



## Executive summary

# An integrity, availability and continuity test method for EGNOS/WAAS

### Problem area

The actual development of satellite navigation systems is crucial and includes after the already introduced EGNOS system (augmentation of GPS), the planned modernization of GPS, and the newly designed European Galileo system. For safety of life applications of the satellite navigation systems reliability in the sense of integrity, availability and continuity is essential. In order to test the integrity, availability and continuity, a test method is required. In practice, it is a requirement to be able to perform the analysis based on a limited amount of data collected within an acceptable observation time. This report presents such a test method analyzing receiver output data of limited duration for SBAS.

### Description of work

During a first research effort, a practical test method has been developed for analyzing receiver output data. As a test case the method has been applied to data gathered during a test campaign of limited duration for EGNOS. With this method an accurate estimate of the integrity, availability and

continuity can be made. In the paper, a detailed discussion of the analysis of availability, integrity and continuity is given. In the paper it is noted that the method is promising indeed, however it is a requirement to continue this investigation in order to improve the confidence which can be put on the method and the results.

### Results and conclusions

From the test case with EGNOS data, as presented in this paper, it can be concluded that the estimation of the integrity, availability and continuity on the basis of test campaign data is possible indeed. The results of the test case show the integrity usually satisfies the requirements, however the availability and continuity for the APV1 and more stringent aeronautical services was usually insufficient.

### Applicability

The method developed is to be used to analyze measurement data gathered during test campaigns in order to estimate the reliability of the navigation satellite system in the sense of integrity, availability and continuity.

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## Summary

For safety of life applications of satellite navigation systems, in e.g. aviation, reliability in the sense of integrity, availability and continuity is essential. In order to test the integrity, availability and continuity, a test method is required. In practice, it is a requirement to be able to perform the analysis based on a limited amount of data collected within an acceptable observation time. This paper presents such a method for analyzing receiver output data of limited duration for SBAS. In the paper, a detailed discussion of the analysis of availability, integrity and continuity is given.

The availability computation starts from the measured protection level data. It is possible to approach the protection level probability distribution by the Weibull distribution as will be shown in the paper.

The HMI (Hazardous Misleading Information) probability depends on two basic parameters: the actual position error and the computed protection level. Usually one presents the test results in the so-called Stanford plots visualizing the occurrence of Hazardous Misleading Information. Since the probability of HMI is very low, it is not practical to state that the HMI probability is the ratio of the number of HMI results divided by the total number of measured samples. Very often, no HMI condition occurs during the tests; the resulting number of HMI conditions will then be zero and the computed HMI probability on this basis will then be zero as well, being obviously incorrect. Therefore, we need to invent a correct way in obtaining a realistic estimate of the HMI probability also called the integrity risk. The method developed for this purpose starts from the determination of two probability density functions: the probability density distribution of the protection level and a probability density function related to the position error, together forming a two dimensional probability density function.

It is possible to compute the continuity risk (or non-continuity) as function of the alert limit. It turns out the resulting non-continuity probability distribution can be approached by a lognormal distribution.



## Contents

<b>1</b>	<b>Introduction</b>	<b>9</b>
<b>2</b>	<b>Description of the method with test case results</b>	<b>10</b>
2.1	Integrity, availability and continuity related to vertical position errors	10
2.1.1	Misleading Information probability computation	11
2.1.2	Availability computation	14
2.1.3	Integrity computation	16
2.1.4	Continuity computation	21
2.2	Integrity, availability and continuity related to horizontal position errors	25
2.2.1	Misleading Information probability computation	27
2.2.2	Availability computation	28
2.2.3	Integrity computation	29
2.2.4	Continuity computation	31
<b>3</b>	<b>Brief overview of test results</b>	<b>31</b>
<b>4</b>	<b>Conclusions</b>	<b>32</b>
	<b>References</b>	<b>33</b>

## Abbreviations

EGNOS	European Geostationary Navigation Overlay Service
GPS	Global Positioning System
HAL	Horizontal Alert Limit
HMI	Hazardous Misleading Information
HPE	Horizontal Position Error
HPL	Horizontal Protection Level
MI	Misleading Information
SBAS	Space Based Augmentation System
UTC	Universal Time, Coordinated
VAL	Vertical Alert Limit
VPE	Vertical Position Error
VPL	Vertical Protection Level
WAAS	Wide Area Augmentation System



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## 1 Introduction

For safety of life applications of satellite navigation systems, in e.g. aviation, reliability in the sense of integrity, availability and continuity is essential. In order to test the integrity, availability and continuity, a test method is required. In practice, it is a requirement to be able to perform the analysis based on a limited amount of data collected within an acceptable observation time. The presented test method for availability, continuity and integrity starts from the determination of the probability density distribution of measured events. For SBAS, these events will be the protection levels. For integrity analysis, these events will be the protection levels as function of the position errors.

### Basic approach

#### *Availability*

The availability computation starts from the measured protection level data. Use is made of the Weibull probability distribution to approach the protection level probability distribution. The use of this Weibull distribution is justified because of its special features as explained later on in this paper. The presented test data do show that it behaves like the Weibull distribution indeed.

#### *Integrity*

The HMI (Hazardous Misleading Information) probability depends on two basic parameters: the actual position error and the computed protection level. Usually one presents the test results in the so-called Stanford plots visualizing the occurrence of Hazardous Misleading Information. Since the probability of such HMI condition generally is very low, it is not practical to compute the HMI probability as the ratio of the number of HMI results divided by the total number of measured samples. So a method is required to determine a realistic estimate of the HMI probability. The method developed starts from the determination of two probability density functions: the probability density distribution of the protection level and a probability density function related to the position error, together forming a two dimensional probability density function. Since the uncertainty in the position error increases approximately in ratio with the computed protection level, no use is made of the probability density distribution of the position error itself, but of the ratio of the position error over the computed protection level. Once the position error is larger than the protection level and thus the ratio between them is larger than one, we interpret the situation as misleading and accordingly we may speak about the MI (Misleading Information) function. In vertical direction, the MI probability is approximately normally distributed. In the horizontal plane the MI probability is approximately Rayleigh distributed. Summarizing, to compute the HMI probability we need to estimate both the protection level and MI probability density distributions.

### *Continuity*

It is possible to compute the continuity risk as function of the alert limit. The paper presents the applied equations and logic to derive the continuity risk. It turns out that the resulting non-continuity probability distribution can be approached by a lognormal distribution as will be shown on the basis of the presented test data.

## **2 Description of the method with test case results**

As test case, use will be made of EGNOS data recorded at a measurement site at Lugano airport with a Septentrio PolaRX2 receiver. The data collected on 21 April 2005 did start at 7.15 hours and ended at 13.45 hours UTC (see Ref. 1).

The probability density functions are partly different for the vertical and horizontal case. Therefore, it is necessary to split up the discussion in two chapters accordingly.

### **2.1 Integrity, availability and continuity related to vertical position errors**

Figure 1 shows time series of the Vertical Position Error (VPE) and the Vertical Protection Level (VPL).

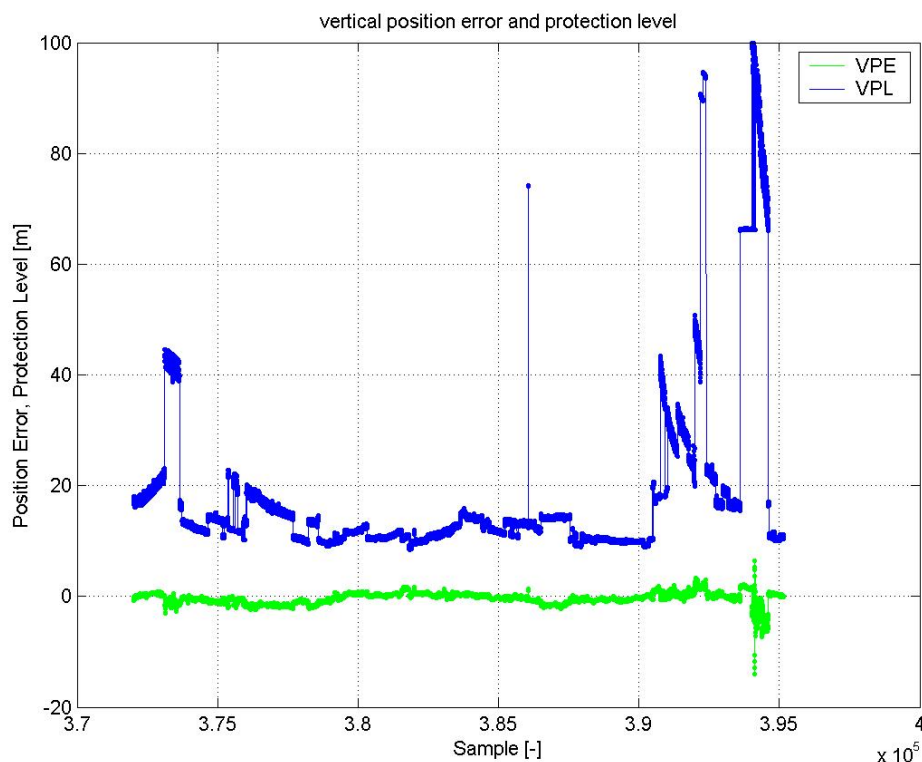
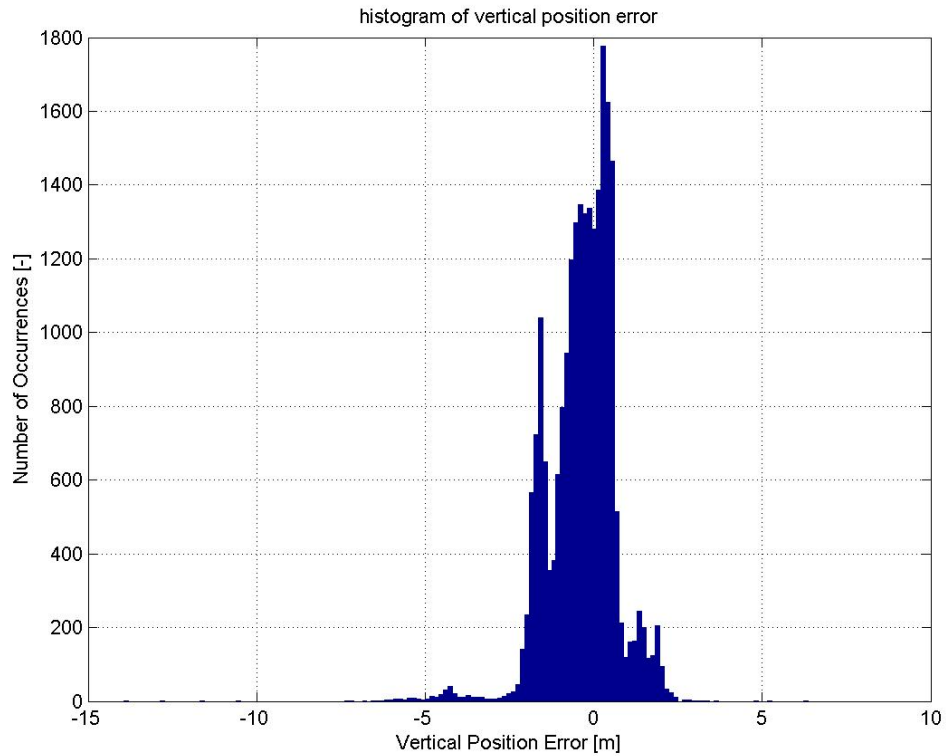


Figure 1: Time series of the vertical position error and the vertical protection level.

Figure 2 shows the histogram of the vertical position error. The 95% percentile vertical error is 1.89 m.



*Figure 2: Histogram of the vertical position error.*

The computation of the MI probability starts from the vertical position error and the vertical protection level.

### **2.1.1 Misleading Information probability computation**

Figure 3 shows a time series of the ratio between the vertical position error and the vertical protection level.

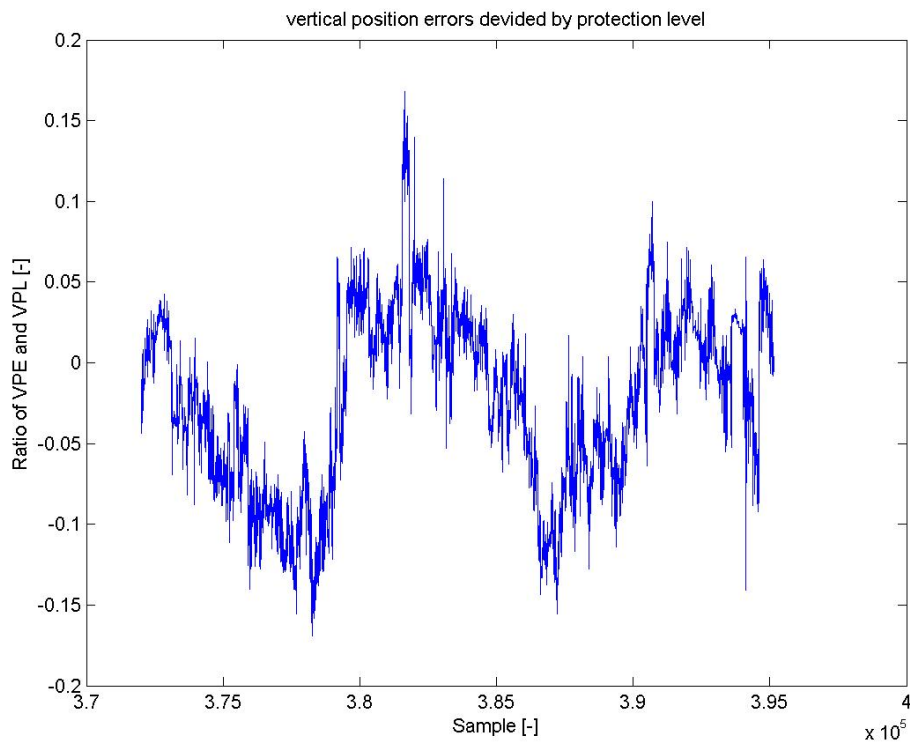


Figure 3: Ratio between the vertical position error and the vertical protection level.

Figure 4 shows that the ratio between the vertical position error and the vertical protection level behaves approximately as a normal probability distribution. Fitting the normal distribution function through the data yields a standard deviation of 0.059. It is possible now to compute the MI probability; it is the chance that the vertical position error is larger than the VPL and consequently the chance their ratio is larger than one. This probability turns out to be  $1.5 \times 10^{-63}$ . Besides this probability one usually is also interested in having a confidence figure of this result. However computing the so-called confidence interval in the classical way leads to a far too optimistic figure due to the fact that the successive samples are usually not independent; in the contrary they are highly correlated. Taking into account a decorrelation time of 2000 seconds (see the autocorrelation function of VPE/VPL in figure 5) and a confidence level of 5%, the MI probability is  $2.3 \times 10^{-33}$ . Note that according to the original Stanford plot, the MI probability is zero and consequently is not correct.

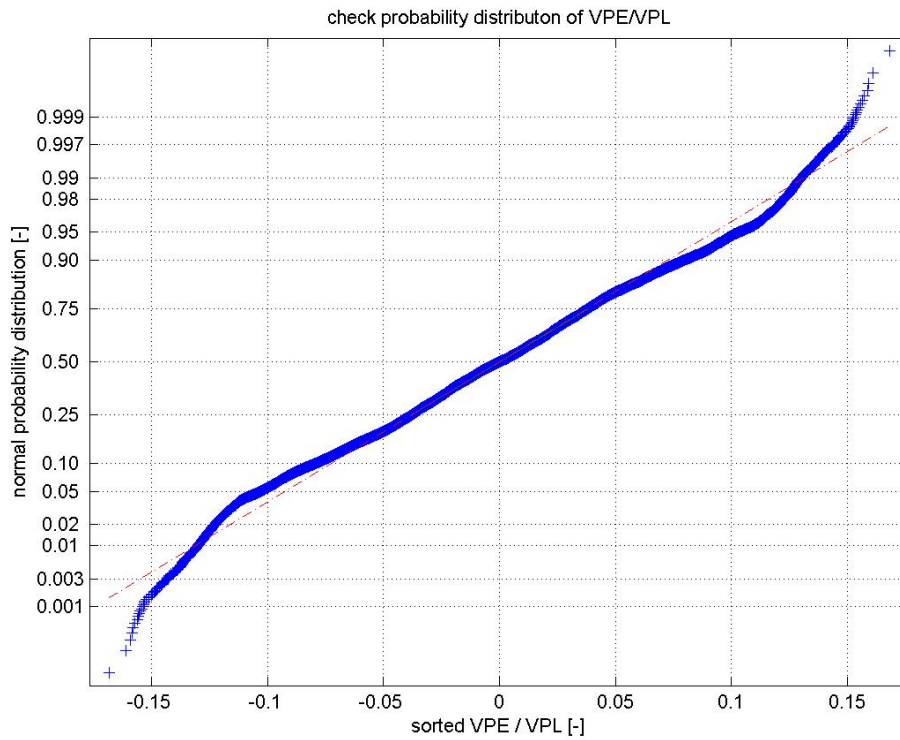


Figure 4: Normal probability distribution check of the ratio between the vertical position error and the vertical protection level.

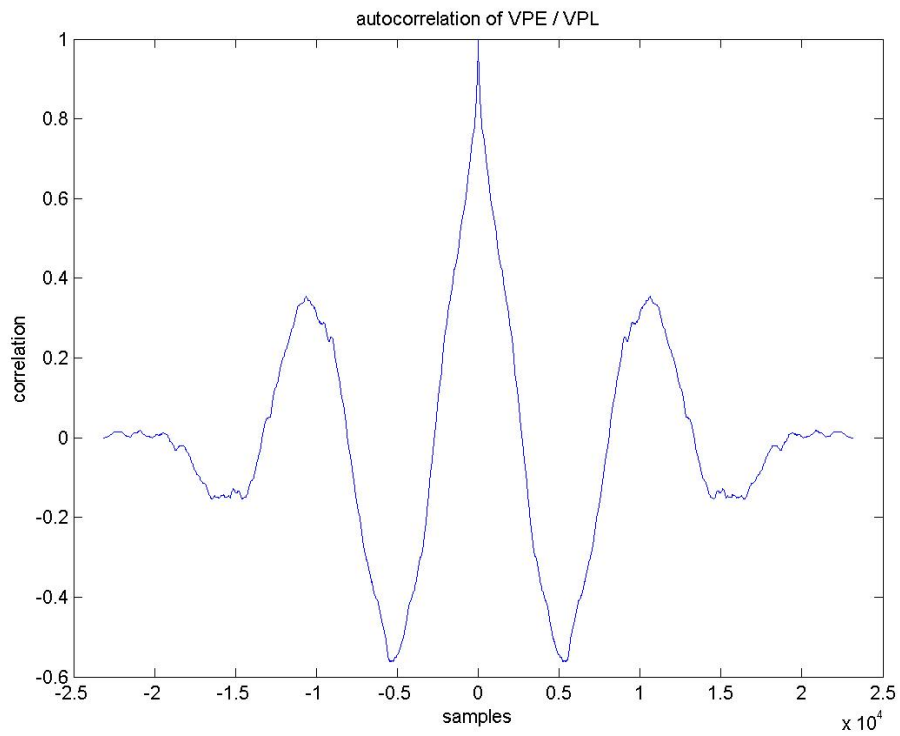


Figure 5: Autocorrelation of VPE/VPL.

**2.1.2 Availability computation**

Figure 6 shows the obtained Weibull probability distribution check of the vertical protection level. From this figure, it is obvious that the VPL behaves roughly similar as the Weibull distribution. In reliability studies (see e.g. Ref. 2); it is normal practice to make use of the Weibull distribution.

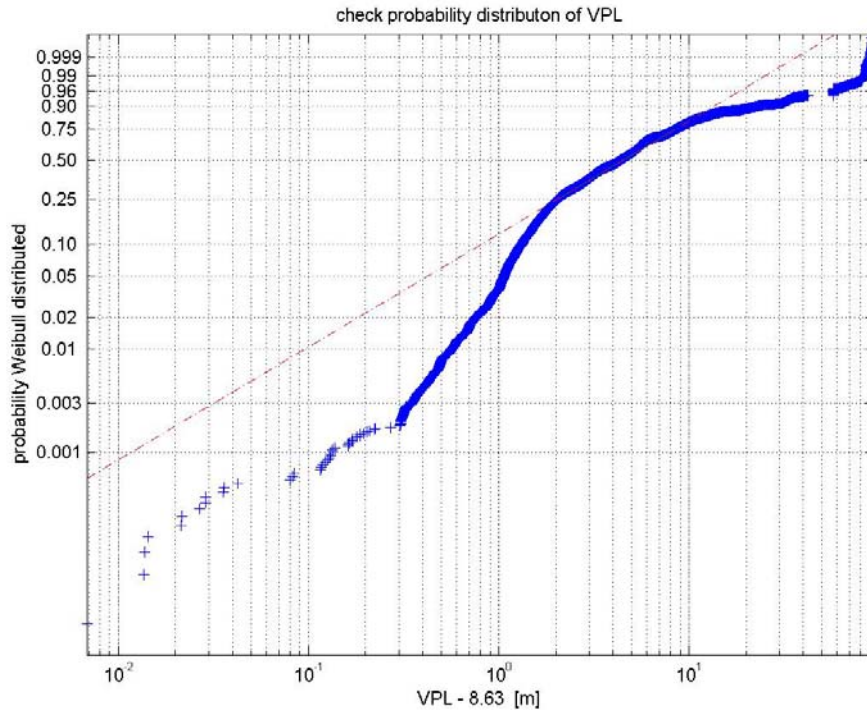
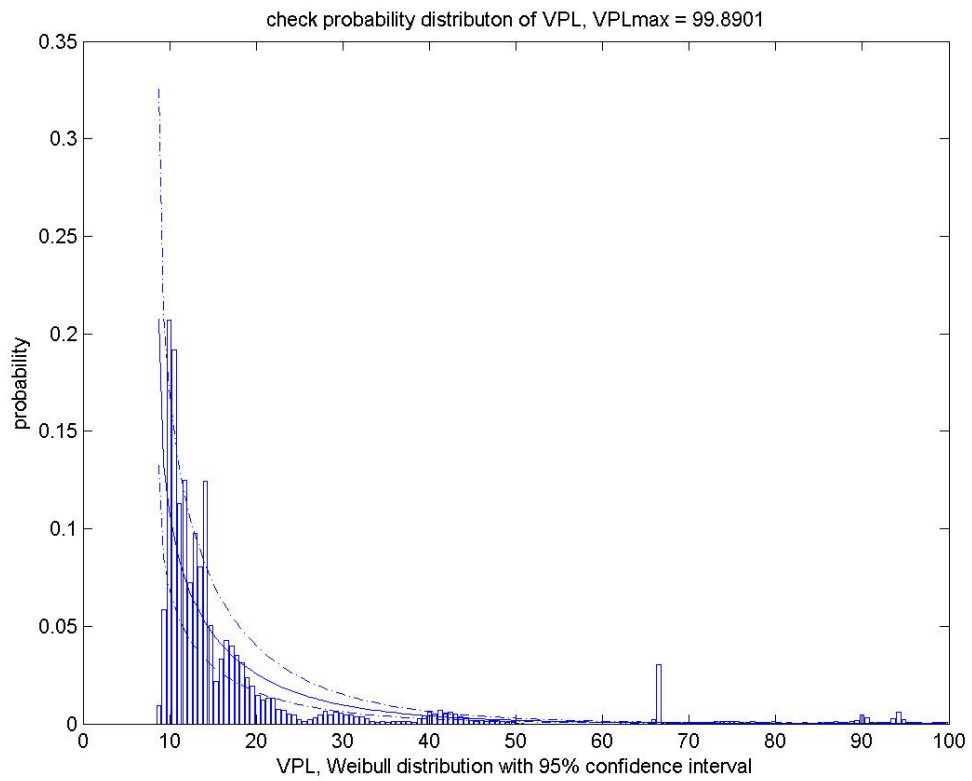


Figure 6: Weibull probability distribution check of the vertical protection level.

The Weibull probability density function is:

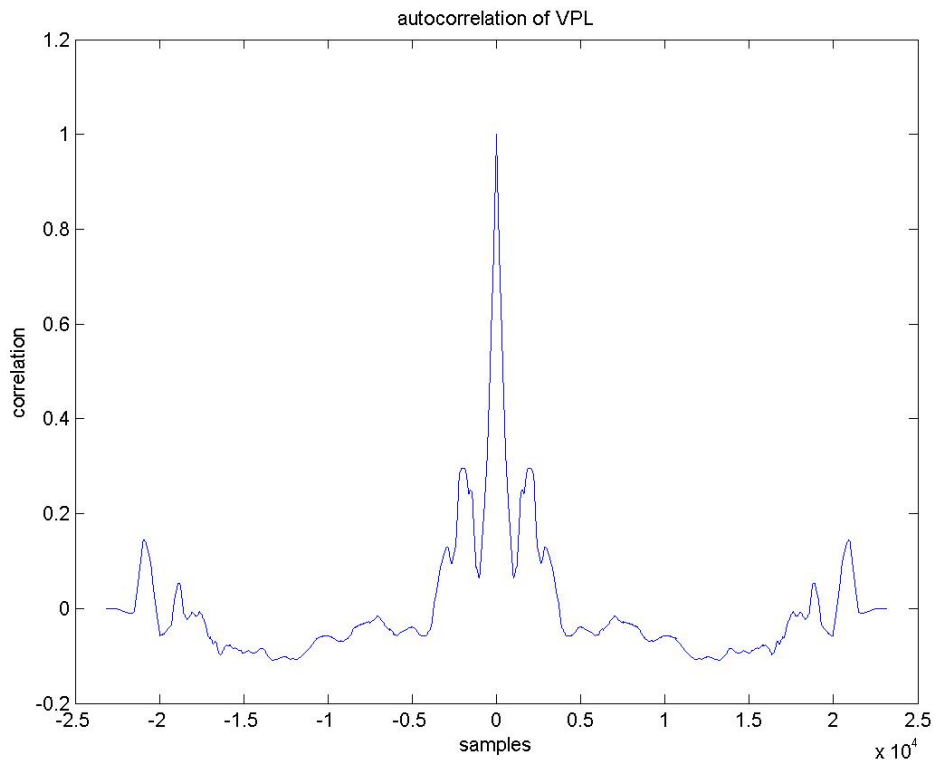
$$y = abx^{b-1}e^{-ax^b}$$

where a and b are the two statistical shape and scale parameters to be estimated. Note that this function essentially consists of the multiplication of two exponential functions; one describing the initial fast grows at the start of the distribution (x relatively small) and the other one describes the tail part of the distribution. Due to the two exponential functions the description of the low values of VPL decouples in relation to the high values of VPL, resulting in an accurate prediction of the availability (occurs in the tail part). This way the probability distribution at the tail at the right is not disturbed by the central bulk of the data at the left (see figure 7). The choice of the Weibull distribution function is justified for two reasons: (1) the start of the distribution function is steep allowing the bulk of the data at low VPL magnitudes and (2) the central bulk of the data do not disturb the tail towards infinity.



*Figure 7: Histogram of VPL with Weibull probability distribution including a confidence interval of 95%.*

To compute the confidence interval in figure 7, a decorrelation of the data is essential. The decorrelation time used for the VPL is 2000 seconds (see figure 8)



*Figure 8: Autocorrelation of VPL.*

It is now possible to compute the unavailability being the probability that the VPL exceeds the Vertical Alert Limit (VAL) using the Weibull distribution fit. For a VAL equals 50 m the unavailability turns out to be 2.5%. This is less than the originally computed, Stanford plot based, unavailability of 5.3%. The cause of this difference is the presence of outliers in the VPL (see the histogram of figure 7). For a confidence interval of 95%, the unavailability turns out to be 3.9%.

### **2.1.3 Integrity computation**

Usually one visualizes the integrity risk using the so-called Stanford plots. In the figures 9a, 9b and 9c the Stanford plots of the APV-I, APV-II and CAT I services are shown.



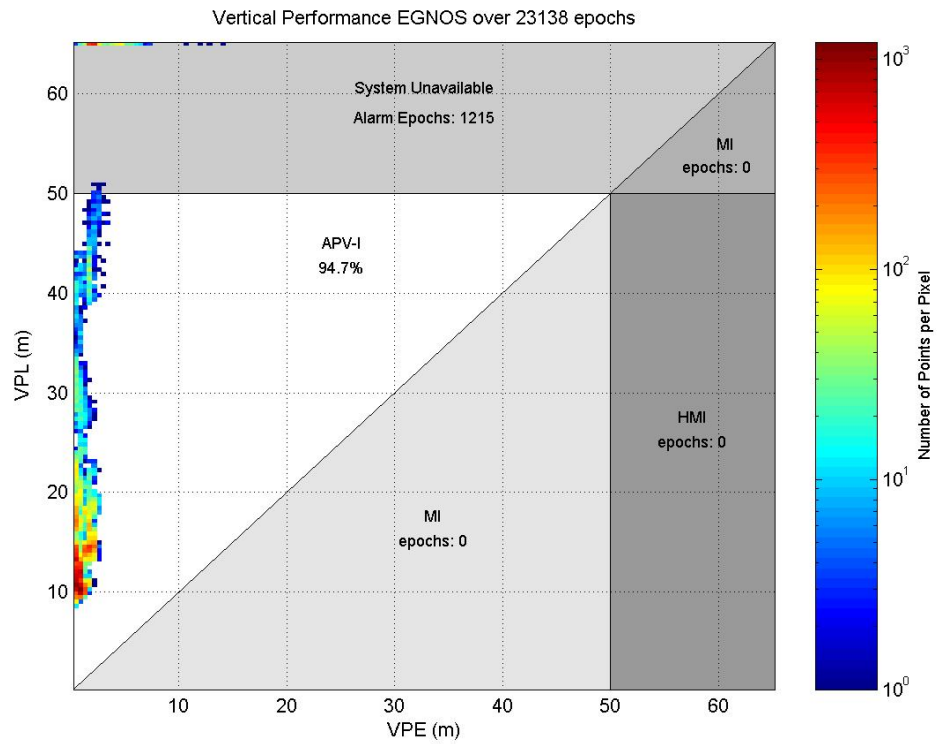


Figure 9a: Stanford plot for the APV-I service.

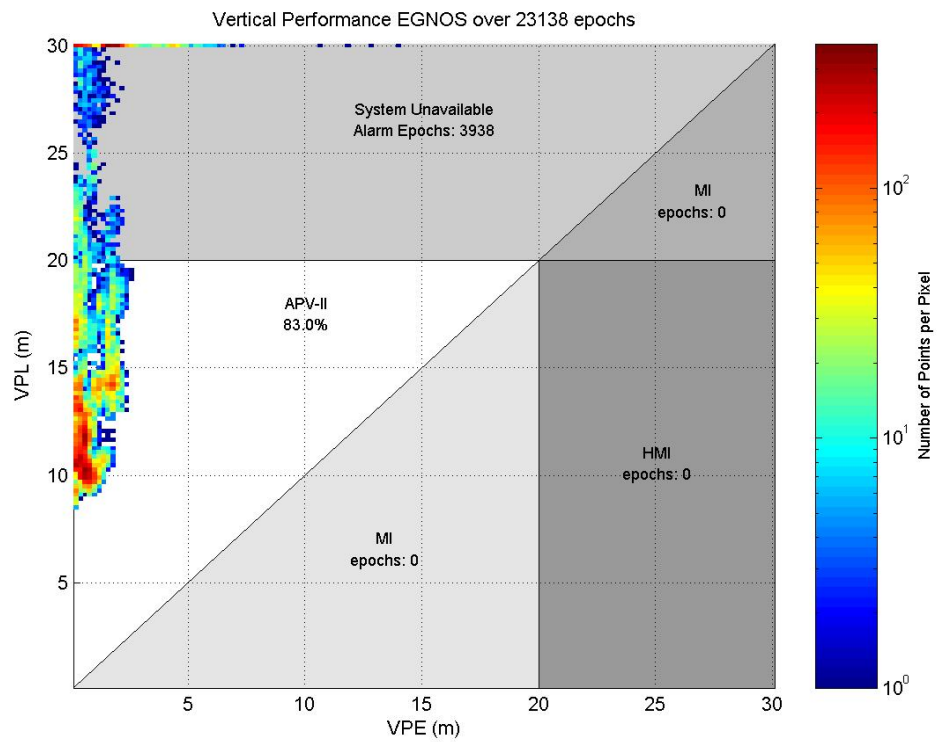


Figure 9b: Stanford plot for the APV-II service.

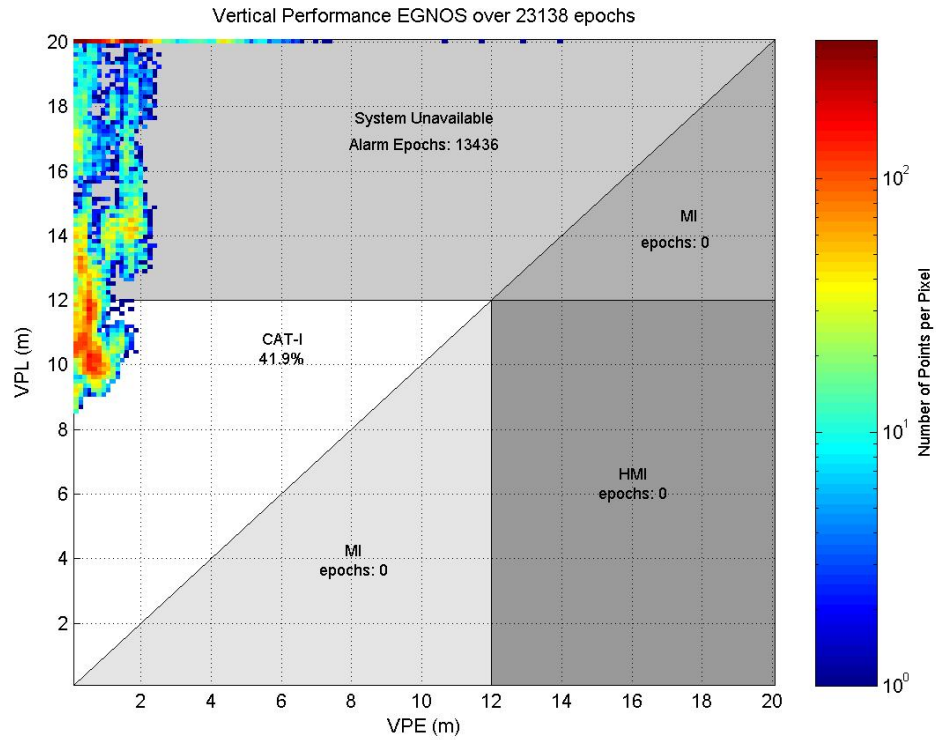


Figure 9c: Stanford plot for the CAT-I service.

Note that the number of observed epochs in the MI region is zero in all cases. This of course does not mean that the MI and HMI probabilities are zero. To estimate these probabilities we have to make use of the probability density distributions described in the previous chapters.

Figure 10 explains the discretized integration scheme to compute the HMI probability  $P_{HMI}$ . The contribution  $dP_{HMI}$  (the yellow colored element in figure 10) is equal:

$$\int_{\frac{VAL}{VPL}}^{\omega} P_{MI} \left( \frac{VPE}{VPL} \right) d \frac{VPE}{VPL} * dVPL$$

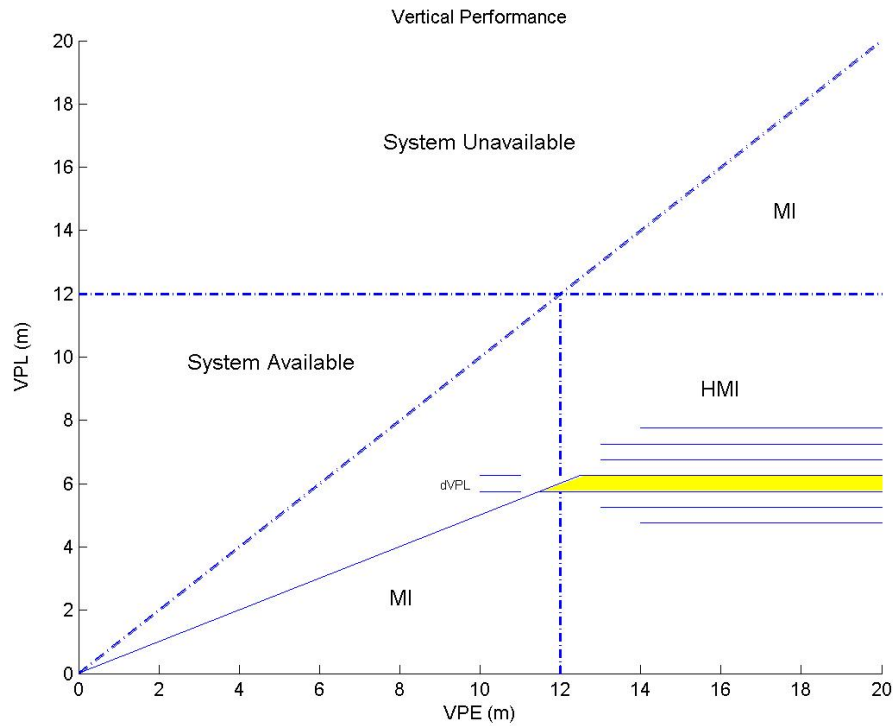


Figure 10: Elements (yellow) to integrate for obtaining the HMI probability.

The computed MI probabilities result by the evaluation of following equations:

$$P_{HMI} = \int_0^{VAL} p_{VPL}(VPL) \int_{VAL}^{\infty} p_{MI}\left(\frac{VPE}{VPL}\right) dVPE * dVPL$$

$$P_{MI(VPL > VAL)} = \int_{VAL}^{\infty} p_{VPL}(VPL) \int_{VPE=VPL}^{\infty} p_{MI}\left(\frac{VPE}{VPL}\right) dVPE * dVPL$$

$$P_{MI(VPL < VAL)} = \int_0^{VAL} p_{VPL}(VPL) \int_{VPE=VPL}^{\infty} p_{MI}\left(\frac{VPE}{VPL}\right) dVPE * dVPL - P_{HMI}$$

In the figures 11a, 11b and 11c the availability, the MI risks and HMI risk for the APV-I, APV-II and CAT-I services are plotted. The contours having the values 1, 0.5, 0.1, 0.01, 0.001,  $10^{-5}$  and  $10^{-10}$  indicate the probability densities relative to the maximum probability density. Figure 11c shows that for the CAT-1 service the HMI risk of  $3.7 \times 10^{-66}$  ( $5.9 \times 10^{-66}$  with confidence interval of 5%) is almost neglectable while the availability of 37.8% (2.4% with confidence interval of 5%) is dramatically low.

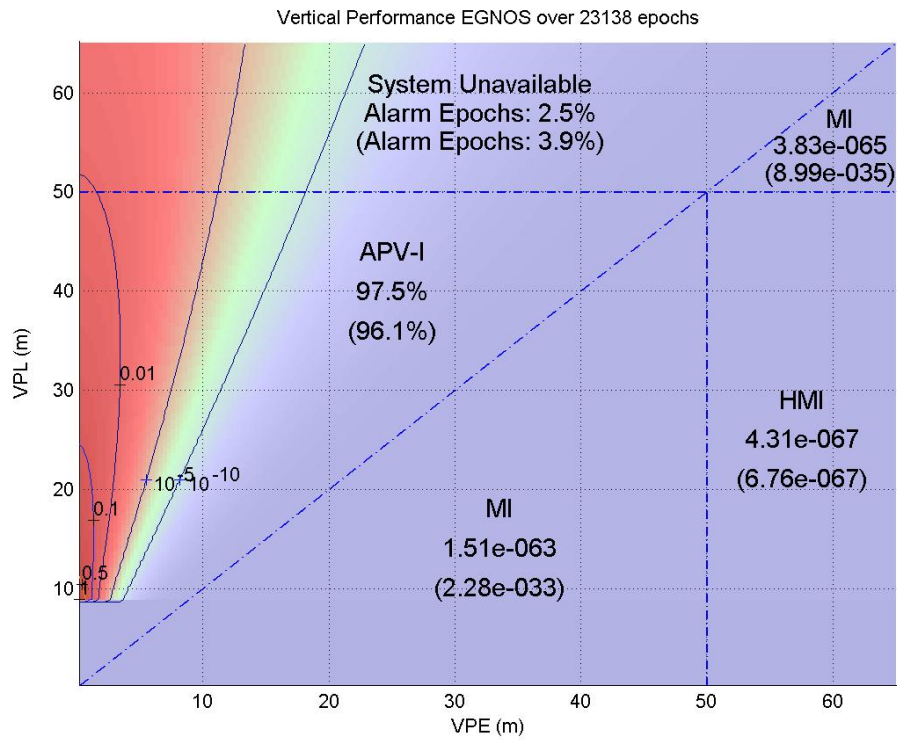


Figure 11a: Availability, MI risks and HMI risk for the APV-I service (the values within brackets are 5% confidence values).

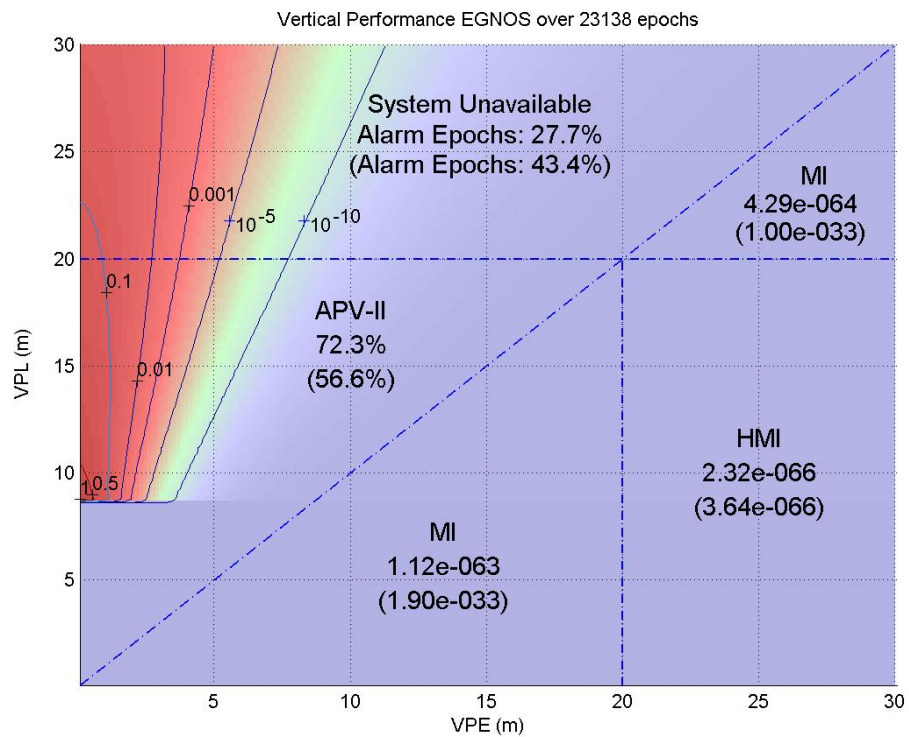


Figure 11b: Availability, MI risks and HMI risk for the APV-II service (the values within brackets are 5% confidence values).

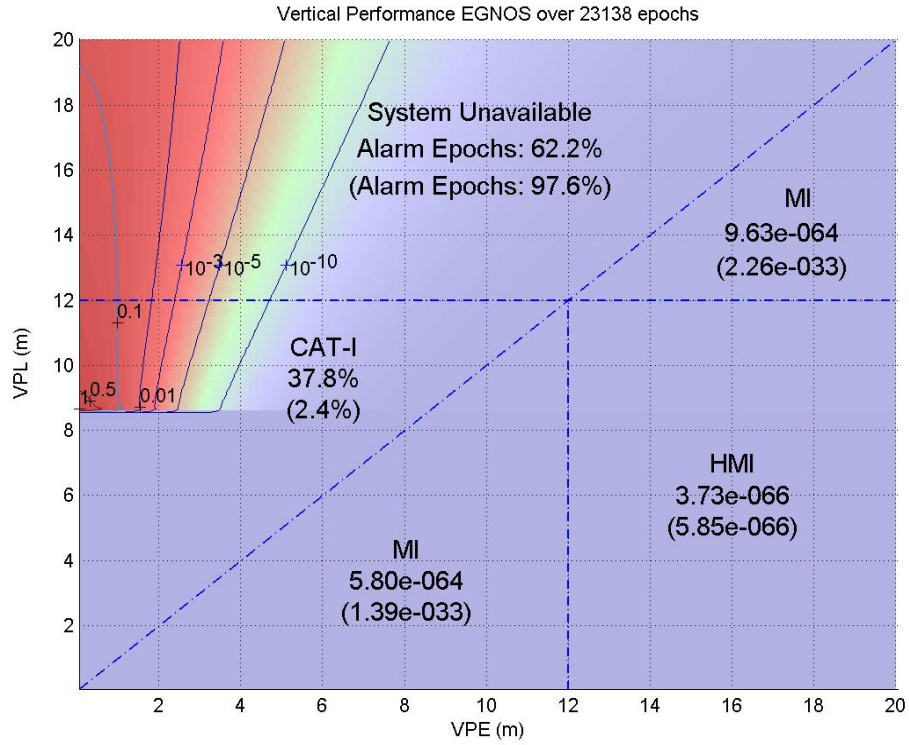


Figure 11c: Availability, MI risks and HMI risk for the CAT-I service (the values within brackets are 5% confidence values).

### 2.1.4 Continuity computation

Continuity risk is the probability that the system will not provide position information with the accuracy and the integrity required during the intended operation, presuming that the system was available at the beginning of that operation and was predicted to operate throughout the operation. During the APV-I, APV-II and CAT-I operation the specified continuity is  $1-8 \times 10^{-6}$  in any 15 seconds. Therefore, each set of 15 consecutive epochs should pass the criteria as described in the definition. It is possible now to compute the continuity risk as follows:

$$N_{success} = \sum_{t=0}^T (VPL < VAL \text{ over the entire periods } t \rightarrow t+15)$$

$$N_{fail} = \sum_{t=0}^T (VPL > VAL \text{ at 1 or more epochs over } t+1 \rightarrow t+15 \text{ while } VPL < VAL \text{ at } t)$$

$$P_{non\_continuity} = \frac{N_{fail}}{N_{success} + N_{fail}}$$

Running these three equations over the VPL registration (see figure 1), results in a continuity risk of  $P_{\text{non-continuity}} = 0.0031$  for  $\text{VAL} = 50$  m (APV-I service), 0.016 for  $\text{VAL} = 20$  m (APV-II service) and 0.11 for  $\text{VAL} = 12$  m (CAT-I service). These results lead to the conclusion that the requirement for continuity is not satisfied for the APV-I, APV-II and CAT-I services (requirement is  $8 \cdot 10^{-6}$ ). It is possible to refine these results and to include confidence intervals; however, this is possible only using statistics. Obtaining these statistics is possible by evaluation of the three equations above for a range of vertical alert limits varying from the minimum VPL up to the maximum VPL occurring in the available data set. Figure 12 shows the non-continuity computed as function of the VAL.

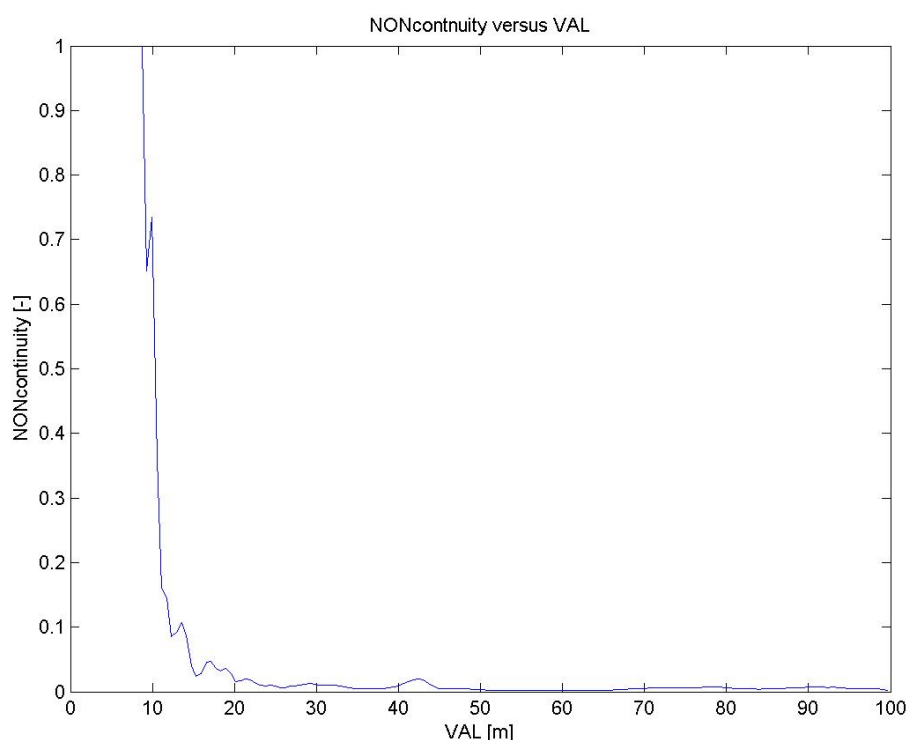


Figure 12: Non-continuity as function of VAL.

Figure 13 does show that a lognormal distribution approximately describes the non-continuity as function of the VAL. To compute the confidence interval a decorrelation is necessary; for this case, a factor 8 was applicable. From the result shown in figure 13 the continuity risk for all vertical alert limits can be computed by evaluation of the equation

$$P_{\text{non-continuity}} = 10^{-2.133-0.012 \cdot \text{VAL}} \quad (\text{VAL} > 0)$$

This equation is the best fit of the non-continuity probability as function of VAL. Since the lognormal distribution of figure 13 is not so well described by a straight line, it is suggested to split up this function into three local intervals for example as indicated in following table:

Service	VAL	lower bound	upper bound	see figure
APV-1	50	40	60	14
APV-2	20	15	25	15
CAT-1	12	7	17	16

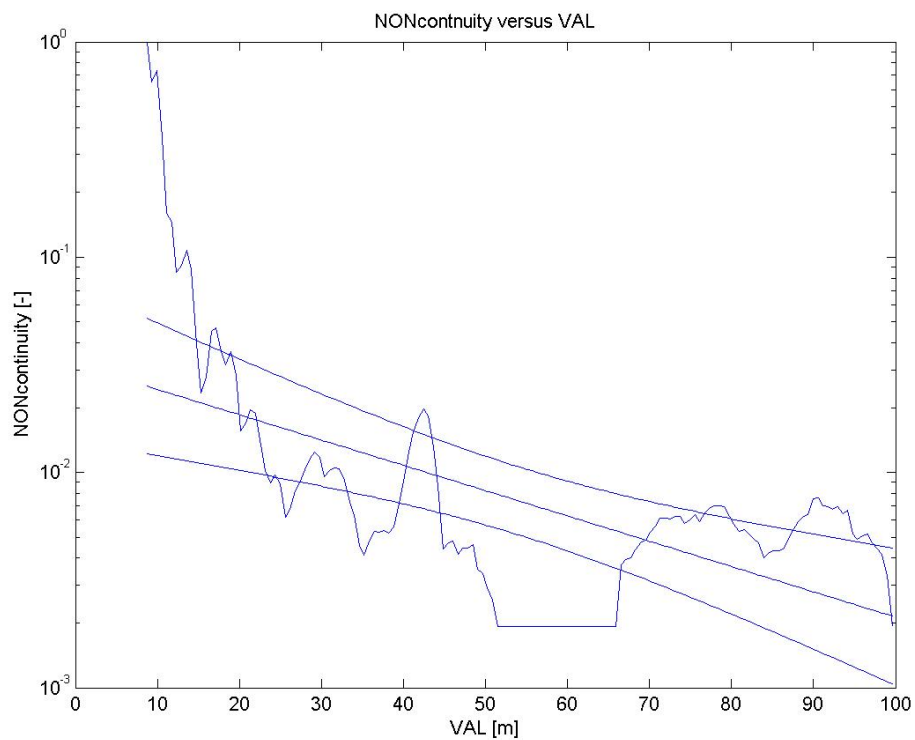


Figure 13: Lognormal distribution of the non-continuity as function of VAL including 5% confidence intervals.

The figures 14, 15 and 16 show the non-continuity versus VAL in the intervals as specified in the table. It is possible now to determine from these intervals the continuity risk for the APV1, APV2 and CAT1 services respectively.

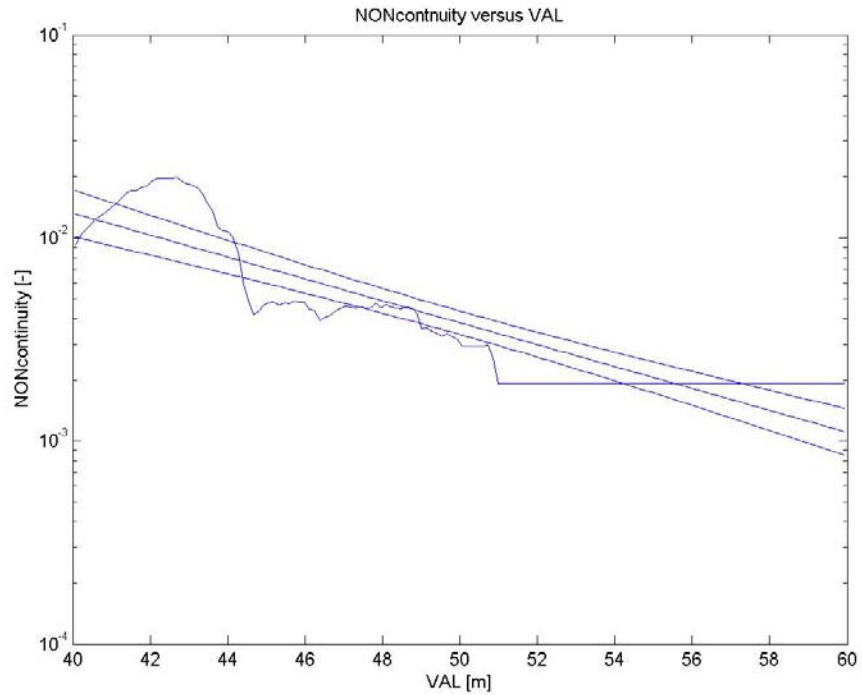


Figure 14: Lognormal distribution of the non-continuity as function of VAL from 40 to 60 m including 5% confidence intervals.

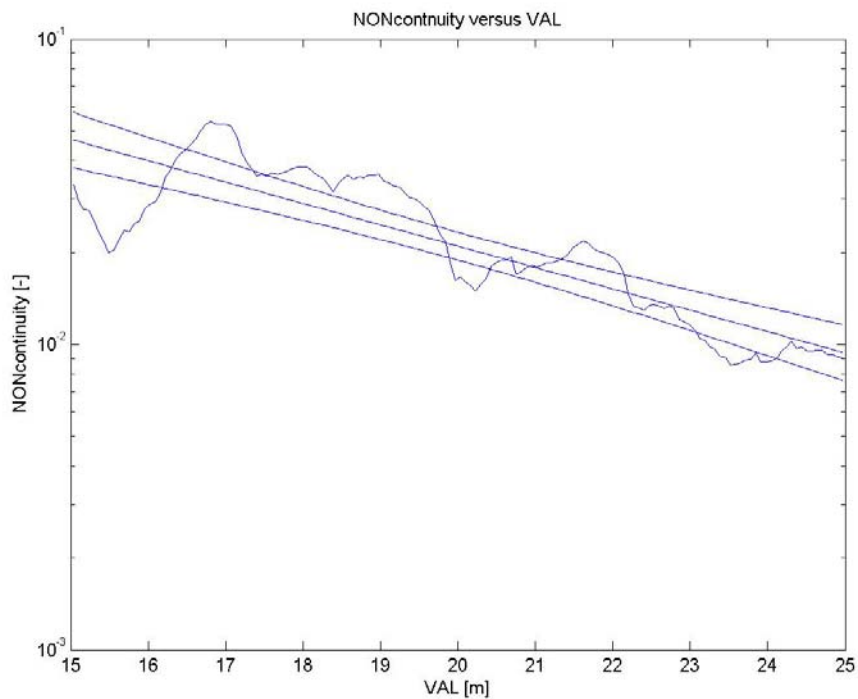


Figure 15: Lognormal distribution of the non-continuity as function of VAL from 15 to 25 m including 5% confidence intervals.



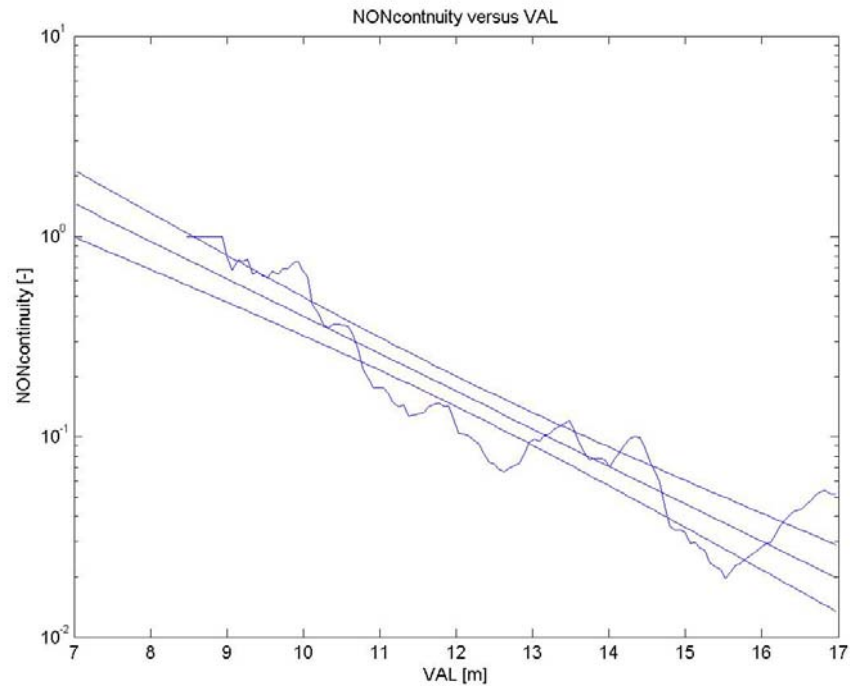


Figure 16: Lognormal distribution of the non-continuity as function of VAL from 7 to 17 m including 5% confidence intervals.

The actual receiver did not compute an estimate of the expected receiver performance during the operation at the start of that operation. The inclusion of such estimate may very well improve the continuity performance significantly.

## 2.2 Integrity, availability and continuity related to horizontal position errors

Figure 17 shows the time series of the Horizontal Position Error (HPE) and the Horizontal Protection Level (HPL).

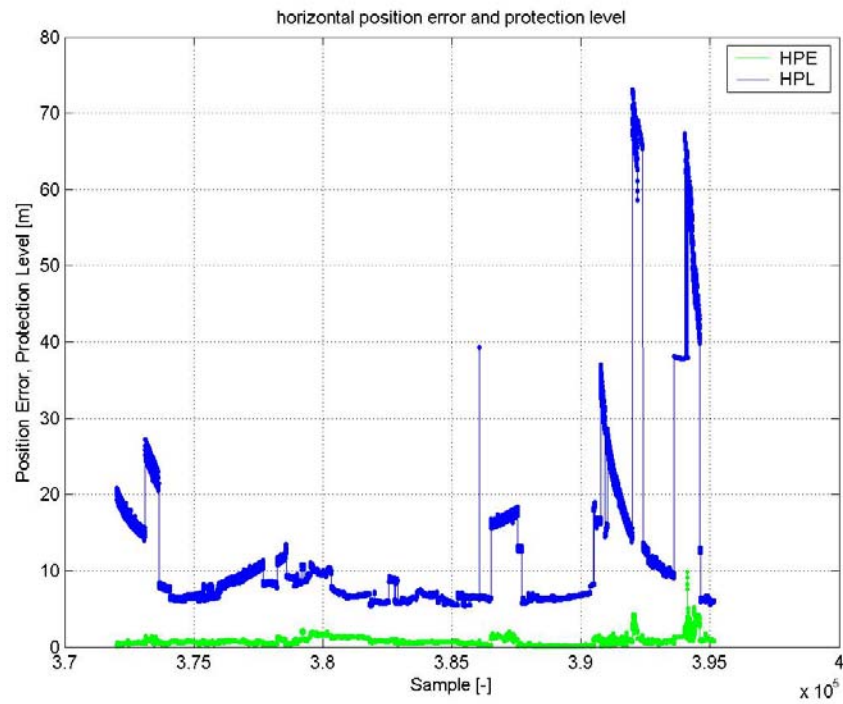
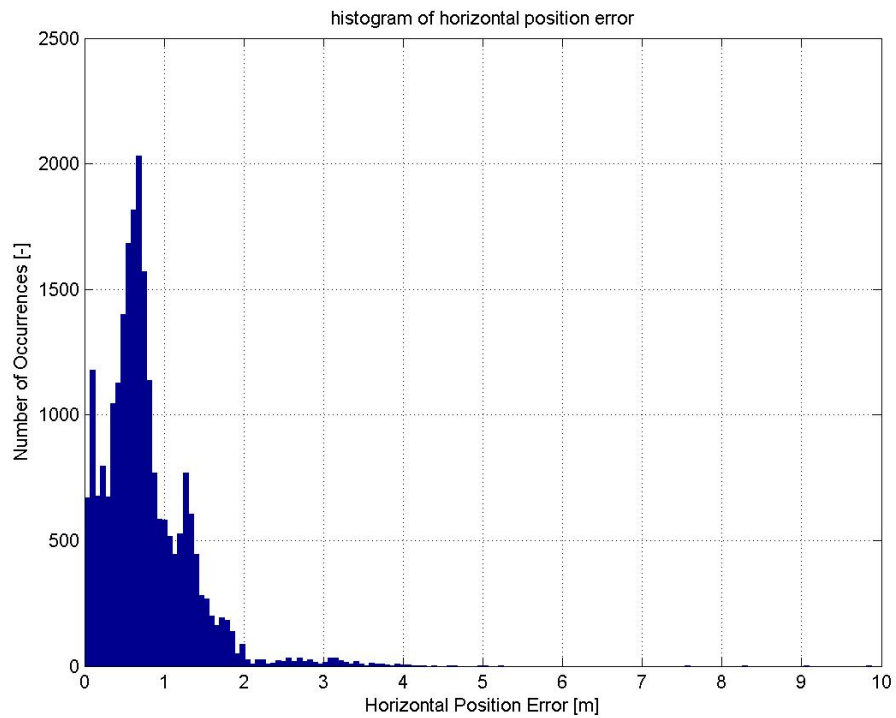


Figure 17: Time series of the horizontal position error and the horizontal protection level.

Figure 18 shows the histogram of the horizontal position error (HPE is the combined error in the latitude and longitude direction). The 95% percentile horizontal error is 1.7 m.



*Figure 18: Histogram of the horizontal position error.*

Based on the horizontal position error and the horizontal protection level it is possible to compute the MI probability.

### **2.2.1 Misleading Information probability computation**

Figure 19 shows a time series of the ratio between the horizontal position error and the horizontal protection level.

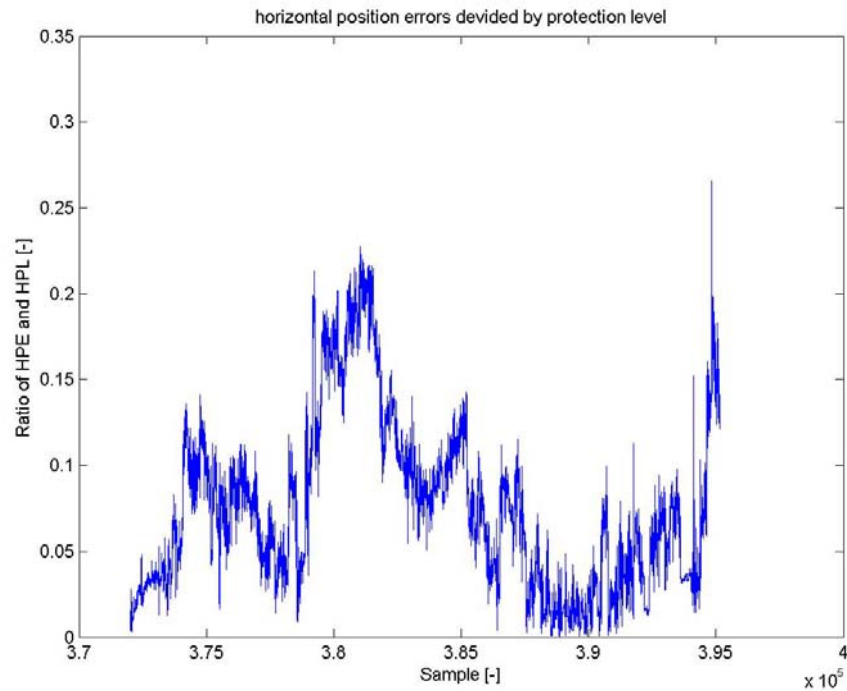
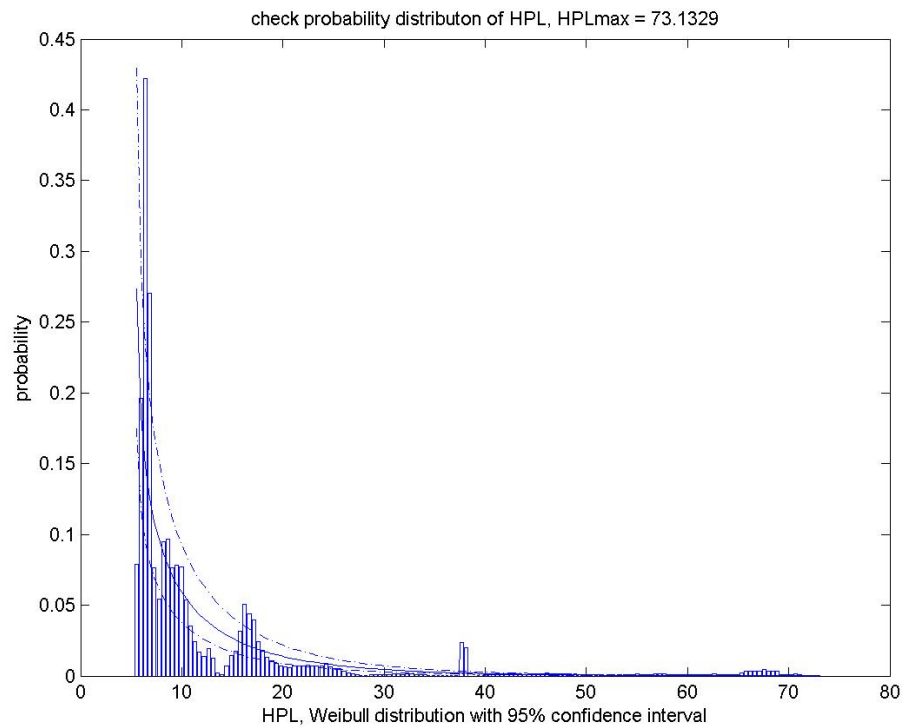


Figure 19: Ratio between the horizontal position error and the horizontal protection level.

Fitting the Rayleigh distribution function through these data (HPE/HPL), yields a Rayleigh standard deviation of 0.065. It is now possible to compute the MI probability; it is the chance that the horizontal position error is larger than the HPL and consequently the chance their ratio is larger than one. This probability turns out to be  $1.1 \times 10^{-52}$ . Taking into account a decorrelation time of 2000 seconds and a confidence level of 5%, the MI probability is  $2.4 \times 10^{-32}$ .

### 2.2.2 Availability computation

Figure 20 shows that the Weibull distribution function describes approximately the horizontal protection level distribution in the same way as for the VPL (see figure 7).



*Figure 20: Histogram of HPL with Weibull probability distribution including a confidence interval of 95%.*

It is possible now to compute the unavailability, being the probability that the HPL exceeds the Horizontal Alert Limit (HAL), using the Weibull distribution fit. For a HAL equals 40 m the unavailability turns out to be 2.0%. The Stanford plot (see figure 21) shows an unavailability equals 4.2%. For a confidence interval of 95%, the unavailability turns out to be 3.1%.

### 2.2.3 Integrity computation

Figure 21 shows the Stanford plot for the APV-II and CAT I services.

Figure 22 shows that for the APV-II and CAT-1 services the HMI risk is of  $2.9 \times 10^{-56}$  ( $4.5 \times 10^{-56}$  with confidence interval of 5%).

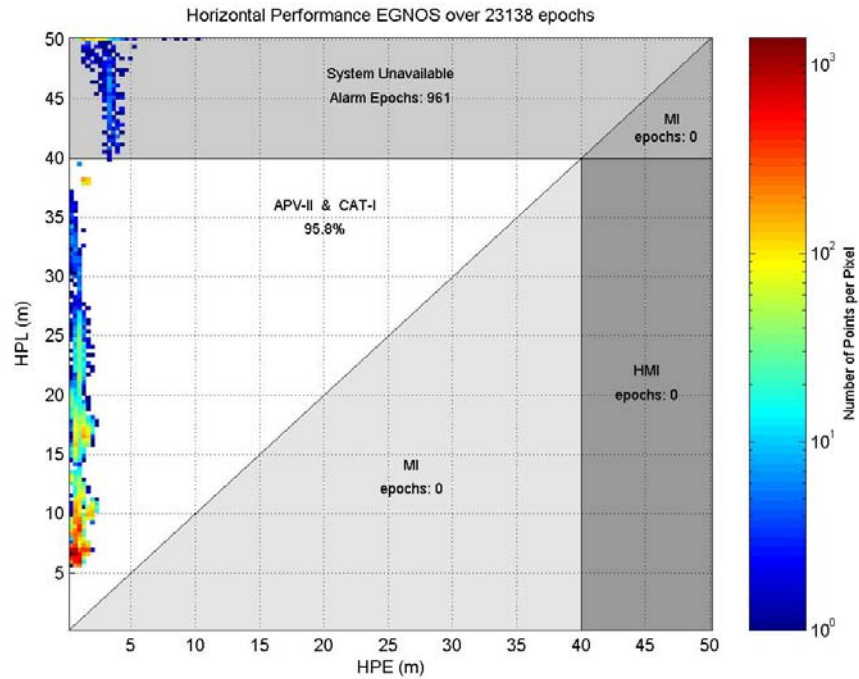


Figure 21: Stanford plot for the APV-II and CAT-I services.

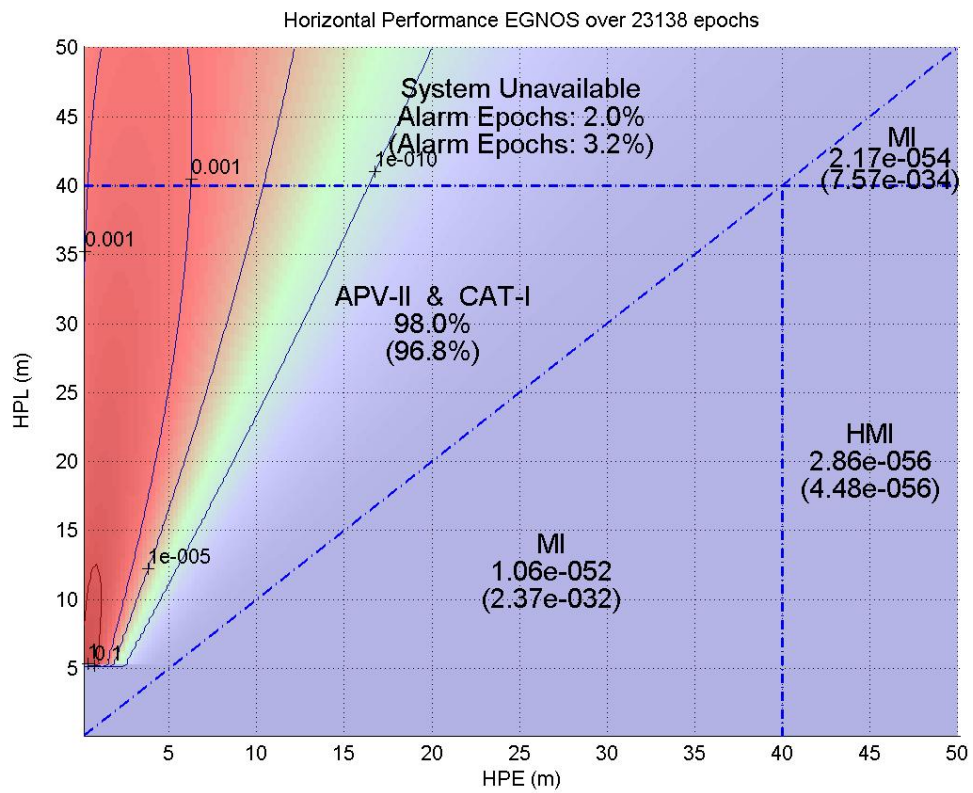


Figure 22: Availability, MI risks and HMI risk for the APV-II and CAT-I services (the values within brackets are 5% confidence values).

### 2.2.4 Continuity computation

Figure 23 shows the non-continuity versus the horizontal alert limit. The continuity risk is thus  $P_{\text{non-continuity}} = 0.0074$  for  $\text{HAL} = 40$  m (the APV-II and CAT-I service requirement is  $8 \cdot 10^{-6}$ ).

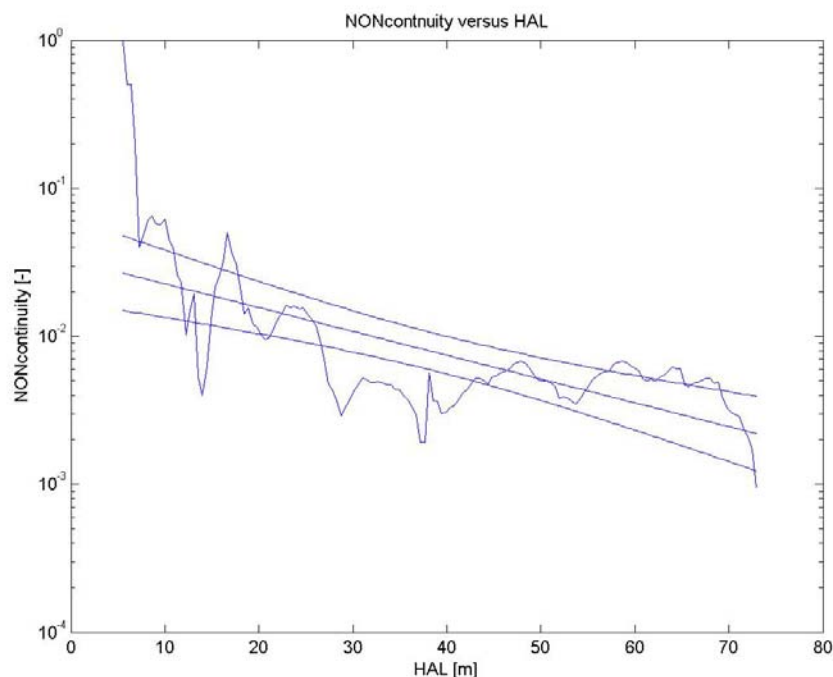


Figure 23: Lognormal distribution of the non-continuity as function of HAL including 5% confidence intervals.

## 3 Brief overview of test results

From the in the paper presented test results for integrity, availability and continuity, as they are related to the vertical as well as the horizontal position errors, the following can be concluded:

1. EGNOS did satisfy the requirements of the APV-I, APV-II and CAT-I services as far as the Hazardous Misleading Information is concerned,
2. EGNOS did not satisfy the requirements of the APV-I, APV-II and CAT-I services as far as availability and continuity is concerned.

#### Remark

EGNOS was still under development at the time, hence these conclusions are not valid for EGNOS when it becomes fully in operation.



## 4 Conclusions

This paper presents a test method to analyze receiver output data gathered during a test campaign of limited duration for SBAS. With this method, it is always possible to make an estimate of the integrity, availability and continuity. The method is applied to an example test case and turned out to be applicable indeed.

It is recommended to continue this investigation including following topics:

- Dependability of results on the data ensemble,
- Minimal required data period for sufficient confidence,
- Further investigation of the assumptions made in the method,
- Setting up, and compare with, alternative statistical approaches wherever possible.





## References

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