Nationaal Lucht- en Ruimtevaartlaboratorium
National Aerospace Laboratory NLR

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Probabilistic Wake Vortex Induced Accident Risk Assessment

Final Issue

J. Kos, H.A.P. Blom, L.J.P. Speijker, M.B. Klompstra and G.J. Bakker

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Abstract

New air traffic management (ATM) concepts for departure and landing on busy airports might have a major impact on capacity if the wake vortex induced risks are better understood. The present wake vortex separation standards for the use of a single runway are often limiting capacity too much. To give ATM concept developers feedback with respect to the safety of new ATM concepts, a novel probabilistic methodology is under development for the assessment of wake vortex induced accident risks. The results of the safety assessment give insight in critical ATM design issues and thereby well-founded advice can be given on improvement of the design of the ATM concept.

The probabilistic wake vortex induced accident risk assessment is based on recent progress in wake vortex research. Commonly accepted models for wake vortex evolution and wake encounter have been adapted and integrated through a stochastic modeling and analysis based accident risk assessment methodology.

This paper outlines this probabilistic wake vortex induced accident risk assessment methodology and illustrates its initial application to the case of multiple aircraft landing on a single runway.

Contents

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Abstract

New air traffic management (ATM) concepts for departure and landing on busy airports might have a major impact on capacity if the wake vortex induced risks are better understood. The present wake vortex separation standards for the use of a single runway are often limiting capacity too much. To give ATM concept developers feedback with respect to the safety of new ATM concepts, a novel probabilistic methodology is under development for the assessment of wake vortex induced accident risks. The results of the safety assessment give insight in critical ATM design issues and thereby well-founded advice can be given on improvement of the design of the ATM concept.

The probabilistic wake vortex induced accident risk assessment is based on recent progress in wake vortex research. Commonly accepted models for wake vortex evolution and wake encounter have been adapted and integrated through a stochastic modeling and analysis based accident risk assessment methodology.

This paper outlines this probabilistic wake vortex induced accident risk assessment methodology and illustrates its initial application to the case of multiple aircraft landing on a single runway.

I. Introduction

New ATM concepts for departure and landing on busy airports with multiple runways might have a major impact on capacity if the wake vortex induced risks are better understood. In particular, there is a need for identifying the conditions under which the present wake vortex separation standards would limit capacity too much. In Ref. [1], an initial probabilistic methodology has been developed to assess such safe separations in case of a single runway.

There are several reasons why there is a need to extend this initial methodology, e.g.:

1. To guide and incorporate ongoing developments in wake vortex induced risk modeling;
2. To generalize its application from a single runway to closely spaced runways;
3. To allow the evaluation of advanced ground and/or airborne procedures that make use of wake vortex detection and decision support systems;
4. To allow the evaluation of safe separation standards for new aerodynamic aircraft designs;

5. To integrate it with a methodology that assesses risk of collision with other aircraft in the air or on the ground.

These reasons provided a clear motivation to extend the accident risk assessment methodology to enable the determination of safe separations for advanced ATM [2]. This extension resulted in a complementary and novel probabilistic methodology for the assessment of wake vortex induced accident and incident risks.

The aim of this paper is to show how the accident risk assessment methodology can be used to support the assessment of wake vortex induced accident and incident risks, and how advantage is taken from recent progress in wake vortex research, e.g. Refs. [1], [3], and [4]. The methodology is also illustrated for an example of a medium-weighted aircraft landing behind a heavy-weighted aircraft on a (single) runway.

The paper is organized as follows. Section II outlines the risk assessment methodology and the supporting tool sets. In Section III the wake vortex specific models are explained in more detail. Section IV illustrates the application to the specific single runway example. Section V draws conclusions.

II. Risk Assessment Methodology

As the basis for the development of a wake vortex risk induced assessment approach use is made of a stochastic analysis and modeling approach based accident risk assessment methodology for advanced ATM operations [2]. This methodology supports the spiral development cycle that is part of modern safety case building for new ATM operational concepts, [5], [6]. Such a cycle is typically of the form:

A. Design of an ATM operational concept.
B. Assessment of the ATM concept, resulting in a cost-benefit overview.
C. Detailed analysis of the assessment results, which results in recommendations for improvements of the ATM concept.
D. Review ATM concept development strategy and plan.
E. Back to A: adapted and/or more detailed ATM concept design using the results from C resulting in a new or optimized ATM concept.

This stochastic analysis and modeling based accident risk assessment methodology has been developed to provide designers of advanced ATM with safety feedback following on a (re)design cycle and is referred to as Traffic Organization and Perturbation AnalyZer (TOPAZ), see Fig. 1.

During the assessment cycle four stages are sequentially conducted:

1. Stage 1: Identification of operation and hazards (upper box in Fig. 1). Information about nominal and non-nominal behavior of the ATM concept or procedure is gathered, through hazard identification sessions with a variety of experts.
2. Stage 2: Mathematical modeling (lower right box in Fig. 1). A stochastic dynamical model of the operation is developed that incorporates both all the nominal and all the non-nominal events of the operation. During this stage, all model assumptions made are systematically specified.
3. Stage 3: Accident risk assessment (middle box in Fig. 1). The mathematical model of stage 2 supports an effective procedure, consisting of a number of steps to be followed, to quantify the accident risk. In addition to such numerical approach, a qualitative analysis of the model assumptions is performed.
4. Stage 4: Feedback to operational experts (lower left box in Fig. 1). The results of the quantitative safety assessment are fed back to and discussed with the designers and operational experts, who can use the results to redesign or optimize their proposed ATM design if necessary.

For the second and third stages use can be made of the following TOPAZ tools:

1. SIMULATOR is a tool set used to specify and implement the mathematical model and to subsequently run Monte Carlo simulations with that implementation. SIMULATOR can simulate all aspects of operations, including the stochastic non-nominal aspects.
2. COLLIR is a methodology and tool set that supports the evaluation of collision risks in the terminal manoeuvring area (TMA) and en route.
3. TAXIR is a methodology and tool set that supports the evaluation of accident risks at the airport.
4. CRITER is a risk criteria framework that is needed to judge the acceptability of the risks that are assessed by COLLIR, WAVIR and TAXIR.

The methodological parts of COLLIR, WAVIR and TAXIR incorporate the evaluation of statistical data that are obtained either through empirical data collections or through Monte Carlo simulations (e.g., SIMULATOR). One should be aware that for each of the tools further extensions are ongoing at the National Aerospace Laboratory NLR.

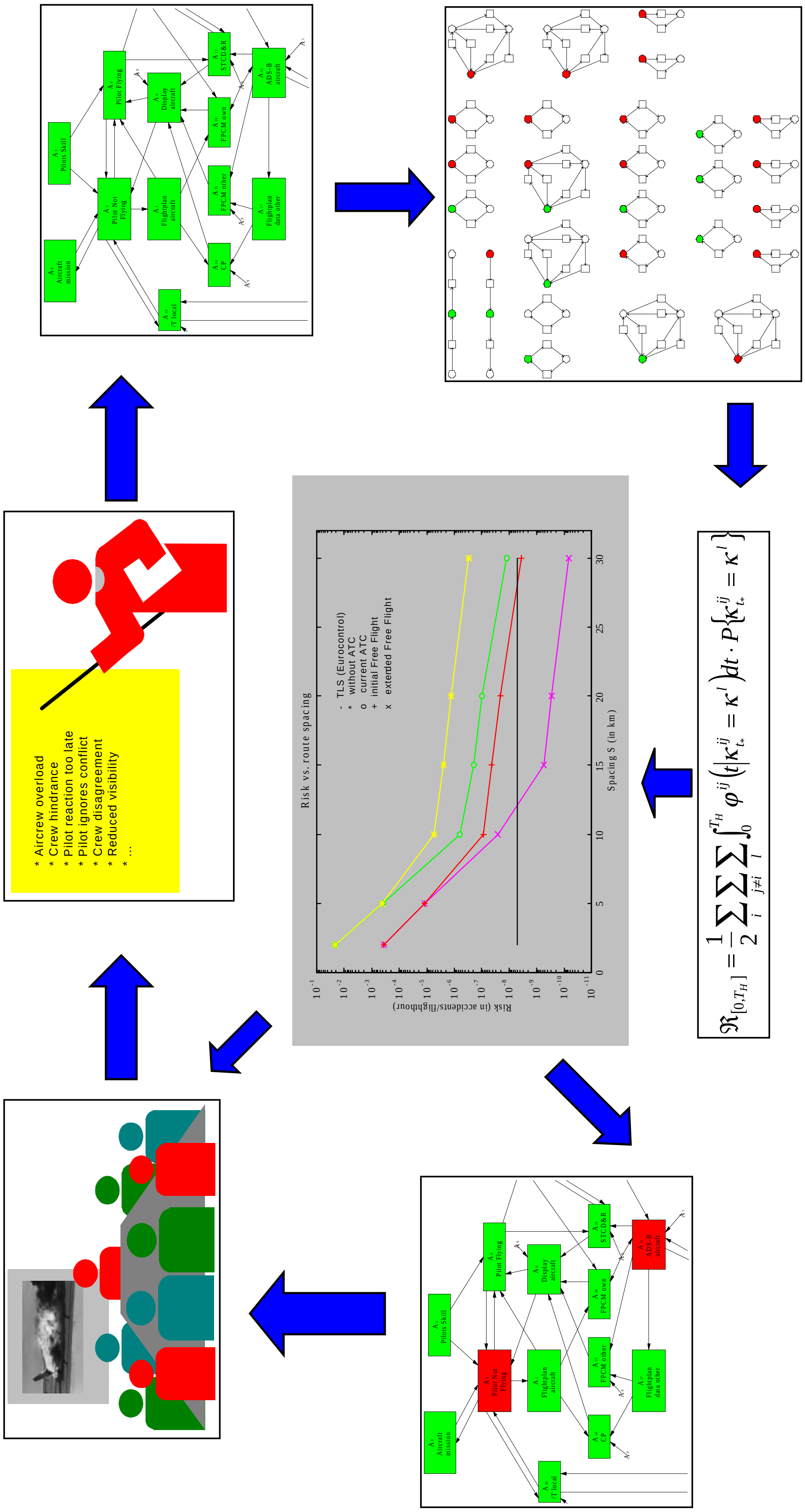

Figure 1: TOPAZ accident risk assessment cycle.

III. Wake Vortex Risk Assessment

A. Overview

For the assessment of the wake vortex induced accident and incident risks the following tools are used:

1. flight path evolution (in SIMULATOR),
2. wake vortex evolution and decay (in WAVIR),
3. wake encounter model (in WAVIR),
4. integration and risk evaluation model (in WAVIR), and
5. risk criteria framework (in CRITER).

These tools are described in Sections III.B-III.E, together with references that give more details about the models used.

B. Flight Path Evolution

The flight path evolution model yields the following stochastic variables:

1. the lateral and vertical coordinates of the leader if its longitudinal co-ordinate x is given,
2. the period of time elapsed between the generation of the wake and the time instant that the trailer has longitudinal position x, and
3. the lateral and vertical coordinates of the trailer when it has longitudinal coordinate x.

The flight path evolution model is a stochastic dynamical model, which incorporates the established International Civil Aviation Organization-Collision Risk Model (ICAO-CRM) [7] as baseline, and which has been further developed to handle the dependent usage of closely spaced runways [8].

The flight path evolution model is represented in a form [9] that allows a straightforward extension of the SIMULATOR tool set for new air and/or ground procedures and advanced vortex detection and decision-support systems.

C. Wake Vortex Evolution and Decay

The wake vortex evolution model yields the position and strength stochastic variables of the wake vortex at any time instant after the generation of the wake vortex. The wake vortex evolution model is mainly based on Refs. [3] and [4]. The models in the latter papers have been probabilitized and they have been completed for application to the wake vortex induced risk assessment.

This wake vortex evolution model is able to take into account probabilistic models for stratification, atmospheric turbulence, ground effects (rebound, divergence), and crosswind (advection, shear). It can also handle probabilistic models for the vertical and horizontal wind fields and their impact on wake evolution.

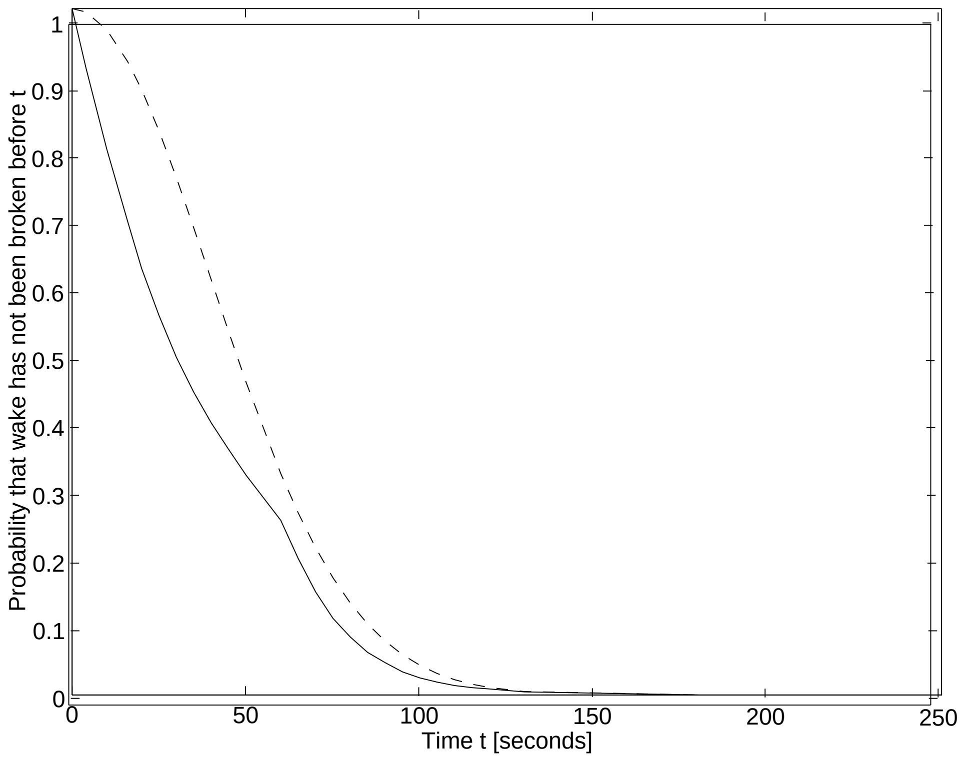

Figure 2: Observed vortex residence time distribution for B-747 vortices with initial height 30 metres (solid line), initial strength of the wake of 600 m^2/s and wingspan 60 metres (from Fig. 2 in Ref. [10]); Rayleigh density adopted in WAVIR for the vortex bursting or linking period (dashed line).

During many landings, the trailer aircraft does not meet any wake vortex. In the wake vortex evolution model, two possible causes are distinguished:

1. the wake vortex has disappeared due to a gradual diminishing of its strength, and
2. the wake vortex has disappeared due to a sudden bursting or linking.

In Ref. [3], for the latter an analytical model has been proposed that assumes bursting and linking to happen in time as a function of some meteorological parameters. To better account for observed data, in WAVIR the probabilistic bursting and linking period is modeled independently of the vortex evolution and decay as a stochastic variable with a Rayleigh density, the mean of which is assumed equal to 50 s. This Rayleigh density is depicted in Fig. 2 together with empirical data for vortex residence period. This Rayleigh density modeling also differs significantly from the theoretical probability density model of Kuzmin [1].

D. Wake Vortex Encounter Model

The wake vortex encounter model yields the probability that the wake vortex induced rolling moment is larger than the maximum control capability – in terms of rolling control moment – of the encountering aircraft.

The wake vortex control capability model is based on Refs. [1] and [11] and accounts for the fact that the encountering aircraft tries to compensate for the wake vortex flow field generated by the leader only. In line with this, the key effect is to reduce the rolling moment calculated with the wake vortex evolution model. The aircraft control capability model is also based on Ref. [1] and assumes a value twice as high as the minimal requirement by the British Civil Airworthiness Requirements (BCAR), which is also adopted by Joint Aviation Authority (JAA), [12].

It is important to realize that this approach inherently involves an important modeling assumption: the pilot is *not* able to anticipate timely on the first signs of a wake vortex. In practice, a pilot of an encountering aircraft might respond with the immediate initiation of a missed approach when its aircraft experiences a slight roll upset, i.e., in a very early stage of a possible wake encounter. Hence, this Kuzmin model is expected to imply a pessimistic effect on the quantified risk near the threshold.

E. Model Integration and Numerical Evaluation

To assess numerically the wake vortex induced accident risk, the models described in Sections III.B-III.D are integrated through a stochastic model for wake vortex induced risk (see Section VI). Based on this model a numerical assessment procedure has been developed, which is carried out in seven steps:

Step 1: The parameters in the wake vortex evolution model are identified and the parameter distributions are based on empirical data and/or state-of-the-art literature. In addition, a set of relevant longitudinal positions x is determined.

Step 2: Run Monte Carlo simulations with the wake vortex evolution model for the case in which the wake vortex is generated when the leading aircraft has longitudinal position x. The position, strength, and core radius of the wake vortex are obtained at the time instant that it has the same longitudinal coordinate as the trailer aircraft. The latter time instant follows from Monte Carlo simulations with the SIMULATOR tool.

Step 3: The simulation results from Step 2 are analyzed. Based on this analysis, a dedicated probability density fitting procedure is identified that accounts for dependencies between the position coordinates, the strength, and the core radius of the wake vortex. The probability density fitting procedure is carried out and the joint distribution of the wake vortex position, strength, and core radius is obtained.

Step 4: Monte Carlo simulations are carried out to simulate the wake vortex encounter. In this step the joint distribution from Step 3 is used and distributions of the position of the trailer aircraft obtained with the SIMULATOR tool set are used.

Step 5: Step 5 concerns the numerical evaluation of the wake induced accident risk due to a wake vortex that is generated when the leading aircraft was at position *x*.

Step 6: The wake induced accident risk is obtained by maximizing over *x* the risk obtained in Step 5.

Step 7: Perform a qualitative evaluation of the influence of the modeling assumptions on the estimated accident risk.

The results for these seven steps are illustrated for the case of a single runway example, in Section IV.

F. Risk Criteria Framework

To judge whether a newly proposed ATM concept is safe or to determine more appropriate safe separation distances, a suitable metric for quantification of wake vortex induced risk is required. Up to now several metrics have been used to quantify the risk imposed by wake vortices, e.g., bank angle, roll angle, roll rate and roll control ratio. However, because they do not relate to the safety perception of involved interest groups (e.g., crew, passengers, controllers, regulators, people living in the airport vicinity), they are felt to be insufficient. Other possible risk metrics are the risk probability per movement and the risk probability per year.

In Ref. [13] some initial guidelines are developed for the assessment of safety requirements. It discusses two possible safety management approaches: the as-low-as-reasonably-practicable (ALARP) approach and the target level of safety (TLS) approach. Ranges are suggested from which to adopt a TLS for the risk event *probabilities per movement*.

For the adoption of applicable risk criteria, it is clear that policy makers definitively have to be involved, and also the relation with existing wake vortex induced incident and accident frequencies should be clearly identified.

G. From Risk to Safe Separation

By assessing accident risks for various separation distances, one arrives at a curve that shows the risk as a function of the separation distance between successive aircraft. Figure 3 illustrates how such a curve subsequently maps an ALARP region in terms of risk into one in terms of separation distances.

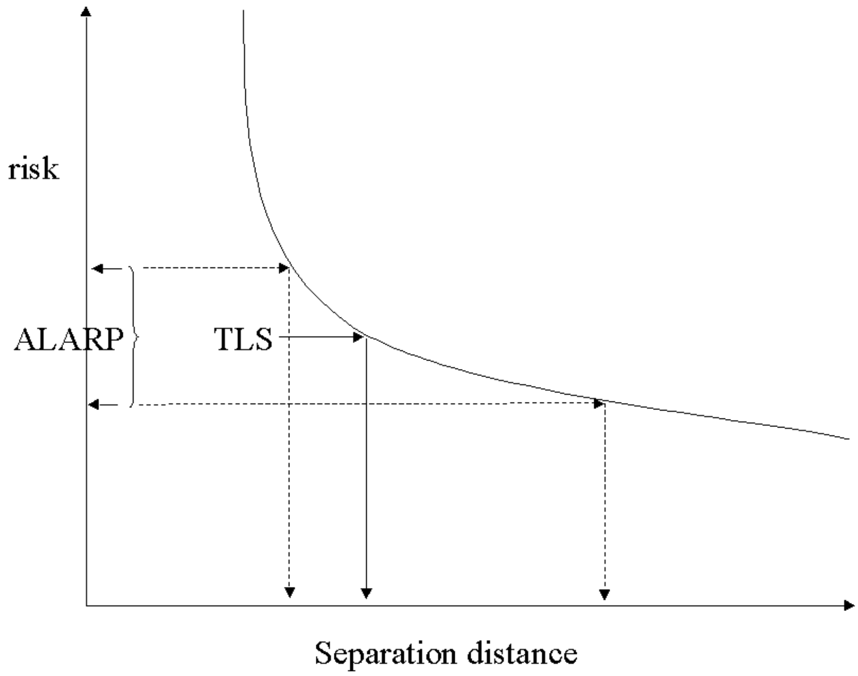

Figure 3: Wake vortex risk vs separation distance.

IV. Single Runway Approach

A. Boeing 737-400 Behind Boeing 747-400

To illustrate the novel wake vortex induced accident risk assessment methodology we consider a (single) runway, on which a Boeing 737-400 aircraft, which is in the ICAO medium-weight class, is landing behind a Boeing 747-400 aircraft, which is in the ICAO heavy-weight class, with controller expected separation distance of 5 n miles when the heavy is at the threshold. For both aircraft, it is assumed that the approach is instrument landing system (ILS) Category I.

The landing phase starts at about 20 km before the threshold, and ends at touchdown, which is 300 m beyond threshold. Figure 4 shows the side view of the runway and glide slope, where the *x*-axis is along the runway centerline and positive in runway direction.

Because of its stochastic dynamical modeling basis, the novel wake induced risk assessment methodology clearly allows to bring the assumptions made to the foreground. For the example considered, the following main assumptions have been adopted:

1. Long landings (landings far beyond threshold) do not happen.
2. A wake vortex induced accident event is characterized by the wake induced rolling moment being larger than the aircraft control capability.

3. A pilot does not respond with the initiation of a missed approach when its aircraft experiences a slight roll upset.
4. Bursting and linking probabilities are modeled by a Rayleigh density with mean 50 s.
5. There is no head wind, no tail wind and no vertical wind. The wind speed in lateral direction is normally distributed with expectation 0 and standard deviation 1.5 m/s.
6. There are no wind shear layers.
7. Turbulence of the air is 10% of the wind speed.

In addition to these main assumptions, several other assumptions have been made. It would go beyond the scope of this paper to list all these assumptions.

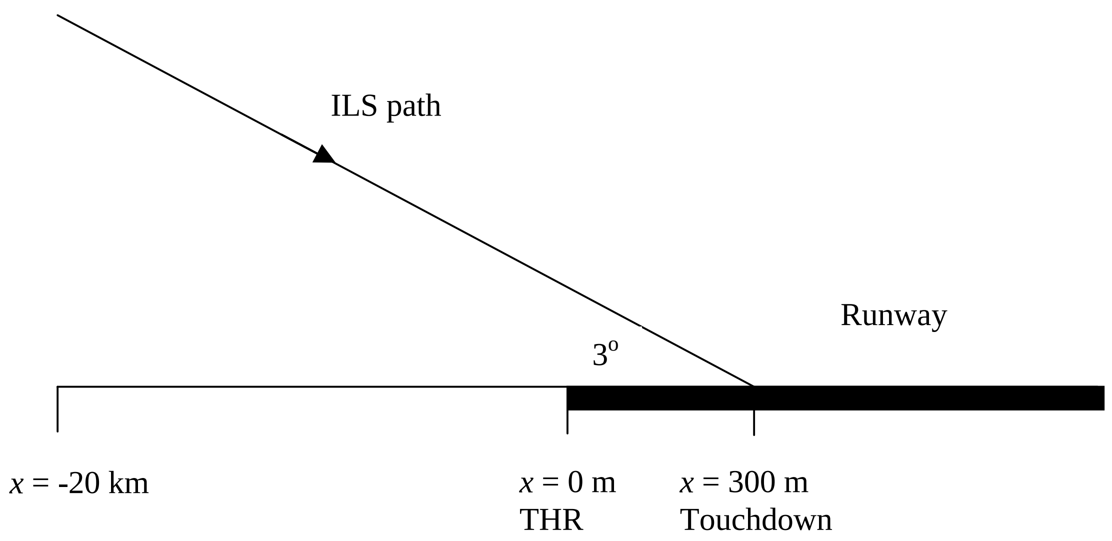

Figure 4: Side view of runway and glide slope.

B. Numerical Results

With support of the SIMULATOR and WAVIR tool sets, the wake vortex induced accident risk is evaluated for the single runway scenario.

Figures 5 and 6 show data plots of the left vortex for the case that the wake vortex is generated at 4 km before threshold (cf. output of Step 3 in Section III.E).

Subsequently, Fig. 7 shows the results for the wake induced accident risk resulting from a wake that is generated at $-x$ km before the threshold. The vertical axis has a logarithmic scale. The + signs indicate the values of x (cf. output of Step 5 in Section III.E).

Figure 7 shows that the estimated values for the accident risk that is instantaneously induced by wake vortices along the glide slope decrease from 20 km until approximately 4 km before the threshold. The decrease is due to the descent of the wake and the higher navigation precision (in height) of the trailer. At shorter distance from the threshold, the instantaneous risk increases due to the rebound of wakes near the ground.

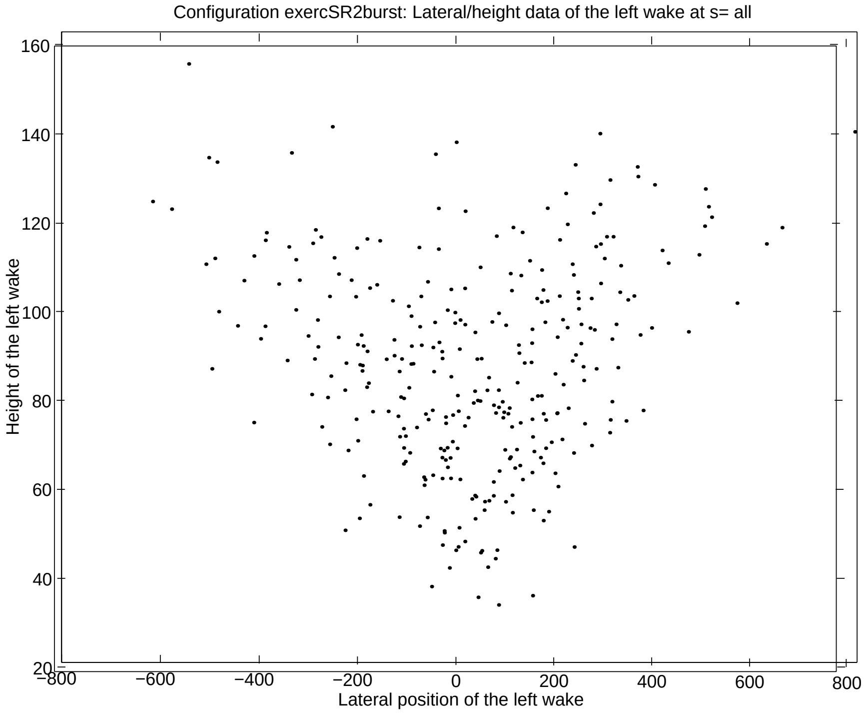

Figure 5: Monte Carlo simulation results of the lateral and vertical coordinate (m) of the center of the left wake that is generated at 4 km before the threshold.

C. Qualitative Uncertainty Analysis

A straightforward maximisation over *x* for the curve in Fig. 7 would lead to an overall maximum risk at the threshold. However, prior to the maximization one should take into account that the calculated wake vortex induced accident risk curve may bear significant bias and/or uncertainty both in positive and negative directions. Usage of such a curve without taking into consideration existing bias and/or uncertainty can inspire undue conclusions.

To understand the impact of the assumptions on the wake vortex induced risk, assumptions 1-7 have been analyzed in a qualitative way. The results are given in Table 1. The first column refers to the assumptions. The second column gives for each assumption the expected direction of the effect on the wake vortex induced risks (optimistic, pessimistic or neutral), and the last column gives the expected magnitude (major or significant). A pessimistic expected direction means that the modeled risk increases due to the assumption. An optimistic expected direction means that the modeled risk reduces due to the assumption. A neutral direction means that there exists uncertainty about the direction.

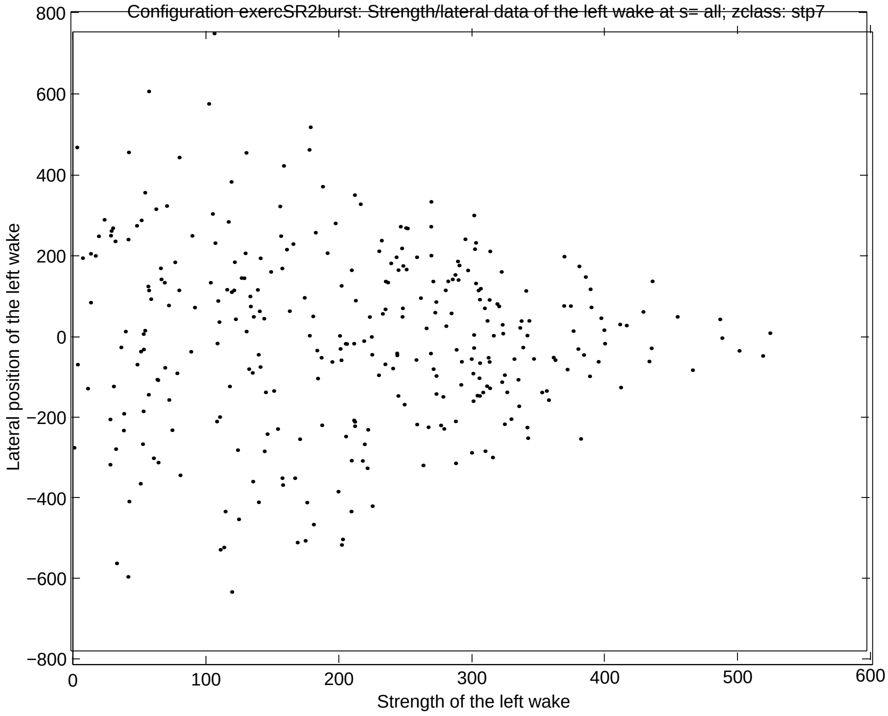

Figure 6: Monte Carlo simulation results of the lateral coordinate (m) and the strength (m^2/s) of the center of the left wake that is generated at 4 km before the threshold.

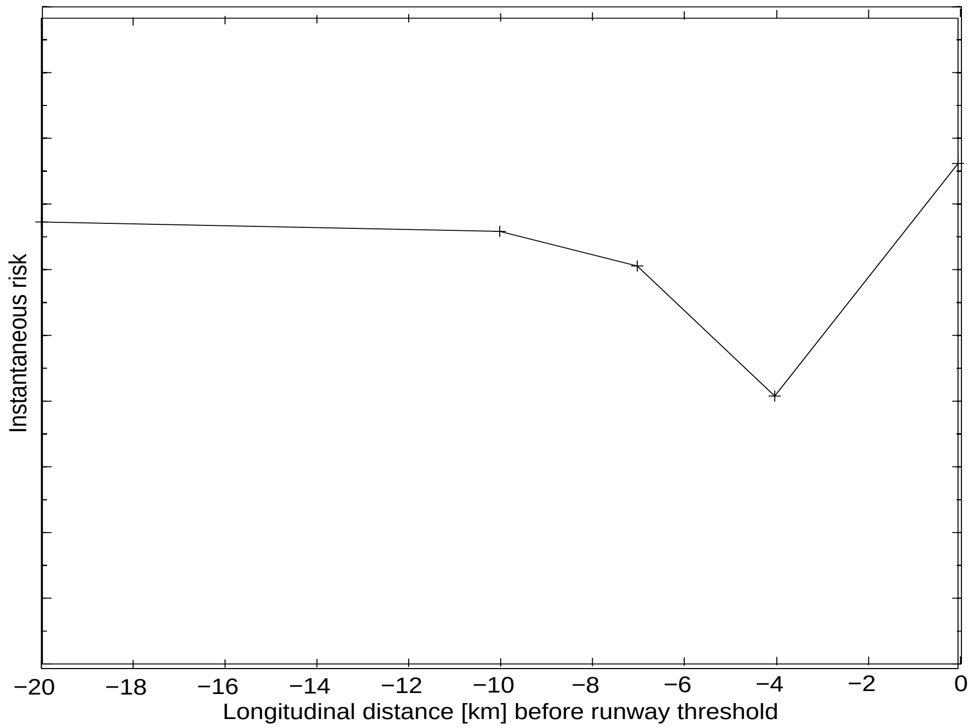

Figure 7: Estimated values for the severe risk that is instantaneously induced by wake vortex along the glide slope. The vertical axis has a logarithmic scale.

Table 1: Expected effects of the main assumptions on assessed risk.

Assumption	Expected direction of effect on wake vortex induced accident risk	Expected magnitude
1	Optimistic	Significant
2	Neutral	Significant
3	Pessimistic	Major
4	Neutral	Significant
5	Neutral	Significant
6	Optimistic	Major
7	Pessimistic	Significant

This qualitative analysis has also been applied to all other assumptions. Because their effect on the wake induced accident risk has been estimated as being either minor or negligible, these assumptions are not listed in this paper.

D. Discussion of Results

If one takes into account the impact of the assumptions 1-7 then the curve in Fig. 7 shows that there are two distinct areas where the instantaneous wake vortex induced accident risks along the ILS are not negligible:

1. near the threshold, this is due to the ground effect on the wake evolution, and
2. at distances larger than 10 km from the threshold, due to larger ILS navigation errors at further distances from the threshold.

Near the threshold, the effect of assumption 6 is negligible. Assumption 3 is the only assumption that has a major impact (pessimistic). The net effect of all assumptions is that the very right part of the curve in Fig. 7 has a major level of uncertainty with a clear bias in the pessimistic direction.

At distances larger than 10 km from the threshold, the effect of assumptions 1 and 3 is negligible. Assumption 6 is the only assumption that has a major impact (optimistic). The net effect of all assumptions is that the left part of the curve in Fig. 7 has a major level of uncertainty with a clear bias in the optimistic direction.

The example shows that there are a few directions that specifically deserve the development of improved wake vortex induced risk models. These directions can be placed in the following two groups:

1. General modeling is needed to mitigate the need for the neutral and pessimistic assumptions 2, 3, 4 and 7.

2. Airport specific modeling is needed to mitigate the need for the neutral and optimistic assumptions 1, 5 and 6.

V. Concluding Remarks

A. Probabilistic Approach

This paper has outlined a probabilistic methodology to assess wake vortex induced accident risks. The aim is to understand the safety evaluation of established separation standards for current operations, and of new separation standards for new operational concepts and aircraft designs for busy airports with closely spaced runways.

The probabilistic methodology, the tool sets, and the models initially adopted have been outlined in Sections II and III respectively. The novelty of this methodology is the probabilistic integration of the models that are available from the complementary domains that play a key role in wake vortex risk assessment:

1. wake vortex evolution and decay model [3], [4];
2. fight path evolution model [7], [8];
3. wake vortex encounter model [1], [11]; and
4. risk criteria framework [13].

The probabilistic integration of these submodels has been accomplished through first developing an integral stochastic model (see Section VI), and subsequently using this to develop a hierarchical Monte Carlo simulation scheme (see Section III.E).

Subsequently, in Section IV, illustrative numerical results of the methodology are given for a Boeing 737-400 aircraft landing on a single runway behind a Boeing 747-400 aircraft, with expected separation distance of 5 n miles at the threshold. The numerical and complementary qualitative results obtained for this example clearly show that the methodology is able to identify the key bottlenecks in developing advanced wake vortex procedures. In the current situation, the safety of the established operations is insufficiently understood in a few key areas. Most of these areas ask for general modeling effort. A few areas only ask for airport specific modeling effort.

B. General Modeling Areas

There are four key general modeling areas that deserve significant modeling effort. A relatively simple one is to improve the probabilistic modeling of navigation performance and long landing models under different navigation modes and various wind conditions. The basis for this activity is one of collecting statistical data of aircraft navigation performance.

The second area is to improve the modeling of autopilot reactions to wake induced roll upset, and missed approaches initiated by pilots as a reaction to experiencing a roll upset during the ILS approach. The basis for this activity seems to be one of analyzing pilot incident reports on missed approaches. In addition, flight simulation data should be used to develop models that represent the pilot behavior during wake vortex encounter.

The third area is to improve the modeling of bursting and linking phenomena. Because of the existing uncertainty about bursting and linking, and their dependency on weather condition, it is strongly recommended to model bursting and linking with appropriate probability distributions. The distributions are to be validated with real experiments or state-of-the-art computational fluid dynamic models for wake vortex evolution.

The fourth area is the further development of a risk management framework for wake vortex induced accident and incident risks such that it becomes clearly connected to existing wake vortex incident and accident data. This modeling effort can only be concluded in discussions with regulatory authorities and other relevant interest groups (e.g., pilots and controllers). In the current European constellation, this process still is very much airport specific.

C. Airport Specific Modeling Areas

The key airport specific modeling area that deserves significant modeling effort is weather. It is important to realize the major influence of specific weather conditions, in particular wind fields, turbulence, stable stratification, and wind shear [14]. These weather conditions are so airport specific that the developed models have to be tuned for the airport under consideration.

Another key airport specific modeling issue is that each particular airport runway geometry may lead to all kinds of dependencies between runway usage. The particular geometry of an airport layout often leads to airport specific dependencies that involve combinations of wake vortex induced accident risks and risk of collision with another aircraft.

It should also be taken into account that, due to these airport specific modeling needs, different appropriate and safe separation distances might result for different airports.

D. Ongoing Developments

The results obtained so far form a clear motivation to continue the development of the TOPAZ/WAVIR methodology toward the assessment of wake vortex induced accident and incident risks. The results obtained with the methodology in its current state already provide clear overall insight into the large variety of wake vortex subproblems.

Apart from the single runway example illustrated in this paper, also a closely spaced runway example has been evaluated with the TOPAZ/WAVIR methodology (this was under a study contract with the DFS). This study also provided valuable overall insight into the wake vortex subproblems in case of parallel flying aircraft.

Since January 2000, the National Aerospace Laboratory NLR is leading a major three-year project [named Assessment of Wake Vortex Safety (S-WAKE)] for the European Commission in which key European wake vortex experts are collaborating to develop solutions for the outstanding modeling areas

VI. Appendix: Stochastic Wake Vortex Model

A. Introduction

This appendix presents the stochastic model that has been developed to allow the integration of the wake vortex submodels from the various domains. We assume a situation of a sequence of aircraft which fly on parallel tracks toward multiple closely spaced parallel runways. For the position and velocity components of aircraft *i*, this means there is a well defined stochastic process $\{x_t^i, y_t^i, z_t^i, \dot{x}_t^i, \dot{y}_t^i, \dot{z}_t^i\}$ for the three-dimensional position and speed components of aircraft *i*'s state. In addition, this means that there is a well-defined stochastic process $\{w_{x,t}^i, w_{y,t}^i, w_{z,t}^i\}$ for the wind speed components acting locally on aircraft *i*.

For the evaluation of the wake vortex induced risk, it is necessary to present an appropriate stochastic dynamical model characterization of the wake vortex induced incident and accident risks. This is done as follows. In subsection B, we present a causal stochastic model for the wake vortex random field. Subsequently, in subsection C, we extend this with a stochastic model for the wake vortex induced roll moment for the following aircraft, together with a model for the compensation of this roll moment. Next, in subsection D, we use our stochastic dynamical model to present a model for the wake vortex induced risk.

B. Wake Vortex Random Field Model

The left and right centres of the vortex at moment *s* which are generated by aircraft *j* at moment *t*, are represented by the two parameter random fields $\delta^{j-}(t,s)$ and $\delta^{j+}(t,s)$, with $s \geq 0$, each of which assumes (y,z) values in $\mathbb{R}^2$. At moment $t+s$, the strengths of the left and right vortices that are generated by aircraft *j* at moment *t* are represented by the two parameter random fields $\gamma^{j-}(t,s)$ and $\gamma^{j+}(t,s)$, each of which assumes strength values in $\mathbb{R}$.

To shorten our notation in the sequel, we place the above components into a joint $\mathbb{R}^6$ -valued random field $v^j(t,s)$:

$$v^j(t,s) \overset{\Delta}{=} \text{column}\{\delta^{j-}(t,s),\ \delta^{j+}(t,s),\ \gamma^{j-}(t,s),\ \gamma^{j+}(t,s)\}$$

Research is ongoing to develop differential equations for the motion and decay of the components of the joint random field $\nu^j(t,s)$. Widely known equations in current literature are the ones given by Ref. [3], which are largely based on those of Refs. [15] and [16]. When adding a straightforward extension for non-zero wind velocity in z direction, these equations are of the form:

$$\frac{d\nu^j(t,s)}{ds} = f\left(\nu^j(t,s), \kappa^j(s)\right)$$

with $\kappa^j_t(s)$ denoting local external influences such as the local cross-wind $w^j_{y,t}$ and the local vertical wind $w^j_{z,t}$ at moment t, the drag coefficient C^j_D of j's "wake oval", the local Brünt-Väisälä frequency $N^j(t+s)$, and the RMS velocity of atmospheric turbulence $q^j(t+s)$.

To uniquely define the solution of the latter differential equation for $s \geq 0$, we have to characterize the components of $\nu^j(t,0)$. From Refs. [5] and [17], we know

$$\delta_y^{j\pm}(t,0) = y_t^j \pm \tfrac{1}{2} b_0^j$$
$$\delta_z^{j\pm}(t,0) = z_t^j$$
$$\gamma^{j\pm}(t,0) = \pm\Gamma_{0,t}^j \equiv \pm \frac{m^j g}{b_0^j \rho_t^j}\left(\dot{x}_t^j - w_{x,t}^j\right)^{-1}$$

with b^j_0 the initial spacing between the primary vortex centres, m^j the mass of aircraft j, g the gravitational acceleration, and ρ^j_t the local air density.

Next we have to characterize the moment in time that a wake generated at longitudinal position x by aircraft j will arrive at the longitudinal position of aircraft i. To do so we assume that the velocity in longitudinal direction of the wind acting on the wake is constant. Then that moment in time is a stopping time τ^{ij}_x, which is defined by:

$$\tau_x^{ij} = \tau_x^j + \inf_s \left\{ s > 0 \,\middle|\, x^i_{\tau_x^j + s} = x + s\, w^j_{x,\tau_x^j} \right\}$$

with

$$\tau_x^j \equiv \inf_t \left\{ t \,\middle|\, x_t^j = x \right\}.$$

By the very nature of the situation we consider, the airspeeds of both aircraft in x direction are bounded and either both strictly positive or both strictly negative. In view of this, the latter equation this means that $\{\tau^{ij}_x\}$ is a strictly increasing continuous process. Hence, we can define an $\mathbb{R}^6$-valued stochastic process $\{\nu^{ij}_t\}$ to represent the actual contribution to aircraft i of the wake vortex generated by aircraft j, as follows:

$$\nu^{ij}_{\tau_x^{ij}} \equiv \nu^j\left(\tau_x^j, \tau_x^{ij} - \tau_x^j\right)$$

Our next step is to characterize how the latter stochastic process $\{v^{ij}_t\}$ induces a rolling moment process $\{\mu^{ij}_t\}$ for aircraft *i*.

C. Induced Roll Moment and Compensation Model

For the characterization of how the vortex stochastic process $\{v^{ij}_t\}$ induces a rolling moment process $\{\mu^{ij}_t\}$ for aircraft *i*, we adopt the approximate model developed by Ref. [1] for a rectangular wing. Then the non-dimensionalized induced rolling moment satisfies:

$$\mu^{ij}_{\tau^{ij}_x} = \frac{C^i_\alpha}{2\pi b^i} \max_{\pm} \left\{ \gamma^{ij\pm}_{\tau^{ij}_x} \; f\left(\delta^{ij\pm}_{\tau^{ij}_x}, y^i_{\tau^{ij}_x}, z^i_{\tau^{ij}_x}, b^i, r^j_{core,\tau^{ij}_x}\right)\right\} \cdot \left(\dot{x}^j_t \; - \; w^j_{x,t}\right)^{-1}$$

with $C_\alpha^{\;i}$ the lift curve slope of aircraft *i* (assuming values between 1.4 and 1.6), $b^{\,i}$ is *i*'s wing span, $r^{\,j}_{core,\tau^{ij}_x}$ is the radius of the vortex core generated by *j* at longitudinal position *x* and arriving at moment $\tau^{ij}_{\;x}$ at the longitudinal position of aircraft *i*, while *f* satisfies:

$$f\left(\delta_y, \delta_z, y, z, b, r\right) \equiv 1 - f'\left(\sqrt{\frac{\left(\delta_y - y\right)^2}{b}}, \sqrt{r^2 + \frac{\left(\delta_z - z\right)^2}{b}}\right)$$

$$f'(y, z) \equiv z \arctan\left(\frac{\frac{1}{2} - y}{z}\right) + z \arctan\left(\frac{\frac{1}{2} + y}{z}\right) - \frac{y}{2} \ln\left\{\frac{z^2 + \left(\frac{1}{2} - y\right)^2}{z^2 + \left(\frac{1}{2} + y\right)^2}\right\}$$

Based on the Corjon and Poinsot models, we adopt the following characterization for the vortex core radius:

$$r^{\,j}_{core,\tau^{ij}_x} = \max\left\{ r^{\,j}_{core,0}, \; \frac{1}{80}\sqrt{\Gamma^{\,j}_{0,\tau^j_x}\left(\tau^{ij}_x - \tau^{\,j}_x\right)}\right\}$$

with $r^{\,j}_{core,0}$ the initial radius of aircraft *j*'s vortex cores.

We get

$$\mu^{ij}_{\tau^{ij}_x} = \frac{C^i_\alpha}{2\pi b^i} \max_{\pm} \left\{ \gamma^{ij\pm}_{\tau^{ij}_x} \; f\left(\delta^{ij\pm}_{\tau^{ij}_x}, y^i_{\tau^{ij}_x}, z^i_{\tau^{ij}_x}, b^i, r^j_{core,\tau^{ij}_x}\right)\right\} \cdot \left(\dot{x}^j_{\tau^{ij}_x} \; - \; w^j_{x,\tau^{ij}_x}\right)^{-1}$$

In Kuzmin [Ref. [1], Eq (3.3)] also gives a characterization of the maximum non-dimensionalized rolling moment $\mu_{\max,\tau^{ij}_x}$ for which it is possible to compensate for by aircraft *i*

$$\mu_{\max,\tau^{ij}_x} = \tfrac{1}{12} b^i \, C^i_\alpha \; p_{\max} \left(\dot{x}^j_{\tau^{ij}_x} \; - \; w^j_{x,\tau^{ij}_x}\right)^{-1}$$

with the maximum steady roll rate $p_{\max}$ assumed to be twice as large as the roll rate p_{REQ} minimal required by the British Civil Airworthiness Requirements:

$$p_{\max} = 2\, p_{\mathrm{REQ}}$$

with

$$p_{\mathrm{REQ}} = \frac{\phi_{\mathrm{BCAR}}}{T_{\mathrm{BCAR}} - 1}$$

where ϕ_{BCAR} and T_{BCAR} are specified by the British Civil Airworthiness Requirements (BCAR) and by JAA [12] to satisfy π/3 and 7 seconds, respectively (i.e., a roll over 60 deg from 30 deg is required to be possible within 7 seconds). Thus $p_{REQ} \approx 0.175\ \mathrm{s}^{-1}$.

As long as the vortex induced rolling moment and downwash can be compensated, there will be no reason for a vortex induced accident. Thus it is reasonable to assume that there is no triggering of a rolling induced accident as long as

$$\left| \mu^{ij}_{\tau^{ij}_x} \right| \le \mu_{\max,\tau^{ij}_x}$$

Through substitution of our characterizations for the right and left terms in this inequality, and subsequent evaluation, we get

$$\max_{\pm} \left\{ \left| \gamma^{ij\pm}_{\tau^{ij}_x} \right| \; f\left(\delta^{ij\pm}_{\tau^{ij}_x}, y^{i}_{\tau^{ij}_x}, z^{i}_{\tau^{ij}_x}, b^{i}, r^{j}_{core,\tau^{ij}_x} \right) \right\} \le \tfrac{\pi}{3}\, p_{\mathrm{REQ}} \left(b^{i} \right)^{2}$$

Thus our test on the possibility to compensate the wake vortex induced rolling moment at a time-moment simplifies to

$$\xi^{ij}_{\tau^{ij}_x} \le 1$$

with

$$\xi^{ij}_{\tau^{ij}_x} \equiv \left(\tfrac{\pi}{3}\, p_{\mathrm{REQ}} \left(b^{i} \right)^{2} \right)^{-1} \max_{\pm} \left\{ \left| \gamma^{ij\pm}_{\tau^{ij}_x} \right| \; f\left(\delta^{ij\pm}_{\tau^{ij}_x}, y^{i}_{\tau^{ij}_x}, z^{i}_{\tau^{ij}_x}, b^{i}, r^{j}_{core,\tau^{ij}_x} \right) \right\}$$

A similar expression for compensation of downwash is not available in this initial model.

D. Induced Risk Model

The risk measures to be characterized are:

1. Probability p^{ij}_{I} of an incident for *i*, induced by *j*'s wake.
2. Probability p^{ij}_{A} of an accident for *i*, induced by *j*'s wake.
3. Probability p^{ij}_{C} of a crash of *i* into terrain, induced by *j*'s wake.

We start the characterization for *accident risk*. As long as ξ^{ij}_t assumes values on the interval [-1,1] then at moment t the vortex induced rolling moment can be compensated, and there will be no reason for a vortex induced accident. Thus

$$p_A^{ij} \equiv \Pr\left\{ \exists\, t \,\middle|\, \left|\xi_t^{ij}\right| > 1 \right\}$$

In terms of probability at moment t we define

$$p_A^{ij}(t) \equiv \Pr\left\{ \left|\xi_t^{ij}\right| > 1 \right\}$$

Evaluation yields

$$p_A^{ij}(t) = 1 - \int_{-1}^{1} p_{\xi_t^{ij}}(\xi)\, d\xi = \int_0^{\infty} \Pr\left\{ \left|\xi_t^{ij}\right| > 1 \,\middle|\, z_t^i < z' \right\} p_{z_t^i}(z')\, dz'$$

where $p_{\xi_t^{ij}}(\xi)$ denotes the density of ξ^{ij}_t.

This means we have the following upper bound characterization for p^{ij}_A

$$p_A^{ij} \le \hat{p}_A^{ij} \equiv \max_t \, p_A^{ij}(t)$$

Because aircraft *i* and *j* are flying into the same direction, both $\{\xi^{ij}_t\}$ and $p^{ij}_A(t)$ have local peaks which are rather flat. This implies that our upper bound characterization also is a rather accurate approximation, thus:

$$p_A^{ij} \approx \hat{p}_A^{ij}$$

Our subsequent characterization for *incidents* follows from a similar approach. As long as ξ^{ij}_t assumes values on the interval $[-\varepsilon,\varepsilon]$, then at moment t the vortex induced rolling moment will not be noticed as being largely uncomfortably for a certain value of ε, and there will be no reason to speak of a vortex induced incident. Thus

$$p_I^{ij} \equiv \Pr\left\{ \exists\, t \,\middle|\, \left|\xi_t^{ij}\right| > \varepsilon \right\}$$

In terms of probability at moment t and its upper bound, this yields

$$p_I^{ij}(t) = 1 - \int_{-\varepsilon}^{\varepsilon} p_{\xi_t^{ij}}(\xi)\, d\xi = \int_0^{\infty} \Pr\left\{ \left|\xi_t^{ij}\right| > \varepsilon \,\middle|\, z_t^i = z' \right\} p_{z_t^i}(z')\, dz'$$

$$p_I^{ij} \le \hat{p}_I^{ij} \equiv \max_t \, p_I^{ij}(t)$$

Again, because aircraft i and j are flying into the same direction, both $\{\xi^{ij}_t\}$ and $p^{ij}_I(t)$ have local peaks which are rather flat. This implies that our upper bound characterization also is a rather accurate approximation, thus

$$p_I^{ij} \approx \hat{p}_I^{ij}$$

Finally, our characterization for *crashes* becomes

$$p_C^{ij} \equiv \Pr\left\{ \exists\, t \,\middle|\, \left|\xi_t^{ij}\right| > 1,\ z_t^i < z_{\min}^i \right\}$$

where $z^i_{\min}$ is a random variable, assuming a Weibull probability density function

$$p_{z_{\min}^i}(z) = \kappa\, z^{-1} \left(\frac{z}{z_{\text{nom}}^i} \right)^{\kappa} \exp\left\{ -\left(\frac{z}{z_{\text{nom}}^i} \right)^{\kappa} \right\}$$

with parameters $\kappa \geq 1$ and $z^i_{\text{nom}} > 0$.

Next, similar as before we define

$$p_C^{ij}(t) \equiv \Pr\left\{ \left|\xi_t^{ij}\right| > 1,\ z_t^i < z_{\min}^i \right\}$$

by which we get

$$p_C^{ij} \leq \hat{p}_C^{ij} \equiv \max_t \ p_C^{ij}(t)$$

with $z^i_{\min}$ the minimal height above ground level at which an escape from a crash is possible for an aircraft of type η_i. Straightforward evaluation yields

$$p_C^{ij}(t) = \int_0^\infty \int_0^z \Pr\left\{ \left|\xi_t^{ij}\right| > 1,\ z_t^i < z' \right\} p_{z_t^i}(z')\, dz'\ p_{z_{\min}^i}(z)\, dz$$

It should be noted that from an incident point of view, the critical moment in time is:

$$\hat{t}_I^{ij} = \arg\max_t \ p_I^{ij}(t)$$

From an accident point of view, the critical moment in time is:

$$\hat{t}_A^{ij} = \arg\max_t \ p_A^{ij}(t)$$

From a crash-into-terrain point of view, the critical moment in time is:

$$\hat{t}_C^{ij} = \arg\max_t \ p_C^{ij}(t)$$

In general, one could expect significant differences between realizations of these three moments in time. In this paper, simulation results for the accident risk characterization have been given.

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