



Executive summary

A Flight Dynamics Helicopter UAV Model For A Single Pitch-Lag-Flap Main Rotor

Model Extensions

Problem area

The current model includes only the main and tail rotor modules, this to allow for an incremental model validation¹ procedure. We present hereunder some candidate model extensions that could potentially be added in the future, to increase the model representativity and validity. These additions exclude

aeroelasticity aspects, but include issues on the fuselage, tail, further comments on the tail rotor, ground effects, engine control and transmission, fuel slosh, and atmospheric disturbances.

Report no.

NLR-TP-2010-286-PT-2

Author(s)

S. Taamallah

Report classification

UNCLASSIFIED

Date

May 2011

Knowledge area(s)

Helikoptertechnologie

Descriptor(s)

Fuselage/Tail

Ground effects

Engine control and transmission

Fuel slosh

Atmospheric disturbances

UNCLASSIFIED

A Flight Dynamics Helicopter UAV Model For A Single Pitch-Lag-Flap Main Rotor
Model Extensions

Nationaal Lucht- en Ruimtevaartlaboratorium, National Aerospace Laboratory NLR

Anthony Fokkerweg 2, 1059 CM Amsterdam,
P.O. Box 90502, 1006 BM Amsterdam, The Netherlands
Telephone +31 20 511 31 13, Fax +31 20 511 32 10, Web site: www.nlr.nl

UNCLASSIFIED



NLR-TP-2010-286-PT-2

A Flight Dynamics Helicopter UAV Model For A Single Pitch-Lag-Flap Main Rotor

Model Extensions

S. Taamallah

Part-I of this report is based on a paper (with same title) accepted for publication at the 36th European Rotorcraft Forum, Paris, September 7-9, 2010.

The contents of this report may be cited on condition that full credit is given to NLR and the authors.
This publication has been refereed by the Advisory Committee AEROSPACE SYSTEMS & APPLICATIONS and AEROSPACE VEHICLES.

Customer	National Aerospace Laboratory NLR
Contract number	----
Owner	National Aerospace Laboratory NLR
Division NLR	Aerospace Systems and Applications
Distribution	Unlimited
Classification of title	Unclassified
	May 2011

Approved by:

Author	Reviewer	Managing department

A Flight Dynamics Helicopter UAV Model For A Single Pitch-Lag-Flap Main Rotor Model Extensions

Skander Taamallah *[†]

* Avionics Systems Department, National Aerospace Laboratory (NLR)
1059 CM Amsterdam, The Netherlands (staamall@nlr.nl).

[†] Delft Center for Systems and Control (DCSC)
Faculty of Mechanical, Maritime and Materials Engineering
Delft University of Technology, 2628 CD Delft, The Netherlands.

Keywords: Unmanned Aerial Vehicle (UAV), fuselage; tail;
ground effects; engine control and transmission,
fuel slosh, atmospheric disturbances

Abstract: The current model includes only the main and tail rotor modules, this to allow for an incremental model validation¹ procedure. We present hereunder some candidate model extensions that could potentially be added in the future, to increase the model representativity and validity. These additions exclude aeroelasticity aspects, but include issues on the fuselage, tail, further comments on the tail rotor, ground effects, engine control and transmission, fuel slosh, and atmospheric disturbances.

1 Model extensions

1.1 Fuselage

The following two paragraphs are derived from [54].

¹The final step of any model development is model validation, through wind tunnel measurements and/or flight tests. It is indeed well known that about 80% of fidelity can be achieved with a physical model, the remaining 20% requires artificial tuning and corrections applied to the physical parameters of the model [55, 54]. But, due to time and space constraints, further aspects relative to model validation such as time- and frequency-domain system identification techniques will not be covered in this paper

The flow around the fuselage is characterized by strong nonlinearities and distorted by the influence of the main rotor wake [54]. Hence the associated forces and moments due to the surface pressures and skin friction are therefore complex functions of flight speed and direction [54]. Further it is important to note that in general the fuselage moments are destabilizing, stemming from the large planform and side area ahead of the vehicle CG.

For low speed sideways flight, the important fuselage characteristics are the sideforce, vertical drag, and yawing moment. While in forward flight, the three most important fuselage characteristics include drag, and pitching and yawing moments variations with incidence and sideslip [54]. The fuselage rolling moment is usually small, except for configurations with deep hulls where the fuselage aerodynamic center may be significantly below the vehicle CG [54].

For a simple analytical model see [34], for an alternative formulation including also pitch and yaw moments see [56]. Often only steady airloads effects on the fuselage are considered. It is however important to know, as mentioned in [54], that important unsteady separation effects also exist, but are rather complex to model.

An expression for rotor downwash over the fuselage, for typical single rotor helicopters, was provided in [75] as a polynomial in wake skew angle. The polynomial coefficients were empirically fit to data presented in [45], but in the case of our UAV, these would potentially need to be readjusted through flight tests.

For fuselage drag calculations, it is estimated that the fuselage may account for up to 50% of the total helicopter drag [24]. Vertical drag penalty in hover and corresponding drag coefficient may be derived from available lookup tables, or for instance from a chart in [62]. Additionally the presence of the fuselage just under the main rotor acts as a so-called pseudo-ground effect, resulting in some thrust recovery. This latter effect may be obtained from a chart in [62]. In forward flight, for parasite drag calculations, and the associated form factor and skin friction coefficients may be derived from [72, 62].

1.2 Tail

The following is mainly derived from [54].

The horizontal tail (called also horizontal fin) and vertical tail (called also vertical fin) form together the tail of a helicopter. Their role is to perform two principal functions. The first one in forward flight, the horizontal tail

generates a trim load that reduces the main rotor fore-aft flapping, while the vertical tail generates a sideforce and yawing moment reducing the tail rotor thrust requirement, in order to increase the fatigue life of the tail rotor [54, 62]. The second aspect during maneuvers, and during wind gusts, the tail surfaces provide pitch and yaw damping and stiffness, and enhance pitch and directional stability [54].

The tails are basically wings, hence refer to [72, 2] for basic aerodynamics characteristics, and to [43, 75, 34, 56] for helicopter applications.

It is also well known that depending on the longitudinal and vertical position of the horizontal tail with respect to the main rotor, erratic longitudinal trim shifts may happen when the helicopter is transitioning from hover to forward flight [62]. This is the case when the main rotor wake impinges on the tail surface, for an overview of main rotor wake skew angle limits see [54].

Regarding the modeling of the main rotor downwash on the tail, good results were obtained by using flat vortex wake theory [7] (valid for small sideslip angles), as presented in [84, 74]. An alternative formulation is to represent the downwash as a polynomial in wake skew angle [75].

Finally the effect of the main rotor downwash on the tail boom should also be considered at low speed, since this may influence yaw damping [54].

1.3 Tail rotor

The tail rotor operates in an adverse aerodynamic environment, a strongly nonuniform flow field, due to the wake of the main rotor, main rotor hub, fuselage, and vertical fin which reduce the aerodynamic efficiency, and control requirement and increase the tail rotor loads and vibrations [46]. This is particularly true in low-speed flight, in-ground effect, sideways flight (potentially operating in the VRS), and in transition to forward flight [54].

Modeling of the main rotor downwash on the tail rotor can be done using flat vortex wake theory, as outlined in the previous section.

Further a vertical tail blockage factor k_{bl} can be added, as in [4], to account for vertical tail interference

$$k_{bl} = (1 - b_{t_1}) \frac{u_{TR}^2}{v_{bl}^2} + b_{t_1} \quad \text{for } u_{TR} \leq v_{bl} \quad (1a)$$

$$k_{bl} = 1 \quad \text{for } u_{TR} > v_{bl} \quad (1b)$$

Where the transition velocity v_{bl} and tail blockage constant b_{t_1} can be derived from flight tests.

1.4 Interactional aerodynamics

Interactional aerodynamics refers to the interaction between several vehicle components. This phenomena is inherently related to a.o. the geometry of the helicopter configuration, the physical relationships between the elements, the relative wind direction and magnitude, and the rotor downwash velocity [70]. It is safe to say that this presents a formidable modeling problem, see [70, 26, 11, 5, 52, 54, 10, 25] and references therein.

1.5 Ground effects

Ground effects can be divided into three domains: static ground effect, dynamic ground effect, and the ground vortex. Static ground effect had already been taken into account in the current model.

1.5.1 Dynamic ground effect

When a helicopter is hovering above a heaving, rolling and pitching surface (such as a ship deck), the lifting rotor is subjected to a so-called dynamic ground effect. The static ground effect models cannot capture the unsteady aerodynamics due to such dynamic ground effect. In [79, 80] the generalized finite-state dynamic inflow [58] was extended to include dynamic ground effects. For the case of helicopter hover above an inclined ground plane or ship deck, in which not only the magnitude but also the distribution of rotor induced velocity are changed, see [82, 41].

Now in some operations, a helicopter has to partially hover above a building top or a ship deck (thus not completely above the ground surface), the case is known as hovering with partial ground effect. Further details about this problem can be found in [81].

1.5.2 Ground vortex

It is well known that when a lifting rotor is operated close to the ground at low advance ratios, under certain conditions, a horseshoe ground vortex forms under the rotor [70].

The earliest identification of the ground vortex phenomenon occurred in [44], followed by theoretical investigations in [20, 38, 28], and experimental results in [71, 18, 17].

The ground vortex is produced by the interaction of the rotor downwash, the ground, and the velocity of translation. And estimates of the ground vortex strength indicated that it is at least an order of magnitude stronger than the blade tip vortex [18, 17]. Additionally its position relative to the helicopter depends on helicopter forward speed [70]. For example, for a full-size helicopter, the ground vortex does not seem to exist from hover to about 5kts. It appears to form at about 5kts, several rotor diameters in front of the helicopter and in the direction of flight [71]. The consequence of such a ground vortex is that such characteristics as the control required to trim the helicopter and the effectiveness of the tail rotor as well as engine performance² can be markedly changed [18].

1.6 Engine, ECU and transmission unit

The propulsion system dynamics can have a profound effect on the helicopter flight dynamics. Dynamic interface problems involving the engine/fuel control and the rotor/drive train/airframe have been encountered in the ground or flight testing of helicopters for a long time [15].

In flight, the power required by a helicopter varies continuously, due to required changes in forward speed, due to maneuvers, due to power recovery following autorotative flight, and due gusts, which result in rapid thrust variations [53, 73, 1]. Fast main rotor RPM compensation by a governor mechanism, i.e. an Engine Control Unit (ECU), is thus required since any discrepancy between the required and the available main rotor torque will cause the rotor to decelerate or accelerate.

For example, assuming a constant rotor RPM may result in poor heave axis dynamics, and poor yaw response to pitch input dynamics, when compared to flight test data [49]. Also specific problems can also be related to the fact that main rotor and drive train systems may have lightly damped torsional dynamic modes, which may be within the bandwidth of the ECU. Further highly responsive engines have the potential of destabilizing the lag dynamics of the main rotor, or may cause large resonant responses in lag through fast rotor speed excitation. This is especially true when the lag mode is only lightly damped, as is the case for hingeless rotors which are usually not equipped with lag dampers [53].

Finally the advantages of variable rotor speed may be worth considering, since in principle optimal rotor speed for a certain flight condition will not be optimal for another one, i.e. being for example function of vehicle air-speed, altitude, load factor, and/or total mass. It has been shown that

²Such as engine inlet ingestion of its own exhaust [71]

performance benefits, i.e. in terms of maneuverability and agility, can be obtained, by varying the main rotor rotor speed during transient maneuvers [42].

We provided here only a very brief introduction on this subject, for further references see for example [6, 53, 1, 9].

1.7 Fuel slosh

Sloshing is the occurrence of any free liquid surface motion inside a container. Sloshing becomes complex during sporadic movements in partially filled liquid containers. Depending on the frequency of the disturbance and the container shape, the free surface of the liquid will undergo a number of complex motions including non-planar, rotational, irregular beating, quasi-periodic and chaotic [69]. The problem of fuel sloshing has received extensive attention, especially in the spacecraft community (satellite and launcher dynamics).

As in the case of vibrations, there are passive and dynamic ways to prevent and lower the effect of fuel sloshing. It starts with proper fuel tank design, often including hydrodynamic damping through so-called baffles. Then the residual sloshing is often dealt with through modeling and adequate design of the flight control computer. Fuel sloshing can be modeled in several ways, the most common approach is to model the sloshing as a pendulum like motion.

For some UAVs, the sloshing phenomenon may particularly be important to consider, since in some cases up to 30% or 40% of the vehicle total mass may be subject to sloshing.

1.8 Atmospheric disturbances

Atmospheric disturbances may be added to a model as additive perturbations, and these come in three different forms: constant linear wind velocity, stochastic linear turbulence velocity, and stochastic rotational turbulence velocity. While linear disturbances will affect the vehicle aerodynamic velocity, rotational turbulence velocities will affect vehicle body roll, pitch, and yaw rates.

Now the main rotor is sensitive to atmospheric disturbances. Wind and wind gusts, induced by atmospheric variations, by local terrain or man-made structures, will change the aerodynamic conditions at the rotor, hence impact blade lift [37, 50].

A low altitude turbulence survey with a huge amount of data can be found in [77], see also [60] for an early description of a nonstationary³ low altitude atmospheric turbulence. Further two atmospheric models have been extensively used in the aerospace community, to investigate fixed- and rotary-wing responses to atmospheric disturbances. The first one is the von Karman model [76], where an isotropic⁴ turbulence model was assumed, which was validated by experimental measurements of low altitude turbulence in [33]. The second one is the Dryden atmospheric model [3]. The difference between the von Karman and Dryden models lies basically in a small variation in the high frequency content [36]. The Dryden model is most frequently used owing to its greater simplicity of implementation, i.e. by passing white noise through linear filters, [8]. On the other hand, the FLIGHTLAB atmospheric model [4] is based on the von Karman spectrum [76], and on results from [66, 68], where only the vertical turbulence velocity is modeled, since it is supposed to have the most important effect on blade aerodynamics.

The literature on transient and steady-state gusts (turbulence) modeling, and aircraft and helicopter response to gusts, is extensive to say the least. We refer the reader to the following influential contributions, for fixed-wing design and response to gusts see [83, 61, 40, 39, 23, 22], for rotary-wing an excellent *tour d'horizon* is provided by Gaonkar in [31, 29, 30], additionally for flapping response to gusts see [78], for coupled flap-torsion dynamics to stochastic vertical turbulence see [32, 47, 27], for flap-lag dynamics to stochastic vertical turbulence in hover see [63, 48], for flap-lag dynamics to stochastic horizontal turbulence in hover see [64] and in forward flight see [65], for the effect of deterministic gusts see [12, 13, 14], for hingeless response to random gusts in forward flight see [21, 22], for stochastic stability analysis see [51], for effects on handling-qualities see [37, 57], and for blade-fixed⁵ atmospheric turbulence see [67, 68, 59, 35, 16, 19].

In the case of our UAV, due to its small scale compared to the large-scale turbulence, we will assume that the entire rotor disk experiences a spatially uniform turbulence velocity, identical to that at the rotor center, hence spatial gradients effects will be judged insignificant.

³A stochastic process whose probability distribution changes when shifted in time or space

⁴Statistical properties invariant with respect to direction

⁵Body-fixed atmospheric turbulence refers to the turbulence experienced by a point fixed on a non-rotating vehicle component such as the vehicles CG, while blade-fixed atmospheric turbulence refers to the turbulence experienced by a component of a rotating rotor blade [67]. Indeed the atmospheric turbulence velocities seen by non-rotating vehicle components and rotating blades may be substantially different

References

- [1] K.B. Amer, R.W. Prouty, G. Korkosz, and D. Fouse. Lessons learned during the development of the ah-64a apache attack helicopter. In *48th Annual Forum of the American Helicopter Society*, 1992.
- [2] J.D. Anderson. *Fundamentals of Aerodynamics. Third Ed.* McGraw-Hill Higher Education, NY, 2001.
- [3] Anonymous. Flying qualities of piloted airplanes. Technical Report MIL-F-8785C, United States DoD, 1980.
- [4] ART. *FLIGHTLAB Theory Manual (Vol. One & Two)*. Advanced Rotorcraft Technology, Inc., Mountain View CA, 2006.
- [5] D.T. Balch. Experimental study of main rotor tail rotor airframe interaction in hover. *Journal of the American Helicopter Society*, pages 49–56, 1985.
- [6] M.G. Ballin. A high fidelity real-time simulation of a small turbo shaft engine. Technical Report TM 100991, NASA Ames Research Center, 1988.
- [7] V.E. Baskin, L.S. Vildgrube, Y.S. Vozhdayev, and C.I. Maykapar. Theory of the lifting airscrew. Technical Report TT-F-823, NASA, 1976.
- [8] T.R. Beal. Digital simulation of atmospheric turbulence for dryden and von karman models. *AIAA Journal of Guidance, Control, and Dynamics*, 16(1), 1993.
- [9] B. Benoit, A.M. Dequin, K. Kampa, W. von Grunhagen, P.M. Basset, and B. Gimonet. Host a general helicopter simulation tool for germany and france. In *56th Annual Forum of the American Helicopter Society*, 2000.
- [10] J. Berry and N. Bettschart. Rotor-fuselage interaction: Analysis and validation with experiment. In *53rd Forum of the American Helicopter Society*, 1997.
- [11] M.D. Betzina, C.A. Smith, and P. Shinoda. Rotor body aerodynamic interactions. In *9th European Rotorcraft Forum*, 1983.
- [12] G.S. Bir and I. Chopra. Gust response of hingeless rotors. In *41st Annual Forum of the American Helicopter Society*, 1984.
- [13] G.S. Bir and I. Chopra. Gust response of hingeless rotors. *Journal of the American Helicopter Society*, 31(2), 1986.

-
- [14] G.S. Bir and I. Chopra. Prediction of blade stresses due to gust loading. *Vertica*, 10(3), 1986.
- [15] R.T.N. Chen. An exploratory investigation of the flight dynamics effects of rotor rpm variations and rotor state feedback in hover. Technical Report TM 103968, NASA Ames Research Center, 1992.
- [16] M. Costello, G.H. Gaonkar, J.V.R. Prasad, and D.P. Schrage. Technical notes - some issues on modeling atmospheric turbulence experienced by helicopter rotor blades. *Journal of the American Helicopter Society*, pages 71–75, 1992.
- [17] H.C.Jr. Curtiss, W. Erdman, and M. Sun. Ground effect aerodynamics. *Vertica*, 11(1):29–42, 1987.
- [18] H.C.Jr. Curtiss, M. Sun, W.F. Putman, and E.J.Jr. Hanker. Rotor aerodynamics in ground effect at low advance ratios. *Journal of the American Helicopter Society*, pages 48–55, 1984.
- [19] Y.Y. Dang, S. Subramanian, and G.H. Gaonkar. Modeling turbulence seen by multibladed rotors for predicting rotorcraft response with three-dimensional wake. *Journal of the American Helicopter Society*, pages 337–349, 1997.
- [20] F.A. DuWaldt. Wakes of lifting propellers (rotors) in ground effect. Technical Report BB-1665-5-3, Cornell Aeronautical Laboratory, 1966.
- [21] A.S. Elliott and I. Chopra. Hingeless rotor response to random gusts in forward flight. In *AIAA*, 1987.
- [22] A.S. Elliott and I. Chopra. Helicopter response to atmospheric turbulence in forward flight. *Journal of the American Helicopter Society*, pages 51–59, 1990.
- [23] B. Etkin. Turbulent wind and its effect on flight. *AIAA Journal of Aircraft*, 18(3):327–345, 1981.
- [24] A. Filippone and J.A. Michelsen. Aerodynamic drag prediction of helicopter fuselage. *AIAA Journal of Aircraft*, 38(2):326–333, 2001.
- [25] T.M. Fletcher and R.E. Brown. Main rotor - tail rotor wake interaction and its implications for helicopter directional control. In *32nd European Rotorcraft Forum*, 2006.
- [26] C.E. Freeman and J.C. Wilson. Rotor-body interference. analysis and test. In *36th Forum of the American Helicopter Society*, 1980.
- [27] Y. Fujimori, Y.K. Lin, and S.T. Ariaratnam. Rotor blade stability in turbulent flows - part ii. *AIAA Journal*, 17(7):673–678, 1979.

-
- [28] Z. Gao and C.J. He. A study of the rotor wake in nap-of-the earth. *Journal of Nanjing Aeronautical Institute*, 7(4), 1986.
- [29] G.H. Gaonkar. Review of nonstationary gust responses of flight vehicles. In *21st AIAA/AHS/ASME Structures, Structural Dynamics and Materials Conference*, 1980.
- [30] G.H. Gaonkar. Review of turbulence modeling and related applications to some problems of helicopter flight dynamics. *Journal of the American Helicopter Society*, 2007.
- [31] G.H. Gaonkar and K.H. Hohenemser. Stochastic properties of turbulence excited rotor blade vibrations. In *AIAA*, 1971.
- [32] G.H. Gaonkar and K.H. Hohenemser. Stochastic properties of turbulence excited rotor blade vibrations. *AIAA Journal*, 9:419–424, 1971.
- [33] J.D. Gault and D.E.Jr. Gunter. Atmospheric turbulence considerations for future aircraft designed to operate at low altitudes. *AIAA Journal of Aircraft*, 1968.
- [34] V. Gavrillets, B. Mettler, and E. Feron. Nonlinear model for a small-size acrobatic helicopter. In *AIAA Guidance, Navigation and Control Conference*, 2001.
- [35] V.V. George, G.H. Gaonkar, J.V.R. Prasad, and D.P. Schrage. Adequacy of modeling turbulence and related effects on helicopter response. *AIAA Journal*, 30(6):1468–1479, 1992.
- [36] O.B. Gerlach, G. van de Moesdijk, and J.C. van der Vaart. Progress in the mathematical modelling of flight in turbulence. In *AGARD Conference Proceedings on Flight in Turbulence*, 1973.
- [37] R.A. Hess. Rotorcraft handling qualities in turbulence. *AIAA Journal of Guidance, Control, and Dynamics*, 18(1):39–45, 1995.
- [38] H.H. Heyson. Theoretical study of conditions limiting v/stol testing in wind tunnels with solid floor. Technical Report TN D-5819, NASA Langley Research Center, 1970.
- [39] J.C. Houbolt. Atmospheric turbulence. *AIAA Journal*, 11:421–437, 1973.
- [40] L.J. Howell and Y.K. Lin. Response of flight vehicles to nonstationary atmospheric turbulence. *AIAA Journal*, 9(11), 1971.
- [41] N. Itoga, N. Iboshi, T. Nagashima, J.V.R. Prasad, H. Xin, and D.A. Peters. A new numerical method for predicting aero-mechanical behaviors of a rotor hovering at close proximity to inclined flat surface. In *55th Annual Forum of the American Helicopter Society*, 1999.

-
- [42] T. Iwata and S.M. Rock. Benefits of variable rotor speed in integrated helicopter/engine control. In *AIAA*, 1993.
- [43] Houck J.A., F.L. Moore, J.J. Howlett, K.S. Pollock, and M.M. Browne. Rotor systems research aircraft simulation mathematical model. Technical Report NTM 78629, NASA Langley Research Center, 1977.
- [44] J.L.Jr. Jenkins. Trim requirements and static stability derivatives from a wind-tunnel investigation of a lifting rotor in transition. Technical Report TN D-2655, NASA Langley Research Center, 1965.
- [45] J.W.Jr. Jewel and H.H. Heyson. Charts of induced velocities near a lifting rotor. Technical Report 4-15-59LY, NASA, 1959.
- [46] W. Johnson. *Helicopter Theory*. Dover Publications Inc., NY, USA, 1994.
- [47] Y.K. Lin and Y. Fujimori. Rotor blade stability in turbulent flows - part i. *AIAA Journal*, 17(6):545–552, 1979.
- [48] Y.K. Lin and J.E. Prussing. Technical notes concepts of stochastic stability in rotor dynamics. *Journal of the American Helicopter Society*, 27(2):73–74, 1982.
- [49] B. Manimala, D. Walker, and G. Padfield. Rotorcraft simulation modeling and validation for control design and load prediction. In *31st European Rotorcraft Forum*, 2005.
- [50] B. Mettler. *Identification Modelling and Characteristics of Miniature Rotorcraft*. Kluwer Academic Publishers, Norwell Mass, USA, 2003.
- [51] N. Sri Namachchivaya and J.E. Prussing. Almost-sure asymptotic stability of rotor blade flapping motion in forward flight in turbulent flow. *Probabilistic Engineering Mechanics*, 6(1), 1991.
- [52] T.R. Norman and G.K. Yamauchi. Full-scale investigation of aerodynamic interactions between a rotor and fuselage. In *47th Forum of the American Helicopter Society*, 1991.
- [53] C.J. Ockier and R. Celi. Dynamics and aeroelasticity of a coupled rotor-propulsion system in hover. In *32nd Structures, Structural Dynamics and Materials Conference*, 1991.
- [54] G.D. Padfield. *Helicopter Flight Dynamics*. Blackwell Science Ltd, Oxford, UK, 1996.
- [55] M. Pavel. Prediction of the necessary degrees of freedom for helicopter real-time simulation models. In *AIAA Modeling and Simulation Technologies Conference and Exhibit*, 2005.

-
- [56] M.D. Pavel. *On the Necessary Degrees of Freedom for Helicopter and Wind Turbine Low-Frequency Mode-Modelling*. PhD thesis, Delft University of Technology, 2001.
- [57] M.G. Perhinschi. A model of atmospheric turbulence for rotorcraft simulation and analysis of stability and performance. In *AIAA Modeling and Simulation Technologies Conference*, 1998.
- [58] D.A. Peters and C.J. He. Finite state induced flow models part ii: Three-dimensional rotor disk. *AIAA Journal of Aircraft*, 32(2):323–333, 1995.
- [59] J.V.R. Prasad, J. Riaz, G.H. Gaonkar, and Y.Y. Dang. Real time implementation aspects of a rotorcraft turbulence simulation method. In *49th Annual Forum of the American Helicopter Society*, 1993.
- [60] H. Press, M.T. Meadows, and I. Hadlock. Estimates of probability distribution of root-mean-square gust velocity of atmospheric turbulence from operational gust load data by random process theory. Technical Report TN 3362, NACA, 1953.
- [61] M.B. Priestly. Evolutionary spectra and nonstationary processes. *Journal of the Royal Statistical Society*, (2), 1965.
- [62] R.W. Prouty. *Helicopter Performance, Stability, and Control*. Krieger Publishing Company, Malabar, Florida USA, 1995.
- [63] J.E. Prussing and Y.K. Lin. Rotor blade flap-lag stability in turbulent flows. *Journal of the American Helicopter Society*, pages 51–57, 1982.
- [64] J.E. Prussing and Y.K. Lin. A closed-form analysis of rotor blade flap-lag stability in hover and low-speed forward flight in turbulent flow. In *AIAA 24th Structures, Structural Dynamics and Materials Conference*, 1983.
- [65] J.E. Prussing, Y.K. Lin, and T.N. Shiau. Rotor blade flap-lag stability and response in forward flight in turbulent flows. *Journal of the American Helicopter Society*, pages 82–87, 1984.
- [66] J. Riaz. *A Simulation Model of Atmospheric Turbulence for Rotorcraft Applications*. PhD thesis, Georgia Institute of Technology, 1992.
- [67] J. Riaz, J.V.R. Prasad, D.P. Schrage, and G.H. Gaonkar. Helicopter response to atmospheric turbulence. In *48th Annual Forum of the American Helicopter Society*, 1992.
- [68] J. Riaz, J.V.R. Prasad, D.P. Schrage, and G.H. Gaonkar. Technical notes - atmospheric turbulence simulation for rotorcraft applications. *Journal of the American Helicopter Society*, pages 84–88, 1993.

-
- [69] H. Shageer and G. Tao. Modeling and adaptive control of spacecraft with fuel slosh: Overview and case studies. In *AIAA Guidance, Navigation and Control Conference*, 2007.
- [70] P.F. Sheridan and R.P. Smith. Interactional aerodynamics a new challenge to helicopter technology. *Journal of the American Helicopter Society*, pages 3–21, 1980.
- [71] P.F. Sheridan and W. Wiesner. Aerodynamics of helicopter flight near the ground. In *33rd Forum of the American Helicopter Society*, 1977.
- [72] R.S. Shevell. *Fundamentals of Flight*. Prentice Hall, Upper Saddle River NJ, 1989.
- [73] F.K. Straub, J.W. Harding, J.M. Harrison, and J.L. Dorman. Flight simulation modeling in support of engine airframe integration. In *17th European Rotorcraft Forum*, 1991.
- [74] M.D. Takahashi. A flight-dynamic helicopter mathematical model with a single flap-lag-torsion main rotor. Technical Report TM 102267, NASA Ames Research Center, 1990.
- [75] P.D. Talbot, B.E. Tinling, W.A. Decker, and R.T.N. Chen. A mathematical model of a single main rotor helicopter for piloted simulation. Technical Report NTM 84281, NASA Ames Research Center, 1982.
- [76] T. von Karman. *Progress in the Statistical Theory of Turbulence*. Turbulence: Classical Papers in Statistical Theory, Interscience Publishers, NY, 1961.
- [77] P.A. Vorontsoz. Turbulence and vertical currents in the boundary layer of the atmosphere. Technical report, Translated by USAF Foreign Technology Division, 1969, 1966.
- [78] F.Y.M. Wan and C. Lakshmikantham. Blade response to random loads: A direct time-domain approach. *AIAA Journal*, pages 24–28, 1973.
- [79] H. Xin, J.V.R. Prasad, and D.A. Peters. Unsteady aerodynamics of helicopter rotor hovering in dynamic ground effect. In *AIAA Atmospheric Flight Mechanics Conference*, 1998.
- [80] H. Xin, J.V.R. Prasad, and D.A. Peters. Dynamic inflow modeling for simulation of a helicopter operating in ground effect. In *AIAA Modeling and Simulation Technologies Conference*, 1999.
- [81] H. Xin, J.V.R. Prasad, and D.A. Peters. An analysis of partial ground effect on the aerodynamics of a helicopter rotor. In *AIAA Aerospace Sciences Meeting*, 2000.

-
- [82] H. Xin, J.V.R. Prasad, D.A. Peters, N. Itoga, N. Iboshi, and T. Nagashima. Ground effect aerodynamics of lifting rotors hovering above inclined ground plane. In *AIAA Applied Aerodynamics Conference*, 1999.
- [83] J.K. Zbrozek. Atmospheric gusts - present state of the art and further research. *Journal of the Royal Aeronautical Society*, 69:27–44, 1965.
- [84] X. Zhao and H.C.Jr. Curtiss. A study of helicopter stability and control including blade dynamics. Technical Report TR 1823T, NASA Ames Research Center, 1988.