Nationaal Lucht- en Ruimtevaartlaboratorium
National Aerospace Laboratory NLR

NLR-TP-2000-688

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This investigation has been carried out under a contract awarded by ESA, contract number 11476/95/NL/NB, and partly as part of NLR´s Basic Research Programme, Workplan Number E.1.A.8.

This report is based on a presentation held at the Millenium Conference on Antennas and Propagation, Davos, Switzerland, 9-14 April 2000.

Division:	Avionics
Issued:	29 December 2000
Classification of title:	Unclassified

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(7 pages in total)

ELECTRO-MAGNETIC ANALYSIS OF GEOMETRICALLY PERTURBED OBJECTS

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INTRODUCTION

The design of antennas on spacecraft and aircraft as well as the prediction of Radar Cross Sections of fighter aircraft require the geometrical modelling and electro-magnetic analysis of antennas and scattering objects. The electro-magnetic analysis can be performed by means of classical computational methods, which are based on the Stratton-Chu integral representations (using the Methods of Moments or Physical Optics approximations). In these methods the scattering objects are geometrically approximated by means of surface meshes (triangular or quadrilateral patches). Geometrical variations of the objects require, in general, the generation of new surface meshes and new impedance matrices, which require complementary computations.

The objective of the present paper is to describe methods and algorithms which reduce the computations considerably. The first method is based on geometrical parameterisation of the integral representation formulas. Here, the geometrical perturbation is introduced as a parameter α and the electric fields are resolved by means of series expansions in terms of α. This method requires the evaluation of the derivatives of the electric field integral equations with respect to the parameter α at the unperturbed geometry. The second method is based on partially updating the impedance matrix (only those parts of the matrix which are related to the perturbations are generated anew) and the surface fields are determined by means of updating solution procedures which are based on Sherman-Morrison-Woodbury formulas. The first method is applicable to electro-magnetic analysis for small perturbations of the whole geometry, while the second method is suited to large changes on parts of the model.

The methods can be applied to analyse the following problems:

- effects of shape deformations of surfaces of conformal antennas on radiation patterns,
- effects of rough surfaces (on thermal blankets of satellites) on scattered fields,
- effects of uncertainties in mutual positions of antennas on spacecraft platforms.

GEOMETRICAL PARAMETERISATION

The surface of the object is assumed to be given by a collection of boundary surfaces S_i, i.e. $S = S_1 \cup S_2 \cdots \cup S_N$. Assume that the geometrical perturbation occurs in the definition of S_1 and that the other surfaces are exactly known. Let S_0 denote the unperturbed surface in the neighbourhood of S_1. Assume that the surface S_0 can be represented by a map $\vec{\Psi}_0$ between a fixed domain Ω and scattering surface S_0, i.e. $\vec{\Psi}_0 : \Omega \to S_0$, so that the boundary points on S_0 follow from $\vec{r}_0 = \vec{\Psi}_0(\vec{u})$ with $\vec{u} \in \Omega$. The boundary points on the perturbed geometry are defined by a similar mapping $\vec{\Psi}_1$, which depends on α. This is illustrated below for vibration modes $\vec{\phi}_\nu(\vec{u}), \nu = 1, \ldots, L,$ which can be obtained from structural finite element calculations. Assume that the perturbation of the surface is given by mode $\vec{\phi}_V$. Then, the boundary points on S_1 follow from

$$\vec{\Psi}_1(\vec{u}) = \vec{\Psi}_0(\vec{u}) + \alpha\vec{\phi}_V(\vec{u}), \tag{1}$$

where α models the magnitude of the perturbation.
The basic idea of modelling the geometrical perturbation in the electric field is to introduce the map $\vec{\Psi}_1$ in the Stratton-Chu representation formula for the radiated field due to surface S_1. This is shown below for perfectly conducting objects. However, the modelling can be extended to non-perfectly conducting objects where the surface impedance is prescribed. In the case of a perfectly conducting surface the radiated electric field can be written as (see [1], p. 86)

$$\vec{E}^s(\vec{r}) = -j\eta\int_{S_1}\{k\vec{J}(\vec{r}')\,G(\vec{r},\vec{r}') - \frac{1}{k}(div_{S_1}\vec{J})(\vec{r}')\nabla G(\vec{r},\vec{r}')\}\,dS'. \tag{2}$$

This representation formula assumes that the electric surface current is tangential with respect to the surface S_1. In this formula $\mathrm{div}_{S_1}\vec{J}$ denotes the surface divergence of $\vec{J}$. The geometrical perturbation of the surface is modeled in (2) by introducing the map (1) in the integral of (2). Then, the radiated field given by (2) can be written as

$$\vec{E}^s(\vec{r}) = -j\eta\int_{\Omega}\{k\vec{J}(\vec{\Psi}_1(\vec{u}))\,G(\vec{r},\vec{\Psi}_1(\vec{u})) - \frac{1}{k}(div_{\alpha}\vec{J})(\vec{u})\nabla G(\vec{r},\vec{\Psi}_1(\vec{u}))\}\sqrt{g}\,du_1 du_2 \tag{3}$$

with $\sqrt{g}$ Jacobian of the mapping. An expression for the surface divergence in terms of the coordinates u_1 and u_2 can be found in [1, p. 49]. The surface divergence depends on α. It will be obvious that the scattered field due to the reference surface S_0 is obtained from (3) by substitution of α = 0. For other values of α, the reflected field can also be computed from (3). In the case, however, that the reflected field has to be determined for many values of α this can become expensive from a computational point of view. Instead, in the present paper, it is proposed to use Taylor expansions around α=0,

$$\vec{E}(\vec{r};\alpha) = \sum_l \frac{1}{l!}\vec{E}^{(l)}(\vec{r};0)\alpha^l \tag{4}$$

The derivatives $\vec{E}^{(l)}$ have to be evaluated at α = 0. The impact of the use of Taylor expansions in Physical Optics approximations and the Methods of Moments will be discussed in the following sections.

IMPACT OF PARAMETERISATION ON PO SOLVERS

In Physical Optics (PO) solvers the surface current $\vec{J}$ is approximated by $2\hat{n}_\alpha \times \vec{H}^i$ on illuminated parts of the surface, while it is put equal to zero on non-illuminated parts. The incident magnetic field $\vec{H}^i$ is assumed to be locally planar on triangular patches Δ_n. The collection of planar patches yields an approximation of the scattering surface. Then, the reflected field $\vec{E}_n$ due to patch *n* can be calculated by

$$\vec{E}_n(\vec{r};\alpha) = -j\eta k\frac{e^{-jkR}}{4\pi R}(\vec{J}_n - (\vec{J}_n\cdot\hat{R})\hat{R})\int_{\Delta_n} e^{jk\vec{r}'\cdot\vec{w}}dS' \tag{5}$$

with $\vec{w}$ the projection of $\hat{R} - \hat{k}^i$ on patch Δ_n, $\vec{J}_n$ the value of $\vec{J}$ at the center of patch, $\hat{k}^i$ the direction of the incident field and $\hat{R}$ the direction of observation. The total scattered field follows from the summation of $\vec{E}_n$. When the geometry is perturbed with respect to S_0, $\vec{J}_n$ and the phase integral (which is denoted by A_n) in (5) depend on α. Hence, one has to determine Taylor expansions for $\vec{J}_n$ and for A_n. A first order expansion for $\hat{n}_\alpha$ of the form $\hat{n}^0 + \alpha\vec{n}^\phi$ can be obtained from the mapping in (1). A first order expansion for $\vec{J}_n$ becomes $\vec{J}_n = \vec{J}_n^0 + \alpha\vec{J}_n^\phi$ with $\vec{J}_n^0 = 2\hat{n}^0 \times \vec{H}^{i0}$ and

$\vec{J}_n^{\phi} = 2\hat{n}^{\phi} \times \vec{H}^{i0} + 2\hat{n}^{0} \times \partial\vec{H}^{i0} / \partial\alpha$. The phase integral A_n can be computed in closed form by the application of Gordon's formula (see [2]). This formula contains the corner points $\vec{a}_1, \vec{a}_2, \vec{a}_3$ of patch Δ_n on the perturbed surface. The mapping of (1) is used to determine first order expansions for $\vec{a}_l$ in terms of α. Then, the first order expansions of $\vec{w}, \hat{n},$ and $\vec{a}_l$ are substituted into Gordon's formula to calculate the derivatives of A_n with respect to α.

IMPACT OF PARAMETERISATION ON MOM SOLVERS

The numerical solution of the Electric Field Integral Equation EFIE (by means of the Method of Moments (MoM) and classical Glisson-Rao vector functions) yields a system of equations of the form $ZI = b$. By the mapping of (1) and the properties of the Glisson-Rao functions (see [3]), the elements of the impedance matrix contain integrals of type

$$\vec{F} = \int_{\Omega} G(\vec{r}, \vec{\Psi}_1(\vec{u};\alpha))\,(\vec{\Psi}_1(\vec{u};\alpha)) - \vec{\Psi}_1(\vec{u}_T;\alpha))\sqrt{g}\,du_1 du_2 \quad \text{and} \quad f = \int_{\Omega} G(\vec{r}, \vec{\Psi}_1(\vec{u};\alpha))\sqrt{g}\,du_1\, du_2, \tag{6}$$

where $\vec{\Psi}_1(\vec{u}_T;\alpha)$ corresponds with one of the coordinates of a triangular patch. Obviously, the elements of Z and b depend on α. Next, introduce Taylor expansions for I analogously to (4). By successive differentiation of $ZI = b$ with respect to α one obtains

$$Z_0 I^1 = b^1 - Z^1 I^0, \qquad Z_0 I^2 = b^2 - Z^1 I^1 - Z^2 I^0, \text{ etc.,} \tag{7}$$

where the superscripts denote differentiation with respect to α. Furthermore, I^0 is the solution of $Z_0\, I^0 = b^0$. The determination of the derivatives of Z and b with respect to α can be carried out in the same way as in the previous section. The applicability of this approach is restricted by the convergence of the Taylor series expansion. In the case that α is large and the perturbation concerns only a part of the geometry, the approach as described in the following section is more suitable.

SHERMAN-MORRISON-WOODBURY (SMW) APPROACH

Let n be the dimension of the original (not perturbed) problem (i.e. n is equal to the order of the matrix Z_0) and let p indicate the number of Glisson-Rao functions which have been changed by perturbing the geometry. Let Z_α be the impedance matrix of the perturbed geometry. Then, Z_α - Z_0 is a rank p matrix. Then, there exist two U and V matrices such that $Z_\alpha = Z_0 - UV^T$. The inverse of the matrix Z_α can be computed in a rather effective way by means of the SMW algorithm, which is based on the following formula for the inverse of Z_α,

$$Z_\alpha^{-1} = Z_0^{-1} + Z_0^{-1} U (I_d - V^T Z_0^{-1} U)^{-1} V^T Z_0^{-1} \tag{8}$$

The algorithm reads:

1. Compute $B = Z_0^{-1} U$
2. Compute the matrix $K = (I_d - V^T B)$ and its LU factors
3. Solve $y = Z_0^{-1} b$ and compute $w = V^T y$
4. Solve $Ks = w$ for s.

5. The solution for $Z_\alpha I = b$ can be computed as $I = y + Bs$.

The computational cost of this algorithm is $O\,(n^2\,p + p^3)$. When these costs are compared with the cost $O(n^3)$ of LU-factorisation of the new matrix Z_α, the algorithm is very advantageous for values of p/n up to 20 %.

RESULTS OF GEOMETRICAL PARAMETERISATION

The geometrical parameterisation and the Taylor series expansion are implemented in a PO solver for EM scattering analysis from conducting surfaces by means of a non-deterministic approach: the scattering surface is described like a random surface and is characterised by a set of statistical parameters. The solver reads two samples of the scattering surface: the nominal surface and the nominal deformed surface. These two reference surfaces allow the generation of the generic perturbed surface by means of a random displacement amplitude α from the unperturbed surface, related to every node of the scattering surface. The code then allows the evaluation of the elementary scattered field contribution with an approximated calculation utilising a Taylor series expansion in terms of α for the elementary scattered field contribution. The Taylor expansion is useful for the reduction of the calculation time needed for the evaluation of the scattered field values related to each configuration of the perturbed surface. Once, the Taylor series coefficients are known, the field values for many different perturbed surfaces can be obtained with simple algebraic operations and statistical computations can be quickly performed.

In the following, a plane square conducting plate with 10 λ border length (λ is the wavelength), placed in the *xy*-plane and centred in the origin, is considered as the nominal unperturbed scattering surface. Two nominal deformed surfaces are considered: $\vec{\phi}_1 = \hat{z}D\,\cos(\pi x/(10\lambda)\cos(\pi y/(10\lambda)$ and $\vec{\phi}_2 = \hat{z}D\,\mathrm{sinc}(\pi\rho/\lambda)$, for $\rho \le 4\lambda$, $\vec{\phi}_2 = 0$ otherwise, with $\rho = \sqrt{x^2 + y^2}$. The frequency is 3 GHz, the source is a short electrical dipole polarised along the *x*-axis and located along the *z*-axis 10 λ above the plane plate. The modulus (in dB) of the scattered electric field in the *yz*-plane has been computed (at a distance of 10 λ from plate centre) by the standard (STD) PO code on the perturbed surface and by the non-deterministic (ND) PO code with Taylor formulation as described before. **Fig. 1**(a) displays the results of the unperturbed plane surface, where the STD PO and ND PO codes yield the same results. All the graphics presented are normalised to the maximum value of scattered field of unperturbed surface. **Fig. 1**(b) and **Fig. 1**(c) show the results of STD PO code and ND PO code for the perturbed surface obtained from ϕ_1, respectively with $D=\lambda$ and $D=-\lambda$. **Fig. 1**(d) shows the comparison between STD PO and ND PO calculations for the perturbed surface obtained from ϕ_2 with $D=\lambda$. Note that this surface is near the limit of applicability of PO techniques. From the figures below it can be concluded that good agreement is obtained for values of θ between -60 and 60 degrees.

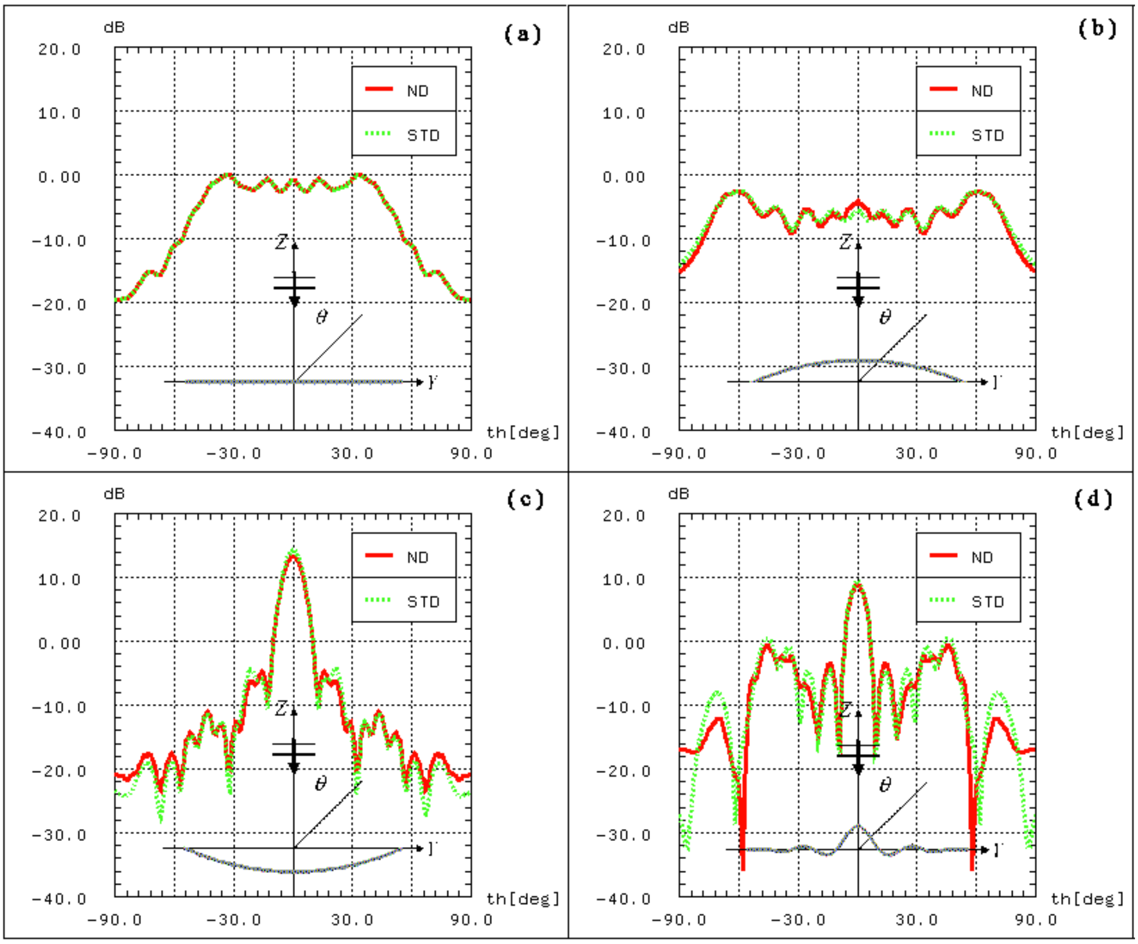

Fig. 1 Comparison between PO scattered field results obtained with non-deterministic (ND) and standard (STD) code.

REFERENCES

[1] S. Silver, *Microwave Antenna Theory and Design,* Peter Peregenius, 1984.
[2] W.B Gordon, "Far-field Approximations to the Kirchhoff-Helmholtz Representations of Scattered", *IEEE Trans. On Antennas and Propagation,* vol. AP-23, pp. 590-592, July 1975.
[3] S.M. Rao, D. R. Wilton, A.W. Glisson, "Electromagnetic Scattering by Surfaces of Arbitrary Shape", *IEEE Trans. On Antennas and Propagation,* vol. AP-30, pp. 409-418, May 1982.