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Interacting Multiple Model Joint Probabilistic Data Association avoiding track coalescence

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List of acronyms

IMM	Interacting Multiple Model
IMMJPDA	Interacting Multiple Model Joint Probabilistic Data Association
IMMJPDA*	Track-coalescence-avoiding IMMJPDA
IMMPDA	Interacting Multiple Model Probabilistic Data Association
JPDA	Joint Probabilistic Data Association
JPDA*	Track-coalescence-avoiding JPDA
MHT	Multiple Hypothesis Tracking
NLR	Nationaal Lucht- en Ruimtevaartlaboratorium
NN	Nearest Neighbour
PDA	Probabilistic Data Association

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Abstract

For the problem of tracking multiple targets the Joint Probabilistic Data Association (JPDA) filter approach has shown to be very effective in handling clutter and missed detections. Recently the problem of track coalescence has been also solved for JPDA. The aim of this paper is to combine this JPDA avoiding track coalescence approach with IMM to track multiple maneuvering targets. The tracking problem is first embedded into one of filtering for a jump linear descriptor system with stochastic coefficients. Next, for this descriptor system, exact filter equations are derived, hypothesis management assumptions are adopted, and IMMJPDA avoiding track coalescence filter equations are developed. Finally, the filter performance is illustrated through Monte Carlo simulations for a simple example.

1 Introduction

We consider the problem of tracking multiple maneuvering targets in clutter with a proper combination of two well known approaches in target tracking: IMM and JPDA. Since each of these two solve complementary tracking problems it is of significant interest to combine these two approaches. In literature the problem of combining IMM (Blom & Bar-Shalom, 1988) and JPDA (Bar-Shalom and Fortmann, 1988) has been studied by Bar-Shalom et al. (1992), De Feo et al. (1997) and Chen and Tugnait (2001). Bar-Shalom et al. (1992) developed an IMMJPDA-Coupled filter for situations where the measurements of two targets are unresolved during periods of close encounter. Blom & Bloem (2000) have shown that these IMMJPDA-Coupled filter equations are rather heuristic. Chen and Tugnait (2001) developed an IMMJPDA-Uncoupled filter; implicitly they assumed that different targets evolve according to modes that are mutualy independent. They also showed that the IMMJPDA of De Feo et al. (1997) does not account for ”interactions” between target modes. All in all, in spite of the significant headway which has been made regarding the combination of IMM and JPDA, there appears to be a lack of insight in the proper choices to be made when combining IMM and JPDA for multiple maneuvering target tracking.

In order to improve this situation, the paper studies the problem of combining IMM and JPDA following an approach that is based on recent new insight gained regarding the derivation of a track coalescence avoiding JPDA version (Blom & Bloem, 2000). The basis for this development is to embed the multi target tracking problem with possibly false and missing measurements into one of filtering for a linear descriptor system with random coefficients. In this paper this embedding approach is extended towards the development of novel IMMJPDA filters.

The paper is organized as follows. Section 2 defines the problem considered. In this way it is ensured that there is no unambiguity which mathematical model is addressed. Section 3 embeds the tracking problem considered into one of filtering for a jump linear descriptor system with stochastic i.i.d. coefficients. Subsequently, in section 4, for this filtering problem an exact Bayesian characterization of the evolution of the conditional density for the state of the multiple targets is developed. In addition second order conditional characterizations are developed. Sections 3 and 4 in particular provide the derivation steps that were announced in Blom & Bloem (2002). Sections 5 and 6 present the steps of novel IMMJPDA filter equations. Section 7 shows Monte Carlo simulation results.

2 Stochastic Modelling

This section describes the target model and the measurement model.

2.1 Target model

Consider M targets and assume that the state of the i-th target is modelled as a jump linear system:

$$x^i_{t+1} = a^i(\theta^i_{t+1})x^i_t + b^i(\theta^i_{t+1})w^i_t, \qquad i = 1, ..., M, \quad (1)$$

where x^i_t is the n-vectorial state of the i-th target, θ^i_t is the mode of the i-th target and assumes values from $\{1, .., N\}$, $a^i(\theta^i_t)$ and $b^i(\theta^i_t)$ are $(n \times n)$- and $(n \times n')$-matrices and w^i_t is a sequence of i.i.d. standard Gaussian variables of dimension n' with w^i_t , w^j_t independent for all $i \neq j$ and w^i_t ,x^i_0, x^j_0 independent for all $i \neq j$. Let $x_t \triangleq \text{Col}\{x^1_t, ..., x^M_t\}$, $\theta_t \triangleq$

$\text{Col}\{\theta_t^1, ..., \theta_t^M\}$, $A(\theta_t) \triangleq \text{Diag}\{a^1(\theta_t^1), ..., a^M(\theta_t^M)\}$, $B(\theta_t) \triangleq \text{Diag}\{b^1(\theta_t^1), ..., b^M(\theta_t^M)\}$, and $w_t \triangleq \text{Col}\{w_t^1, ..., w_t^M\}$. Then we can model the state of our M targets as follows:

$$x_{t+1} = A(\theta_{t+1})x_t + B(\theta_{t+1})w_t \tag{2}$$

with A and B of size $Mn \times Mn$ and $Mn \times Mn'$ respectively.

2.2 Measurement Model

A set of measurements consists of measurements originating from targets and measurements originating from clutter. Firstly the measurements originating from targets are treated. Subsequently the clutter measurements are randomly inserted between the target measurements.

A Measurements originating from targets

We assume that a potential measurement associated with state x_t^i (which we will denote by z_t^i) is modelled as a jump linear system:

$$z_t^i = h^i(\theta_t^i)x_t^i + g^i(\theta_t^i)v_t^i \qquad , i = 1, ..., M \tag{3}$$

where z_t^i is an m-vector, $h^i(\theta_t^i)$ is an $(m \times n)$-matrix and $g^i(\theta_t^i)$ is an $(m \times m')$-matrix, and v_t^i is a sequence of i.i.d. standard Gaussian variables of dimension m' with v_t^i and v_t^j independent for all $i \neq j$. Moreover v_t^i is independent of x_0^j and w_t^j for all i,j. Next with $z_t \triangleq \text{Col}\{z_t^1, ..., z_t^M\}$, $H(\theta_t) \triangleq \text{Diag}\{h^1(\theta_t^1), ..., h^M(\theta_t^M)\}$, $G(\theta_t) \triangleq \text{Diag}\{g^1(\theta_t^1), ..., g^M(\theta_t^M)\}$, and $v_t \triangleq \text{Col}\{v_t^1, ..., v_t^M\}$, we obtain:

$$z_t = H(\theta_t)x_t + G(\theta_t)v_t \tag{4}$$

with H and G of size $Mm \times Mn$ and $Mm \times Mm'$ respectively.

We next introduce a model that takes into account that not all targets have to be detected at moment t, which implies that not all potential measurements z_t^i have to be available as true measurements at moment t. To this end, let P_d^i be the detection probability of target i and let $\phi_{i,t} \in\{0,1\}$ be the detection indicator for target i, which assumes the value one with probability $P_\text{d}^i > 0$, independently of $\phi_{j,t}$, $j \neq i$. This approach yields the following detection indicator vector ϕ_t of size M:

$$\phi_t \triangleq \text{Col}\{\phi_{1,t}, ..., \phi_{M,t}\}.$$

Thus, the number of detected targets is $D_t \triangleq \sum_{i=1}^M \phi_{i,t}$. Furthermore, we assume that $\{\phi_t\}$ is a sequence of i.i.d. vectors.

In order to link the detection indicator vector with the measurement model, we introduce the following operator Φ: for an arbitrary (0,1)-valued M'-vector ϕ' we define $D(\phi') \triangleq \sum_{i=1}^{M'} \phi_i'$ and the operator Φ producing $\Phi(\phi')$ as a $(0,1)$-valued matrix of size $D(\phi') \times M'$ of which the ith row equals the ith non-zero row of $\text{Diag}\{\phi'\}$. Next we define, for $D_t > 0$, a vector that contains all measurements originating from targets at moment t in a fixed order.

$$\tilde{z}_t \triangleq \underline{\Phi}(\phi_t)z_t, \text{ where } \underline{\Phi}(\phi_t) \triangleq \Phi(\phi_t) \otimes I_m,$$

with I_m a unit-matrix of size m, and $\otimes$ the tensor product.

In reality we do not know the order of the targets. Hence, we introduce the stochastic $D_t \times D_t$ permutation matrix χ_t, which is conditionally independent of $\{\phi_t\}$. We also assume that $\{\chi_t\}$ is a sequence of independent matrices. Hence, for $D_t > 0$,

$$\tilde{\tilde{z}}_t \triangleq \underline{\chi}_t \tilde{z}_t, \text{ where } \underline{\chi}_t \triangleq \chi_t \otimes I_m,$$

is a vector that contains all measurements originating from targets at moment t in a random order.

B Measurements originating from clutter

Let the random variable F_t be the number of false measurements at moment t. We assume that F_t has Poisson distribution:

$$p_{F_t}(F) \begin{array}{ll} = \frac{(\lambda V)^F}{F!}\exp\big(-\lambda V\big), & F = 0, 1, 2, \ldots \\ = 0, & \text{else} \end{array}$$

where λ is the spatial density of false measurements (i.e. the average number per unit volume) and V is the volume of the validation region. Thus, λV is the expected number of false measurements in the validation gate. We assume that the false measurements are uniformly distributed in the validation region, which means that a column-vector v_t^* of F_t i.i.d. false measurements has the following density:

$$p_{v_t^*|F_t}(v^*|F) = V^{-F}$$

where V is the volume of the validation region. Furthermore we assume that the process $\{v_t^*\}$ is a sequence of independent vectors, which are independent of $\{x_t\}, \{w_t\}, \{v_t\}$ and $\{\phi_t\}$.

C Random insertion of clutter measurements

Let the random variable L_t be the total number of measurements at moment t. Thus,

$$L_t = D_t + F_t$$

With $\tilde{\text{y}}_t \triangleq \text{Col}\{\tilde{\tilde{z}}_t, v_t^*\}$, it follows with the above defined variables that

$$\tilde{\text{y}}_t = \begin{bmatrix} \underline{\chi}_t \underline{\Phi}(\phi_t) z_t \\ \cdots\cdots\cdots \\ v_t^* \end{bmatrix}, \text{ if } L_t > D_t > 0 \tag{5}$$

whereas the upper and lower subvector parts disappear for $D_t = 0$ and $L_t = D_t$ respectively. With this equation, the measurements originating from clutter still have to be randomly inserted between the measurements originating from the detected targets. To do so, we first define target indicator and clutter indicator processes, denoted by $\{\psi_t\}$ and $\{\psi_t^*\}$, respectively. Let the random variable $\psi_{i,t} \in\{0,1\}$ be a target indicator at moment t for measurement i, which assumes the value one if measurement i belongs to a detected target and zero if measurement i comes from clutter. This approach yields the following target indicator vector ψ_t of size L_t:

$$\psi_t \triangleq \text{Col}\{\psi_{1,t}, ..., \psi_{L_t,t}\}.$$

Let the random variable $\psi_{i,t}^* \in\{0,1\}$ be a clutter indicator at moment t for measurement i, which assumes the value one if measurement i comes from clutter and zero if measurement i belongs to an aircraft (thus $\psi_{i,t}^* = 1 - \psi_{i,t}$). This approach yields the following clutter indicator vector ψ_t^* of size L_t:

$$\psi_t^* \triangleq \text{Col}\{\psi_{1,t}^*, ..., \psi_{L_t,t}^*\}.$$

In order to link the target and clutter indicator vectors with the measurement model, we make use of the operator Φ introduced before. With this the measurement vector with clutter inserted reads as follows:

$$\mathrm{y}_t = \left[\underline{\Phi}(\psi_t)^T \vdots \underline{\Phi}(\psi_t^*)^T\right] \tilde{\mathrm{y}}_t \;\text{ if } L_t > D_t > 0 \qquad (6)$$

Substituting (5) into (6) yields the following model for the observation vector y_t at moment t:

$$\mathrm{y}_t = \left[\underline{\Phi}(\psi_t)^T \vdots \underline{\Phi}(\psi_t^*)^T\right] \begin{bmatrix} \underline{\chi}_t \underline{\Phi}(\phi_t) z_t \\ \cdots\cdots\cdots \\ v_t^* \end{bmatrix} \;\text{ if } L_t > D_t > 0 \qquad (7)$$

This, together with equations (2) and (4), forms a complete characterization of our tracking problem in terms of stochastic difference equations.

3 Embedding into a descriptor system with stochastic coefficients

Because $\left[\underline{\Phi}(\psi_t)^T \vdots \underline{\Phi}(\psi_t^*)^T\right]$ is a permutation matrix for $L_t > D_t > 0$, its inverse satisfies

$$\left[\underline{\Phi}(\psi_t)^T \vdots \underline{\Phi}(\psi_t^*)^T\right]^T = \begin{bmatrix} \underline{\Phi}(\psi_t) \\ \cdots \\ \underline{\Phi}(\psi_t^*) \end{bmatrix} \qquad (8)$$

Premultiplying (7) by such inverse yields

$$\begin{bmatrix} \underline{\Phi}(\psi_t) \\ \cdots \\ \underline{\Phi}(\psi_t^*) \end{bmatrix} \mathrm{y}_t = \begin{bmatrix} \underline{\chi}_t \underline{\Phi}(\phi_t) z_t \\ \cdots\cdots\cdots \\ v_t^* \end{bmatrix} \;\text{ if } L_t > D_t > 0 \qquad (9)$$

From (9), it follows that

$$\underline{\Phi}(\psi_t)\mathrm{y}_t = \underline{\chi}_t \underline{\Phi}(\phi_t) z_t \;\text{ if } D_t > 0 \qquad (10)$$

Substitution of (4) into (10) yields:

$$\underline{\Phi}(\psi_t)\mathrm{y}_t = \underline{\chi}_t \underline{\Phi}(\phi_t) H(\theta_t) x_t + \underline{\chi}_t \underline{\Phi}(\phi_t) G(\theta_t) v_t \;\text{ if } D_t > 0 \qquad (11)$$

Notice that (11) is a linear Gaussian descriptor system (Dai, 1989) with stochastic i.i.d. coefficients $\underline{\Phi}(\psi_t)$ and $\underline{\chi}_t \underline{\Phi}(\phi_t)$. Because χ_t has an inverse, (11) can be transformed into

$$\underline{\chi}_t^T \underline{\Phi}(\psi_t)\mathrm{y}_t = \underline{\Phi}(\phi_t) H(\theta_t) x_t + \underline{\Phi}(\phi_t) G(\theta_t) v_t \;\text{ if } D_t > 0 \qquad (12)$$

Next we introduce an auxiliary indicator matrix process $\tilde{\chi}_t$ of size $D_t \times L_t$, as follows:

$$\tilde{\chi}_t \triangleq \chi_t^T \Phi(\psi_t) \;\text{ if } D_t > 0.$$

With this we get a simplified version of (12):

$$\underline{\tilde{\chi}}_t \mathrm{y}_t = \underline{\Phi}(\phi_t) H(\theta_t) x_t + \underline{\Phi}(\phi_t) G(\theta_t) v_t \;\text{ if } D_t > 0 \qquad (13)$$

Size of $\underline{\tilde{\chi}}_t$ is $D_t m \times L_t m$ and size of $\underline{\Phi}(\phi_t)$ is $D_t m \times M m$.

4 Exact filter equations

In this section a Bayesian characterization of the conditional density $p_{x_t,\theta_t| Y_t}(x,\theta)$ is given where Y_t denotes the σ-algebra generated by measurements y_t up to and including moment t. Subsequently, characterizations are developed for the mode probabilities and the mode conditional means and covariances.

From (14), it follows that for $D_t > 0$ all relevant associations and permutations can be covered by $(\phi_t, \tilde{\chi}_t)$-hypotheses. We extend this to $D_t = 0$ by adding the combination $\phi_t = \{0\}^M$ and $\tilde{\chi}_t = \{\}^{L_t}$. Hence, through defining the weights

$$\beta_t(\phi, \tilde{\chi}, \theta) \triangleq \text{Prob}\{\phi_t = \phi, \tilde{\chi}_t = \tilde{\chi}, \theta_t = \theta \mid Y_t\},$$

the law of total probability yields:

$$p_{x_t \theta_t | Y_t}(x,\theta) = \sum_{\tilde{\chi},\phi} \beta_t(\phi,\tilde{\chi},\theta) p_{x_t|\theta_t,\phi_t,\tilde{\chi}_t,Y_t}(x \mid \theta,\phi,\tilde{\chi}) \qquad (14)$$

And thus, our problem is to characterize the terms in the last summation. This problem is solved in two steps, the first of which is the following proposition.

Proposition 1 *For any $\phi \in\{0,1\}^M$, such that $D(\phi) \triangleq \sum_{i=1}^M \phi_i \leq L_t$, and any $\tilde{\chi}_t$ matrix realization $\tilde{\chi}$ of size $D(\phi) \times L_t$, the following holds true:*

$$p_{x_t|\theta_t,\phi_t,\tilde{\chi}_t,Y_t}(x \mid \theta,\phi,\tilde{\chi}) = \\ = \frac{p_{\tilde{z}_t|x_t,\theta_t,\phi_t}(\underline{\tilde{\chi}} y_t \mid x,\theta,\phi) \cdot p_{x_t|\theta_t,Y_{t-1}}(x \mid \theta)}{F_t(\phi,\tilde{\chi},\theta)} \qquad (15)$$

$$\beta_t(\phi,\tilde{\chi},\theta) = F_t(\phi,\tilde{\chi},\theta)\lambda^{(L_t - D(\phi))} \cdot \\ \cdot[\prod_{i=1}^M (1 - P_d^i)^{(1-\phi_i)} (P_d^i)^{\phi_i}] \cdot p_{\theta_t|Y_{t-1}}(\theta)/c_t \qquad (16)$$

where $\underline{\tilde{\chi}} \triangleq \tilde{\chi} \otimes I_m$ *, and* $F_t(\phi, \tilde{\chi}, \theta)$ *and* c_t *are such that they normalize* $p_{x_t|\theta_t,\phi_t,\tilde{\chi}_t,Y_t}(x \mid \theta, \phi, \tilde{\chi})$ *and* $\beta_t(\phi, \tilde{\chi}, \theta)$ *respectively.*

Proof: Omitted. It is similar to the proof of Proposition 1 in Blom & Bloem (2000). The specialty of this proof is because of the derivation of Bayesian equations for the descriptor system (13). ∎

Our next step is given by the following Theorem.

Theorem 1 *For each* $\theta \in \{1, ..., N\}^M$*, let* $p_{x_t|\theta_t,Y_{t-1}}(x \mid \theta)$ *be Gaussian with mean* $\bar{x}_t(\theta)$ *and covariance* $\bar{P}_t(\theta)$ *and let* $\beta_t(\phi, \tilde{\chi}, \theta)$ *and* $F_t(\phi, \tilde{\chi}, \theta)$ *be defined by Proposition 1. Then* $F_t(\{0\}^M, \{\}^{L_t}, \theta) = 1$*, whereas for* $\phi \neq \{0\}^M$*:*

$$F_t(\phi, \tilde{\chi}, \theta) = [(2\pi)^{mD(\phi)} Det\{Q_t(\phi, \theta)\}]^{-\frac{1}{2}} \cdot \cdot \exp\{-\frac{1}{2}\mu_t^T(\phi, \tilde{\chi}, \theta) Q_t(\phi, \theta)^{-1} \mu_t(\phi, \tilde{\chi}, \theta)\} \quad (17)$$

where

$$\begin{aligned} \mu_t(\phi, \tilde{\chi}, \theta) &\triangleq \underline{\tilde{\chi}} y_t - \underline{\Phi}(\phi) H(\theta) \bar{x}_t(\theta) \\ Q_t(\phi, \theta) &\triangleq \underline{\Phi}(\phi)\big(H(\theta)\bar{P}_t(\theta)H(\theta)^T + \\ &\quad + G(\theta)G(\theta)^T\big)\underline{\Phi}(\phi)^T \end{aligned}$$

Moreover, $p_{x_t|\theta_t,Y_t}(x \mid \theta)$ *is a Gaussian mixture, with overall weight* $p_{\theta_t|Y_t}(\theta)$*, mean* $\hat{x}_t(\theta)$ *and its overall covariance* $\hat{P}_t(\theta)$ *satisfying:*

$$p_{\theta_t|Y_t}(\theta) = \sum_{\phi,\tilde{\chi}} \beta_t(\phi, \tilde{\chi}, \theta) \quad (18)$$

$$\hat{x}_t(\theta) = \bar{x}_t(\theta) + \sum_{\substack{\phi \\ \phi \neq 0}} K_t(\phi, \theta) \left(\sum_{\tilde{\chi}} \beta_{t|\theta}(\phi, \tilde{\chi}) \mu_t(\phi, \tilde{\chi}, \theta) \right) \quad (19)$$

$$\begin{aligned} \hat{P}_t(\theta) &= \bar{P}_t(\theta) + \\ &- \sum_{\substack{\phi \\ \phi \neq 0}} K_t(\phi, \theta) \underline{\Phi}(\phi) H(\theta) \bar{P}_t(\theta) \left(\sum_{\tilde{\chi}} \beta_{t|\theta}(\phi, \tilde{\chi}) \right) + \\ &+ \sum_{\substack{\phi \\ \phi \neq 0}} K_t(\phi, \theta) \left(\sum_{\tilde{\chi}} \beta_{t|\theta}(\phi, \tilde{\chi}) \mu_t(\phi, \tilde{\chi}, \theta) \mu_t^T(\phi, \tilde{\chi}, \theta) \right) \cdot \\ &\cdot K_t^T(\phi, \theta) + \\ &- \left(\sum_{\substack{\phi \\ \phi \neq 0}} K_t(\phi, \theta) \left(\sum_{\tilde{\chi}} \beta_{t|\theta}(\phi, \tilde{\chi}) \mu_t(\phi, \tilde{\chi}, \theta) \right) \right) \cdot \\ &\cdot \left(\sum_{\substack{\phi' \\ \phi' \neq 0}} K_t(\phi', \theta) \left(\sum_{\tilde{\chi}'} \beta_{t|\theta}(\phi', \tilde{\chi}') \mu_t(\phi', \tilde{\chi}', \theta) \right) \right)^T \end{aligned} \quad (20)$$

with:

$$\begin{aligned} K_t(\phi, \theta) &\triangleq \bar{P}_t(\theta) H(\theta)^T \underline{\Phi}(\phi)^T Q_t(\phi, \theta)^{-1} \quad \text{if } \phi \neq 0, \\ &\triangleq 0 \quad \text{else} \end{aligned} \quad (21.a)$$

$$\beta_{t|\theta}(\phi, \tilde{\chi}) \triangleq \beta_t(\phi, \tilde{\chi}, \theta) / p_{\theta_t|Y_t}(\theta) \quad (21.b)$$

Proof: (*Outline*) If $p_{x_t|\theta_t,Y_{t-1}}(x|\theta)$ is Gaussian with mean $\bar{x}_t(\theta)$ and covariance $\bar{P}_t(\theta)$, then the density $p_{x_t|\phi_t,\tilde{\chi}_t,\theta_t,Y_t}(x|\phi, \tilde{\chi}, \theta)$ is Gaussian with mean $\hat{x}_t(\phi, \tilde{\chi}, \theta)$ and covariance $\hat{P}_t(\phi, \theta)$ satisfying for $\phi \neq 0$,

$$\begin{aligned} \hat{x}_t(\phi, \tilde{\chi}, \theta) &= \bar{x}_t(\theta) + K_t(\phi, \theta)[\underline{\tilde{\chi}} \mathrm{y}_t - \underline{\Phi}(\phi) H(\theta) \bar{x}_t(\theta)] \\ \hat{P}_t(\phi, \theta) &= \bar{P}_t(\theta) - K_t(\phi, \theta) \underline{\Phi}(\phi) H(\theta) \bar{P}_t(\theta) \end{aligned}$$

and for $\phi = 0$:

$$\begin{aligned} \hat{x}_t(\phi, \tilde{\chi}, \theta) &= \bar{x}_t(\theta) \\ \hat{P}_t(\phi, \theta) &= \bar{P}_t(\theta) \end{aligned}$$

Hence, $p_{x_t|\theta_t,Y_t}(. \mid \theta)$ is a Gaussian mixture, and all equations follow from a lengthy but straightforward evaluation of this mixture. ∎

Theorem 1 provides some characterizations for the joint targets modes and states. The subsequent step is to further evaluate these equations under the assumption of prior decomposition between individual targets. This is done in Theorem 2.

Theorem 2 *Let* $p_{\theta_t|Y_{t-1}}(\theta) = \prod_{i=1}^M p_{\theta_t^i|Y_{t-1}}(\theta^i)$ *and let* $p_{x_t|\theta_t,Y_{t-1}}(x|\theta)$ *be Gaussian with mean* $\bar{x}_t(\theta) = Col\{\bar{x}_t^1(\theta^1), ..., \bar{x}_t^M(\theta^M)\}$ *and covariance* $\bar{P}_t(\theta) = Diag\{\bar{P}_t^1(\theta^1), ..., \bar{P}_t^M(\theta^M)\}$*, then* $\beta_t(\phi, \tilde{\chi}, \theta)$ *of Proposition 1 satisfies:*

$$\beta_t(\phi, \tilde{\chi}, \theta) = \lambda^{(L_t - D(\phi))} \cdot \cdot \prod_{i=1}^M \left[f_t^i(\phi, \tilde{\chi}, \theta^i)(1 - P_d^i)^{(1-\phi_i)} (P_d^i)^{\phi_i} \cdot p_{\theta_t^i|Y_{t-1}}(\theta^i) \right] / c_t \quad (22)$$

with: $f_t^i(\phi, \tilde{\chi}, \theta^i) = 1$ *for* $\phi = \{0\}^M$ *and for* $\phi \neq \{0\}^M$

$$f_t^i(\phi, \tilde{\chi}, \theta^i) = [(2\pi)^m Det\{Q_t^i(\theta^i)\}]^{-\frac{1}{2}\phi_i} \cdot \cdot \exp\{-\frac{1}{2} \sum_{k=1}^{L_t} \left([\Phi(\phi)]_{*i}^T \tilde{\chi}_{*k} \mu_t^{ik}(\theta^i)^T [Q_t^i(\theta^i)]^{-1} \mu_t^{ik}(\theta^i) \right)\} \quad (23.a)$$

where:

$$\mu_t^{ik}(\theta^i) \triangleq y_t^k - h^i(\theta^i) \bar{x}_t^i(\theta^i) \quad (23.b)$$

$$Q_t^i(\theta^i) \triangleq h^i(\theta^i) \bar{P}_t^i(\theta^i) h^i(\theta^i)^T + g^i(\theta^i) g^i(\theta^i)^T \quad (23.c)$$

whereas $[\Phi(\phi)]_{*i}$ *and* $\tilde{\chi}_{*k}$ *are the* i*-th and* k*-th columns of* $\Phi(\phi)$ *and* $\tilde{\chi}$*, respectively. Moreover,* $p_{x_t^i|\theta_t^i,Y_t}(x^i|\theta^i)$*,* $i \in$

$\{1, ..., M\}$, is a Gaussian mixture, while its overall mean $\hat{x}_t^i(\theta^i)$ and its overall covariance $\hat{P}_t^i(\theta^i)$ satisfy:

$$p_{\theta_t^i|Y_t}(\theta^i) = \sum_{\substack{\phi,\tilde{\chi},\eta \\ \eta^i=\theta^i}} \beta_t(\phi,\tilde{\chi},\eta) \tag{24.a}$$

$$\hat{x}_t^i(\theta^i) = \bar{x}_t^i(\theta^i) + W_t^i(\theta^i)\left(\sum_{k=1}^{L_t} \beta_t^{ik}(\theta^i)\mu_t^{ik}(\theta^i)\right) \tag{24.b}$$

$$\begin{aligned}\hat{P}_t^i(\theta^i) &= \bar{P}_t^i(\theta^i) - W_t^i(\theta^i)h^i(\theta^i)\bar{P}_t^i(\theta^i)\left(\sum_{k=1}^{L_t}\beta_t^{ik}(\theta^i)\right) + \\ &+W_t^i(\theta^i)\left(\sum_{k=1}^{L_t}\beta_t^{ik}(\theta^i)\mu_t^{ik}(\theta^i)\mu_t^{ik}(\theta^i)^T\right)W_t^i(\theta^i)^T + \\ &-W_t^i(\theta^i)\left(\sum_{k=1}^{L_t}\beta_t^{ik}(\theta^i)\mu_t^{ik}(\theta^i)\right)\cdot \\ &\cdot\left(\sum_{k'=1}^{L_t}\beta_t^{ik'}(\theta^i)\mu_t^{ik'}(\theta^i)\right)^T W_t^i(\theta^i)^T\end{aligned} \tag{24.c}$$

with:

$$\begin{aligned} W_t^i(\theta^i) &\triangleq \bar{P}_t^i(\theta^i)h^i(\theta^i)^T[Q_t^i(\theta^i)]^{-1} \\ \beta_t^{ik}(\theta^i) &\triangleq \mathit{Prob}\{[\Phi(\phi)]_{*i,t}^T\tilde{\chi}_{*k,t} = 1 \mid \theta_t^i = \theta^i, Y_t\} = \\ &= \sum_{\substack{\phi,\tilde{\chi},\eta \\ \phi\neq 0 \\ \eta^i=\theta^i}} \Phi(\phi)_{*i}^T\tilde{\chi}_{*k}\beta_t(\phi,\tilde{\chi},\eta)]/p_{\theta_t^i|Y_t}(\theta^i)\end{aligned}$$

Proof: Omitted. It follows from Proposition 1 and Theorem 1 in a similar way as in Blom & Bloem (2000). ■

5 IMMJPDA filter

In this section the IMMJPDA filter algorithm is specified. To do so use is made of the IMM filter algorithm and of Theorem 2. One cycle of the IMMJPDA filter algorithm consists of the following six steps.

IMMJPDA Step 1: For each target this comes down to the mixing/interaction step of the IMM algorithm (Blom & Bar-Shalom, 1988) for all $i \in \{1, . . . , M\}$: Starting with the weights

$$\hat{\gamma}_{t-1}^i(\theta^i) \triangleq p_{\theta_{t-1}^i|Y_{t-1}}(\theta^i), \quad \theta^i \in \{1, ..., N\}$$

the means

$$\hat{x}_{t-1}^i(\theta^i) \triangleq E\{x_{t-1}^i|\theta_{t-1}^i = \theta^i, Y_{t-1}\}, \quad \theta^i \in \{1, ..., N\}$$

and the associated covariances

$$\begin{aligned}\hat{P}_{t-1}^i(\theta^i) \triangleq E\{[x_{t-1}^i - \hat{x}_{t-1}^i(\theta^i)][x_{t-1}^i - \hat{x}_{t-1}^i(\theta^i)]^T \mid \\ \mid \theta_{t-1}^i = \theta^i, Y_{t-1}\}, \quad \theta^i \in \{1, ..., N\}\end{aligned}$$

one evaluates the mixed initial condition for the filter matched to $\theta_t^i = \theta^i$ as follows:

$$\begin{aligned}\bar{\gamma}_t^i(\theta^i) &= \sum_{\eta^i=1}^N \Pi_{\eta^i,\theta^i}\cdot\hat{\gamma}_{t-1}^i(\eta^i) \\ \hat{x}_{t-1|\theta_t^i}^i(\theta^i) &= \sum_{\eta^i=1}^N \Pi_{\eta^i,\theta^i}\cdot\hat{\gamma}_{t-1}^i(\eta^i)\cdot\hat{x}_{t-1}^i(\eta^i)/\bar{\gamma}_t^i(\theta^i) \\ \hat{P}_{t-1|\theta_t^i}^i(\theta^i) &= \sum_{\eta^i=1}^N \Pi_{\eta^i,\theta^i}\cdot\hat{\gamma}_{t-1}^i(\eta^i)\cdot \\ &\cdot\Big(\hat{P}_{t-1}^i(\eta^i) + [\hat{x}_{t-1}^i(\eta^i) - \hat{x}_{t-1|\theta_t^i}^i(\theta^i)]\cdot \\ &\cdot[\hat{x}_{t-1}^i(\eta^i) - \hat{x}_{t-1|\theta_t^i}^i(\theta^i)]^T\Big)/\bar{\gamma}_t^i(\theta^i)\end{aligned}$$

with

$$\begin{aligned}\Pi_{\eta^i,\theta^i} &\triangleq Pr\{\theta_t^i = \theta^i \mid \theta_{t-1}^i = \eta^i\} \\ \bar{\gamma}_t^i(\theta^i) &\triangleq p_{\theta_t^i|Y_{t-1}}(\theta^i) \\ \hat{x}_{t-1|\theta_t^i}^i(\theta^i) &\triangleq E\{x_{t-1}^i \mid \theta_t^i = \theta^i, Y_{t-1}\} \\ \hat{P}_{t-1|\theta_t^i}^i(\theta^i) &\triangleq E\{[x_{t-1}^i - \hat{x}_{t-1}^i(\theta^i)]\cdot \\ &\cdot[x_{t-1}^i - \hat{x}_{t-1}^i(\theta^i)]^T|\theta_t^i = \theta^i, Y_{t-1}\}\end{aligned}$$

IMMJPDA Step 2: Prediction for all $i \in \{1, . . . ,M\}$, $\theta^i \in \{1, . . . ,N\}$:

$$\bar{x}_t^i(\theta^i) = a^i(\theta^i)\hat{x}_{t-1|\theta_t^i}^i(\theta^i) \tag{25.a}$$

$$\bar{P}_t^i(\theta^i) = a^i(\theta^i)\hat{P}_{t-1|\theta_t^i}^i(\theta^i)a^i(\theta^i)^T + b^i(\theta^i)b^i(\theta^i)^T \tag{25.b}$$

IMMJPDA Step 3: Gating, which is based on Bar-Shalom & Li (1995).
Evaluate for each i and θ^i the covariance as follows:

$$Q_t^i(\theta^i) = h^i(\theta^i)\bar{P}_t^i(\theta^i)h^i(\theta^i)^T + g^i(\theta^i)g^i(\theta^i)^T$$

Subsequently identify for each target the mode for which Det $Q_t^i(\theta^i)$ is largest:

$$\theta_t^{*i} = \underset{\theta^i}{\text{Argmax}}\ \{\text{Det}\, Q_t^i(\theta^i)\}$$

and use this to define for each target i a gate $G_t^i \in I\!R^m$ as follows:

$$\begin{aligned}G_t^i \triangleq \{z^i \in I\!R^m; [z^i - h^i(\theta_t^{*i})\bar{x}_t^i(\theta_t^{*i})]^T\cdot \\ \cdot Q_t^i(\theta_t^{*i})^{-1}[z^i - h^i(\theta_t^{*i})\bar{x}_t^i(\theta_t^{*i})] \leq \gamma\}\end{aligned}$$

with γ the gate size. If the j-th measurement y_t^j falls outside gate G_t^i; i.e. $\mathrm{y}_t^j \notin G_t^i$, then the j-th component of the i-th row of $[\Phi(\phi)^T\tilde{\chi}_t]$ is assumed to equal zero. This reduces the set of possible detection/permutation hypotheses to be evaluated at moment t for various ϕ to $\tilde{\mathcal{X}}_t(\phi)$.

IMMJPDA Step 4: Evaluation of the detection/association/mode hypotheses is based on Theorem 2.

For all $\phi \in \{0,1\}^M$, $\tilde{\chi} \in \{0,1\}^{D(\phi)\times D(\phi)}$, $\theta \in \{1,...,N\}^M$:

$$\begin{aligned}\beta_t(\phi,\tilde{\chi},\theta) &= \lambda^{(L_t - D(\phi))}\prod_{i=1}^M [f_t^i(\phi,\tilde{\chi},\theta^i)\cdot \\ &\quad \cdot(1-P_{\rm d}^i)^{(1-\phi_i)}(P_{\rm d}^i)^{\phi_i}\cdot\bar{\gamma}_t^i(\theta^i)/c_t \\ &\qquad\qquad \text{for } \tilde{\chi}\in\tilde{\mathcal{X}}_t(\phi), \\ &= 0 \qquad\qquad \text{else}\end{aligned} \tag{26.a}$$

with $f_t^i(\{0\}^M, \{\}^{L_t}, \theta^i) = 1$ and for $\phi \neq \{0\}^M$:

$$\begin{aligned}f_t^i(\phi,\tilde{\chi},\theta^i) &\cong [(2\pi)^m \mathrm{Det}\{Q_t^i(\theta^i)\}]^{-\frac{1}{2}\phi_i} \\ &\cdot \exp\{-\frac{1}{2}\sum_{k=1}^{L_t}[\Phi(\phi)^T_{*i}\tilde{\chi}_{*k}\mu_t^{ik}(\theta^i)^T[Q_t^i(\theta^i)]^{-1}\mu_t^{ik}(\theta^i)]\}\end{aligned} \tag{26.b}$$

where

$$\mu_t^{ik}(\theta^i) = \mathrm{y}_t^k - h^i(\theta^i)\bar{x}_t^i(\theta^i) \tag{26.c}$$

IMMJPDA Step 5: Measurement update equations are based on Theorem 2. For all $i \in \{1,...,M\}$, $\theta^i \in \{1,...,N\}$:

$$\hat{\gamma}_t^i(\theta^i) \cong \sum_{\substack{\phi,\tilde{\chi},\eta \\ \eta^i=\theta^i}} \beta_t(\phi,\tilde{\chi},\eta) \tag{27.a}$$

$$\hat{x}_t^i(\theta^i) \cong \bar{x}_t^i(\theta^i) + W_t^i(\theta^i)\left(\sum_{k=1}^{L_t}\beta_t^{ik}(\theta^i)\mu_t^{ik}(\theta^i)\right) \tag{27.b}$$

$$\begin{aligned}\hat{P}_t^i(\theta^i) &\cong \bar{P}_t^i(\theta^i) - W_t^i(\theta^i)h^i(\theta^i)\bar{P}_t^i(\theta^i)\left(\sum_{k=1}^{L_t}\beta_t^{ik}(\theta^i)\right) + \\ &+W_t^i(\theta^i)\left(\sum_{k=1}^{L_t}\beta_t^{ik}(\theta^i)\mu_t^{ik}(\theta^i)\mu_t^{ik}(\theta^i)^T\right)W_t^i(\theta^i)^T + \\ &-W_t^i(\theta^i)\left(\sum_{k=1}^{L_t}\beta_t^{ik}(\theta^i)\mu_t^{ik}(\theta^i)\right)\cdot \\ &\cdot\left(\sum_{k'=1}^{L_t}\beta_t^{ik'}(\theta^i)\mu_t^{ik'}(\theta^i)\right)^T W_t^i(\theta^i)^T\end{aligned} \tag{27.c}$$

with:

$$W_t^i(\theta^i) \stackrel{\triangle}{=} \bar{P}_t^i(\theta^i)h^i(\theta^i)^T[Q_t^i(\theta^i)]^{-1} \tag{27.d}$$

$$\beta_t^{ik}(\theta^i) = \sum_{\substack{\phi,\tilde{\chi},\eta \\ \phi\neq 0 \\ \eta^i=\theta^i}} \Phi(\phi)^T_{*i}\tilde{\chi}_{*k}\beta_t(\phi,\tilde{\chi},\eta)]/\hat{\gamma}_t^i(\theta^i) \tag{27.e}$$

with $[.]_{*k}$ the k-th column of $[.]$.

IMMJPDA Step 6: Output equations:

$$\hat{x}_t^i = \sum_{\theta^i=1}^N \hat{\gamma}_t^i(\theta^i)\cdot\hat{x}_t^i(\theta^i)$$

$$\hat{P}_t^i = \sum_{\theta^i=1}^N \hat{\gamma}_t^i(\theta^i)\big(\hat{P}_t^i(\theta^i) + [\hat{x}_t^i(\theta^i) - \hat{x}_t^i]\cdot \\ \cdot[\hat{x}_t^i(\theta^i) - \hat{x}_t^i]^T\big)$$

Remark 1: It can be verified that the above IMMJPDA filter algorithm is similar to the IMMJPDA filter algorithm of Chen & Tugnait (2001). The main new element is that the above specification of IMMJPDA Steps 4 and 5 explicitly shows the relation to the processes $\{\tilde{\chi}_t\}$ and $\{\phi_t\}$. In the sequel this relation is exploited for the development of a track-coalescence-avoiding IMMJPDA filter, for short IMMJPDA* filter.

6 IMMJPDA* filter

A shortcoming of JPDA is its sensitivity to track coalescence. With the JPDA* approach, Blom & Bloem (2000) have shown that this is due to JPDA's merging over permutation hypotheses, and that a suitable hypothesis pruning may provide an effective countermeasure. The JPDA* filter equations can be obtained from the JPDA algorithm by pruning per (ϕ_t,ψ_t)-hypothesis all less likely χ_t-hypotheses prior to measurement updating. In order to apply this approach to IMMJPDA the JPDA* hypothesis pruning strategy is now extended: evaluate all (ϕ_t,ψ_t,θ_t) hypotheses and prune per (ϕ_t,ψ_t,θ_t)-hypothesis all less-likely χ_t-hypotheses. To do so, define for every ϕ, ψ and θ, satisfying $D(\psi) = D(\phi) \leq$ Min$\{M, L_t\}$, a mapping $\hat{\chi}_t(\phi,\psi,\theta)$:

$$\hat{\chi}_t(\phi,\psi,\theta) \stackrel{\triangle}{=} \underset{\chi}{\text{Argmax}}\ \beta_t(\phi,\chi^T\Phi(\psi),\theta)$$

where the maximization is over all permutation matrices χ of size $D(\phi)\times D(\phi)$.
The pruning strategy of evaluating all (ϕ,ψ,θ)-hypotheses and only one χ-hypothesis per (ϕ,ψ,θ)-hypothesis implies that for $D(\phi) > 0$ we adopt the following pruned hypothesis weights $\hat{\beta}_t(\phi,\psi,\theta)$:

$$\begin{aligned}\hat{\beta}_t(\phi,\psi,\theta) &= \beta_t(\phi,\hat{\chi}(\phi,\psi,\theta)^T\Phi(\psi),\theta)/\hat{c}_t \\ &\qquad \text{if } D(\phi)=D(\psi)\leq \text{Min}\{M,L_t\} \\ &= 0 \qquad \text{else}\end{aligned}$$

with $\hat{c}_t$ a normalization constant for $\hat{\beta}_t$; i.e. such that

$$\sum_{\substack{\phi,\psi,\theta \\ D(\psi)=D(\phi)}} \hat{\beta}_t(\phi,\psi,\theta) = 1$$

By inserting these particular weights within IMMJPDA, we get IMMJPDA*. One cycle of the IMMJPDA* filter algorithm consists of 6 steps, all except step five are equivalent to the IMMJPDA steps. IMMJPDA* step 5 reads as follows:

IMMJPDA Step 5:* Track-coalescence hypothesis pruning and measurement update equations.
First evaluate for every (ϕ,ψ,θ) such that $0 < D(\psi) = D(\phi) \leq$ Min$\{M, L_t\}$:

$$\hat{\chi}_t(\phi,\psi,\theta) = \underset{\chi}{\text{Argmax}}\ \beta_t(\phi,\chi^T\Phi(\psi),\theta)$$

Next evaluate all $\hat{\chi}_t(\phi,\psi,\theta)$ hypothesis weights:

$$\begin{aligned}\hat{\beta}_t(\phi,\psi,\theta) &= \beta_t(\phi,\hat{\chi}_t(\phi,\psi,\theta)^T\Phi(\psi),\theta)/\hat{c}_t \\ &\qquad \text{if } 0 < D(\psi) = D(\phi) \leq \text{Min}\{M, L_t\} \\ &= \beta_t(\{0\}^M, \{\}^{L_t}, \theta)/\hat{c}_t \\ &\qquad \text{if } D(\psi) = D(\phi) = 0 \\ &= 0 \qquad \text{else}\end{aligned}$$

where $\hat{c}_t$ is a normalizing constant for $\hat{\beta}_t$.

Measurement update equations for all $i \in \{1,...,M\}$, $\theta^i \in \{1,...,N\}$:

$$\hat{\gamma}_t^i(\theta^i) \cong \sum_{\substack{\phi,\psi,\eta \\ \eta^i=\theta^i}} \hat{\beta}_t(\phi,\psi,\eta) \tag{28.a}$$

$$\hat{x}_t^i(\theta^i) \cong \bar{x}_t^i(\theta^i) + W_t^i(\theta^i)\left(\sum_{k=1}^{L_t} \hat{\beta}_t^{ik}(\theta^i)\mu_t^{ik}(\theta^i)\right) \tag{28.b}$$

$$\begin{aligned}\hat{P}_t^i(\theta^i) &\cong \bar{P}_t^i(\theta^i) - W_t^i(\theta^i)h^i(\theta^i)\bar{P}_t^i(\theta^i)\left(\sum_{k=1}^{L_t}\hat{\beta}_t^{ik}(\theta^i)\right) + \\ &+W_t^i(\theta^i)\left(\sum_{k=1}^{L_t}\hat{\beta}_t^{ik}(\theta^i)\mu_t^{ik}(\theta^i)\mu_t^{ik}(\theta^i)^T\right)W_t^i(\theta^i)^T + \\ &-W_t^i(\theta^i)\left(\sum_{k=1}^{L_t}\hat{\beta}_t^{ik}(\theta^i)\mu_t^{ik}(\theta^i)\right)\cdot \\ &\cdot\left(\sum_{k'=1}^{L_t}\hat{\beta}_t^{ik'}(\theta^i)\mu_t^{ik'}(\theta^i)\right)^T W_t^i(\theta^i)^T\end{aligned} \tag{28.c}$$

with:

$$W_t^i(\theta^i) \triangleq \bar{P}_t^i(\theta^i)h^i(\theta^i)^T[Q_t^i(\theta^i)]^{-1} \tag{28.d}$$

$$\hat{\beta}_t^{ik}(\theta^i) = \sum_{\substack{\phi,\psi,\eta \\ \phi,\psi\neq 0 \\ \eta^i=\theta^i}} [\Phi(\phi)_{*i}^T[\hat{\chi}_t(\phi,\psi,\eta)^T\Phi(\psi)]_{*k}\cdot \cdot\hat{\beta}_t(\phi,\psi,\eta)]/\hat{\gamma}_t^i(\theta^i) \tag{28.e}$$

where $[.]_{*k}$ is the k-th column of $[.]$.

7 Monte Carlo simulations

In Blom & Bloem (2002) Monte Carlo simulation results are given for the IMMJPDA and IMMJPDA* filter algorithms, and for an IMMPDA which updates an individual track using PDA by assuming the measurements from the adjacent targets as false. The simulations primarily aim at gaining insight into the behavior and performance of the filters when objects move in and out close approach situations, while giving the filters enough time to converge after a manoeuvre has taken place. In the example scenarios there are two targets, each modelled with two possible modes. The first mode represents a constant velocity model and the second mode represents a constant acceleration model. Both objects start moving towards each other, each with constant initial velocity V_{initial} (i.e. the initial relative velocity $V_{\text{rel, initial}} = 2V$). At a certain moment in time both objects start decelerating with -0.5 m/s^2 until they both have zero velocity. The moment at which the deceleration starts is such that when the objects both have zero velocity, the distance between the two objects equals d. After spending a significant number of scans with zero velocity, both objects start accelerating with 0.5 m/s^2 away from each other without crossing until their velocity equals the opposite of their initial velocity. From that moment on the velocity of both objects remains constant again (thus the final relative velocity $V_{\text{rel, final}} = V_{\text{rel, initial}}$). Note that $d < 0$ implies that the objects have crossed each other before they have reached zero velocity. Each simulation the filters start with perfect estimates and run for 40 scans. An example of the trajectories for $d > 0$ is depicted in figure 1.

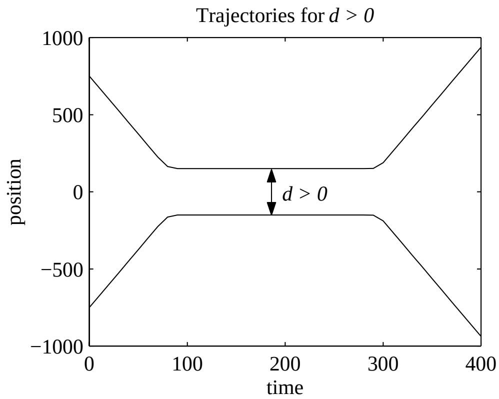

Figure 1: Trajectories example for $d > 0$

For each target, the underlying model of the potential target measurements is given by (1) and (3)

$$x_{t+1}^i = a^i(\theta_{t+1}^i)x_t^i + b^i(\theta_{t+1}^i)w_t^i \tag{1}$$

$$z_t^i = h^i(\theta_t^i)x_t^i + g^i(\theta_t^i)v_t^i \tag{3}$$

Furthermore for $i = 1,2$ and $\theta_t^i \in \{1,2\}$:

$$a^i(1) = \begin{bmatrix} 1 & T_s & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \qquad a^i(2) = \begin{bmatrix} 1 & T_s & \frac{1}{2}T_s^2 \\ 0 & 1 & T_s \\ 0 & 0 & 1 \end{bmatrix}$$

$$b^i(1) = \sigma_a^i \cdot \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \qquad b^i(2) = \sigma_a^i \cdot \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$h^i = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}, \qquad g^i = \sigma_m^i$$

$$\Pi = \begin{bmatrix} 1 - T_s/\tau_1 & T_s/\tau_1 \\ T_s/\tau_2 & 1 - T_s/\tau_2 \end{bmatrix}$$

where σ_a^i represents the standard deviation of acceleration noise and σ_m^i represents the standard deviation of the measurement error. For simplicity we consider the situation of similar targets only; i.e. $\sigma_a^i = \sigma_a$, $\sigma_m^i = \sigma_m$, $P_{\text{d}}^i = P_{\text{d}}$. With this, the scenario parameters are P_{d}, λ, d, V_{initial}, T_s,

σ_m, σ_a, τ_1, τ_2, and the gate size γ. We used fixed parameters $\sigma_m = 30$, $\sigma_a = 0.5$, $\tau_1 = 500$, $\tau_2 = 50$, and $\gamma = 25$. Table 1 gives the other scenario parameter values that are being used for the Monte Carlo simulations.

Table 1: Scenario parameter values.
IMMPDA's $\lambda = 0.00001$ for scenarios 1 and 3

Scenario	P_{d}	λ	d	V_{initial}	T_s
1	1	0	Variable	7.5	10
2	1	0.001	Variable	7.5	10
3	0.9	0	Variable	7.5	10
4	0.9	0.001	Variable	7.5	10

During our simulations we counted track i "O.K.", if

$$| h^i \hat{x}_T^i - h^i x_T^i | \leq 9\sigma_m$$

and we counted track $i \neq j$ "Swapped", if

$$| h^i \hat{x}_T^i - h^j x_T^j | \leq 9\sigma_m$$

Furthermore, two tracks $i \neq j$ are counted "Coalescing" at scan t, if

$$| h^i \hat{x}_t^i - h^j \hat{x}_t^j | \leq \sigma_m \wedge | h^i x_t^i - h^j x_t^j | > \sigma_m$$

For each of the scenarios Monte Carlo simulations containing 100 runs have been performed for each of the tracking filters. To make the comparisons more meaningful, for all tracking mechanisms the same random number streams were used. The results of the Monte Carlo simulations for the three scenarios are pictured in Blom & Bloem (2002) and summarized in Tables 2 through 4 below:

- The average percentage of Both tracks "O.K." (Table 2).
- The average number of "coalescing" scans (Table 3).
- The percentage of Both tracks "O.K." or "Swapped" (Table 4).

Table 2: Average % Both tracks "O.K.".

Scenario	IMMPDA	IMMJPDA	IMMJPDA*
1	18	67	74
2	8	56	67
3	8	63	69
4	3	42	49

For the example considered, the simulation results show that both IMMJPDA* and IMMJPDA perform much better than IMMPDA. Moreover the results show that IMMJPDA* avoids track coalescence and is less sensitive to track loss than IMMJPDA is.

Table 3: Average number of coalescing scans.

Scenario	IMMPDA	IMMJPDA	IMMJPDA*
1	9.8	1.5	0.4
2	11.1	1.9	0.3
3	18.6	1.7	0.5
4	14.7	2.6	0.5

Table 4: Average % Both tracks "O.K." or "swapped".

Scenario	IMMPDA	IMMJPDA	IMMJPDA*
1	27.2	99.6	99.9
2	17.8	93.2	97.4
3	9.8	99.8	100
4	6.4	75.2	79.4

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