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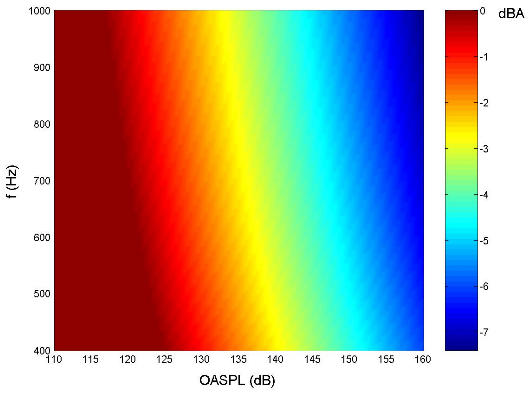
Executive summary

On the effect of nonlinear propagation on perceived jet noise levels



Problem area
It is common practice to determine the noise impact of aircraft (both civil and military) by using computations, rather than by measurements. Noise impact computations usually make use of noise levels, measured at a relatively small distance from the aircraft, combined with a propagation model. These propagation models are based on linear acoustics and account for effects like atmospheric absorption and lateral attenuation.
In the past decades, however, evidence has been found that linear acoustics is inadequate to describe the propagation of high-intensity jet noise. More specifically, it is found that the high frequency part of the spectrum is decaying much slower than predicted by linear acoustics. This behaviour is attributed to nonlinear effects in the propagation.

Description of work
In this paper a computation method is described that is based on a frequency-domain approach for a broadband time signal.The method is used to conduct a parameter study on the effects of nonlinear propagation in terms of overall sound pressure levels.

Results and conclusions
Although measured data are not reproduced accurately in a quantitative sense, the effects attributed to nonlinear propagation are qualitatively the same in both

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computed and measured results: acoustic energy is transferred from the centre frequencies to the higher frequencies, leading to a significantly lower decay of this part of the spectrum.
For typical values of the noise produced by a fighter jet in the near field (~ 10 m), the nonlinear effect may be of the order of 6 dB. Even if the starting point of the propagation is taken at a more practical distance (~ 50 m), the nonlinear effect still makes a considerable difference in the outcome of noise impact studies.

Applicability
The method presented here, can be used to improve the noise impact assessment related to military jet aircraft.

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Contents

On the effect of nonlinear propagation on perceived jet noise levels

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Abstract

It is common practice to determine the noise impact of aircraft (both civil and military) by using computations, rather than by measurements. Noise impact computations usually make use of noise levels, measured at a relatively small distance from the aircraft, combined with a propagation model. These propagation models are based on linear acoustics and account for effects like atmospheric absorption and lateral attenuation.

In the past decades, however, evidence has been found that linear acoustics is inadequate to describe the propagation of high-intensity jet noise. More specifically, it is found that the high frequency part of the spectrum is decaying much slower than predicted by linear acoustics. This behaviour is attributed to nonlinear effects in the propagation.

In this paper a method is described that is based on a frequency-domain approach for a broadband time signal. Example calculations are presented, on the simulation of an experiment on an F/A-18E/F aircraft, during a static engine run-up test. Although the measured data are not reproduced accurately in a quantitative sense, the effects attributed to nonlinear propagation are qualitatively the same in both the computed and the measured results: acoustic energy is transferred from the centre frequencies to the higher frequencies, leading to a significantly lower decay of this part of the spectrum.

In addition, a parameter study is presented on the effects of nonlinear propagation in terms of overall sound pressure levels. The main conclusions of this study are:

- The sound levels at large distances, predicted by a nonlinear propagation method are smaller than those predicted by a linear method.
- For the same shape of the spectrum, the difference between ‘nonlinear’ and ‘linear’ results mainly depends on the product of the sound level and the peak frequency at the starting point of the computation.
- For typical values of the noise produced by a fighter jet in the near field (~ 10 m), the nonlinear effect may be of the order of 6 dB. Even if the starting point of the propagation is taken at a more practical distance (~ 50 m), the nonlinear effect still makes a considerable difference in the outcome of noise impact studies.

Nomenclature

A	=	dimensionless diffusivity
c_0	=	small-signal speed of sound
f	=	frequency
f_s	=	sample rate
f_c	=	peak frequency
L_A	=	A-weighted Overall Sound Pressure Level
M	=	ratio of peak frequency to frequency resolution
N	=	number of Fourier coefficients
P	=	dimensionless acoustic pressure
PSD	=	Power Spectral Density
p	=	acoustic pressure
Q	=	parameter in simulation of broadband spectrum
r	=	radial distance
t	=	time
x	=	distance in one-dimensional problems
$\bar{x}$	=	plane-wave shock formation distance

Greek:

α	=	atmospheric absorption coefficient
$\overline{\alpha}$	=	dimensionless atmospheric absorption coefficient
α_0	=	thermoviscous attenuation coefficient
β	=	dimensionless coefficient of nonlinearity
δ	=	sound diffusivity for a thermoviscous fluid
λ	=	integer parameter, equals 0 for a one-dimensional problem and 1 for a three-dimensional problem
θ	=	dimensionless retarded time
ρ_0	=	density of air
σ	=	dimensionless distance
τ	=	retarded time
ω	=	angular frequency

subscripts:

0	=	reference
l	=	linear propagation model
nl	=	nonlinear propagation model
rms	=	root-mean-square
s	=	source

1. Introduction

It is common practice to determine the noise impact of aircraft (both civil and military) by using computations, rather than by measurements. Noise impact computations usually make use of noise levels, measured at a relatively small distance from the aircraft, combined with a propagation model. These propagation models are based on linear acoustics and account for effects like atmospheric absorption and lateral attenuation.

In the past decades however, evidence has been found that linear acoustics is inadequate to describe the propagation of high-intensity jet noise, as generated by fighter aircraft like the F-22 or the F/A-18 [1]. More specifically, it is found that the high frequency part of the spectrum is decaying much slower than predicted by linear acoustics. This behaviour is generally attributed to nonlinear effects in the propagation. In order to verify this assumption, and to enable a quantitative assessment of the nonlinear effects, computational methods are required for the numerical simulation of the nonlinear propagation of broadband noise. In the recent past two such methods have been published[2,3].

Computational methods for nonlinear propagation are usually based on the (generalised) Burgers equation for the acoustic pressure (e.g. Ref. 4). In the case of the original, one-dimensional Burgers equation, analytical approximations are known for an initial sinusoidal waveform. No such solution is known, however, for the generalised Burgers equation, which incorporates 3-dimensional spherical spreading. A second, even more complicating aspect is that a main component of jet noise (in the case of subsonic jets the only significant one) is mixing noise. This broadband noise is generated by stochastic, turbulent processes, and solutions for deterministic, periodic time signals seem useless for this application.

Therefore, most prediction methods are focused on the Power Spectral Density (PSD), a quantity that contains meaningful time-averaged spectral information, but no phase information. The PSD is usually the quantity in which experimental results are reported. For the PSD, being the Fourier transform of the second order moment of the pressure time signal, an equation can be derived by using the Burgers equation for the pressure. However, in the right hand side of this equation a third order moment appears, which again can be inserted into the Burgers equation, leading to a right hand side with a fourth order moment. By continuation of this scheme an infinite hierarchy of equations is obtained. In some papers the analysis is based on a truncated series, but it appears that such a series cannot predict the nonlinear propagation over longer distances[5]. In Ref. 6 an attempt is made to remedy this shortcoming by transforming the second term of the series into a new differential equation, valid for the whole domain. In this approach it is assumed that the initial signal is Gaussian, and stays almost Gaussian. Another method has been proposed by Menounou and Blackstock[7]. Their approach is based on including ever-higher order moments, all evaluated at the source, while marching the numerical solution forward along the spatial variable. At present their model does not incorporate spherical spreading nor atmospheric absorption. Furthermore it can only be used if the moment of any order can be evaluated at the source. The authors give two examples where this is possible: a sinusoidal waveform and a Gaussian stochastic process.

In the present paper a method is described that is based on solving the generalised Burgers equation in the frequency-domain, starting from a measured or simulated time signal. A simulated time series is used in case only an initial (source) PSD is known (i.e. no phase information). By using a time signal no assumptions are needed with respect to the higher order averaged moments of the acoustic pressure. The way in which the data, representing a pressure time series, are handled in the presented method is similar to the way measured data are handled in digital signal processing.

In section 2 of the paper the basic equations are derived and the solution method is explained. The results of the application to a jet noise experiment are presented in section 3. In section 4 a parameter study is presented, which gives some insight in the expected magnitude of nonlinear effects, depending on noise level and characteristic frequency. Results in this section are given in terms of the A-weighted Overall Sound Pressure Level, to facilitate an estimate of the effect of nonlinearity on environmental impact studies. Finally, conclusions and recommendations are given in section 5.

2. Basic equations and solution procedure

In Refs. 4 and 8 it is shown that the weakly nonlinear propagation of acoustic waves is described by the Burgers equation. ‘Weakly nonlinear’ means here that terms of the relative order of $(u_0/c_0)^2$ are neglected, where u_0 is the velocity amplitude of a plane wave and c_0 is the small-signal speed of sound. In Ref. 8 it is argued that for a plane wave the relative error is less than 0.5% at a sound level of 154 dB.

The (one-dimensional) Burgers equation is given by:

$$\frac{\partial p}{\partial x} = \frac{\beta p}{\rho_0 c_0^3}\frac{\partial p}{\partial \tau} + \frac{\delta}{2c_0^3}\frac{\partial^2 p}{\partial \tau^2} \tag{1}$$

where p is the acoustic pressure, x the spatial variable, β the coefficient of nonlinearity, ρ_0 the ambient density of air, $\tau = t - x/c_0$ is the retarded time, and δ is the sound diffusivity for a thermoviscous fluid. Equation (1) does not include spherical spreading nor the absorption by molecular relaxation. A generalised form of the Burgers equation that does include these effects is given by:

$$\frac{\partial p}{\partial r} + \frac{\lambda}{r} p = \frac{\beta p}{\rho_0 c_0^3}\frac{\partial p}{\partial \tau} + \frac{\delta}{2c_0^3}\frac{\partial^2 p}{\partial \tau^2} - \alpha p \tag{2}$$

where r is the radial distance and α is the atmospheric absorption coefficient. The parameter λ equals 0 for a one-dimensional problem and 1 for a three-dimensional problem. To solve this equation we follow a procedure outlined in Ref. 8, pp. 312-314. Note that details, e.g. in the scaling, may not be the same. First, we introduce the following dimensionless variables:

$$P = p / p_0, \qquad \sigma = r / \bar{x}, \qquad \theta = \omega_0 \tau, \qquad A = \alpha_0 \bar{x}, \qquad \bar{\alpha} = \alpha \bar{x} \tag{3}$$

where p_0 is a reference pressure, ω_0 a reference angular frequency, $\bar{x} = \rho_0 c_0^3 / \beta p_0 \omega_0$ is the lossless plane-wave shock formation distance for a signal with source condition $p = p_0 \sin(\omega_0 t)$, and $\alpha_0 = \delta \omega_0^2 / 2c_0^3$ the thermoviscous attenuation coefficient at frequency ω_0. Substituting this into Eq. (2) we find:

$$\frac{\partial P}{\partial \sigma} = -\frac{\lambda}{\sigma} P + P \frac{\partial P}{\partial \theta} + A \frac{\partial^2 P}{\partial \theta^2} - \bar{\alpha} P \tag{4}$$

To solve Eq. (4) we assume that the pressure can be written as a finite Fourier sum:

$$P(\sigma, \theta) = \sum_{n=-N}^{N} P_n(\sigma) e^{in\theta / M} \tag{5}$$

The integer number M (which is not present in Ref. 6) is introduced because we intend to apply the method to broadband noise. The frequency resolution determined by Eq. (5) should be much smaller than the frequencies of main physical interest (i.e. $\Delta\omega << \omega_0$), which necessitates the factor M. In practice, this factor is determined by the frequency at which the initial spectrum has its maximum. Note that $P_{-n} = P_n^*$, where * denotes complex conjugate, because P is real. Substitution of Eq. (5) into Eq. (4) and rearranging terms according to Ref. 8 yields:

$$\frac{dP_n}{d\sigma} = -\left(\frac{\lambda}{\sigma} + \frac{n^2}{M^2} A + \bar{\alpha} \right) P_n + i \frac{n}{2M} \left(\sum_{m=1}^{n-1} P_m P_{n-m} + 2 \sum_{m=n+1}^{N} P_m P_{m-n}^* \right) \tag{6}$$

where it is assumed that P_0 (the time-averaged pressure) vanishes. So we end up with N coupled ordinary differential equations.
Starting from a time series representing a broadband noise signal (either from measurement or simulation) at some distance r_0 (or equivalently σ_0), the initial Fourier coefficients P_n can be determined, and the system of Eq. (6) can be integrated numerically to any other distance. Input to the algorithm is a pressure time series, i.e. a set of pressure values (samples) p_j, in Pascal, denoting the pressure values at a given sample rate f_s (in s^{-1}), i.e. $p_j = p(t_j)$ with $t_j = j/f_s$. First, the largest number (smaller than the total number of samples) is determined which is a power of 2, in order to enable the use of Fast Fourier Transforms (FFT's). The result determines the number N. Note that the maximum frequency in the system is given by $f_s/2$, and the frequency resolution by $\Delta f = f_s/N$, which are relations well-known from digital signal analysis. The reference pressure p_0 is set to $\sqrt{2} p_{rms}$ where the root-mean-square value is determined by:

$$p_{rms} = \frac{1}{N} \sqrt{\sum_{j=1}^{N} p_j^2} \tag{7}$$

Next an FFT routine is applied to the time series, yielding the complex amplitudes of the $N/2$ frequency components. The reference parameter that reflects the level of nonlinearity, is the lossless plane-wave shock formation distance $\bar{x}$. If there is no nonlinearity this parameter is infinite. In the algorithm the value of $\bar{x}$ is determined for each frequency component, with the corresponding amplitude as reference pressure (note that summing the positive and negative frequency terms yields a real value). Out of this set of $N/2$ values for $\bar{x}$, the smallest one is selected; the corresponding index is chosen as M, and the corresponding frequency determines ω_0. This procedure ensures that, if the input signal is a single pure tone, it satisfies $p = p_0\sin(\omega_0 t)$ (apart from a possible phase shift) and $\bar{x}$ satisfies its original definition. Now we have determined all the reference parameters necessary to make the equations dimensionless.

Also input to the algorithm is a step size $\Delta\sigma$. At each step in σ, the right hand side of Eq.(6), denoted here by $RHS_n(\sigma)$, is determined for n=1,2,3,….$N/2$ successively, by using the *current* values of $\{P_n\}$. Before moving to n+1, the new value of P_n is determined by $P_n = P_n + \Delta\sigma \times RHS_n(\sigma)$. This means that in the calculation of $RHS_n(\sigma)$, the new values of P_m are used if $m < n$, whereas the old values are used if $m \geq n$. This amounts to the application of an *explicit* Euler scheme at the low-frequency end (n=1), gradually evolving into an *implicit* Euler scheme at the high-frequency end ($n=N/2$). Stepping from a given initial value of σ to a given end value, results into the final Fourier coefficients P_n, which can be transformed again, if desired, into a pressure time series.

3. Application to test cases

In reference 3 the method outlined above is applied to both a single pure tone and the jet noise experiment published in reference 1. The application to a single pure tone showed that for distances smaller than the lossless plane-wave shock formation distance, the results from the method presented here agree almost exactly with the analytic solution. For larger distances discrepancies occur, as finite Fourier sums cannot adequately represent shocks.

The jet noise experiment described in reference 1 consisted of the measurement of the noise of an F/A-18E/F aircraft. In this experiment noise data were acquired at distances of 18 m, 74 m, and 150 m from the nozzle of this aircraft, during a static engine run-up test. In this section we will try to simulate the propagation from 18 m to 150 m, for the military thrust case. The spectrum measured at 18 m (red curve of Fig. 1) is used to create an input time signal.

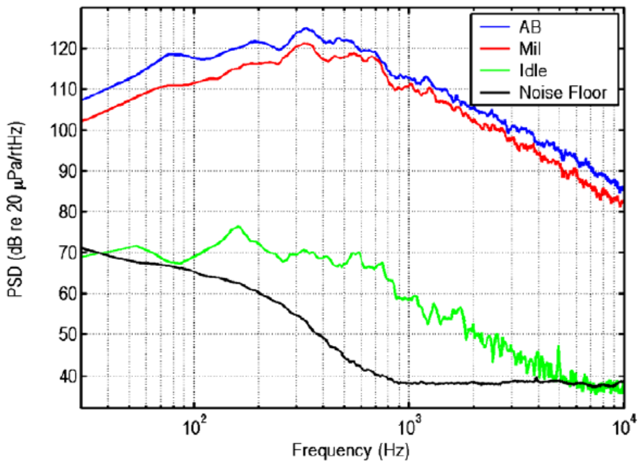

Figure 1. PSD of F/A-18E/F noise at 18 m (from Ref. 1).

The measured time series, needed as input of the algorithm, was not available to the author. Therefore, a broadband time signal was constructed, starting from the plotted Power Spectral

Density, written as $PSD_s(f)$. The objective thus is to construct a time series of N samples corresponding to a sample rate f_s, the Power Spectral Density of which equals $PSD_s(f)$. This is achieved in the following steps:

- A sequence of N random numbers satisfying a Gaussian distribution with zero mean and unit variance is generated. This sequence represents band-limited white noise.
- This white noise time series is Fourier-transformed.
- The Fourier coefficients are scaled to the starting spectrum $PSD_s(f)$.

By application of an inverse Fourier transformation, a broadband time signal is obtained with the desired PSD.

It is observed that the low frequency part of the (Mil) spectrum at 18 m behaves as $f^{1.7}$, whereas the high frequency part behaves as $f^{-2.7}$. Therefore, the following approximation for the PSD was adopted:

$$\mathrm{PSD_s}(f) = 10\ {}^{10}\log\left(\frac{2.5\times10^{19}\,f^{1.7}}{4.21\times10^{11}+f^{4.4}}\right) \qquad (8)$$

with f in Hz. With this function as input, a time signal was generated, using the procedure described above. This time signal consists of $N = 2^{16}$ samples, with a sample rate of $f_s = 2^{16}$ s^{-1}, leading to a time record of 1 s, a frequency resolution of 1 Hz, and a maximum analysis frequency of 32768 Hz. The Overall Sound Pressure Level (OASPL) of the simulated time signal is 148 dB, close to the measured result 147 dB.

The time signal is fed into the computer program outlined in reference 3, with r = 18 m as starting point and r = 150 m as end point. The atmospheric absorption coefficients are determined according to the method by Bass et al.[10,11].

The result (averaged over 32 blocks of 1024 data) is shown in figure 2, together with the measured results, the extrapolation from 18 m based on linear acoustics, and the results computed by Gee et al. (Ref. 2) based on a time-domain method.

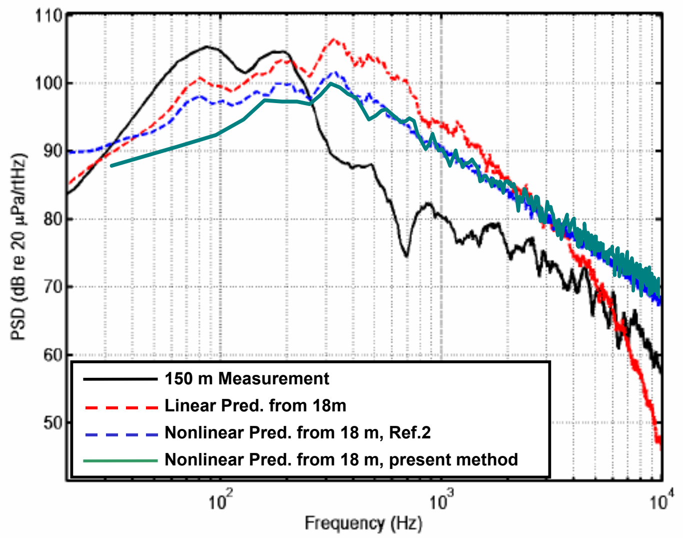

Figure 2. Measured and predicted PSD of F/A-18E/F noise at 150 m (partly from Ref. 2).

The effects attributed to nonlinear propagation are present in both predicted curves and the measured data: acoustic energy is transferred from the centre frequencies to the higher

frequencies, leading to a significantly lower decay of this part of the spectrum. Quantitatively however, both predicted curves deviate significantly from the measured data. The cause of this is unknown; the effect of ground reflections on the measured data may be part of the cause. In the lower frequency region the predicted curves are different as well; it is not clear whether this is caused by differences in the method or in the input data. At and above the centre frequencies the agreement between both predicted curves is excellent.
Although the application to these test cases do not constitute a definite validation, the results seem to justify the use of the method as a research tool, e.g. to study the effect of nonlinearity on noise impact computations, which will be done in the next section.

4. Parameter study on the results of noise impact computations

To study the effects of noise level and characteristic frequency on the magnitude of nonlinear effects, we will focus on the differences in the accumulated A-weighted Overall Sound Pressure Level at infinity, $L_A(r=\infty)$ in dBA, computed with linear propagation and with nonlinear propagation, from a given starting point r_0:

$$\delta L_{A,nl}(r_0) = L_{A,nl}(r=\infty; r_0) - L_{A,l}(r=\infty; r_0) \tag{9}$$

In practice 'infinity' means a distance r_{max} where $\delta L_{A,nl}(r_0)$ has become negligible. Starting with a given initial time signal, the propagated time signal will be calculated for a series of distances (i.e. values of r_0), up to r_{max}, by using the nonlinear propagation model. From each of these distances the resulting time signal will also be propagated to r_{max} with the linear model, and $\delta L_{A,nl}(r_0)$ can be evaluated from both time signals at r_{max} . The definition of $\delta L_{A,nl}(r_0)$ is illustrated in figure 3.

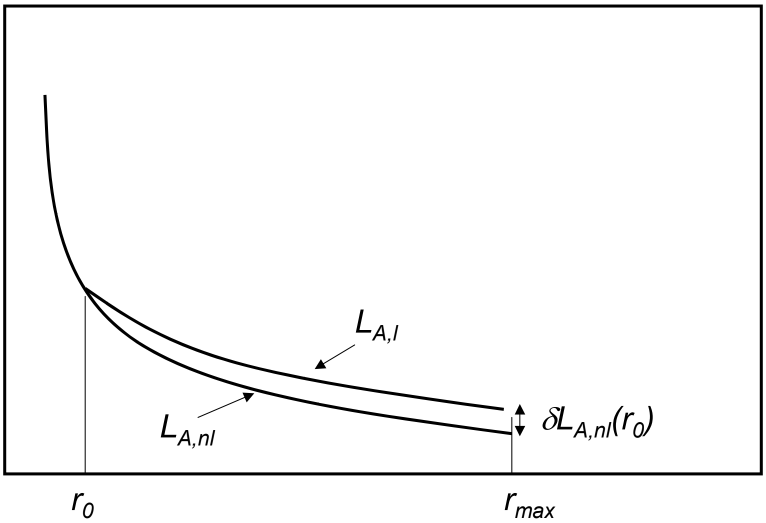

Figure 3. Definition of the accumulated nonlinear effect on the A-weighted Overall Sound Pressure Level, as function of starting distance r_0 .

For the present study we will make use of a simulated generic jet noise time signal. For the initial PSD_s of this signal we make the choice:

$$\mathrm{PSD}_s(f) = 10\ {}^{10}\log\left(\frac{Q f^2}{f_c^4 + f^4}\right) \tag{10}$$

This choice means that the spectrum is proportional to f^2 at low frequencies and to f^{-2} at high frequencies, not uncommon to what is observed in test data or resulting from theoretical models (see e.g. reference 12). The maximum sound pressure level occurs at $f = f_c$. The Overall Sound Pressure Level of this spectrum can be evaluated analytically:

$$\mathrm{OASPL} = 10\ {}^{10}\log\left(\frac{\pi}{2\sqrt{2}}\frac{Q}{f_c}\right) \tag{11}$$

The expression given in eq. (10) is used to create a broadband time signal, following the procedure presented in the previous section, with a sample rate f_s = 81920 Hz, number of samples N = 8192, yielding a frequency resolution of 10 Hz. As baseline case we choose an OASPL of 150 dB at a distance of 10 m from the source, and a peak frequency of 400 Hz, leading to $Q = 3.6 \times 10^{17}$. If the OASPL is determined numerically from the simulated broadband time signal, up to a frequency of 20 kHz, the result is 149.65 dB. The A-weighted level is L_A = 148.28 dBA. In the sequel the baseline case will be denoted by case I.
For the second case (II), we simply double the initial pressure values, i.e. $Q = 1.44 \times 10^{18}$, leading to an increase of the sound pressure level of 6 dB. In the third case (III) we take the same level as in the baseline case, but with a peak frequency which is twice as high: $Q = 7.2 \times 10^{17}$, f_c = 800 Hz. For all these cases it appears that at 250 m the nonlinear effects become negligible, so we take r_{max} = 250 m. The accumulated difference $\delta L_{A,nl}$ is computed for the distances r_0 = 10 m, 25 m, 50 m, 100 m, 150 m, and 200 m. The results are plotted in figure 4.

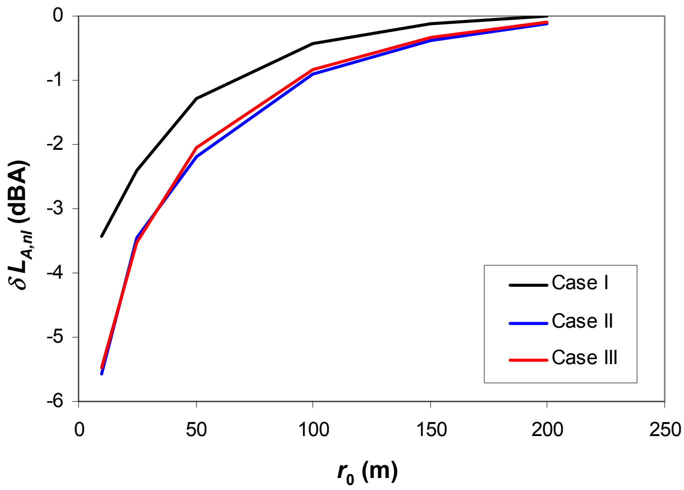

Figure 4. Computed results for the accumulated nonlinear effect, as function of starting distance r_0 .

The first observation is that the differences are negative: acoustic energy is scattered to higher frequencies, which results into a lower total acoustic energy. This can be understood by comparing a *N*-shaped pressure wave to a sinusoidal wave of the same amplitude: the latter has a higher *rms* value. This effect is enhanced by atmospheric damping, which is stronger at higher frequencies. A second observation is that doubling the amplitude yields virtually the same result as doubling the peak frequency, and the effect of both corresponds approximately to rescaling the distance. Indeed, it can be deduced easily from eq.(4) that the rescaling would be exact without the damping terms (i.e. the last two terms at the right hand side). To show this more explicitly, the results are plotted against the local value of $\bar{x}$, where $\bar{x}$ is the lossless plane-wave shock formation distance, defined below eq.(3). In this definition p_0 refers to the amplitude of a sinusoidal wave. For a broadband noise we adopt the following definition:

$$p_0 = \sqrt{2} p_{rms} \tag{12}$$

which is consistent with the definition for a sinusoidal wave. The value of $\bar{x}$ is evaluated at each starting distance, i.e. based on the *local* sound pressure level.

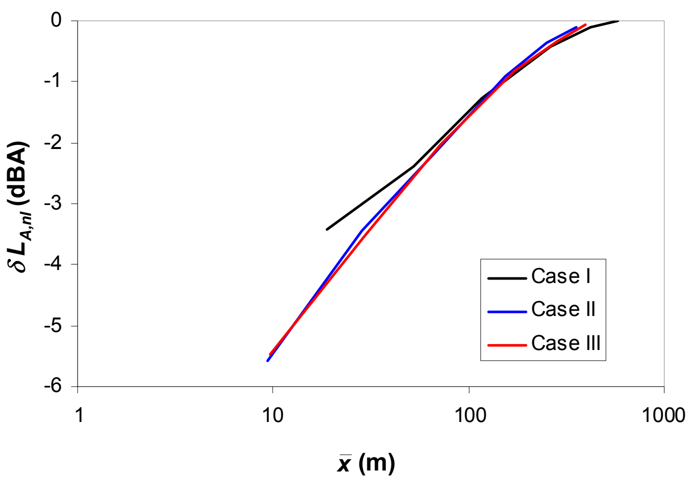

Figure 5. Computed results for the accumulated nonlinear effect, as function of the local value of $\bar{x}$.

Only the value at the lowest value of $\bar{x}$ for the baseline case is somewhat deviant (by less than 1 dB), otherwise the curves coincide.
Plotted on a logarithmic scale, the curves follow a straight line for the largest part, which gives us the opportunity to give a simple approximate expression for the accumulated nonlinear effect, as a function of $\bar{x}$ at the starting distance:

$$\begin{aligned} \delta L_{A,nl} &= 3.4 \times {}^{10}\log \bar{x} - 8.6, \quad \bar{x} < 338.4\,m \\ \delta L_{A,nl} &= 0, \quad \bar{x} > 338.4\,m \end{aligned} \tag{13}$$

with $\bar{x}$ given in meters.
For our baseline case (f_c = 400 Hz) the value of $\delta L_{A,nl}$ becomes zero for p_0 = 46.2 Pa, corresponding to L_A = 121 dBA, meaning that for this and lower noise levels the nonlinear effect is negligible. Note that the approximate relation given by eq.(13) may not be generally

valid; it only applies to a spectrum of the form of eq.(10), and the form it takes after (nonlinear) propagation.
In figure 6 the value of $\delta L_{A,nl}$, calculated from eq.(13), is presented in a color plot, as function of the Overall Sound Pressure Level and peak frequency f_0. Note that the OASPL is not A-weighted; the value for L_A is usually a few dB's lower, depending on the spectrum.

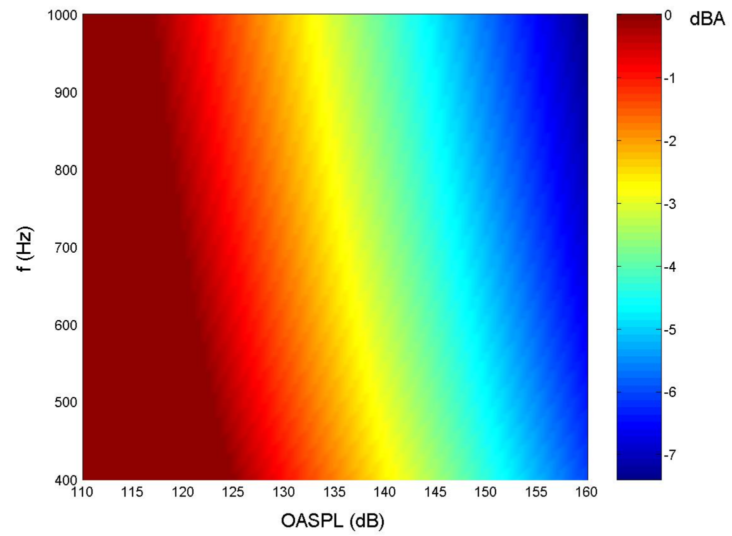

Figure 6. The accumulated nonlinear effect at infinity from eq.(13), as function of OASPL and the peak frequency at the starting point.

The values obtained for the accumulated nonlinear effect in the present analysis could have a significant effect on the result of noise impact studies. For example, if an aircraft produces the noise level of the baseline case defined above at a distance of 10 m (which is comparable to that of the F-18 measurement described in section 3), and the A-weighted noise level L_A would be measured at 50 m, an analysis based on linear acoustics would introduce an error of 1.3 dBA. In a noise impact study expressed in, say, L_{den}, 1.3 dBA corresponds to a difference in number of flights of 35%. When measurements are performed nearer to the aircraft or if the spectrum peaks at higher frequencies, this difference will even be larger.

5. Conclusions

In this paper a method is presented for the numerical simulation of the nonlinear propagation of broadband noise. The method is based on a frequency-domain approach. Results of this method applied to a published test on the noise of a F/A-18E/F aircraft have been compared to the measured data and the results of an alternative, time-domain method. The results of both computation methods compare well; there is only qualitative agreement with the test results. Although the method has not been validated yet, it was applied to a parameter study on a simulated broadband, jet noise-like sound spectrum.
The main conclusions are:

- The sound levels at large distances, predicted by a nonlinear propagation method are smaller than those predicted by a linear method.
- For the same shape of the spectrum, the difference between 'nonlinear' and 'linear' results mainly depends on the product of the sound level and the peak frequency at the starting point of the computation. An simple, approximate relation between the plane-

wave shock formation distance at the starting point, and the nonlinear effect at infinity is given.
- For typical values of the noise produced by a fighter jet at in the near field (~ 10 m), the nonlinear effect may be of the order of 6 dB. Even if the starting point of the propagation is taken at a more practical distance (~ 50 m), the nonlinear effect still makes a considerable difference in the outcome of noise impact studies.

For future research it is recommended to obtain suitable test data for the validation of this (and other) computation methods for nonlinear propagation.

6. References

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