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National Aerospace Laboratory NLR

NLR-TP-2003-609

An alternative fully stochastic approach to determine the lifetime and inspection scheme of a component

SLAP (Stochastic Life APproach)

F.P. Grooteman

This report is based on a presentation held at the ASIP 2003 Conference, Savannah, Paper presented at the ASIP 2003 Conference in Savannah, Georgia, USA on 2-4 December 2003.

Customer:	National Aerospace Laboratory NLR
Working Plan number:	S.1.B.3
Owner:	National Aerospace Laboratory NLR
Division:	Structures and Materials
Distribution:	Unlimited
Classification title:	Unclassified
	December 2003

Approved by author:	Approved by project manager:	Approved by project managing department:
5/1	06/01-'04	21/1

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(34 pages in total)

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AN ALTERNATIVE FULLY STOCHASTIC APPROACH TO DETERMINE THE LIFETIME AND INSPECTION SCHEME OF A COMPONENT

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ABSTRACT

Currently, most aircraft components are designed according to two different philosophies: the *Safe-Life* or *Damage Tolerance* approach. Both concepts cover a different part of the lifetime and are based on so-called deterministic models, in which the model parameters are constants (single-valued). In order to compensate for neglecting the natural variability of the model parameters (e.g. scatter in material parameters) and other uncertainties, scatter and safety factors are applied explicitly and implicitly (e.g. by means of an assumed initial crack length). The results obtained with both approaches can be very conservative, although the safety level (reliability) of the design remains unknown.

Another better way of dealing with this variability of the model parameters is by means of a stochastic analysis, adding an extra dimension to the deterministic analysis, by introducing a range of values that can occur with their chance on occurrence. However, performing a stochastic Damage Tolerance or Durability analysis does not make much sense, since the most important stochastic parameter, initial crack length distribution, is unknown.

In this paper, an alternative life philosophy named ***SLAP*** (*S*tochastic *L*ife *AP*proach) will be presented by which the lifetime and inspection scheme of a component can be determined in a fully stochastic manner, covering the crack initiation period as well as the crack growth period in a realistic way. The approach can serve as an alternative for the current approaches, especially the Safe-Life and Damage Tolerance approaches, resulting in more realistic predictions of the lifetime and inspection scheme.

The method is demonstrated by a realistic example based on in-service inspection data gathered for the Upper Longeron of the F-16.

1 INTRODUCTION

Before introducing the SLAP philosophy in the next chapter, first some background information is provided describing the current two lifing philosophies as applied for military aircraft and subsequently some issues related to deterministic and stochastic analysis in general.

Airworthiness regulations require proof that aircraft can be operated safely. This implies that critical components must be replaced or repaired before safe operation can no longer be guaranteed. Different approaches can be followed to prove that a component is safe. For military aircraft the approach followed depends on the customer requirements, the type of component and the possibilities for inspection during service. The fatigue philosophies underlying the approaches for guaranteeing safety are called Safe-Life and Damage Tolerance.

The Safe-Life philosophy is based on the concept that significant damage, i.e. fatigue cracking, will not develop during the service life of a component. The actual life for which this is "true" is calculated and then checked by a suitable test programme. Then the design safe-life is obtained by factoring the determined life by an appropriate safety factor. When the service life equals the design safe-life the component has to be replaced. The US Navy and US Army, for example, apply this philosophy. However, there is a major drawback, since components are taken out of service when they may still have substantial remaining lives. Also, despite all precautions, it is still possible for cracks to occur prematurely, and this fact has led the US Air Force to introduce the Damage Tolerance philosophy.

The Damage Tolerance philosophy not only recognises that damage can occur and develop during the service life of a component, but it also stipulates that the possibility of cracks or flaws in a new structure should be accounted for. Safety is incorporated into this approach by the requirements either that (1) any damage be detected by routine inspection before it results in a dangerous reduction of the static strength (inspectable components), or (2) the damage shall not grow to a dangerous size during the service life (non-inspectable components).

Two requirements are necessary for this approach to be successful. First, it must be possible to define either a minimum crack length a_d that will not go undetected during routine inspections, or else an initial crack length a_i based on pre-service inspection. Second, one should be able to predict the growth of such cracks during the time until the next inspection or until the design service life is reached. The result of a Damage Tolerance analysis is a curve presenting the crack length as function of the number of cycles, starting from the initial crack length a_i up to the critical crack length a_{cr}. (When a component is not safety-critical a similar analysis can be done with a significantly smaller initial crack length. This is called a Durability analysis, which is directed to assessing the economic life of a component. The US Air Force makes use of both Damage Tolerance and Durability analyses.)

From fracture surface investigations it is known that for virgin undamaged materials a considerable time is spend to initiate a crack. After initiation the crack will grow till failure. The ratio between crack initiation life and crack growth life depends on the geometry and loading. Material property data used in a Safe-Life analysis is often obtained from tests on narrow specimen. Such specimens fail shortly after a crack initiates. This implies that for a Safe-Life analysis the crack growth part of the life is ignored. The Damage Tolerance philosophy on the other hand starts with a crack of minimal detectable size therefore ignoring the crack initiation life. This notion encouraged civil aircraft manufactures to combine both approaches. They often use the Safe-Life approach (with the proper material property data) to determine the crack initiation life and from that the initial inspection. The Damage Tolerance approach is used to determine the repeat inspection intervals.

In chapter 2 an alternative approach will be presented. This approach includes both the initiation and crack growth life simultaneously and in a stochastic way. A first example in which this approach is applied to a structural component of the F-16 will be presented in chapter 3.
What follows first are some general remarks and considerations related to deterministic and stochastic analysis in general.

1.1 Deterministic Analysis

The above analyses are based on a so-called deterministic analysis. This means that the variability of the parameters used in the model, introducing variability in the results of the model, is not taken into account. In order to account for the variability in material properties, manufacturing quality, et cetera: scatter and safety factors are introduced (e.g. Ref. 1). For material parameters often conservative lower bound values are used, such as the A- and B-value,

which represent the 95 % confidence lower limit on the first, respectively, tenth percentile of their distribution (Ref. 2). Furthermore, often a scatter factor of 1.5 is put on the limit load. Finally, often a safety factor of two to four is set on the obtained lifetime.
In general the result will be a very conservative life estimate and inspection scheme. The reliability of the structural design, i.e. one minus the probability that the structure will fail, is unknown. Moreover, the safety factors applied (e.g. 1.5) are quite arbitrary (Ref. 3), although historically successful, probably caused by the high degree of conservatism.

Elements of uncertainty are inherent in almost any engineering system. Therefore, these systems will always behave to a certain extent in an unpredictable way, no matter how much is known about the systems behaviour. It is known in general that absolute safe systems do not exist, since even the safest systems sometimes fail unexpectedly. Moreover, the demand for lighter more economical structures forces an ongoing optimisation of engineering structures. Also the introduction of new materials (e.g. composites, fibre metal laminates) and design methodologies might require other safety factors than the historically applied ones. Applying the same safety factors as before will therefore in general not result in a similar safety level and could lead to unsafe designs.

Another related issue is the ongoing increase in computer power, which allows engineers to further expand their computational models by introducing more and more structural details, believing that this will improve the accuracy and reliability of the design. However, this will only be the case to a certain extent. Errors (e.g. modelling errors) remain or are even introduced (the chance on input errors increases with complexity of the numerical model). Besides this, often only a small part of the generated data is used, debating the usefulness of this approach. More useful would be to use a less detailed, but correct, model and use the computing power for a stochastic (reliability) analysis on top of this model. This is further discussed in the next section. For this purpose powerful stochastic methods have been developed (e.g. Ref. 4) in the last decades.

A more worrying fact is that application of safety factors afterwards might even lead to unconservative results. For example taking into account scatter in non-linear systems during the analysis stage can have a considerable effect on the outcome of the model. The sensitivity of such systems for even slight changes in the input parameters is a well-known phenomenon. A correction afterwards on a deterministically obtained result might therefore result in an unsafe design. Non-linear models are more and more applied in the design stage of structures, due to increased computational power and availability of these numerical tools.

Once a certain deterministic analysis has been performed the results often are compared with experimental data. However, the outcome is questionable, certainly in cases with substantial scatter and limited experimental results. In these cases correlation between the numerical and experimental results can be pure coincidence. The only correct approach is to take into account the scatter in the analysis by performing a stochastic analysis and compare the results with experimentally obtained ones, schematised in figure 1.1. The conclusion that the numerical result (denoted by a 90 % scatter band) correlate with the experimental ones (denoted by circles) is valid and gives real confidence in the numerical analysis. Moreover, extra information can be obtained, such as: the safety level of the design (reliability) and the directions to most efficiently modify the design in order to increase reliability.

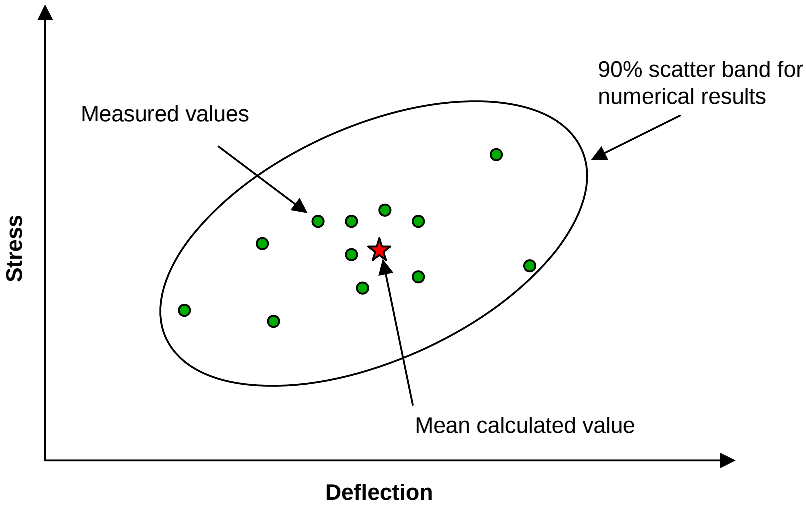

Figure 1.1 Schematised ideal relation between the scatter found in measurements and analysis

Another important point to mention is that stochastic methods can not be used to improve the deterministic model. If the outcome of the deterministic model for all mean parameter values (denoted by a star) is not close enough to the mean of the experimental results, the conclusion can be drawn that the deterministic model is lacking some physical phenomenon and therefore is not reliable in predicting reality.

1.2 Stochastic Analysis

Bearing in mind the reasons mentioned in the previous section, it becomes increasingly important to replace the deterministic by a stochastic approach, including the variability of the model parameters in a 'natural way'.
Good stochastic tools are available nowadays. What basically is lacking is the proper knowledge, familiarity with the subject, having enough confidence in the approach to apply them, and to a certain extent the will to change the current deterministic design philosophy (why change something what has proven itself to work). Adopting a new design philosophy requires an open mind to begin with.
If the above conditions are fulfilled application of a stochastic design philosophy can become common practice. Computing power is often no longer an issue and good stochastic tools are available. The stochastic tool functions as a shell on top of any existing deterministic tool or can be integrated with it. The latter, for example stochastic FEM, is not attractive here since it requires a rewrite of the deterministic code.

Another argument not to use stochastic tools is the lack of sufficient data to generate proper distribution functions. This can certainly be a problem, however, with available data and some engineering judgement, in many of these cases a useful stochastic analysis can be made providing all the advantages discussed above. For example, the distribution type can often be allocated based on physics, together with a mean value and an upper bound of the coefficient of variation. In this way a lower bound of the distribution can be fully characterised. Nevertheless, experimental programs should be set-up to collect sufficient statistical data, which is not common practice at the moment.

In the field of fatigue analysis a vast amount of tests have been performed and are still performed, generating huge amounts of data. In this field stochastic methods already have been applied for a while. Due to, amongst others, the large amount of scatter seen in fatigue experiments. Mostly the Monte-Carlo method is applied due to its simplicity and versatility, but much more efficient and accurate methods exist, e.g. Ref. 4.

The following steps can be distinguished in a stochastic analysis:
- Choice of random variables and their distribution functions
- Choice of failure function
- Solution of the stochastic problem, requiring a stochastic method
- Interpretation of the results

These steps will not be detailed here, see for example reference 1, but roughly outlined.

Based on available data, goodness-of-fit tests and engineering judgement, distribution functions can be assigned to every model parameter (excluding non-physical parameters, such as tuning parameters necessary for the correct operation of numerical algorithms). By means of a **sensitivity analysis** it can be determined whether a model parameter or combination of parameters should be treated as stochastic (random) variable(s). That is: is the scatter in these parameters causing any significant scatter in the results of interest? If not so, these variables can be treated deterministically.

The choice of failure function(s) is in most cases straightforward, although a more or less continuous behaviour of this function in the stochastic domain is preferred.

There exist numerous numerical techniques to solve the stochastic problem (e.g. Ref. 4). The most simple and well-known method is the Monte Carlo method, but is not very efficient especially when dealing with smaller probabilities of failure ($< 10^{-3}$), which is the case for engineering structures. More efficient methods, such as FORM, SORM and Importance Sampling methods, have been developed in the past decades. There exist commercial tools (FPI, ST-ORM) or in-house developed tools having implemented several of these methods. At NLR a stochastic tool has been implemented over the past six years called RAP (Reliability Analysis Program, Ref. 5) including methods as, FORM, SORM and several Importance Sampling methods: radial based Monte-Carlo, Latin-Hypercube, Directional sampling, Adaptive Directional Importance Sampling (ADIS). These stochastic tools operate on top of any deterministic tool and need no modifications to the deterministic tool what-so-ever to operate. An interface between the stochastic and deterministic tool is provided by the stochastic tool. The extra input compared to the deterministic analysis, consists of specification of the random variables and their distribution functions, specification of the failure function(s) and selection of the stochastic method to be applied.

An important issue in interpretation of the results obtained is determining the allowed probability of failure. Table 1.1 (Ref. 6) depicts some target probability values for lifetimes, based on the relative costs of safety measures against the consequences of failure. In general, the target lifetime probability of failure will be in the order of 10^{-3} for engineering problems.

Table 1.1 Lifetime probability of failure targets according to ISO 2394, Ref. 6

Relative costs of safety measures	**Consequences of failure**			
	Small	Some	Moderate	Great
High	0	10^{-1}	10^{-2}	10^{-3}
Moderate	10^{-1}	10^{-2}	10^{-3}	10^{-4}
Low	10^{-2}	10^{-3}	10^{-4}	10^{-5}

According to reference 3 this probability of failure should be of the order 10^{-7} per flight hour for aircraft structures, i.e. approximately 10^{-3} for the lifetime (assuming 10^4 flight hours during a lifetime).

As mentioned before it is a prerequisite that the underlying deterministic model is a good representation of reality, not lacking important phenomena.

2 ALTERNATIVE *S*TOCHASTIC *L*IFE *AP*PROACH (SLAP)

The Damage Tolerance approach has an inherent strange assumption in it that leads to very conservative results. The initial damage size is based on the *Probability Of Detection* (**POD**) of the inspection method. In other words the inspection method determines the initial crack length and not the damage that actually may be present in the structural component. This observation has lead to the concept of *Equivalent Initial Flaw Size* (**EIFS**). The critical crack length and life of the component are used to determine the crack length at time zero (start of functional life) by means of a backward crack growth calculation. This approach leads to an equivalent initial flaw size, which is not the true flaw size as the name already indicates, due to the fact that the crack growth model used is not valid anymore before the long crack length region. Performing this backward crack growth calculation for many crack length/life combinations found in similar components results in an EIFS distribution. The only way the EIFS concept can be used is by assuming that the EIFS distribution obtained is representative for the structural detail under design, which often is more complex.

The idea of a backward crack growth analysis can be used in another way. In the figures 2.1 to 2.5 an alternative fully stochastic approach called **SLAP** (Stochastic Life APproach) is presented by which the lifetime and inspection scheme of a structural component can be determined, covering the crack initiation as well as the crack growth period in a much more realistic way. The approach consists of the following three steps:

1. Construct the failure distribution
2. Backward crack growth analyses
3. Forward crack growth analyses including inspections

The basic principle related to these steps will be discussed underneath. In chapter 3 more details will be presented, discussing the approach for a realistic application.

Step 1: Construct the failure distribution
First, the failure distribution has to be obtained, e.g. by means of a Weibull analysis. This failure distribution should be a conservative estimate (lower bound) based on a limited set of experiments and will be updated during the lifetime of the component with in-service data (failure and non-failure data) that become available, to enhance safety. This will be discussed in more detail in chapter 3 and appendix A.

An important notion is that all the scatter introduced by material properties, load spectrum, et cetera is included in this one failure distribution and therefore need not to be characterised separately. Moreover, the scatter present in the component can be updated easily afterwards in this way. This is a very important advantage over using the scatter in all the model parameters (material properties, loads, etc.), since these data values are often hard to acquire and can often not be acquired afterwards. Furthermore, a limited number of random variables is a very attractive concept especially for use in industry.

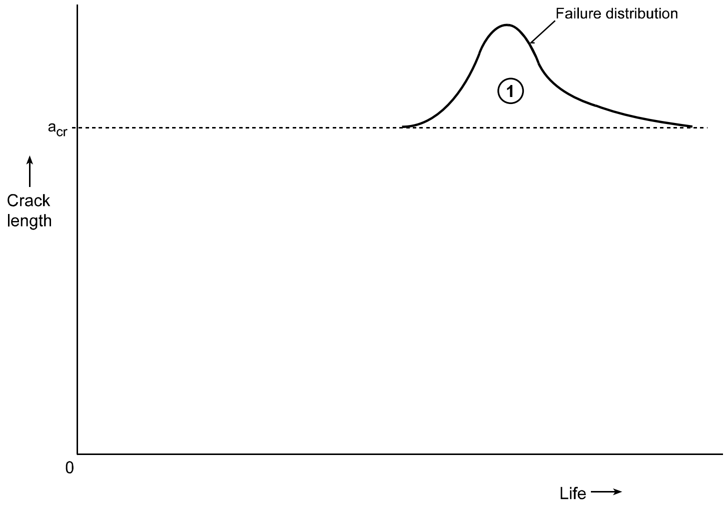

Figure 2.1 Step 1 of the SLAP philosophy

Step 2: Backward crack growth analyses*: Determine initial inspection time and corresponding crack length distribution*

a) Secondly, a backward calculation is performed starting from the failure distribution of the component. However, this backward calculation is not proceeded until the time zero as in the EIFS approach, but until a detectable crack length (a_{det} in figure 2.2) has been reached, determined by the inspection method used (comparable with the Damage Tolerance philosophy). The resulting distribution (*PDF*-a_{det} in figure 2.2) corresponds to an estimate of the distribution describing the time it takes before a crack of length a_{det} is present in a certain percentage of the components.

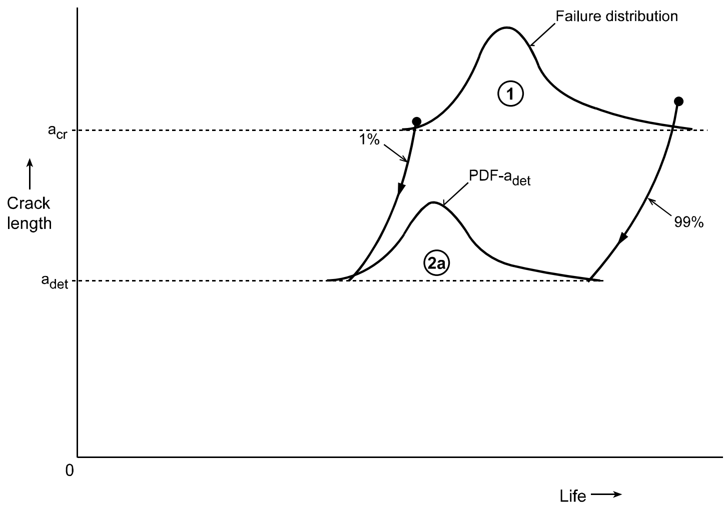

Figure 2.2 Step 2a of the SLAP philosophy

b) When a certain threshold percentage (p_{th}, e.g. 1 %, Fig. 2.3) of these detectable cracks are present, the initial (threshold) inspection becomes opportune. An earlier inspection does not make much sense, since cracks that exist are hard to find anyway with a reasonable chance on detection. In this way the time for the initial inspection $t_{initial}$ can be obtained in a realistic conservative way, covering the crack initiation, micro and short crack period, without the need to explicitly model these phenomena. Thus, not requiring micro-crack and initiation models, for which good models are "currently" lacking. Even short crack models are not required, since the detectable crack length will be a long crack (> 1 mm) in general for the inspection techniques in use today.

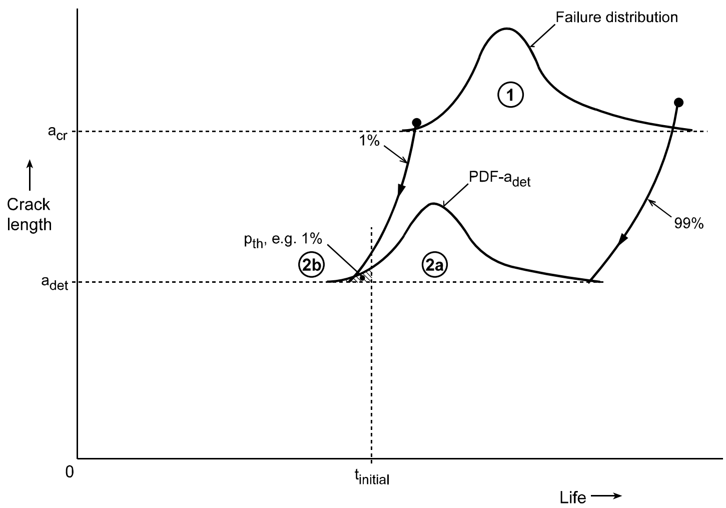

Figure 2.3 Step 2b of the SLAP philosophy

c) Next, the crack length distribution function (*PDF-$t_{initial}$* in figure 2.4) at $t_{initial}$ can be obtained simultaneously by extending the back crack growth analysis a bit further. Also now there is no need for initiation, micro and short crack models. In some cases this latter might not be completely true for the lower tail of this distribution, however, this part of the PDF will not contribute to the probability of failure (discussed in the next step) and is therefore irrelevant here. The above will be demonstrated and more detailed in the example given in chapter 3.

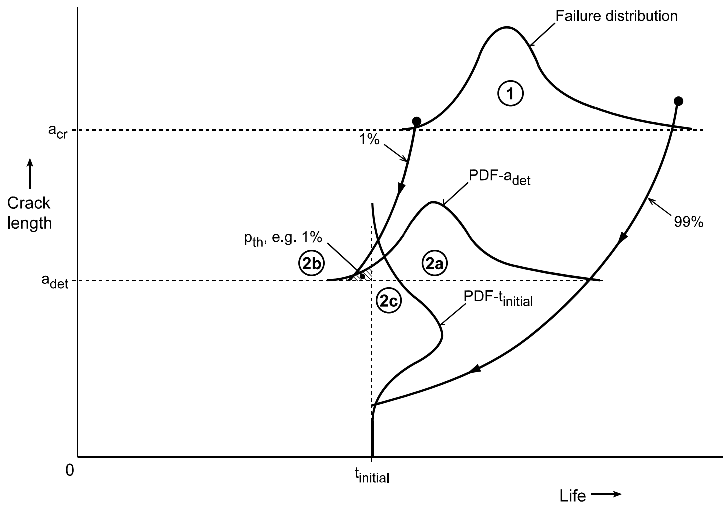

Figure 2.4 Step 2c of the SLAP philosophy

Step 3: Forward crack growth analyses including inspections: *Determine repeat inspections*
In the third and last step a stochastic upward crack growth analysis is performed, starting from the time $t_{initial}$ and the crack distribution (*PDF-$t_{initial}$* in figure 2.5), in which a certain repeat inspection scheme (denoted by the crosses in the figure) is simulated by means of the Probability Of Detection (POD) function corresponding to the inspection method applied.

Once a crack has been found in the numerical simulation of the crack growth, the component is being replaced (or repaired) and the new component will have again a period $t_{initial}$ in which a new crack will initiate and grow. From this time again a crack growth analysis can be performed using a new crack length value drawn from the crack length distribution function (*PDF-* $t_{initial}$). Often it can be assumed that a replaced or repaired component will survive until the economical life of the overall structure is reached, since this will normally be less than two times the crack initiation period.

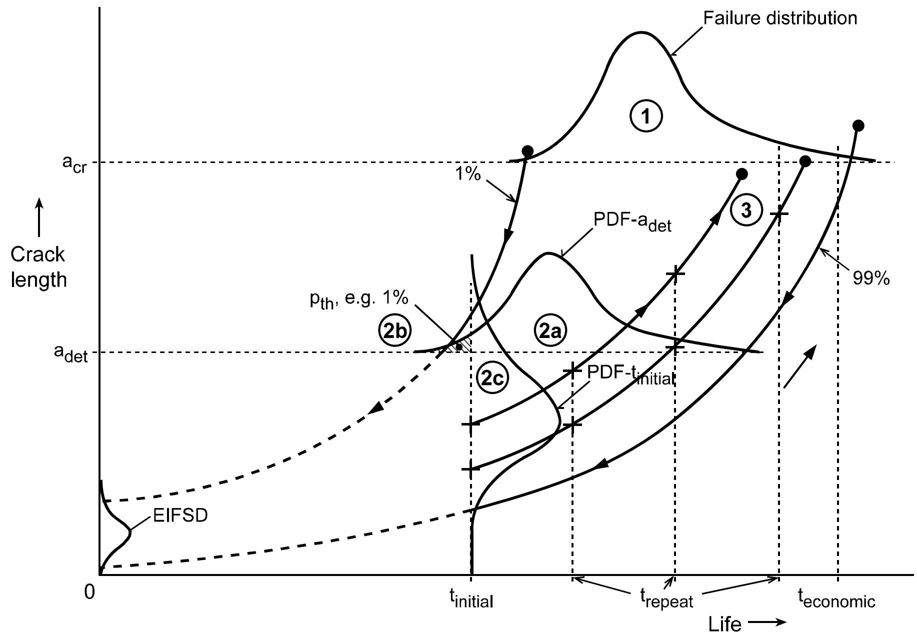

Figure 2.5 Schematised overview of the SLAP philosophy

The crack growth calculation stops when the component has failed, when cracks have been missed in all inspections, or when the economic lifetime of the component has been reached, depicted by $t_{economic}$ in the figure 2.5.

Performing many of such crack growth simulations (with the number of simulations depending on the stochastic method applied) finally results in a *Probability Of Failure* (**POF**) value. This POF value can be obtained for different repeat inspection schemes till a required safety level is obtained. For a given inspection method and inspection scheme, the upward stochastic crack growth analysis thus results in a probability of failure of the component. Inversely, for a required safety level a repeat inspection scheme can thus be determined in an iterative manner. This inspection scheme can consist of constant repeat inspection intervals or variable intervals. In this way an optimal inspection scheme can be determined.

With this approach the lifetime and inspection scheme can be determined in a fully stochastic way, covering the crack initiation and crack growth period, to ensure the safe operation of the component reflected by an upper bound of the probability of failure and the inspection scheme applied. The approach serves as such as an alternative for the deterministic Safe-Life and Damage Tolerance approach.

The backward and upward crack growth calculation is done deterministically, because all the variability is already included in the failure distribution. In this way the upward crack growth calculation leads to the same failure distribution as originally started with, which is a prerequisite.

3 FIRST APPLICATION

3.1 Introduction

As a first application, the approach presented in chapter 2 will be applied to the Upper Longeron of the F-16, having part number 16B5120. The longeron is a tee-extrusion machined from 2024-T62 aluminium and its function is to distribute flight loads from the fuselage upper skin to the center fuselage structure. High positive g-loads may cause fatigue cracking in the tab radii of the longeron, see figure 3.1. This longeron is one of the airframe inspection points (point 3005) within the F-16 Aircraft Structural Integrity Program (**ASIP**). When cracks are detected during an inspection the information obtained about crack lengths are registered in the Core Automated Maintenance System (**CAMS**) database. For this particular point a lot of inspection data is available and thus serves as a good initial starting point to demonstrate the alternative stochastic life approach. Table 3.1 depicts the cracks found during subsequent inspections (boldfaced) for different longerons.

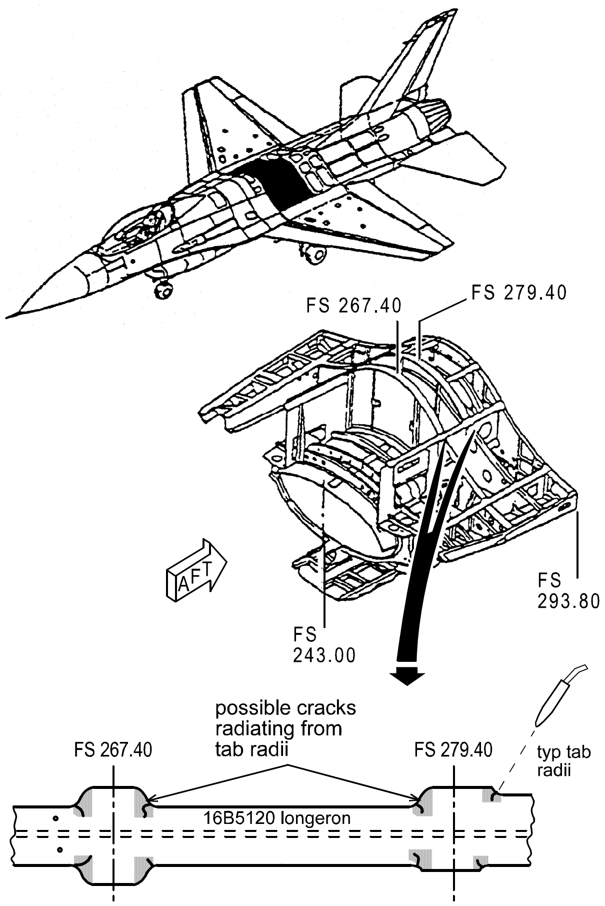

Figure 3.1 F-16 ASIP 3005 inspection point. Manual eddy current inspection of the tab radii in the center fuselage longeron (Figure from Ref. 8)

3.2 Conventional deterministic analysis

Before presenting the results of the alternative stochastic approach, first the inspection scheme will be presented resulting from the conventional deterministic approach. The normal procedure to determine the initial and repeat inspection scheme is depicted in figure 3.2. A crack growth curve is required, which is determined with the crack growth model as described in the Durability and Damage Tolerance Analyses (**DADTA**) handbook of Lockheed Martin (Ref. 8).

This crack growth model has been correlated by Lockheed Martin on data gathered on a test aircraft (Ref. 8) and therefore represents a realistic crack growth model for this component. The load spectrum applied here is a measured load spectrum representative for the average usage of the Royal Netherlands Air Force (**RNLAF**), denoted here as the baseline load spectrum.
Since the longeron is not a safety-of-flight structure a Durability analysis, instead of a Damage Tolerance analysis, has been performed to determine the inspection scheme. Therefore, the starting crack length was selected to be a corner crack with a length of 0.007 inch in both directions, instead of the 90/95 % detectable crack length a_d. Functional impairment has been defined (Ref. 8) to occur at a crack length of 0.187 inch. The resulting baseline crack growth curve is presented in figure 3.3. Application of the procedure schematised in figure 3.2 to the crack growth curve depicted in figure 3.3 leads to the following inspection scheme:

Initial inspection	**2655**	**Flight hours**
Repeat inspection	**62**	**Flight hours**

It is assumed here that the reliably detectable crack length a_d equals 0.1 inch, which is regarded a safe value for the manual eddy current inspection technique used to inspect this location. In the next section a value will be determined for the detectable crack length based on inspection data, altering the above inspection scheme.

Table 3.1 Available CAMS field inspection data (boldfaced numbers) of the tab radii of the F-16 center fuselage longeron. Normal faced numbers are estimated values (see section 3.3). Crack lengths are in inches and lifetimes are in flight hours (see section 3.4).

Longeron	Phased inspection times (Flight Hours; inches)									Failure time
	1200	1400	1600	1800	2000	2200	2400	2600	2800	
1	0.023	0.026	0.029	0.034	0.040	**0.049**				2741
2	0.018	0.019	0.021	0.024	0.027	**0.030**				3194
3	0.024	0.027	0.030	0.035	0.041	**0.050**				2736
4	0.019	0.020	0.023	0.026	0.028	0.033	0.038	**0.047**		3189
5	0.019	0.021	0.024	0.027	**0.030**					2994
6	0.021	0.024	0.026	0.029	0.034	0.040	**0.050**			2920
7	0.019	0.021	0.024	0.027	**0.030**					2948
8	0.019	0.021	0.024	0.027	**0.030**					2957
9	0.019	0.020	0.023	0.027	**0.030**					2931
10	0.018	0.019	0.021	0.024	0.027	**0.030**				3166
11	0.020	0.023	0.025	0.028	**0.030**					3234
12	0.020	0.022	0.025	0.027	**0.030**					3177
13	0.019	0.020	0.023	0.025	0.028	0.033	**0.039**			3140
14	0.018	0.019	0.021	0.023	0.026	0.029	0.034	**0.040**		3330
15	0.020	0.023	0.026	0.029	0.034	**0.040**				2880
16	0.021	0.023	0.025	0.028	**0.030**					3296
17	0.022	0.024	0.027	**0.030**						2878
18	0.021	0.024	0.027	**0.030**						2845
19	0.023	0.025	0.028	**0.030**						3049
20	0.020	0.022	0.024	0.027	**0.030**					3078
21	0.021	0.024	0.027	0.031	0.037	0.046	0.058	0.082	**0.236**	2764
22	0.020	0.022	0.025	0.027	0.030	0.036	0.043	0.053	**0.070**	3074
23	0.019	0.020	0.023	0.026	0.029	0.034	**0.040**			3087
24	0.023	0.026	0.028	0.032	0.038	0.045	0.054	**0.070**		2896

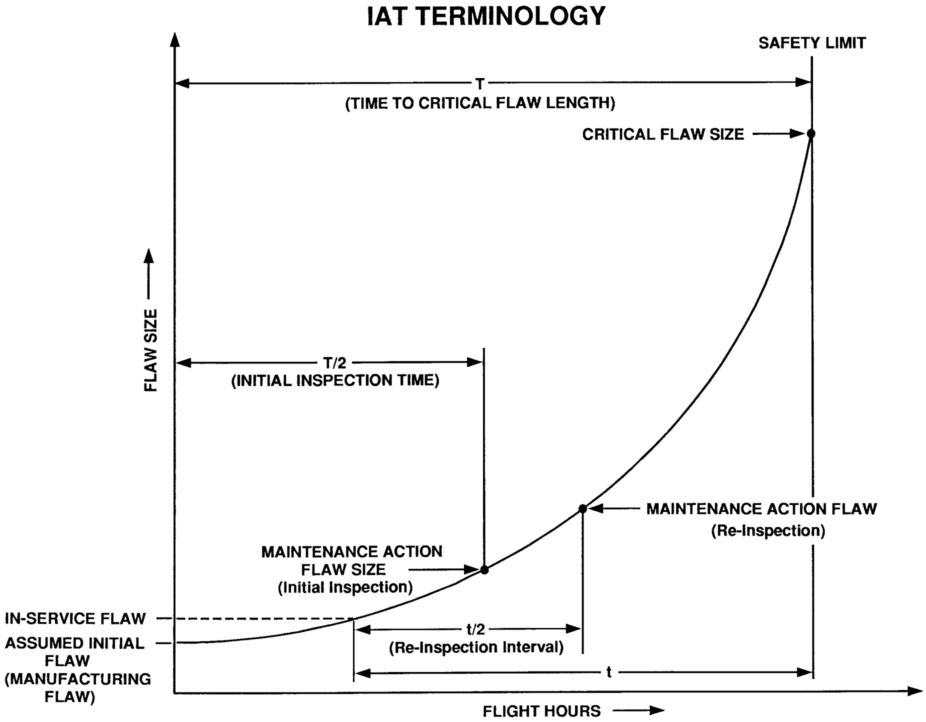

Figure 3.2 Schematic procedure to determine the initial and repeat inspection interval

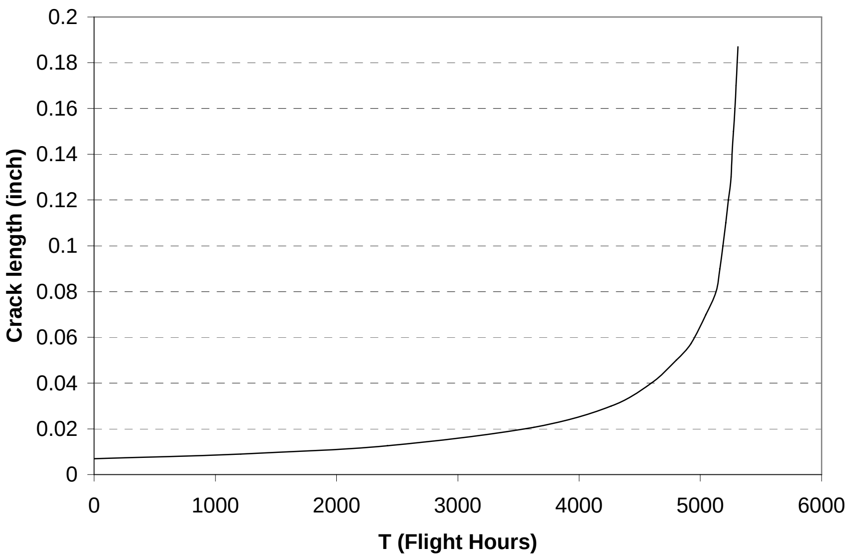

Figure 3.3 Mean crack growth curve for F-16 16B5120 center fuselage longeron representative for the average usage of RNLAF

3.3 Probability of detection

Non-destructive inspection (**NDI**) is an integral part of aircraft maintenance. The inspection method used to inspect this ASIP point is a manual eddy current inspection technique using a standard eddy current phase-analysis instrument and a 50-200 kHz shielded pencil-probe. A value of 0.1 inch was set as the reliably detectable crack length a_d for this ASIP point.

If sufficient hit/miss field data is available a Probability Of Detection (**POD**) curve can be constructed, representing a cumulative distribution function reflecting the chance on detecting a crack of certain length, see for example figure 3.5. With the detected (hit) and undetected (miss) crack length data, presented in table 3.1, enough data was available to construct a reliable POD function.
The most appropriate POD models have been evaluated in reference 9 and it was concluded that the log-normal distribution function provides the most realistic POD curve. The log-normal distribution has also been recommended by AGARD (Ref. 10). The parameters of the log-normal distribution (mean and variance) are determined by means of the Maximum Likelihood Estimators (**MLE**) method, see appendix A.2, with $F(t_i)$ equal to P_i representing the probability of detecting a crack of length a_i. Thus (1- P_i) the probability of not detecting a crack of length a_i, n is the number of hits and N-n the number of misses.

Since the detected crack length and the number of flight hours at detection are known, an estimate can be obtained of the missed crack lengths at the previous inspections, using the baseline crack growth curve presented in the previous section. Since the load spectrum of each individual aircraft is tracked, resulting in a CSI value, this information can be used to improve the individual crack growth curves. The CSI value is a measure of the severity of the load spectrum (Ref. 11) seen by the aircraft. Figure 3.4 depicts the scatter found in the load spectrum. The data follow a normal distribution.

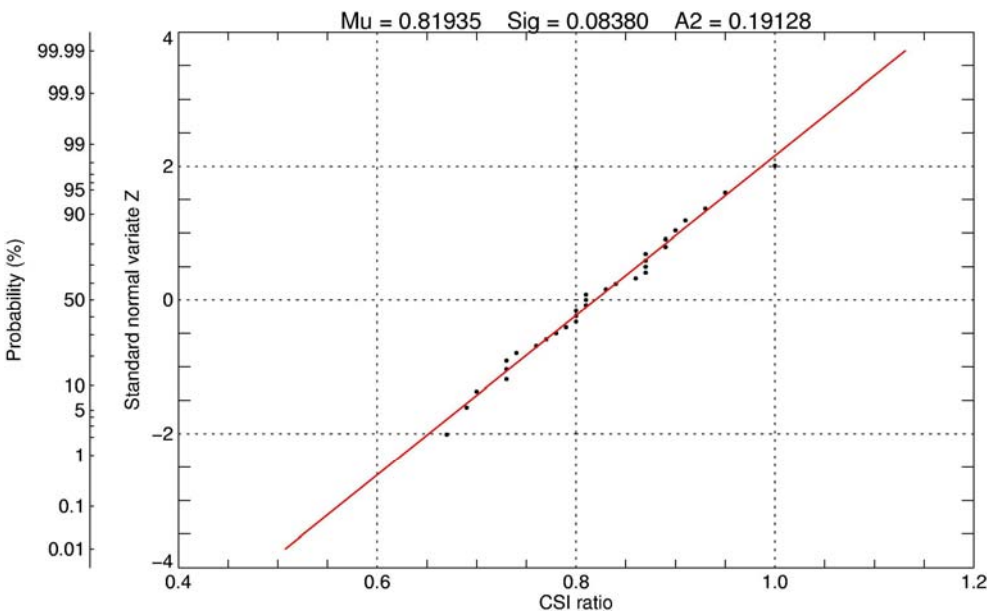

Figure 3.4 Normal distribution of the CSI ratio (CSI value divided by the baseline CSI value) on normal probability paper

The crack lengths thus obtained are also given in table 3.1 (normal faced). The total data set consisted of 24 hits and 121 misses and resulted in a MLE fit having the following parameter values of the log-normal POD function:

μ = 0.0528 inch
σ = 0.0254 inch

The corresponding cumulative probability plot for the mean POD is depicted in figure 3.5.

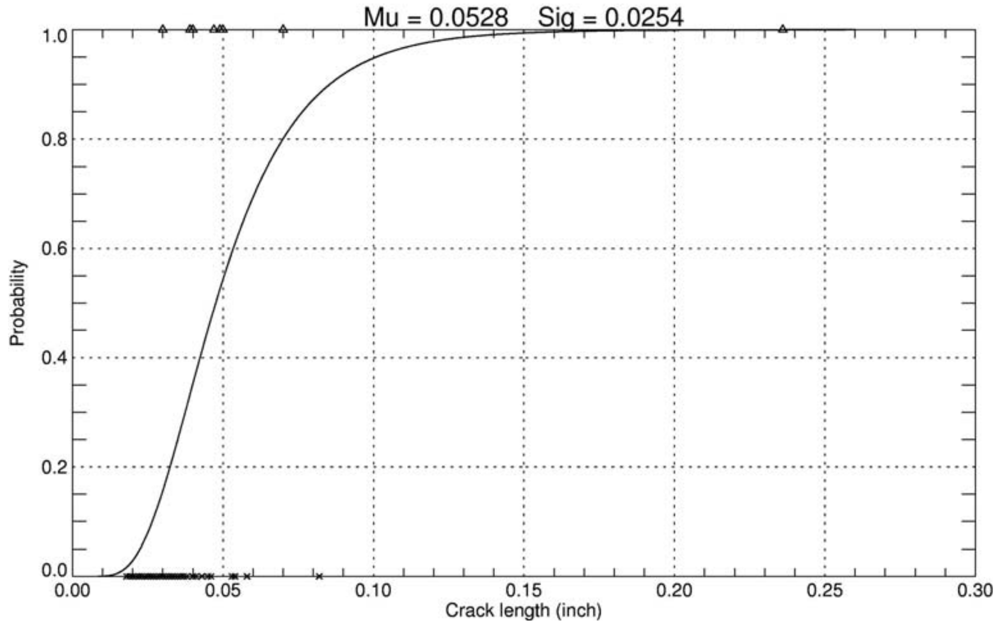

Figure 3.5 Mean POD curve for hit (Δ) and miss (x) data of the manual eddy current inspection of the F-16 fuselage longeron tab radii

The reliably detectable crack length a_d used in the deterministic analysis is defined at 90 % probability with a confidence of 95 %. From figure 3.5 a 90/50 % value instead can be obtained, yielding **0.085 inch**. Furthermore, the figure indicates that the original assumed 90/95 % value of 0.1 inch is realistic. A 90/50 % value is used here to yield a better comparison with the stochastic approach. Based on this smaller value the initial and repeat inspection would yield:

Initial inspection	**2655**	**Flight hours**
Repeat inspection	**82**	**Flight hours**

More details concerning the construction of this POD function can be found in reference 12. Many POD functions can be found in reference 13.

In the stochastic approach, presented in the next section, use is made of the whole POD curve instead of the single valued a_d to simulate an inspection.

3.4 Application of alternative stochastic life approach SLAP

So far, the deterministic Durability approach has been applied resulting in an initial and repeat inspection of 2655, respectively, 82 flight hours. The corresponding safety level (reliability) is unknown. In practice these values are adjusted such that they fit in the inspection policy of the air force. The effect hereof on the reliability level is unknown as well.

This section will present the results obtained with the alternative stochastic life approach SLAP, as described in chapter 2. The approach is implemented at NLR in a computer code.

Step 1: Construct failure distribution: *Weibull analysis*

The first step in the approach is the determination of the failure distribution by means of a Weibull analysis (see appendix A and Ref. 7 for details). No test data was available, however sufficient in-service data is available, demonstrating the use of this approach. The cracks found (table 3.1) do not represent a true failure, defined for this component as functional impairment at

a crack length of 0.187 inch, Ref. 8. The failure times are determined by means of a crack growth analysis starting from the crack length found up to functional impairment. The number of flight hours obtained for each crack is added to the number of flight hours at detection of the crack resulting in extrapolated failure times (last column table 3.1). The crack growth analysis used again the CSI corrected crack growth curves, correcting for the severity of the load spectrum experienced by each individual aircraft with respect to the baseline load spectrum.
Since sufficient failure data is available to fit a reliable failure distribution, addition of non-failure data (current accumulated flight hours for non-cracked longerons) is not necessary. Figure 3.6 depicts the results of the Weibull analysis on so-called Weibull probability paper, resulting in a straight line for any cumulative Weibull distribution function, due to the special axis of this Weibull probability paper (see figure 3.6). The same distribution in normal co-ordinates is plotted in figure 3.10 as a red line. The drawn straight line represents the MLE fit, with coefficients as depicted on top of the plot in figure 3.6. The last coefficient (A2-value) denotes the Anderson-Darling test statistic value (see appendix B). This is a goodness-of-fit test, and the best statistic test for this application, since the Anderson-Darling test is sensitive to deviations in the tails of the distribution, which are important here. The obtained A2-value of 0.315 is much less than the upper bound threshold value 0.757, necessary to accept the fit as being a Weibull distribution with a 5 % significance level. The goodness-of-fit is also illustrated by the fact that the failure times, denoted by dots in figure 3.6, are lying well along the fit (drawn line). The dashed lines represent the 95 % confidence bounds on the fit, which are narrow, indicating that the fit is close to the real one.
The location parameter t_0 (see appendix A) represents the life (**2624 Flight Hours**) below which no failures are to be expected. The summation of the location t_0 and scale parameter η represent the life (**3072 Flight Hours**) when 63.2 % of the longerons will have failed. Finally, the shape parameter β represents the speed of the failure process. A value of 2.57 indicates a mild failure mechanism.

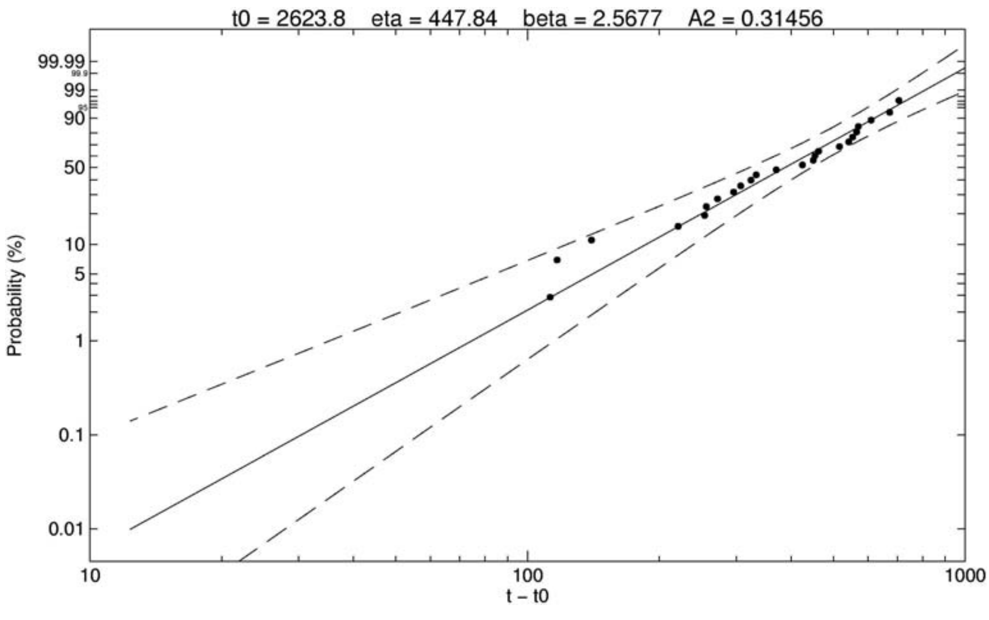

Figure 3.6 Plot of the fitted Weibull on Weibull probability paper and its 95 % confidence bounds (dashed lines). The dots represent the longeron failure times

The following 95 % confidence bounds where obtained for the distribution parameters:

$$t_0: \quad 2297 < 2624 < 2736$$
$$\eta: \quad 380.2 < 447.8 < 527.5$$
$$\beta: \quad 1.854 < 2.568 < 3.557$$

The confidence bounds are relatively small due to the amount of data, the mean values are used in the stochastic analysis, since these values represent reality best.

Step 2: Backward crack growth analyses*: Determine initial inspection time and corresponding crack length distribution*

The results of the backward analysis are:

- Initial inspection time
- Crack length distribution at the initial inspection time

As explained in chapter 2 the backward analysis is controlled by means of two parameters: a detectable crack length a_{det} and a threshold probability value p_{th}. Based on the detectable crack length the distribution describing the time it takes before a crack of length a_{det} is present in a certain percentage of the components (PDF-a_{det} in figure 2.5) is obtained. When a threshold percentage (p_{th}) of the cracks have become detectable with a reasonable probability, the initial (threshold) inspection becomes opportune. The value of p_{th} is set here at a fixed value of 0.1 %. A lower value will not lead to a noticeable earlier initial inspection (see also figure 3.10). A larger value can be chosen, however, the shift in initial inspection can also be accomplished by selection of another detectable crack length. To study this effect a range of a_{det} values has been analysed.

As mentioned before all the scatter present in material properties, load spectrum, et cetera is included in the Weibull failure distribution. The scatter in the crack length at a time instance is derived from this distribution in the backward analysis using a deterministic mean crack growth curve. This of course is not completely correct, since in general the crack growth curves will be steeper than the mean curve for the shorter lives and less steep for the longer lives, see for example figure 3.7. The real crack growth curves, however, are unknown and thus a stochastic backward crack growth analysis is not possible, even if the scatter in material properties and loading would be known. The crack length determined with the mean curve will therefore, in general, be too large for the shorter lives and too small for the longer lives. Since the probability of failure is determined mainly by the mean part and lower tail of the failure distribution, use of the mean curve gives an overestimate of the crack length, which is conservative, However, the chance on detection (defined by the POD) will not be conservative. Whether the sum of both will always lead to a conservative estimate still remains to be examined. However, the Weibull distribution function can be chosen as a conservative lower bound, which also accounts for the POD curve, introducing the necessary conservatism.

Also, it is enormously appealing to have an approach in which all the scatter is concentrated in only two distribution functions: failure distribution and inspection distribution. Instead of having the need for a distribution for all the model parameters (see for example Ref. 1). Moreover, both can be determined based on in-service data. This is a very important fact, since this data can be determined afterwards as shown here, in contrary to distributions for material properties and loading which information should be gathered right away.

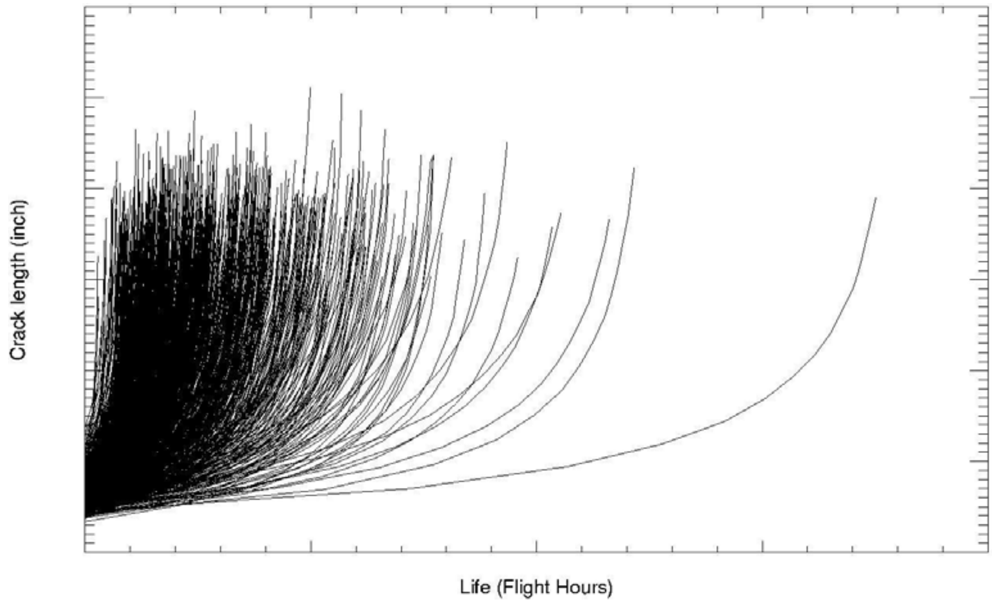

Figure 3.7 Schematised crack growth curves, representing scatter in material properties and loading (Ref. 1)

Figure 3.8 shows the minimum and maximum crack lengths found in a sample of 100,000 simulations, in which failure times are randomly drawn from the failure distribution and all calculated backwards to crack lengths obtained at $t_{initial}$ (represented by the distribution PDF-$t_{initial}$). This has been done for different values of the detectable crack length a_{det}. From this figure it can be concluded that even for very small values of a_{det} cracks can be mainly regarded as long cracks and a long crack model suffices, since the lower tail is of no importance to the probability of failure. All crack lengths in the sample fall also within the range of the baseline crack growth curve applied in the original deterministic analysis.

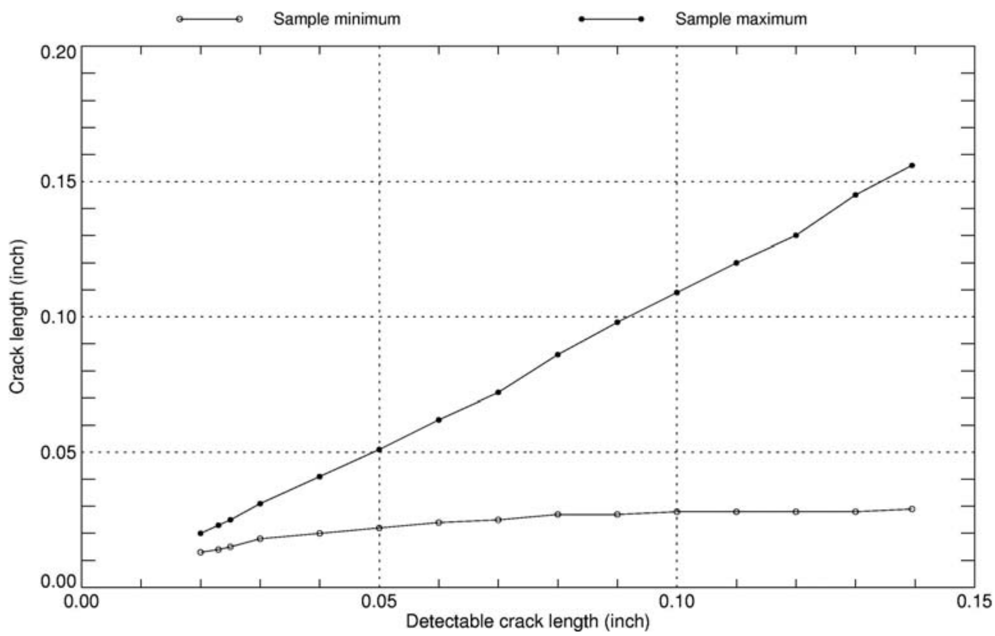

Figure 3.8 Minimum and maximum crack length found in the sample (100,000 simulations) for various detectable crack length values

For one value of the detectable crack length a_{det} a distribution fit is shown in figure 3.9 of the resulting crack length distribution at the initial inspection time, which forms the starting point in the forward analysis. However, this fitted distribution is not used in the stochastic analysis, but the calculated crack lengths themselves, to exclude possible errors caused by fitting the data points. In this case the crack lengths at initial inspection is represented very well by an Extreme value type 4 maximum distribution (see reference 14), which is a distribution having a lower (a*0 in plot*) and an upper bound (*a1 value in plot*) for the crack lengths. Although the small negative value of a0 is not realistic, it indicates possible negative crack lengths caused by using the mean crack growth curve for slowly growing cracks. The corresponding PDF distribution is depicted in figure 3.10 (green curve) in which all the results of the backward analysis are depicted also. Besides the crack distribution at $t_{initial}$ these consists of: Failure distribution (red curve); Time to reach detectable crack length distribution (purple curve); POD distribution (blue curve); crack growth curves at 1 %, mean and 99 % of failure times. The detectable crack length for which the results are presented in figures 3.9 and 3.10 was set at a_{det} = 0.06 inch, giving the most optimal inspection scheme as will be shown later on. Similar results have been obtained for a range of a_{det} values, discussed next.

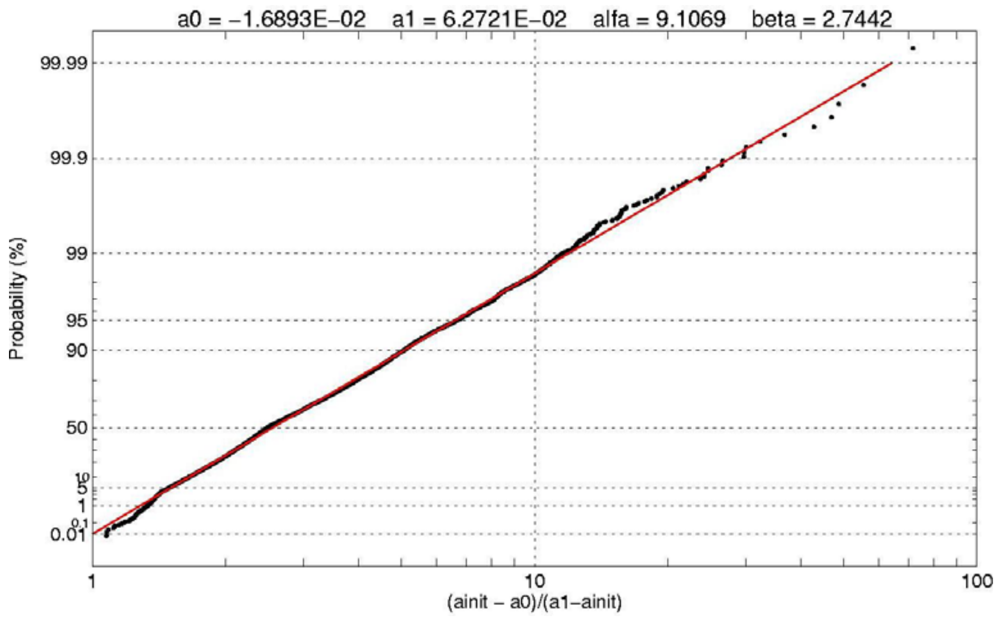

Figure 3.9 Extreme value type 4 max (Ref. 14) probability fit of crack lengths at initial inspection at 2293 Flight Hours (detectable crack length a_{det} = 0.06 inch), for 100,000 values. Corresponds with the green PDF plot in figure 3.10

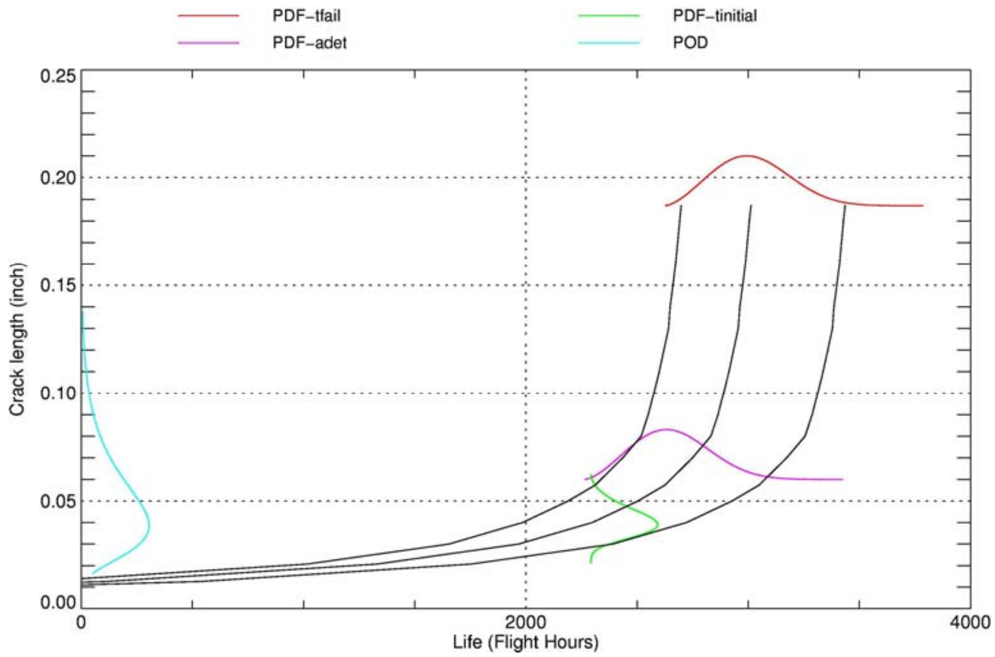

Figure 3.10 Backward analysis results plot, consisting of: Failure distribution (red curve); Time to reach detectable crack length distribution (purple curve); Crack length distribution at initial inspection (green curve); POD distribution (blue curve), 1 %, mean and 99 % crack growth curves. Detectable crack length a_{det} = 0.06 inch, initial inspection at 2293 Flight Hours.

Step 3: Forward crack growth analyses including inspections: *Determine repeat inspections*
In the third and last step, the crack length distribution at $t_{initial}$ obtained from the backward analysis is used to perform a stochastic crack growth analysis, while simulating the inspection scheme. The only random variable now is the crack length at $t_{initial}$ represented by the distribution PDF-$t_{initial}$ (green curve in figure 3.10). The result of the upward stochastic analysis of course is again the failure distribution, however in this step an inspection scheme is simulated.
As discussed above, this starting crack length distribution is a much more realistic crack length distribution than the one used in the EIFS approach. The selected inspection scheme is simulated by means of the POD distribution that is determined here with field inspection data and therefore realistic (see section 3.3). For each inspection a detectable crack length a_{det} is randomly drawn from the POD distribution. When a crack is found ($a \geq a_{det}$) the component is replaced and it is assumed that the new component is of similar quality (belonging to the same failure distribution). During all the inspections a crack may be missed, which leads to a failure of the component (here functional impairment). A probability of failure can thus be determined by performing an upward analysis for many of these initial crack lengths. The stochastic analysis is stopped when the economic life of the component has been reached. An economic lifetime $t_{economic}$ of 5310 Flight Hours is selected here, equal to the deterministic lifetime (Fig. 3.3), to make a more meaningful comparison with the deterministic results.

Stochastic analyses have been performed for detectable crack length values a_{det} in the range 0.02 up to 0.14 inch, the range more or less covered by the POD distribution function, and various number of repeat inspections. The results of the analyses are presented in figures 3.11 to 3.15.
Lowering the detectable crack length will result in an earlier initial inspection, depicted in figure 3.11, which can easily be understood from figure 3.10. The probability of failure then, in case of only this first inspection, subsequently will increase, depicted in figure 3.12, since these cracks will be smaller and more difficult to detect. Therefore, starting to inspect too early (small detectable crack length) will result in more inspections to obtain a certain safety level than for

later initial inspection (larger detectable crack length) as depicted in figure 3.13 for a number of POF values. The first number of inspections will in fact be for nothing.
On the other hand more inspections during the economic life will be required to reach a certain safety level when the initial inspection time is postponed too long (large detectable crack length), as depicted in figure 3.13. This, because cracks are growing faster and less time is available to detect them requiring a shorter repeat inspection interval.
An optimal initial inspection time, with a corresponding detectable crack length, thus exists minimising the total number of inspections. From figures 3.13 and 3.14 it can be observed that the optimum for this case lies more or less at a_{det} = 0.06 inch, which is the 69/50 % percentile of the POD function, contrary to the 90/95 % percentile used in the deterministic analysis (see section 3.2).
In summary, it is thus economical to start inspecting at a time when the chance on detecting a crack is still low, but as mentioned above it is uneconomical to start too early. This is once more illustrated in figure 3.14, giving the probability of failure as function of the detectable crack length for a number of values of the total number of inspections.

Finally, figure 3.15 depicts curves of equal safety level as function of the initial inspection time and the total number of inspections. This is the most important figure expressing the results of the stochastic analysis. The repeat inspection interval Δt_{insp}, which is kept constant in this case, can easily be determined from the total number of inspections by:

$$\Delta t_{insp} = \frac{t_{econ} - t_{init}}{n_{insp}}$$

with $t_{economic}$ equal to 5310 Flight Hours and $t_{initial}$ and n_{insp} obtained from the figure.
The optimal inspection schemes for different POF values (marked by red dots in the figure) are given in table 3.2.

Table 3.2 Optimal inspection schemes for different safety levels

POF	$t_{initial}$ (FH)	n_{insp}	Δt_{insp} (FH)
10^{-3}	2300	19	158
$5*10^{-3}$	2300	16	188
10^{-2}	2300	14	215
$5*10^{-2}$	2400	9	323

Remains the question which safety level to select. A probability of failure of 10^{-3} for the lifetime of a safety-of-flight structure more or less corresponds with a 10^{-7} value per flight hour (1 failure in 10^{7} flight hours), which is regarded as the safety level in a deterministic analysis for historical reasons, see section 1.2. For a not-safety-of-flight structure it is reasonable to select a probability value of at least one order lower , i.e. 10^{-2}. This results in the following inspection scheme:

Initial inspection **2300 Flight hours**
Repeat inspection **215 Flight hours**

In table 3.3 these values are compared against the deterministically obtained ones. Compared with the deterministically determined values of 2655 FH and 82 FH, this has resulted in an earlier (355 FH) initial inspection and much less repeat inspections, **14** instead of **32** = (5310-2655)/82. In principle this means that in the deterministic case during many inspections no cracks will be found since they are too small to be detected and these inspections are in fact ineffective.

Table 3.3 Comparison with deterministic results

	Deterministic		**Stochastic**			
a_{det} (inch)	0.1	0.085	0.06	0.06	0.06	0.06
POF	Unknown	Unknown	10^{-3}	$5*10^{-3}$	10^{-2}	$5*10^{-2}$
$T_{initial}$ (FH)	2655	2655	2300	2300	2300	2400
Δt_{insp} (FH)	62	82	158	188	215	323
n_{insp}	43	32	19	16	14	9

From figure 3.15 it can be seen that the probability of failure in case of the deterministic analysis, denoted by the green star, is lower than 10^{-3}, indicating a too severe inspection scheme. Furthermore, the scheme is far from optimal.
Again, in practice these values will be rounded off to more suitable values fitting in the overall inspection policy applied.
An even more optimal inspection scheme can be found by including the repair and replacement costs and allow for a variable repeat inspection period.

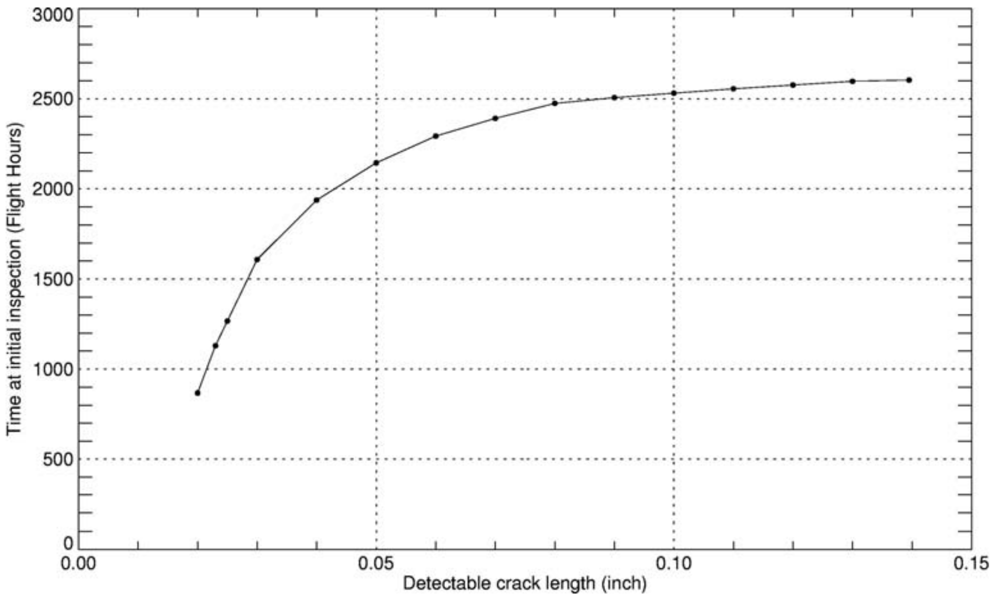

Figure 3.11 Time at initial inspection for different values of the detectable crack length

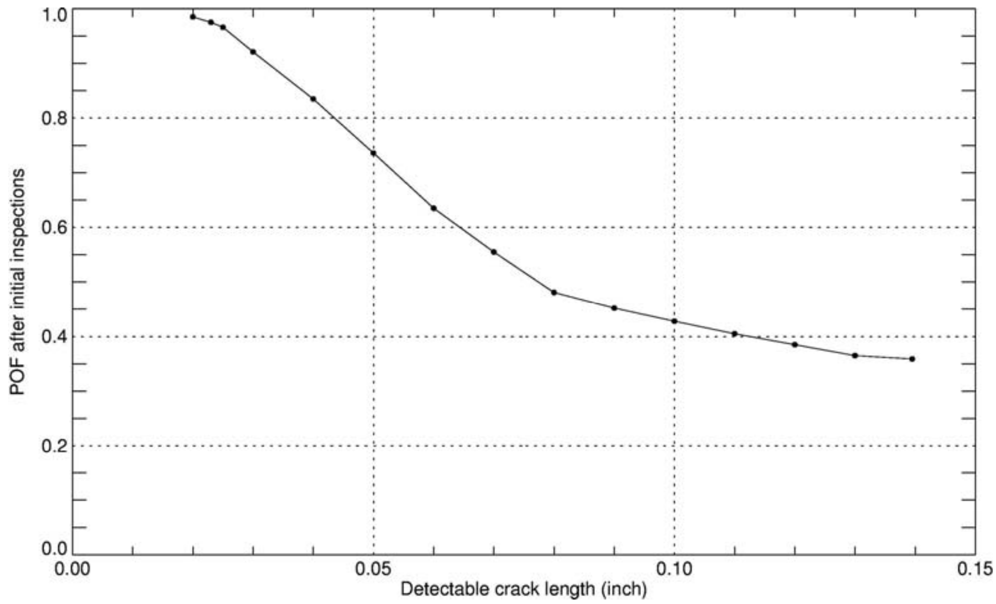

Figure 3.12 Probability of failure in case of only one inspection

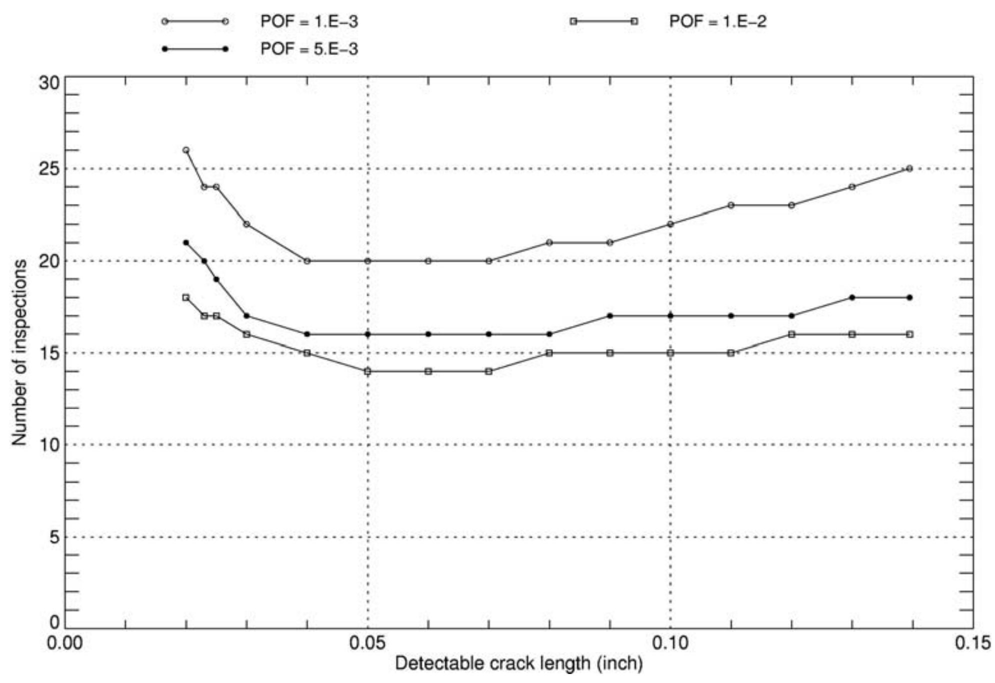

Figure 3.13 Required number of inspections as function of the detectable crack length for different values of the probability of failure

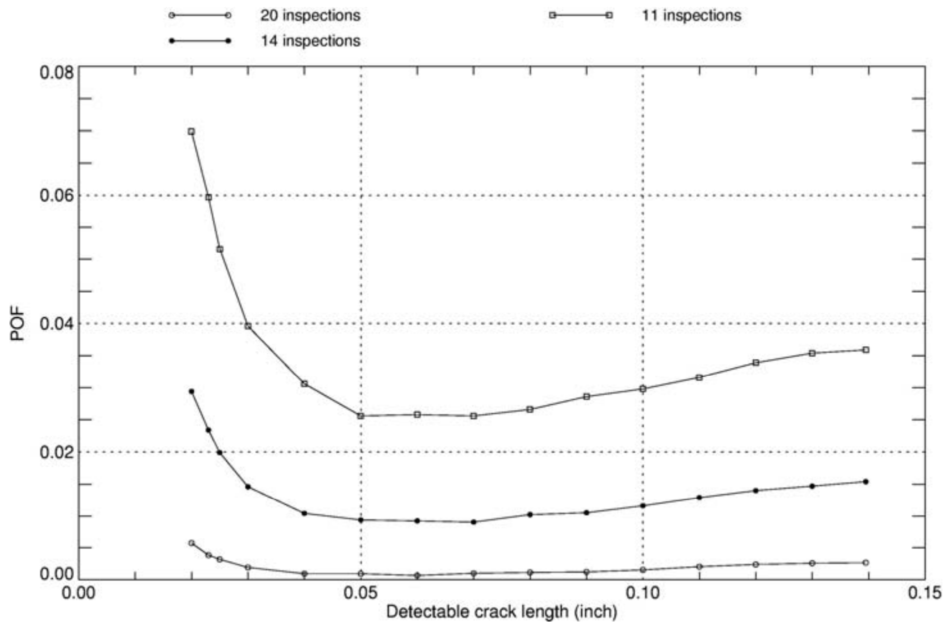

Figure 3.14 Probability of failure as function of the detectable crack length for a number of values of the total number of inspections

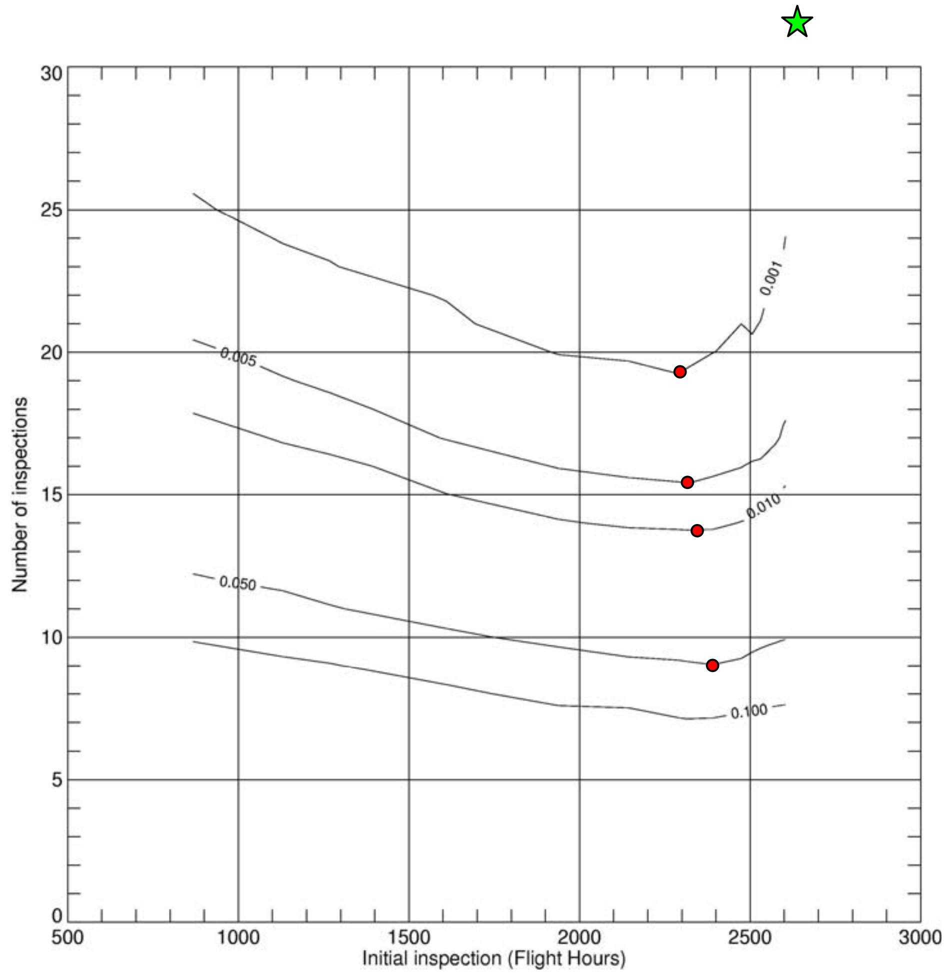

Figure 3.15 Contour plot for different values of the probability of failure as function of the initial inspection time and total number of inspections until $t_{economic}$ = 5310 FH

4 DISCUSSION AND CONCLUSIONS

In this paper, an alternative life philosophy SLAP was presented by which the lifetime and inspection scheme of a component can be determined in a fully stochastic manner, covering the crack initiation period as well as the crack growth period in a realistic way. The approach can serve as an alternative for the Safe-Life and Damage Tolerance approaches, resulting in more realistic predictions of the lifetime and inspection scheme.

The method was demonstrated by a realistic example based on in-service inspection data of the Upper Longeron of the F-16.

Advantages of the proposed approach over existing approaches are:

- One unified approach covering the whole lifetime, including crack initiation, micro, short and long crack growth period.
- No initiation, micro and short crack models are required. Realistic models are still lacking and may even never be obtained (Ref. 15). They require the modelling of the microstructure of the material, such as: grain size, grain orientation, grain boundaries, dislocations. Moreover, the unknown statistics of the crack/material interaction is important to take into account. All this makes modelling of initiation and micro cracks very complicated and computation time intensive, requiring all kinds of new model parameters, which makes it hard for application by aircraft industry. All this is indirectly accounted for by the starting failure distribution.
- Fully stochastic, introducing the concept of reliability based design. This will become more important in the near future, given the continuous optimisation of aircraft structures making them less reliable, due to the unknown reliability obtained by applying a standard scatter/safety factor(s).
- Only a very limited number of random variables are introduced. In principal all the scatter in the structure is covered by the failure distribution, which can be determined on a limited number of tests and improved by in-service data. Furthermore, a POD distribution should be known (can be obtained from Ref. 13) to simulate inspections. It has to be examined whether scatter in the critical crack length should be taken into account (by means of scatter in K_c). Realistic load sequences should be applied in the experiments, as is common practise.
- The methodology is reasonably straightforward and can be easily applied in an engineering environment, based on the **same** deterministic tools as currently used. This is an important prerequisite to become accepted.
- The approach suits very well in the currently applied inspection philosophy applied by military and civil operators, and in more recent Prognostic Health Management (PHM) systems.
- More realistic flaw sizes are obtained contrary to the EIFS approach.

Possible disadvantages are:

- The applicability range and validity of the methodology has yet to be demonstrated.
- The methodology requires more realistic testing.
  Much less or no coupon testing is foreseen. The reliability of coupon testing versus application of these test results on complex component scale can however be questioned in the current deterministic design philosophies.
- Requires introduction of stochastic design philosophy in the engineering environment

Some issues that still need to be addressed:

- Minimum number of tests required (e.g. determined by reliability testing).
- Prove the starting distribution can be selected conservative (see appendix A and also Ref. 7).

- Demonstrate the updating of the failure function during service life.
- Examine the necessity to include separate “rogue flaws”. This can be done by means of a predefined chance on occurrence that can easily be integrated in the stochastic process.

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Appendix A Weibull Analysis

A.1 Introduction

The failure distribution function can be determined from experimental data by fitting a distribution function to a sufficient number of failure points. Since the failure probability will be largely determined by the lower tail of this failure distribution, a large number of tests on the component under view are required in order to determine the lower tail of the distribution function with sufficient accuracy. It is superfluous to say that this is unwanted in general. If however, a lower bound curve of this failure distribution could be found, requiring a limited number of tests, this would be acceptable. In that case the design could be based on this distribution function, which would be conservative, and in-service data gathered during the lifetime of the component can then be used to improve the estimate of the real failure distribution.

The in-service data consist of non-failure and failure data, which both are used to improve the estimated failure distribution as will be discussed below. The non-failure data consists of usage times at the moment of inspection in case of non-detects and failure data will consist of the time and crack lengths found during inspections. The latter are therefore no real failures. The lifetime at the moment of crack detection can be easily extrapolated by means of a (long) crack growth analysis to obtain a realistic failure time.

The Weibull analysis method (Ref. 7) is very suitable in this respect, for the following reasons:

- The Weibull distribution function plays an important role in failure analysis and often is the most suitable distribution to describe a failure mode.
- An initial conservative lower bound estimate of the real failure distribution can be obtained with only a few failures (Chapter 2, Ref. 7). If even only non-failure data is available, an initial conservative lower bound of the real failure distribution can be determined by means of a **Weibays** or **Weibest analysis**.

A.2 Parameter estimation

Mathematically the Weibull distribution function is defined as:

$$F(t) = 1 - e^{-\left(\frac{t-t_0}{\eta}\right)^{\beta}} \tag{A.1}$$

where

t = time to failure
t_0 = location parameter
η = scale parameter
β = shape parameter

F(t) is the so-called **cumulative distribution function** and *f(t)* the **probability density function** obtained by differentiating *F(t)*:

$$f(t) = \left(\frac{\beta}{\eta}\right)\left(\frac{t-t_0}{\eta}\right)^{\beta-1} e^{-\left(\frac{t-t_0}{\eta}\right)^{\beta}} \tag{A.2}$$

F(t) represents the fraction of components that has failed at time *t*, which can also be expressed as *R(t) = 1 - F(t)* called the **reliability**, being the fraction that has survived at time *t*.
The location parameter t_0 represents the starting point of the distribution (no failures are present before this time) and can be selected zero in a number of cases.
The scale parameter η is a measure of the **characteristic life** of the component and represents the time when 63.2 % of the components have failed.
The shape parameter β is a measure of the **speed of the failure mechanism** and normally lies in the range of 2 to 6. A value larger than 1 denotes a wearout failure mode and a value of 4 or higher denotes a very fast failure mechanism (rapid wearout). A value equal to 1 indicates a constant failure rate, which is independent of the elapsed time. In other words the failures are random and lack memory of the past. A value less than 1 denotes a decreasing failure rate, or in other words an increasing reliability when the component ages. This is referred to as infant mortality rate.

The unknown values of the parameters t_0, β and η are determined from a set of available data points, which may consist of a mix of failed and non-failed sample times, so-called **multiple censored data**. Various methods exist to fit these three parameters, of which the method of maximum likelihood is the preferred one. This method is based on a so-called **likelihood function** *L*, describing the probability of obtaining the observed data and it can handle multiple censored data.

$$L = \prod_{i=1}^{n} f(t_i) \prod_{i=n+1}^{N} \left(1 - F(t_i)\right) \tag{A.3}$$

where:
- n = number of failed samples
- N = total number of samples

F(t) and *f(t)* are the cumulative distribution function, respectively, probability density function given by equations A.1 and A.2 in case of the Weibull distribution function.

The three parameters t_0, β and η are now found by maximising this likelihood function, which can be obtained by differentiating the log-normal of equation A.3 with respect to t_0, β and η and equate the result to zero, resulting in equations A.4, A.5 and A.6.

$$\frac{\hat{\beta}}{\hat{\eta}^{\hat{\beta}}} \sum_{i=1}^{N} (t_i - t_0)^{\hat{\beta}-1} - (\hat{\beta} - 1) \sum_{i=1}^{n} (t_i - t_0)^{-1} = 0 \tag{A.4}$$

$$\frac{\sum_{i=1}^{N} (t_i - t_0)^{\hat{\beta}} \, ln(t_i - t_0)}{\sum_{i=1}^{N} (t_i - t_0)^{\hat{\beta}}} - \frac{1}{n} \sum_{i=1}^{n} ln(t_i - t_0) - \frac{1}{\hat{\beta}} = 0 \tag{A.5}$$

$$\hat{\eta} = \left(\frac{\sum_{i=1}^{N} \left(t_i - t_0\right)^{\hat{\beta}}}{n} \right)^{\frac{1}{\hat{\beta}}} \tag{A.6}$$

The values for t_0, β and η, (determined iteratively with the above equations A.4, A.5 and A.6), are estimates of the real values and therefore denoted by the symbol ^. The more failure and non-failure data is available the better the estimates. A so-called **confidence interval** can be determined (Ref. 7) which contains the true value of the parameters. In order to ensure that a conservative estimate is obtained for the failure distribution, the lower bound value for the location t_0 and scale parameter η can be selected. For the shape parameter β an upper bound value can be selected, since this is a measure of the speed of the failure mechanism.

Even in the case only non-failure data is available an estimate of the parameters can be made. Then, for β a value is assumed, based on historical failure data of similar components or engineering judgement. A lower confidence bound of 63.2 % for η can be found by setting n equal to 1 in equation A.6. This resembles the situation where the first failure is assumed to be imminent and is called the **Weibayes** method. Assuming that the first failure is imminent is often very conservative. A less conservative approach is selecting n=0.693 resulting in a 50 % lower confidence bound on the true Weibull, the so-called **Weibest** method. For more details see reference 7.

Appendix B Anderson-Darling goodness-of-fit test

There are many goodness-of-fit tests, for example Komolgorov-Smirnov, Anderson-Darling, Shipiro-Wilk. The Anderson-Darling test is more sensitive to deviations in the tails of the distribution than the older Komolgorov-Smirnov test. The Anderson-Darling test can be applied to any distribution. The Anderson-Darling test (or Komolgorov-Smirnov or Shipiro-Wilk) does *not* tell you that you *do have* a certain type of distribution function, it only tells you when the data make it (*un*)likely that you do.
The Anderson-Darling test statistic value is determined by:

$$A^2 = -\left[\sum_{i=1}^{n} (2i-1)\left(\ln\left(F\left(x_i\right)\right) + \ln\left(1 - F\left(x_{n+1-i}\right)\right)\right) \right] / n - n \tag{B.1}$$

where *F()* is the cumulative distribution function and *n* the number of observations.
The result of this formula needs to be modified for small sampling values. In case of a **normal** and **log-normal** distribution this is:

$$A_m^2 = A^2 \left[1 + \frac{0.75}{n} + \frac{2.25}{n^2} \right] \tag{B.2}$$

The value then must be compared with the following values, where α is the significance level, to accept (less than) or reject (larger than) the distribution function.

$\boldsymbol{\alpha}$	0.1	**0.05**	0.025	0.01
$\boldsymbol{A^2_{crit}}$	0.631	**0.752**	0.873	1.035

In case of a **Weibull** or **Gumbel** ($X_G = ln(1/X_W)$) distribution the modified value yields:

$$A_m^2 = A^2\left[1+\frac{0.2}{\sqrt{n}}\right] \qquad \text{(B.3)}$$

The value then must be compared with the following values to accept (less than) or reject (larger than) the distribution function.

$\boldsymbol{\alpha}$	0.1	**0.05**	0.025	0.01
$\boldsymbol{A^2_{crit}}$	0.637	**0.757**	0.877	1.038