tag{14}$$

With $P_{U,0}$ and $V_{U,0}$ and $T_{U,0}$ are the initial pressure, volume and temperature respectively of the Ullage at the start. In both cases the system function can be shortly written as:

$$\beta_M = f = a\,(1 - b) \text{ with} \tag{15}$$

$$a = \frac{V_T\rho_L(T_L)}{M_{AIT}} \approx 1.1 \quad \text{and} \quad b = \frac{P_{U,0}}{P_U}\frac{T_U}{T_{U,0}}\frac{V_{U,0}}{V_T} \tag{16}$$

Note that b is a measurement related to the empty ratio $b \sim 1-\beta_M/a$ and the Ullage pressure drops with $P_U \sim (a-1)P_{U,0}/(a-\beta_M)$. For and Ullage pressure of ~20 Bar (**Table 3**) during AIT the pressure drop down to ~1.8 Bar EOL. The relative error function for PVT becomes:

<table>
<tr><th colspan="2">Partial derivatives system function</th><th>Linear error function</th></tr>
<tr><td>$\frac{\partial f_{PVT}}{\partial V_T} =$</td><td>$\frac{a}{V_T}$</td><td rowspan="7">$a = \frac{V_T\rho_L(T_L)}{M_{AIT}} \approx 1.1$, $b = \frac{P_{U,0}}{P_U}\frac{T_U}{T_{U,0}}\frac{V_{U,0}}{V_T} \approx 1 - \frac{\beta_M}{a}$

$d\beta_M = a\left|\frac{dV_T}{V_T}\right| + ab\left\{\left|\frac{dV_u}{V_{U,0}}\right| + \left|\frac{dP_U}{P_{U,0}}\right| + \left|\frac{dT_U}{T_{U,0}}\right| + \left|\frac{dP_U}{P_U}\right| + \left|\frac{dT_U}{T_U}\right|\right\} + a(1-b)\left\{\left|\frac{dM_{AIT}}{M_{AIT}}\right| + \left|\frac{1}{\rho}\frac{d\rho}{dT_L}\right|\right\}$

$P_{U,0}$~20 Bar (*typical, table 3*), $P_U \sim (a-1)P_{U,0}/(a-\beta_M)$,

$\Delta\beta_M \le a\frac{\varepsilon V_T}{V_T} + (a-\beta_M)\left\{\frac{\varepsilon V_U}{V_{U,0}} + \frac{\varepsilon T_U}{T_{U,0}} + \frac{\varepsilon T_U}{T_U} + \frac{\varepsilon P_U}{P_{U,0}} + \frac{\varepsilon P_U}{P_U}\right\} + \beta_M\left\{\frac{\varepsilon M_{AIT}}{M_{AIT}} + \left|\frac{1}{\rho}\frac{d\rho}{dT_L}\right|\right\}$

$\frac{\varepsilon T_U}{T_{U,0}} = \frac{\varepsilon T_U}{T_U}, \frac{\varepsilon P_U}{P_{U,0}} = \frac{\varepsilon P_U}{P_U}$

For AIT (β_M=1, P_U~ 20 Bar)</td></tr>
<tr><td>$\frac{\partial f}{\partial V_{U,0}} =$</td><td>$\frac{-ab}{V_{U,0}}$</td></tr>
<tr><td>$\frac{\partial f}{\partial M_{AIT}}$</td><td>$-\frac{a(1-b)}{M_{AIT}}$</td></tr>
<tr><td>$\frac{\partial f}{\partial P_{U,0}}$</td><td>$-\frac{ab}{P_{U,0}}$</td></tr>
<tr><td>$\frac{\partial f}{\partial P_U} =$</td><td>$\frac{ab}{P_U}$</td></tr>
<tr><td>$\frac{\partial f}{\partial T_U} =$</td><td>$\frac{-ab}{T_U}$</td></tr>
<tr><td>$\frac{\partial f}{\partial T_{U,0}} =$</td><td>$\frac{ab}{T_{U,0}}$</td></tr>
</table>

Partial derivatives system function		Linear error function
$\frac{\partial f}{\partial T_L} =$	$\frac{a(1-b)}{\rho}\frac{\partial \rho}{\partial T_L}$	$\Delta\beta_M \le 1.1\frac{\varepsilon V_T}{V_T} + 0.1\left\{\frac{\varepsilon V_U}{V_{U,0}} + 2\frac{\varepsilon T_U}{T_{U,0}} + 2\frac{\varepsilon P_U}{P_{U,0}}\right\} + \left\{\frac{dM_{AIT}}{M_{AIT}} + \left\lvert\frac{1}{\rho}\frac{d\rho}{dT_L}\right\rvert\right\}$ *For EOL (β_M=0, P_U~ 1.8 Bar)* $\Delta\beta_M \le 1.1\left\{\frac{\varepsilon V_T}{V_T} + \frac{\varepsilon V_U}{V_{U,0}} + 2\frac{\varepsilon T_U}{T_{U,0}} + 2\frac{\varepsilon P_U}{P_{U,0}}\right\}$

Table 6: **Linear error function for PVT**

Conclusion

For PVT the EOL gauging accuracy is related to the error in the Tank and Ullage volume during AIT an the in-flight Ullage pressure and temperature sensor accuracies

C. Linear error analysis method II: Bookkeeping (BK)

Bookkeeping [ref 9] is based on summing the expelled propellant mass M_n after n burns measured with a flow meter from the propellant mass in the tank after orbit injection (BOL) giving the momentary propellant M_L in the tank. An essential parameter for the propellant system function is the anticipated total number of burns N needed for EOL station keeping to empty the propellant tank. This is related to the anticipated number of burns per day and the mission duration D_m (days).

$$N = \text{burns/day} * D_m \tag{17}$$

However *on average* the propellant mass m_n expelled per burn for station keeping should be $M_{L,BOL}/N$ to fully empty the tank at EOL.

$$\overline{\mathrm{m}_n} = \frac{M_{L,BOL}}{N} = \frac{M_{AIT}}{N}\beta_{M,BOL} \tag{18}$$

The mass fill factor $\beta_{M,BOL}$ after GTO is typically between 0.09-0.24 (table 3). The total expelled propellant M_n after n burns is estimated by summation over the number of burns (n), the burn time Δt_i and the mass flow rate (φ_i) of burn number *i*.

$$M_n = \sum_1^n \phi_i \Delta t_i = n\overline{\phi_n}\,\overline{\Delta t_n} = n\overline{m_n} \tag{19}$$

For this example it is assumed that the flow rate φ_i and burn time Δt_i is (nearly) constant for each burn and that no errors are made in burn counting giving a relative accuracy of the flow meter per burn. For actual systems these issues will need to be investigated and tested in more detail.

$$dM_n = ndm_n = \left\{\left\lvert\frac{d\phi}{\overline{\phi_n}}\right\rvert + \left\lvert\frac{d\Delta t}{\overline{\Delta t_n}}\right\rvert\right\}M_n = \left\{\frac{\varepsilon_\phi}{\phi_n} + \frac{2\varepsilon_t}{\Delta t_n}\right\}M_n = n\varepsilon_{m_n} \tag{20}$$

The momentary propellant mass in the tank after n burns thus becomes:

$$\mathrm{M_L}(n) = \mathrm{M_{L,BOL}} - \mathrm{M_n} = N\overline{m_n} - \sum_1^{\mathrm{n}} m_i \Delta t_i = (N-n)\overline{m_n} = \beta_M M_{AIT} \tag{21}$$

With mass ratio factor β_M becomes:

$$\beta_M = \beta_{M,BOL}\left(1-\frac{n}{N}\right) = \frac{M_L(n)}{M_{AIT}} \tag{22}$$

Note that the uncertainty of the propellant load is related to the measurement errors made in the BOL propellant mass and flow-meter accuracy. The required relative accuracy (ε_m/m_n) of the flow-meter should be about M_{BOL}/N to ensure that the overall accuracy is sufficient up to blow down. In this case the dimensionless system function for estimation of the momentary propellant after GTO using book-keeping for gauging becomes:

$$\beta_M = \frac{M_L}{M_{AIT}} = f_{BK}(M_{AIT}, M_{BOL}, \overline{nm_n}) = \frac{M_{BOL}}{M_{AIT}}\left(1-\frac{\overline{nm_n}}{M_{BOL}}\right) = a(1-b) \tag{23}$$

With $a=M_{BOL}/M_{AIT}$ and $b=nm_n/M_{BOL.}$ The partial derivatives and linear error function for Bookkeeping now becomes.

Partial derivatives system function		**Linear error function**
$\frac{\partial f_{BK}}{\partial M_{AIT}} =$	$\frac{-a(1-b)}{M_{AIT}}$	$a=\beta_{BOL}, b=\frac{n}{N}=1-\frac{\beta_M}{\beta_{BOL}}, \beta_M \geq \beta_{BOL}$ $d\beta_M = a\left\lvert\frac{dM_{BOL}}{M_{BOL}}\right\rvert + a(1-b)\left\lvert\frac{dM_{AIT}}{M_{AIT}}\right\rvert + ab\left\lvert\frac{dm_n}{m_n}\right\rvert$
$\frac{\partial f_{BK}}{\partial M_{BOL}} =$	$\frac{a}{M_{BOL}}$	$\Delta\beta_M \leq \beta_{BOL}\frac{\varepsilon_{M_{BOL}}}{M_{BOL}} + \beta_M\frac{\varepsilon_{M_{AIT}}}{M_{AIT}} + (\beta_{BOL}-\beta_M)\frac{\varepsilon_{m_n}}{\overline{\overline{m_n}}}$
$\frac{\partial f_{BK}}{\partial \overline{\overline{nm_n}}} =$	$-\frac{ab}{\overline{\overline{nm_n}}}$	$\frac{\varepsilon_{M_{BOL}}}{M_{BOL}} = \frac{\varepsilon_{m_n}}{\overline{\overline{m_n}}}\frac{\Delta M_{AIT\to BOL}}{M_{BOL}} = \frac{\varepsilon_{m_n}}{\overline{\overline{m_n}}}(1-\beta_{BOL})\frac{M_{AIT}}{M_{BOL}} = \frac{\varepsilon_{m_n}}{\overline{\overline{m_n}}}(1-\beta_{BOL})\frac{1}{\beta_{BOL}}$ $\Delta\beta_M \leq \beta_M\frac{\varepsilon_{M_{AIT}}}{M_{AIT}} + (1-\beta_M)\frac{\varepsilon_{m_n}}{\overline{\overline{m_n}}}$ *For BOL (β_M=β_{BOL})* $\Delta\beta_M \leq \frac{\varepsilon_{m_n}}{\overline{\overline{m_n}}} + \beta_{BOL}\left(\frac{\varepsilon_{M_{AIT}}}{M_{AIT}} - \frac{\varepsilon_{m_n}}{\overline{\overline{m_n}}}\right)$ *For EOL (β_M=0)* $\Delta\beta_M \leq \frac{\varepsilon_{m_n}}{\overline{\overline{m_n}}}$

Table 7: Linear error function for Bookkeeping with flow meter

Conclusion

For book-keeping the accuracy the EOL gauging accuracy is related to the error in the tank mass during AIT and the relative error of the flow-meter per burn.

D. Linear error analysis method III: Thermal Nocking (TPGS)

This method [ref 5] is based on an estimation of the thermal mass of the propellant tank including using electrical heaters that are attached to the tank and a couple of temperature sensors. The tank design should be such that it is thermally insulated from the space-craft by using blankets wrapped around the tank to limit IR radiation and poor conductive supports. The heater power should be sufficient with respect to the overall heat leakages to be able to raise the tank temperature in a short time (say a couple of minutes) and to allow for uniform spreading of the heat inside the tank. It must be investigated in detail if under zero-gravity conditions how the heat is spreading efficiently enough into the liquid propellant for instance by using capillary material at the walls. From the slope of the temperature response after heating-up the heat capacity of the system is estimated as follows, which is the sum of the tank and propellant heat capacity. The temperature response as function T(t) when applying a power (P_e) to a system with heat capacity (H_S) and effective conduction to the spacecraft C can be calculated with the following differential equation:

$$\mathrm{P_e} - \mathrm{H_S}\frac{\mathrm{dT}}{\mathrm{dt}} - \mathrm{CdT} = 0 \tag{24}$$

Note that the temperature stabilizes after some time (e.g. for t->∞, dT/dt=0) where $P_e=C\Delta T_\infty$ The general solution, starting from temperature T(0) with a thermal time constant ($\lambda=C/H_S$) becomes:

$$\mathrm{T(t)} = \mathrm{T(0)} + \Delta T_\infty\left(1 - \mathrm{e}^{-\lambda t}\right) = \mathrm{T_0} + \frac{\mathrm{P_e}}{\mathrm{C}}\left(1 - \mathrm{e}^{-\left(\frac{C}{H_S}\right)t}\right) \tag{25}$$

In this case the slope of the temperature response at t=0 (when the heater is switched on) becomes:

$$\frac{dT}{dt} = \lambda\Delta T_\infty e^{-\lambda t} \Rightarrow \frac{dT}{dt}(0) \approx \frac{P_e}{H_S} \tag{26}$$

The system heat-capacity H_S which is the sum of the heat capacity of the propellant H_L and the tank H_T is estimated from the system temperature response shortly after applying heater power which is derived from the slope of the temperature curve. This is simply the ratio between the difference between two temperatures versus two time measurements and the applied heater power P.

$$H_S = H_T + H_L \approx \frac{\Delta t}{\Delta T}P_e \tag{27}$$

When switching on the heater at t = 0 the slope of the temperature increase equals the ratio between the heater power (Pe) and the system heat capacity H_s making it possible to estimate the propellant mass from the sum of the propellant and tank heat capacity. The solution might be improved by curve fitting of function (26) and/or the slope function (27). Note that the heat capacity of the (empty) tank and total heat leak C to the spacecraft must be accurately measured during ground testing.

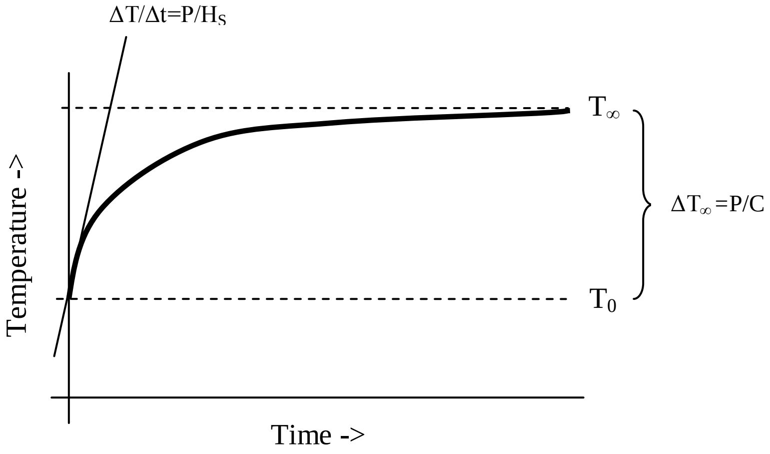

Figure 4: Temperature respons of a system during heating

It is recommended that the electrical power P_e is proportional with the amount of propellant over life to get a sufficient temperature response ΔT within a sample period Δt.

$$\frac{P_e}{C_{P_L} M_{L_{AIT}}} \approx \frac{\Delta T}{\Delta t} \frac{H_S}{H_{L_{AIT}}} \approx \frac{\Delta T}{\Delta t} \beta_M \tag{28}$$

To illustrate this the required heater power over life has been calculated in **Table 8** for 1000kg propellant tank with C_P =1560 J/Kg/K (N2H4) a temperature response of ΔT=10°C with Δt =15 minutes. The environmental heat leak via conduction and radiation has been neglected for this calculation. A smaller temperature step or longer response times reduce the power requirement.

Mission Phase	AIT	BOL	MID	EOL
Mass fill ratio β_M	1	0.2	0.1	0.01
Required Heater Power (kWatt)	17.3	3.4	0.34	0.034

Table 8: Example of the required heater power for TPGS over life for a 1000Kg Hydrazine tank

The residual propellant mass M_L is calculated from its specific heat capacity of the propellant Cp_L which is calculated from the measured system heat capacity H_S minus the tank heat capacity H_T by:

$$M_L = \frac{H_L}{Cp_L} = \frac{H_S - H_T}{Cp_L} \tag{29}$$

With the momentary heat capacity $H_S = P_e\Delta t/\Delta T = H_L + H_T$ estimated from the slope of the temperature increase after heating-up. The error function for TPGS now becomes:

$$dH_S = \left\{ \left|\frac{dP}{P_e}\right| + \left|\frac{d\Delta t}{\Delta t_S}\right| + \left|\frac{d\Delta T}{\Delta T_S}\right| \right\} H \quad = \left\{ \frac{\varepsilon_p}{P_e} + \frac{2\varepsilon_t}{\Delta t_S} + \frac{2\varepsilon_T}{\Delta T_S} \right\} H_S \tag{30}$$

The tank heat capacity $H_T = M_T C_{P,T}$ is estimated during AIT on an empty tank from the dry mass and specific heat capacity with an error of:

$$dH_T = \left\{ \left|\frac{dM_T}{M_T}\right| + \left|\frac{dC_{P_T}}{C_{P_T}}\right| \right\} H_T = \left\{ \frac{\varepsilon_M}{M_T} + \frac{\varepsilon C_{P_T}}{C_{P_T}} \right\} H_T \tag{31}$$

The propellant heat capacity is estimated from H_S on a full tank during AIT subtracting the tanks heat capacity.

$$dH_{L_{AIT}} = \left\{\left|\frac{dM_L}{M_{L_{AIT}}}\right| + \left|\frac{dC_{P_L}}{C_{P_L}}\right|\right\} H_{L,AIT} = \left\{\frac{\varepsilon_M}{M_{L_{AIT}}} + \frac{\varepsilon C_{P_L}}{C_{P_L}}\right\} H_{L_{AIT}} \tag{32}$$

The system function for TPGS now becomes:

$$\beta_M = \frac{M_L}{M_{L_{AIT}}} = f_{TPGS}(M_{L_{AIT}}, H_S, H_T, Cp_L(T_L)) = \frac{H_S - H_T}{Cp_L(T_L) M_{L_{AIT}}} = \frac{H_S - H_T}{H_{L_{AIT}}} = \frac{H_S}{H_{L_{AIT}}}\left(1 - \frac{H_T}{H_S}\right) = a(1-b) \tag{33}$$

With $a = H_S/H_{L,AIT} \sim \beta_M$ (AIT) and $b = H_T/H_S \sim 0$ (AIT) and b=1 for EOL. In case it is assumed that the tank is full $H_S \sim H_L$ and the tank has a cylindrical shape with diameter D and height h and wall thickness t the ratio of the heat capacities (tank wall/tank volume) approaches to zero when D>> t which will be usually the case for large tank systems.

$$b = \frac{H_T}{H_S} \approx \frac{M_T Cp_T}{M_{L,AIT} Cp_L} = \frac{Cp_T}{Cp_L}\frac{\pi D h t \rho_T}{\frac{\pi}{4} D^2 h \rho_L} = \frac{Cp_T}{Cp_L}\frac{\rho_T}{\rho_L}\frac{4t}{D} \approx 0 \text{ when D>>t (for a full tank)} \tag{34}$$

The partial derivatives for the system function for TPGS are:

<table>
<tr><th colspan="2">Partial derivatives system function</th><th>Linear error function</th></tr>
<tr><td>$\dfrac{\partial f}{\partial M_{AIT}} =$</td><td>$-\dfrac{a(1-b)}{M_{AIT}}$</td><td rowspan="4">$a = \dfrac{H_S}{H_{L,AIT}} \approx \beta_M, b = \dfrac{H_T}{H_S} \approx 1 - \beta_M$

$d\beta_M = a\left|\dfrac{dH_S}{H_S}\right| + a(1-b)\left\{\left|\dfrac{dM_{AIT}}{M_{AIT}}\right| + \left|\dfrac{dC_{PL}}{C_{PL}}\right|\right\} + b\left|\dfrac{dH_T}{H_T}\right|$

$\Delta\beta_M \le \beta_M\left\{\dfrac{\varepsilon_P}{P_S} + \dfrac{2\varepsilon_t}{\Delta t_S} + \dfrac{2\varepsilon_T}{\Delta T_S}\right\} + \beta_M{}^2\left\{\dfrac{\varepsilon_M}{M_{L_{AIT}}} + \dfrac{\varepsilon_{C_{P_L}}}{C_{P_L}}\right\} + (1-\beta_M)\left\{\dfrac{\varepsilon_M}{M_T} + \dfrac{\varepsilon_{C_{P_T}}}{C_{P_T}}\right\}$

For AIT (β_M=1)

$\Delta\beta_M \le \dfrac{\varepsilon_P}{P_S} + \dfrac{2\varepsilon_t}{\Delta t_S} + \dfrac{2\varepsilon_T}{\Delta T_S} + \dfrac{\varepsilon_M}{M_{L_{AIT}}} + \dfrac{\varepsilon_{C_{P_L}}}{C_{P_L}}$

For EOL (β_M=0)

$\Delta\beta_M \le \dfrac{\varepsilon_M}{M_T} + \dfrac{\varepsilon_{C_{P_T}}}{C_{P_T}}$</td></tr>
<tr><td>$\dfrac{\partial f}{\partial H_s} =$</td><td>$\dfrac{a}{H_S}$</td></tr>
<tr><td>$\dfrac{\partial f}{\partial H_T} =$</td><td>$\dfrac{-b}{H_T}$</td></tr>
<tr><td>$\dfrac{\partial f}{\partial T_L} =$</td><td>$-\dfrac{a(1-b)}{C_{P_L}}\dfrac{\partial C_{P_L}}{\partial T}$</td></tr>
</table>

Table 9: Linear error function for a TPGS system

Conclusion

For TPGS the EOL gauging accuracy is related to the error in the tank mass and heat capacity which both can be measured on ground. Disadvantage of TPGS is that a thermal analysis is required to model the system response as well as having capillary material located at the tank walls to ensure a uniform temperature distribution in the propellant during heating-up.

E. Error analysis summary

The linear error functions as derived for PVT, BK and TPGS are basically a summation of error sources which weighting factor mostly depends on the tank fill and corresponding life phase. In **Table 10** a summary of the system and error functions are given.

System	System parameters	System function	Linear error function
PVT (Table 6)	V_T, $V_{U,0}$, M_{AIT}, $\Delta P_{U,0}$, ΔP_U, T_U, $T_{U,0}$, T_L	$\beta_M = \frac{V_T \rho_L(T_L)}{M_{AIT}} \cdot \left(1 - \frac{P_{U,0}}{P_U}\frac{T_U}{T_{U,0}}\frac{V_{U,0}}{V_T}\right)$	$\Delta\beta_M \le 1.1\frac{\varepsilon V_T}{V_T} + (1.1-\beta_M)\left\{\frac{\varepsilon V_U}{V_{U,0}} + \frac{\varepsilon T_U}{T_{U,0}} + \frac{\varepsilon T_U}{T_U} + \frac{\varepsilon P_U}{P_{U,0}} + \frac{\varepsilon P_U}{P_U}\right\} + \beta_M\left\{\frac{\varepsilon M_{AIT}}{M_{AIT}} + \left\lvert\frac{1}{\rho}\frac{d\rho}{dT_L}\right\rvert\right\}$
BK (Table 7)	M_{AIT}, M_{BOL}, $n\overline{m}_n$	$\beta_M = \frac{M_{BOL}}{M_{AIT}}\left(1 - \frac{n\overline{m}_n}{M_{BOL}}\right)$	$\Delta\beta_M \le \beta_M \frac{\varepsilon_{M_{AIT}}}{M_{AIT}} + (1-\beta_M)\frac{\varepsilon_{m_n}}{\overline{m}_n}$
TPGS (Table 9)	M_{LAIT}, H_S (P_s, t_s, ΔT_s), H_T (M_T, Cp_T), $C_{PL}(T_L)$	$\beta_M = \frac{H_S}{H_{L_{AIT}}}\left(1 - \frac{H_T}{H_S}\right)$	$\Delta\beta_M \le \beta_M\left\{\frac{\varepsilon_p}{P_S} + \frac{2\varepsilon_t}{\Delta t_S} + \frac{2\varepsilon_T}{\Delta T_S}\right\} + \beta_M{}^2\left\{\frac{\varepsilon_M}{M_{L_{AIT}}} + \frac{\varepsilon_{C_{P_L}}}{C_{P_L}}\right\} + (1-\beta_M)\left\{\frac{\varepsilon_M}{M_T} + \frac{\varepsilon_{C_{P_T}}}{C_{P_T}}\right\}$

Table 10: Summary of the system function and linear error function over life for three gauging systems PVT, BK and TPGS

To illustrate the error calculation the system and error functions with repect to the accuracy of the gauging methods over life the following system uncertainties have been applied (**Table 11**) which are based on values extracted from the literature [1-10] and on the high accuracy pressure sensor (HAPT) and improved Ultrasonic Flow Meter (UFM) of Moog Bradford. See the Appendix for more information about these sensors.

System Parameter	PVT	BK	TPGS	Remark
Propellant				
Density (T)	0.1%	-	-	
Heat capacity	-	-	0.5%	
Mass	0.1%	0.1%	0.1%	
Temperature	-	-	1.0%	*=>0.5K/10K*
Ullage (He)				
Pressure (mbar)	13	-	-	*=> Bradford HAPT 0.05% FS=25 Bar*
Volume	0.3%	-	-	
Temperature	0.2%	-	-	*=> 0.5K/300K*
Propellant Tank				
Heat capacity (T)	-	-	0.1%	
Mass	-	-	0.1%	
Temperature	-	-	1.0%	*=> 0.5K/10K*
Volume	0.3%	-	-	
Flow meter				
Mass flow	-	0.5%	-	*=> Bradford UFM 0.5% FS*
Heater				
Power	-	-	0.1%	
Time	-	-	0.01%	

Table 11: Summary of uncertainties (literature extracted values) used as inputs to the error analysis

These literature values are used as input for the linear error function calculation giving the results in **Figure 5** for the gauging accuracy over life for a 15 years optimistic (depletion of 0.04% per month) and a 10 years pessimistic (depletion of 0.2% per month) mission scenario.

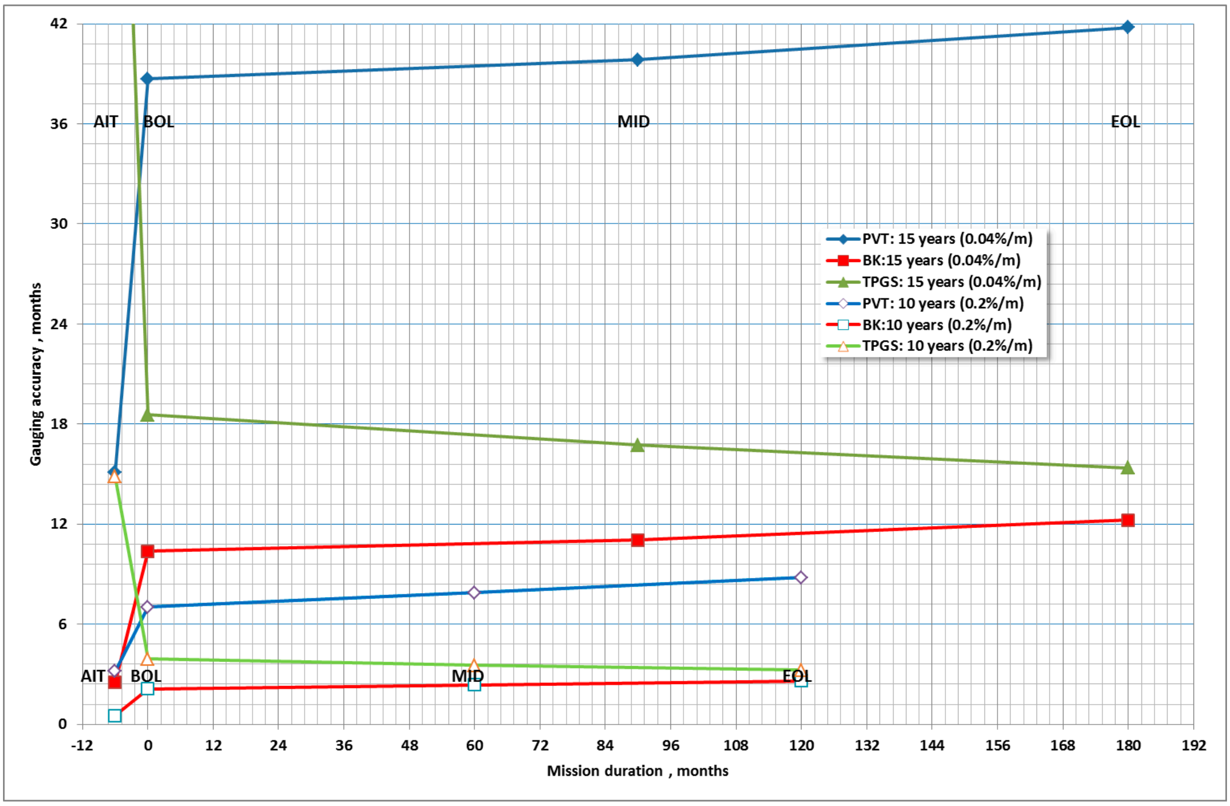

Figure 5: Gauging analysis of three gauging systems (based on literature estimates of errors) versus 10-15 years mission scenario

From Figure 5 Bookkeeping with the improved UFM gives the highest EOL gauging accuracy between ±3 and ± 12 months for 10 and 15 years mission scenarios respectively. Second best is TPGS giving ±4 and ±14 months EOL gauging accuracy for 10 and 15 years respectively. PVT is the least accurate giving between ±8 months to ±3.5 years EOL gauging accuracy for 10 and 15 years. Conclusion is that Bookkeeping with the improved UFM gives the best performance at EOL followed by TPGS. Note that for this analysis TPGS outperforms PVT already from the start of the operations at BOL. A combination of TPGS and BK should give the best overall accuracy. Conclusion is that at the current state of technology an EOL gauging accuracy (see **Table 12**) was found between ±3-12 months (for 10 to 15 years mission duration) achieved with bookkeeping using the improved UFM follow by TPGS with ±4-14 months EOL.

Gauging method	**Measurement accuracy (literature)**	**Predicted EOL gauging accuracy [months]** *[kg]**	
		10 years *Depletion rate: -0.19%/m* *[-0.95kg/m]**	**15 years** *Depletion rate: -0.04%/m* *[-0.2 kg/m]**
BK (Table 7)	Propellant Mass 0.1% Propellant Mass flow 0.5%	± 3 *[± 2.8 kg]**	± 12 *[± 2.4 kg]**
TPGS (Table 9)	Propellant Heat capacity 0.5% Propellant & tank Mass 0.1% Tank Temperature 1.0% Tank Heat capacity (T) 0.1% Heater Power 0.1% Switch on Time 0.01%	± 4 *[± 3.8 kg]**	± 14 *[± 2.8 kg]**

Gauging method	Measurement accuracy (literature)	Predicted EOL gauging accuracy [months] *[kg]**	
		10 years *Depletion rate: -0.19%/m [-0.95kg/m]**	**15 years** *Depletion rate: -0.04%/m [-0.2 kg/m]**
PVT (Table 6)	Propellant Density (T) 0.1% Propellant Mass 0.1% Ullage Pressure (Bar) 0.013 Ullage Volume 0.3% Ullage Temperature 0.2%	± 7 *[± 6.7 kg]**	± 42 *[± 8.4 kg]**

Table 12: Summary of gauging accuracies over life based on state-of-the-art data for three gauging systems PVT, BK and TPGS

**Example calculation for a 500 kg tank*

The linear error functions for the three gauging methods can also be used to distribute the required accuracies given the target gauging accuracy of ±1 month. A straightforward way to do so is to distribute the errors introduce by each sensor/system parameter given its relative to the system error function. In **Table 13** the required accuracies are presented to achieve a target uncertainty at EOL of ±1 months.

	PVT		BK		TPGS		
	15y (opt)	10y (pes)	15y (opt)	10y (pes)	15y (opt)	10y (pes)	
Propellant							
Density (T)	0.083%	0.190%	-	-	-	-	
Heatcapacity (T)	-	-	-	-	1.59%	7.54%	
Mass	0.083%	0.190%	0.33%	0.76%	1.59%	7.54%	
Temperature	-	-	-	-	0.005	0.023	^{o}C (for dT=10^{o}C)
Ullage (He)							
Pressure (mbar)	0.159	0.170	-	-	-	-	
Volume	0.005%	0.024%	-	-	-	-	
Temperature	0.014	0.073	-	-	-	-	K (for T=300K)
Propellant Tank							
Heatcapacity (T)	-	-	-	-	0.006%	0.029%	
Mass	-	-	-	-	0.006%	0.029%	
Volume	0.005%	0.022%	-	-	-	-	
Temperature	-	-	-	-	0.005	0.023	^{o}C (for dT=10^{o}C)
Flowmeter							
Mass flow	-	-	0.021%	0.109%	-	-	
Heater							
Power	-	-	-	-	0.095%	0.452%	
Time	-	-	-	-	0.048%	0.226%	

Table 13: Required error contribution to achieve an gauging accuracy of ±1 month for PVT, BK and TPGS after 10 to 15 years

It is concluded from **Table 13** the required accuracy is the most demanding for 15 years missions – as expected. The propellant mass measurement (0.083%, PVT) requires a slight improvement since it is already close to the currently achieved value. The required accuracies for the tank volume (0.005%, PVT) and tank mass and heat capacity (0.006%, TPGS) require significant improvements during manufacturing and the ground operations. The required accuracies for in-flight sensors for temperature (0.005^{o}C, TPGS), pressure (0.16mbar, PVT) and flow meter (0.02%, BK) measurements are ambitious and require significant improvements of the (in-flight) sensor technology.

VI. Conclusion

Having accurate gauging on board of satellites is vital for system design optimization, life time extension and cost reduction. However, since gauging interacts with the propellant system design, tank fill and applied sensors, its accuracy varies over life and it is hard to be optimized on its own. In this paper state-of-the-art data about gauging requirements, system parameter and applied sensors have been collected based on three common gauging methods: PVT, BK and TPGS. An analysis approach is followed defining the system functions (=mass fill ratio over life) and its corresponding linear error functions for each method. A linear error analysis is recommended because systematic errors are neglected when using statistical error analysis. State-of-the-art data has been applied to tot illustrate the linear error propagation throughout the system. It is concluded that an gauging accuracy between ± 3 months (10 years) and ±12 months (15 years) is possible using BK with the improved UFM of Moog Bradford. See **Table 12**. The ambitious target of ± 1 month gauging accuracy for future satellites requires significant improvements of both in-flight sensor technology and ground operations as indicated in **Table 13**. Extensive system analysis is recommended to improve gauging accuracy in a cost effective way. Secondary influences such as gas compressibility, propellant vapor pressure, pressurant solvability, tank stretch, thermal expansion of tank and the propellant might impose limits to the gauging accuracy.

Acknowledgments

The research activity was financially sponsored by the Netherlands Space Office (NSO), Moog Bradford Engineering and the National Aerospace Laboratory, NLR. Special thanks to our retired NLR colleague Koos Prins for proposing and initiating this work.

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Appendix

UFM Accuracy Breakdown

The unit is calibrated in a flowbench (with H_2O), where the reference is provided by a catch-and-weigh mass scale method. Calibration data is provided in volumetric flowrate (cc/s) as calibrated with water. As the output is medium independent, the mass flowrate of the actual medium can be calculated by multiplication with actual density (temperature/pressure corrected). This is considered customer responsibility and as such not included in the error budget. The UFM output is approximated by a higher order polynomial and a LUT/actual polynomial is provided.

An example of the polynomial and residual error wrt the polynomial approximation is provided below. Data is taken from a recent UFM calibration for a chemical propulsion system application. The reflected error includes repeatability of the unit and test set up as well. The LUT interpolation error has been derived from thermal calibration data at different temperatures as provided in Figure 7 below.

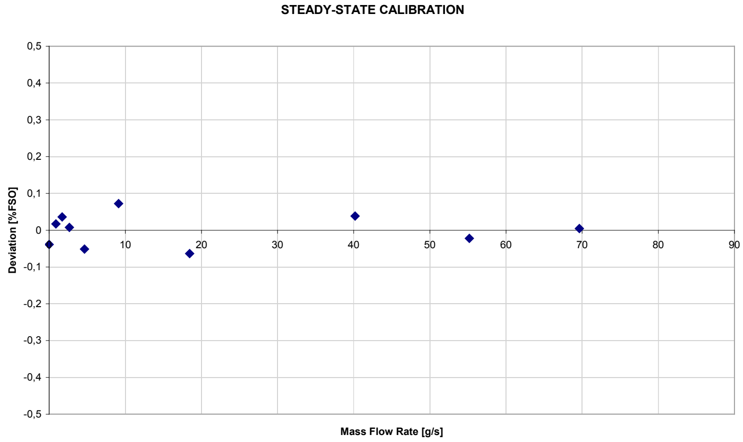

Figure 6: UFM residual calibration/repeatability error

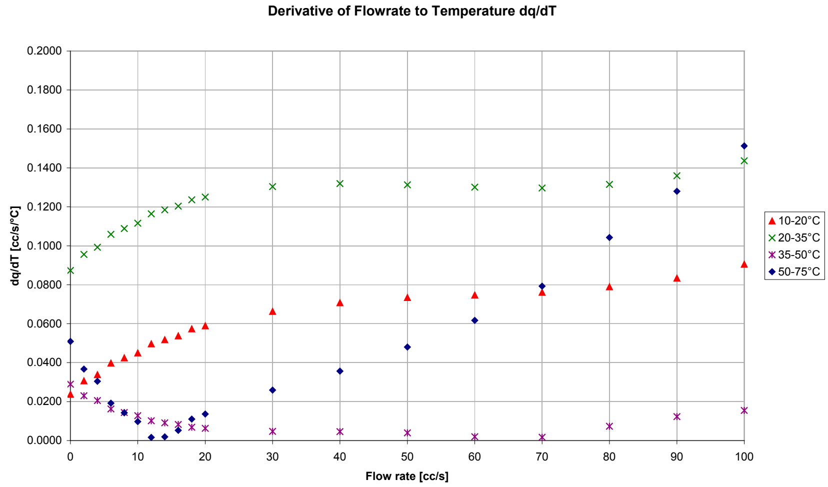

Figure 7: LUT interpolation error

The UFM error sources are summarized in Table 14. The reflected error values reflect worst-case heritage values of recent UFM calibrations.

Error source	Error	Remarks
Long-term stability (End of Life) of the electronics/transducers and radiation drift effects	0.1% FS	Estimate based on 15 yrs lifetime.
Thermal error (LUT interpolation error)	0.15 % FS	Max 0.15 cc/s/°C up to 100g/s. Temperature sensor accuracy assumed +/- 1 °C. For a temperature range from +10 to +50 °C.
Residual calibration / Repeatability errors	0.3 % FS	Repeatability of unit and test set up
Calibration reference	0.1 % FS	Accuracy of flow measurement reference
Density multiplication error	Not included	Dependent on accuracy of propellant density/temperature tabulations employed by flow meter user.
Total Error Budget	0.4 % FS (with LUT)	For instantaneous flow rate output. Determined by square root summing of individual contributions.

Table 14: Flow Meter Error Sources

Remarks

- The full scale range of the flow meter is assumed to be 100g/s for this analysis.
- The LUT error strongly depends upon the accuracy of the temperature measurement and the size of the LUT (# of data points).
- The error of the totalizer is depending upon the totalizer approach, i.e. time of totalization and possibility for zero point offset correction. This can be assessed upon request.

HAPT Accuracy Breakdown:

The HAPT is calibrated against RUSKA 7250 Digital Pressure Controller (DPC) reference.
With the calibration data, digital mapping over the operational pressure and temperature range is performed, as illustrated in the next figure. The unit outputs the pressure and temperature values as input for the polynomial calculation:

$$p = \sum R_B{}^j \cdot S^i \cdot B_{ij}$$

where: B_{ij} = Polynomial coefficients (from digital compensation)
R_B = Bridge resistance [Ω @ 1mA]
S = Bridge imbalance [mV/mA]
P = Pressure [bar]

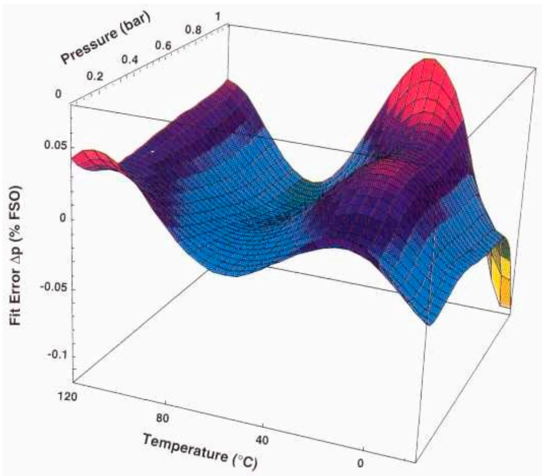

Figure 8: Digital Compensation Mapping

The Total Error Band (TEB) is given in the table below for different temperature ranges. The TEBs are calculated by the root-sum-square of random (uncorrelated) errors. The respective values of the error contributors are based upon actual data from ongoing or past deliveries.

HAPT	**+15/+45°C**	**-10/+65°C**	Error type
Calibration	0,015	0,015	Random
Digital compensation residual error	0,023	0,080	Random
Hysteresis MEOP-0-MEOP	0,011	0,011	Random
Repeatability	0,020	0,020	Random
Noise digital output	0,00136	0,00136	Random
Power supply variation (+/-1VDC)	0,010	0,010	Random
Electronics/sensor ageing	0,01	0,01	Random
HAPT TEB BOL [%FS]	± 0,05	± 0,10	
HAPT TEB EOL [%FS]	± 0,05	± 0,10	

Table 15: HAPT Accuracy Breakdown