Nationaal Lucht- en Ruimtevaartlaboratorium

National Aerospace Laboratory NLR

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Scaling of compressor and turbine maps on basis of equal flow Mach numbers and static flow parameters

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Executive summary

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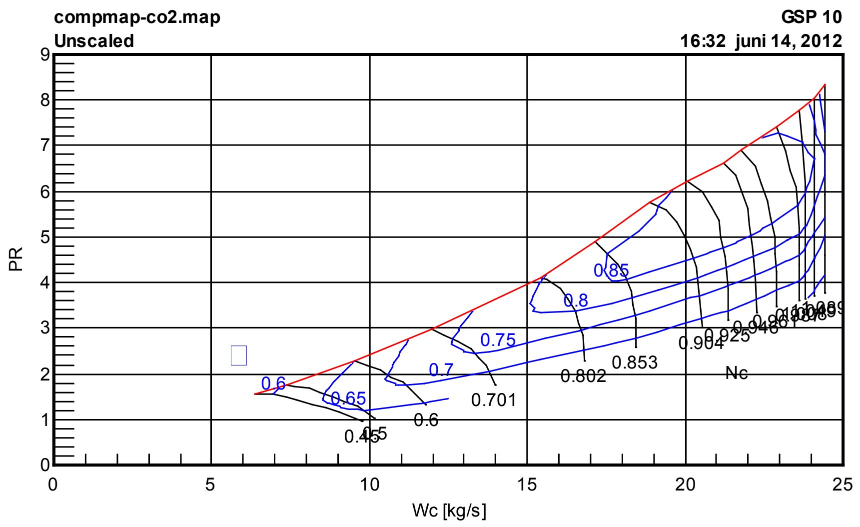

Re-scaled (air) compressor map for the compression of CO_2

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Knowledge area(s)
Gasturbinetechnologie

Descriptor(s)
Gasturbines
Compressor map
Turbine map
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Problem area
Gasturbineprestatieberekeningen is een belangrijk vakgebied, dat veelvuldig zijn toepassing vindt in de luchtvaart en de industriële wereld, met name in de energiesector. Het NLR beschikt over een in eigen huis ontwikkelde gasturbineprestaties-berekeningsprogramma GSP (Gas turbine Simulation Program), welke ook in het publieke domein beschikbaar is.

Bij het modelleren van een gasturbine moeten de eigenschappen van de turbocomponenten zo nauwkeurig mogelijk bekend zijn om betrouwbare prestatieberekeningen te verkrijgen. De karakteristieken van de turbocomponenten (compressor en turbine) worden door zogeheten "*maps*" beschikbaar gemaakt voor gebruik door de software. Om tot een goed werkend rekenmodel te komen, worden *map*-schalingsmethoden voor de turbocomponenten veelvuldig toegepast. Enerzijds is dit een gevolg van een gebrek aan gedetailleerde data van de turbocomponenten. Anderzijds wordt dit geïnitieerd door onderzoek naar de inzet van turbocomponenten (bijv.

compressoren ontworpen voor lucht als medium) voor een ander medium (CO_2) of een medium, waarbij de samenstelling langzaam verandert (aardgas). Een belangrijke karakteristieke grootheid welke verandert, is de verhouding tussen de soortelijke warmten bij constante druk en volume γ. In het algemeen is het vanuit theoretisch oogpunt onmogelijk om turbocomponenten te schalen met gassen met verschillende γ. Deze essentiële parameter wordt daarom veelal met de term "*similarity parameter*" aangeduid. Het is om deze redenen dat er in meer detail is gekeken naar schalingsmethoden.

Description of work
Een schalingsmethode voor compressors en turbines met verschillend werkmedium is uitgewerkt, waarbij de overeenkomstige parameters de stromings-Machgetallen en de statische stromingsparameters (druk en temperatuur) zijn. Deze methode leidt tot gemodificeerde schalingsfactoren in vergelijking tot welke standaard zijn in de gasturbinewereld en die gebaseerd zijn op de schaling op basis van totale stromingsparameters.

Results and conclusions
Het schalen op basis van statische stromingsparameters heeft een duidelijke link met de fysische en aerodynamische verschijnselen, welke in de turbocomponenten van een gasturbine optreden. Deze schalingsmethode verdient dan ook de voorkeur, zeker wanneer hardware en meetgegevens geanalyseerd moeten worden. Het nadeel van deze schalingsmethode is dat de schalingsfactoren afhankelijk zijn van het "operating point" in de compressor- en turbinemap. Bij de standaardmethode voor *maps* wordt volstaan met een enkele schaalfactor per as. Bij schaling met media met verschillende γ's geldt dat de schalingsregels voor relatief beperkte drukratio's (<3) gelden, waarvoor een maat is afgeleid op basis van de ratio's voor de uittreedoorsneden van de turbocomponenten. Voor grotere drukratio's kunnen de schalingsregels trapsgewijs of "*stage wise*" toegepast worden, wat echter hoge eisen stelt aan de database.

Applicability
De voorgestelde methode kan met het NLR gasturbine simulatie programma GSP gebruikt worden om nauwkeurigere voorspelllingen te doen voor de gasturbineprestaties.

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Scaling of compressor and turbine maps on basis of equal flow Mach numbers and static flow parameters

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Summary

Normal practice in engine performance simulations is the lack of detailed (map) data of turbo components, which are regarded as highly proprietary. Furthermore the rare available data are not matched, which is essential if hardware components as compressors and turbines in a gas turbine are co-operating. Therefore to obtain virtual (with no involvement of real hardware) working performance simulation models, the map data of the individual components are scaled (different dimensions and various flow media) and transformed (or morphed without physical background). In the gas turbine community it is common practice to scale the maps on total flow properties. It is argued in the paper that it is better to scale the maps based on equal flow Mach numbers and static flow properties, which relate more to the physics and aerodynamic phenomena. Main disadvantage of this type of scaling is that it requires a minimum amount of additional data and that the scaling factors are dependent on individual operating points in the maps. The paper based on elementary analysis gives the modified scaling rules for the compressor and turbine maps made dimensionless on total flow properties. It emphasizes the most difficult part of the process, the scaling of the component efficiencies, which requires detailed loss models. A scaling example of an air compressor is included with CO_2 as working medium. A validity parameter for the scaling is derived based on turbo component (virtual) exit area ratio when scaling is applied on two media with different $c_p/c_v=\gamma$.

Contents

List of symbols

A	=	flow area [m^2]
c_p	=	gas heat capacity [J/kg/K]
c_v	=	gas heat capacity [J/kg/K]
$c_L(\alpha)$	=	blade lift coefficient [-]
D	=	characteristic rotor diameter [m]
M	=	Mach number [-]
P	=	Power [Watt]
P_s	=	flow static pressure [Pa]
P_s	=	flow total pressure [Pa]
T_s	=	flow static temperature [K]
T_t	=	flow total temperature [K]
R	=	gas constant [J/kg/K]
W	=	mass flow [kg/s]
X	=	axial co-ordinate [m]

Greek symbols

γ	=	heat capacity ratio c_p/c_v [-]
δ_p	=	pressure ratio based on P_{ref} [-]
η	=	efficiency [-]
ρ	=	density of medium [kg/m^3]
θ	=	circumferential co-ordinate [rad]
θ_T	=	temperature ratio based on T_{ref} [-]

Sub-scripts

$_1$	=	medium 1
$_2$	=	medium 2
$_{\text{ref}}$	=	reference conditions (T=288.15 K, P=1 atm.)
$_{\text{c}}$	=	compressor
$_{\text{t}}$	=	turbine
$_{\text{c}}$	=	"corrected" on basis of total flow properties
$_{\text{DP}}$	=	Design Point

Abbreviations

NGV	=	nozzle guide vane
GSP	=	Gas turbine Simulation Program

1 Introduction

The characteristics of engine components as compressors and turbines in many gas turbine performance simulation programs are expressed in so-called component maps, which give the corrected mass flow, the pressure ratio and the efficiency as function of two input variables. In the NLR engine performance simulation program GSP as for others the input parameters are the corrected shaft speed and a dummy variable called β, the both of which return the three above mentioned physical quantities. Normally modellers of gas turbine engine performances don't have access to specific component maps, which are regarded as OEM proprietary. Therefore gas turbine performance simulation programs use map transformation and scaling techniques to be able to coop with the lack of data. The difference between these two techniques will be discussed in the sequel. Furthermore for real applications the gas properties may vary (f.i. the ratio of air heat capacities at constant pressure and volume γ) or the gas itself. Typical examples are compressors which compress natural gasses of slowly varying composition and air compressors, which are used to compress CO_2 while a compressor map is only available for air at standard ISA-conditions. A map scaling technique is discussed, which uses available maps at air ISA-standard conditions to be used for gases with different heat capacity ratio and/or variable composition.

1.1 Real compressor and turbine

The working point of an actual compressor coupled to an electric driver is determined by two quantities, the shaft speed and the (virtual) sonic exhaust area, where the nozzle is contracted until the exhaust flow is choked, which also directly determines the required torque. Each set of these two parameters corresponds with a single point in the map. A turbine connected to a pressure vessel is controlled by the variation in pressure ratio and the exhaust area. For an ideal turbine, where the speed lines in the turbine map coincidence or for a general turbine with the NGV choked and overlapping speed lines, the actual shaft speed is determined by the applied torque. When turbine speed lines overlap, shaft speed and torque are nearly interchangeable parameters except for a slight dependency on turbine efficiency. This is of the reasons that in GSP the power turbine is chosen to operate at constant shaft speed (an alternative is the prescription of the torque leaving the turbine speed as free variable).

1.2 Gas turbine Simulation Program GSP

NLR's primary tool for gas turbine engine performance analysis is the 'Gas turbine Simulation Program' (GSP), a component based modelling environment. GSP's flexible object-oriented architecture allows steady-state and transient simulation of virtually any gas turbine configuration using a user-friendly 'drag and drop' interface with on-line help running under the

Microsoft® Windows operating system. GSP has been used for a variety of applications such as various types of off-design performance analysis, emission calculations, control system design and diagnostics of both aircraft and industrial gas turbines. More advanced applications include analysis of recuperated turbo-shaft engine performance, lift-fan STOVL propulsion systems, control logic validation and analysis of thermal load calculation for hot section life consumption modelling.

Visser et al. [1] describe GSP's object-oriented architecture, which consists of a structured class hierarchy corresponding to a certain component modelling approach. The class structure allows for rapid development of new components. During off-design analyses, the performance of the gas turbine is calculated by solving a set of non-linear differential equations using off-design characteristic maps for compressors and turbines.

These compressor and turbine maps within GSP are automatically scaled or transformed based on the design point operating condition. Compressor or turbine scaling refers to keeping the component geometry identical while sizing the component. Map transformation is applied to match the compressor and turbine characteristics by deforming the maps in such a way that a working gas performance simulation model is obtained. Effectively the "virtual hardware represented by the maps" is changed without paying much attention to the physical background and/or aerodynamics. Otherwise the GSP user would not get a valid performance simulation model unless matched component maps were used with tuned hardware. Map transformation uses linear scale factors for the pressure ratio π, the corrected mass flow W_c and the efficiency η, which are derived from the following equations:
$\frac{\pi - 1}{\pi_{DP} - 1} = \frac{\pi_{map} - 1}{\pi_{map,DP} - 1}$ {1} , $\frac{\eta}{\eta_{DP}} = \frac{\eta_{map}}{\eta_{map,DP}}$ {2} and $\frac{W_c}{W_{c,DP}} = \frac{W_{c,map}}{\eta_{c,map,DP}}$ {3}, where the variables with sub-script *map* denote the variables in the map file and those without the variables in the simulation; *DP* denotes the design operating point. Note that this automatic (can also be disabled) transforming technique within GSP makes an appeal to the technical judgement of the engine modeller since a working performance simulation model is obtained by combining a compressor map of a large industrial gas turbine with the turbine map of a micro-turbine. Compressor and turbine maps therefore have to be critically judged before use and the reliability of the simulation results increases by the use of maps of nearly similar hardware. Finally it is noted that the analysis in the sequel is based on analytical thermodynamic models, whereas the thermodynamic models in GSP make use of real gas models (with chemical equilibrium and varying c_p/c_v ratio or γ).

2 Scaling method

2.1 Standard map scaling method

In the gas turbine performance simulation world it is common to use component maps based on total thermodynamic and aerodynamic parameters, i.e. total pressure P_t and total temperature T_t. GSP also uses the GASTURB format defined by Kurzke [2]. Fig. 1 shows an example of a compressor map, where the abscissa is the corrected or referred mass flow $W_c = \frac{W\sqrt{\theta_T}}{\delta_p}$ {4}, the ordinate the pressure π and the speed lines for which $\frac{N}{\sqrt{\theta_T}}$ {5} with $\delta_p = \frac{P_t}{P_{t,ref}}$ {6} and $\theta_T = \frac{T_t}{T_{t,ref}}$ {7}. The commonly used scaling rules in engine performance simulations are based on total properties.

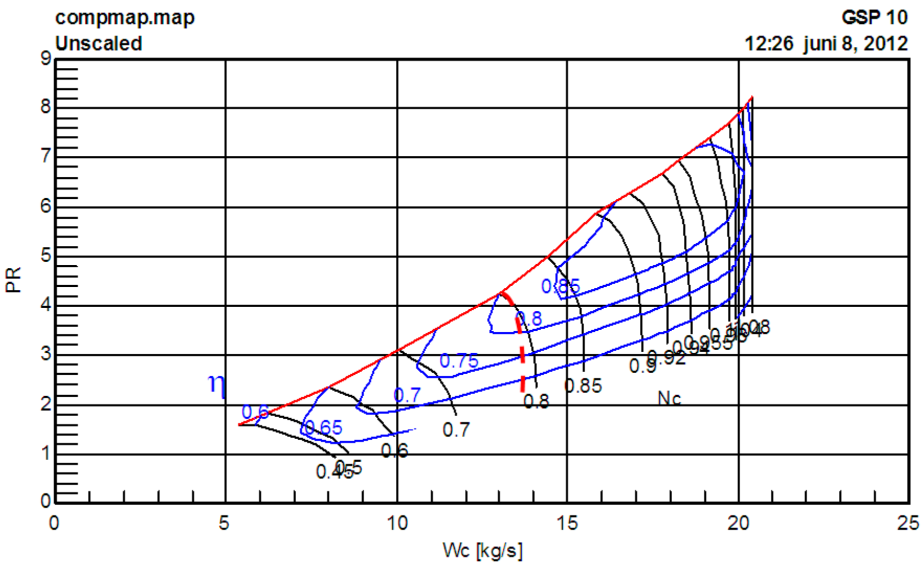

Fig. 1: Typical compressor map with corrected air mass flow W_c versus pressure ratio π and corrected speed lines N_c based on total flow properties T_t and P_t

2.2 Media with different γ

From a theoretical point of view scaling turbo components with media of different ratios of $c_p/c_v = \gamma$ is not possible, since the Mach numbers and velocity triangles will vary through the turbo component since the gasses need different flow areas when travelling through the turbo components. Though initially the Mach numbers and velocity triangles of the different media are equal, a mismatch evolves depending on the pressure ratio of the turbo component. The scaling rules will only be valid for turbo components with a relatively small pressure ratio (π<3 depending on the differences in γ-values: $\Delta\gamma$ smaller, π larger). For large pressure ratios and different γ's the turbo components have to be scaled stage wise. In the sequel a measure for the validity of the scaling is given by a turbo component exit area ratio parameter.

2.3 Alternative scaling method

From a physical (aerodynamic) point of view characterising the compressible flow turbo components as Mach number machines, it would be more logical to express the component characteristics in terms of axial and circumferential flow Mach numbers with physical quantities made dimensionless using static gas properties T_s and P_s (static temperature and pressure). The reason is that the flow velocities are made dimensionless with the speed of sound, which is directly related to T_s and the flow density and pressure distribution on the blades directly relate to P_s (blade surface pressure) and $\rho_s(P_s,T_s)$ (aerodynamic forces, see Fig. 2). The axial Mach number M_x is a measure of the amount of flow and M_θ relates to the shaft speed N. The both of which determine the flow angles to the compressor or turbine stages.

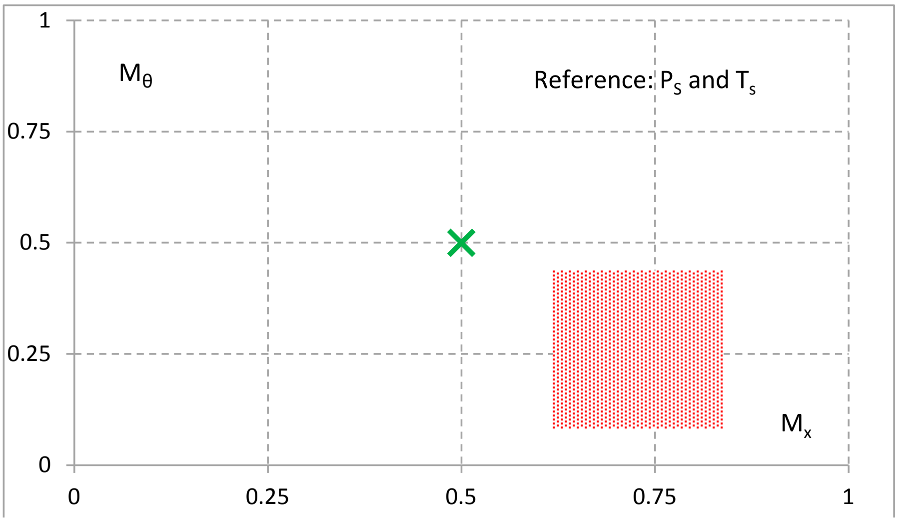

Fig. 2: Alternative compressor map representation with axial and circumferential Mach numbers based on static flow properties T_s and P_s

In the sequel the following assumptions in the analysis are made:

1. The references temperatures and pressures are: $P_{t,ref}=P_{s,ref}=1$ atm. and $T_{t,ref}=T_{s,ref}=288.15$ K.
2. The gas in the modelling has ideal properties to derive the scaling rules. In reality and in the maps the effect of a varying γ is included. The ideal gas equations are: $P = \rho R T$ {8} $c_p = c_v + R$ {9} and $\gamma = \dfrac{c_p}{c_v}$ {10}.

3. Starting point for the calculations are the 1-D compressible flow equations:

   $$P_t = P_s(1+\frac{\gamma-1}{2}M_x{}^2)^{\frac{\gamma}{\gamma-1}} \quad \{11\},\; T_t = T_s(1+\frac{\gamma-1}{2}M_x{}^2) \quad \{12\} \text{ and}$$

   $$W = \sqrt{\frac{\gamma}{R}}\frac{M_x}{(1+\frac{\gamma-1}{2}M_x{}^2)^{\frac{\gamma+1}{2(\gamma-1)}}}\frac{P_t A}{\sqrt{T_t}} = \sqrt{\frac{\gamma}{R}}M_x\frac{P_s A}{\sqrt{T_s}} \quad \{13\}.$$
4. Scaling rules derived from corresponding operating conditions based on equal static flow properties (T_s and P_s) and equal flow Mach numbers.
5. Either the flow area (compressor inlet or NGV turbine area) or the characteristic Mach number for 1 point on the corrected flow axis (M_x for the compressor or M_{NGV} for the turbine) is known. The total Mach number in the throat area of the NGV's for turbines is a more logical scaling parameter in combination with the circumferential Mach number of the rotor. An advantage of the turbine is that a characteristic Mach number or the NGV throat area directly can be determined from the choked flow area in the map.

The method in detail is derived for a compressor but the scaling rules can also be applied to a turbine when instead of M_x the characteristic Mach number in the throat area of the turbine M_{NGV} is chosen. The force on a compressor blade (or n blades by enlarging A_b) is:

$$F_b = A_b c_l(\alpha)\tfrac{1}{2}\rho V^2 = A_b c_l(\alpha)\tfrac{1}{2}\gamma P_s M_t{}^2 \quad \{14\} \text{ with } M_t{}^2 = M_x{}^2 + M_\theta{}^2 \quad \{15\}.$$

The power delivered from the shaft is given by:

$$P = A_b c_l(\alpha)\tfrac{1}{2}\rho V^2 U = A_b c_l(\alpha)\tfrac{1}{2}\gamma P_s M_t{}^2 M_\theta \cos(\varphi)\sqrt{\gamma R T_s} \quad \{16\}.$$

The power delivered per unit flow to the compressor reads:

$$\frac{P}{W} = \frac{A_b c_L(\alpha)\tfrac{1}{2}\gamma R T_s M_t{}^2 M_\theta \cos(\varphi)}{M_x A} = \frac{c_p}{\eta_c}T_s(1+\frac{\gamma-1}{2}M_x{}^2)(\pi^{\frac{\gamma-1}{\gamma}}-1) \; \{17\}.$$

The compressor pressure ratio (for total pressures) is given by:

$$\pi_c = \left(\frac{A_b c_L(\alpha)\tfrac{1}{2}(\gamma-1)M_x{}^2(1+\frac{M_\theta{}^2}{M_x{}^2})M_\theta\cos(\varphi)\eta_c}{M_x A(1+\frac{\gamma-1}{2}M_x{}^2)}+1\right)^{\frac{\gamma}{\gamma-1}} \quad \{18\},$$

which can be written as a function of the axial Mach number M_x, the compressor efficiency and γ:

$$\pi_c = \left(\frac{c(\frac{M_\theta}{M_x})(\gamma-1)M_x{}^2\eta_c}{(1+\frac{\gamma-1}{2}M_x{}^2)}+1\right)^{\frac{\gamma}{\gamma-1}} \quad \{19\} \text{ with } c(\frac{M_\theta}{M_x}) \text{ a function of the (fixed)}$$

flow angle.

The equivalent equations for the turbine are with NGV and circumferential Mach numbers in the nozzle guide vane $M_x = k_x M_{NGV}$ {20} and $M_y = k_y M_{NGV}$ {21}:

$$\pi_t = \left(1 - \frac{A_b c_L(\alpha)\tfrac{1}{2}(\gamma-1)M_{NGV}^{\ 2}(k_x^{\ 2} + (k_y - \frac{M_\theta}{M_{NGV}})^2)M_\theta \cos(\varphi)}{\eta_t M_{NGV} A_{NGV}(1+\frac{\gamma-1}{2}M_{NGV}^{\ 2})}\right)^{\frac{-\gamma}{\gamma-1}} \quad \{22\}$$

and

$$\pi_t = \left(1 - \frac{c(\frac{M_\theta}{M_{NGV}})(\gamma-1)M_{NGV}^{\ 2}}{\eta_t(1+\frac{\gamma-1}{2}M_{NGV}^{\ 2})}\right)^{\frac{-\gamma}{\gamma-1}}$$

{23}, where M_x.and M_y refer to the Mach numbers in the NGV $M_{NGV}^{\ 2} = M_x^{\ 2} + M_y^{\ 2}$ {24}. The scaling of the turbine therefore is based on equal values of M_θ and M_{NGV}. It is remarked that the true speed at the location of the rotor may be higher due to further expansion at supersonic conditions, especially for turbines of the impulse (instead of reaction) type.

2.4 Scaling based on equal Mach numbers and static properties

As mentioned in the header, the scaling rules in the sequel are on based on equal static pressure and temperatures for the two media: $P_{s,1}=P_{s,2}$ and $T_{s,1}=T_{s,2}$.

It is noted that the constant speed lines in figure 1 don't correspond to lines of equal circumferential Mach number, which is indicated by the red dashed speed line. The corresponding speed lines of figure 1 in the M_x,M_θ-plane are not straight but curved (Fig. 3).

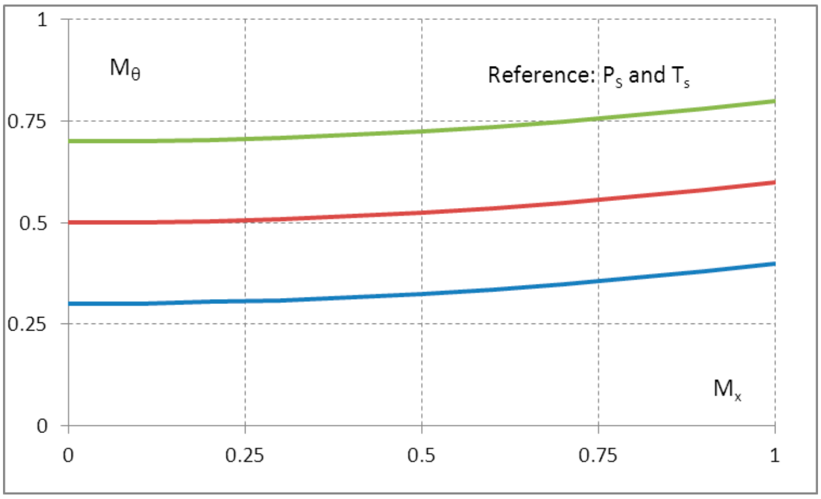

Fig. 3: Curved constant speed lines in the M_x,M_θ-plane corresponding to those with N_C is constant in the compressor map (Fig. 1)

2.4.1 Pressure

The pressure is scaled as follows: calculate for medium (1) per point in the compressor/turbine map c(M_θ/M_x) or c(M_θ/M_{NGV}) using equations {19} and {23} for $M_{x,1}$,γ_1 and π_1 and calculate π_2 with medium properties γ_2 and $M_{x,2}$=$M_{x,1}$. Eliminating c(M_θ/M_x), the pressure ratio $\pi_{2,c}$ for the compressor is given by

$$\pi_{2,c} = \left(\frac{\eta_2(\gamma_2-1)(1+\frac{\gamma_1-1}{2}M_x^{\ 2})}{\eta_1(\gamma_1-1)(1+\frac{\gamma_2-1}{2}M_x^{\ 2})}(\pi_{1,c}^{\frac{(\gamma_1-1)}{\gamma_1}}-1)+1 \right)^{\frac{\gamma_2}{(\gamma_2-1)}}$$

{25} and for the turbine $\pi_{2,t}$ eliminating c(M_θ/M_{NGV})

$$\pi_{2,t} = \left(1-\frac{\eta_1(\gamma_2-1)(1+\frac{\gamma_1-1}{2}M_{NGV}^{\ 2})}{\eta_2(\gamma_1-1)(1+\frac{\gamma_2-1}{2}M_{NGV}^{\ 2})}(1-\pi_{1,t}^{\frac{(1-\gamma_1)}{\gamma_1}}) \right)^{\frac{-\gamma_2}{(\gamma_2-1)}} \qquad \{26\}$$

2.4.2 Shaft speed

The assumption that the circumferential Mach numbers are equal $\frac{N_1 D_{0.75}}{\sqrt{\gamma_1 R_1 T_{s,1}}} = \frac{N_2 D_{0.75}}{\sqrt{\gamma_2 R_2 T_{s,2}}}$ {27} leads to the following scaling rule $N_2 = \sqrt{\frac{\gamma_2 R_2}{\gamma_1 R_1}} N_1$ {28).

2.4.3 Flow

Equal flow Mach numbers $M_{x,1} = M_{x,2}$ {29} leads to the following equation $W_2 = \sqrt{\frac{\gamma_2 R_1}{\gamma_1 R_2}} W_1$ {30} at equal static flow parameters. For completeness it is noted that the dimensionless flow parameter $W_{\dim}$ includes T_s, P_s and an area A_c: $W_{\dim} = M_x = \frac{W}{P_s A_c}\sqrt{\frac{RT_s}{\gamma}}$ {31}.

2.4.4 Efficiency

The scaling rule without the inclusion of detailed loss models reads $\eta_2 = \eta_1$ {32}. It is remarked that the scaling rules for the efficiency are the essential and most difficult part of the process. A lot of work in the past is focussed to the modelling of turbo-component losses [3 and 4].

2.4.5 Energy

The scaling rule for required energy per kg medium at equal static flow parameters is

$$\frac{P_2}{W_2} = \frac{P_1}{W_1}\frac{\gamma_2 R_2}{\gamma_1 R_1} \qquad \{33\}$$

and per turbo-component (compressor or turbine at equal static flow parameters)

$$\frac{P_2}{P_1} = \frac{\gamma_2}{\gamma_1}\sqrt{\frac{\gamma_2 R_2}{\gamma_1 R_1}} \qquad \{34\}.$$

2.4.6 Only geometric scaling

The geometric scaling is clear (except for the turbo component efficiency) and included for completeness. Enlarging the component dimensions with a geometric factor k_g gives a k_g squared larger mass flow and a shaft speed, which equals $1/k_g$ times the original speed.

2.4.7 Scaling summary

Generally the main difference with the map transformation/scaling as described in the introduction is that each point of the compressor and turbine map has its own scale factor based on the axial Mach number. The scaling method is based on shock-free flow with smooth pressure distribution on the blade suction and pressure surfaces. The red area indicated in fig 2 however indicates a flow regime region, where shocks may occur in the first blade row of the compressor (see fig. 4 from [5]). The flow through the first stage rotor blades at high shaft speeds and low back pressure may have various shock and expansion wave phenomena including flow passing 2 oblique shocks and 1 normal shock, 1 oblique and 1 normal shock or 1 normal shock depending on the back pressure. One may expect to get better scaling results in this part of the M_x,M_θ-plane, if these phenomena in some kind of parametric manner are included. Note that the scaling method in this area will give you calculated characteristic values for the second medium.

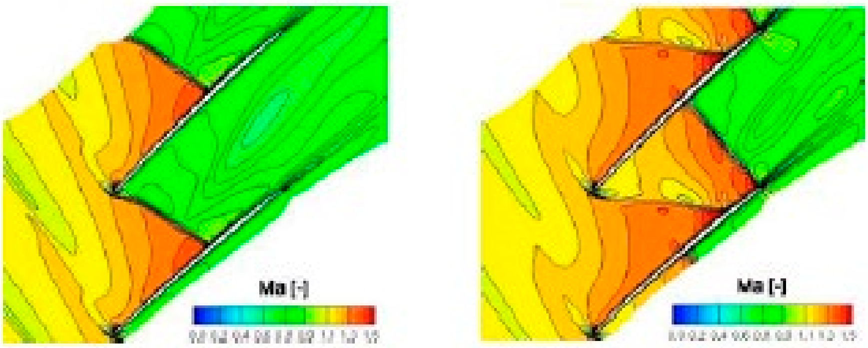

Fig. 4: Shock and expansion wave phenomena at high shaft speeds and relatively high (left) and low (right) back pressure in the compressor map (red area indicated in figure 2), from Kurzke [5]

Implications for the compressor and turbine map based on total flow properties T_t and P_t and equal flow Mach numbers M_x and M_θ. It is noted that the line M_θ=constant does not coincidence with the line N_c=constant.

2.5 Consequences for map data based on total flow properties

In the sequel the consequences of the scaling rules based on static flow properties are translated due to the common practice in the gas turbine community to refer the turbo component characteristics to total flow properties T_t and P_t. In the new scaling rules (still based on equivalent conditions at equal static conditions) additional terms arise, which are a function of the axial Mach number M_x and the ratio of heat capacities γ of both media. This is an expected result since these parameters determine the relations between static and total flow parameters. It is noted that for a turbine M_x must be replaced by M_{NGV}.

2.5.1 Corrected speed on T_t

The scaling rule for corrected speed becomes $N_{c,2} = \sqrt{\dfrac{\gamma_2 R_2 (1+\frac{\gamma_1 - 1}{2} M_x{}^2)}{\gamma_1 R_1 (1+\frac{\gamma_2 - 1}{2} M_x{}^2)}} N_{c,1}$ {35} from which it follows that in the compressor map of the second medium a "semi-speed line" appears where the corrected speed on total temperature slightly varies if $\gamma_1 \neq \gamma_2$, whereas the corrected speeds along the speed line in the map of the first medium are constant. The reason is obvious since the static temperature along the speed lines in media 1 and 2 are not constant. This occurrence requires an additional interpolation step to collect second medium map data at true equal corrected (on T_s) shaft speeds at $P_{t,ref}$ and $T_{t,ref}$.

2.5.2 Corrected flow on T_t and P_t

The scaling rule for corrected flow is $W_{c,2} = \sqrt{\dfrac{\gamma_2 R_1}{\gamma_1 R_2}} \dfrac{(1+\frac{\gamma_1 - 1}{2} M_x{}^2)^{\frac{\gamma_1+1}{2(\gamma_1-1)}}}{(1+\frac{\gamma_2 - 1}{2} M_x{}^2)^{\frac{\gamma_2+1}{2(\gamma_2-1)}}} W_{c,1}$ {36}

2.5.3 Efficiency

As stated before the scaling of the efficiency requires detailed loss models, which impacts are expected to be much larger than the corrections which follow from the differences in static and total flow temperatures and pressures.

2.5.4 Energy on T_t and P_t

The scaling rule for required energy per kg medium at equal total flow parameters is

$$\frac{P_2}{W_2}=\frac{P_1}{W_1}\frac{\gamma_2 R_2}{\gamma_1 R_1}\frac{(1+\frac{\gamma_1-1}{2}M_x^{\ 2})}{(1+\frac{\gamma_2-1}{2}M_x^{\ 2})}$$

{37} and per turbo-component (compressor or turbine at equal static flow parameters and Mach numbers)

$$\frac{P_2}{P_1}=\frac{\gamma_2}{\gamma_1}\sqrt{\frac{\gamma_2 R_2}{\gamma_1 R_1}}\frac{(1+\frac{\gamma_1-1}{2}M_x^{\ 2})^{\frac{3(\gamma_1-\frac{1}{3})}{2(\gamma_1-1)}}}{(1+\frac{\gamma_2-1}{2}M_x^{\ 2})^{\frac{3(\gamma_2-\frac{1}{3})}{2(\gamma_2-1)}}}$$

{38}.

It is remarked that the occurrence of the additional terms as $(1+\frac{\gamma-1}{2}M_x^{\ 2})$ in the equations seems rather unfamiliar since these terms are normally not found in the standard literature [6]. However they appear as a consequence of scaling rules based on equal static flow properties carried out for flows with equal total properties, which is the standard for compressor and turbine maps. The occurrence of these terms at equal total properties can be clarified from the steps in the scaling sequence: (1) calculate $P_{s,1}$ and $T_{s,1}$ for each point in the map (with unique M_x and M_θ) from $P_{t,1}= P_{t,ref}$ and $P_{t,1}= P_{t,ref}$, (2) scale the map on basis of $P_{s,2}=P_{s,1}$ and $T_{s,2}=T_{s,1}$ and equal Mach numbers, (3) calculate new total properties $P_{t,2}$ and $T_{t,2}$ and (4) correct these parameters to $P_{t,ref}$ and $T_{t,ref}$ to get the commonly used so-called “referred parameter results”.

2.6 CO_2 as medium instead of air

The described scaling method is used to alter the compressor map of figure 1 for air to be used for CO_2. The gas properties of bot media are shown in table 1.

Table 1 Gas properties of air and CO_2

Medium	Gas constant R [J/kg/K]	Heat capacity C_p [J/kg/K]	$C_p/C_v=\gamma$
Air	287.04	1004	1.4
CO_2	188.9	832	1.304

The scaling method is based on the knowledge of one axial Mach number M_x on the W_c-axis (or the area). In the current analysis the Mach number is taken to be 0.6 at a corrected flow of 20 kg/s (red cross in figure 1), which delivers the compressor inlet area (A=0.0985 m^2).

GSP is run to determine the efficiencies of the un-scaled compressor at the surge line. These efficiencies are normally not included in the surge line data, which only consists of W_c versus π data The efficiencies for the operating points on the surge line are obtained from map interpolations in the N_c,β-plane.

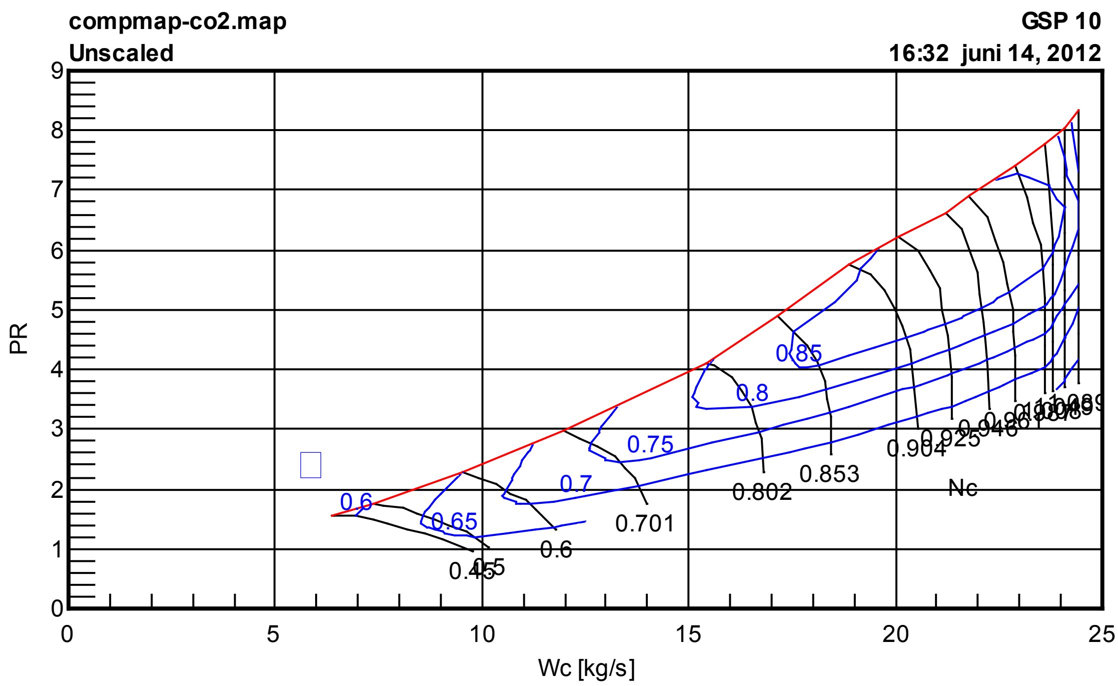

Fig. 5: Re-scaled compressor map with corrected CO_2 mass flow W_c versus pressure ratio π and corrected speed lines N_c based on total flow properties $T_{t,ref}$ and $P_{t,ref}$

The results of the scaling method based on static properties and equal Mach numbers for an air compressor (see for map figure 1) working now with CO_2 as medium are shown in figure 5. The mass flows and pressure ratios are increased by 20% and 1.5% respectively. The relative speed has also increased somewhat with a maximum difference of about 1% at the highest shaft speed ($N_{c,max,air}$=1.08, $N_{c,max,CO2}$=1.089), which may attribute to the slightly increased pressure ratio. Since the shaft speed by the scaling will vary somewhat, the averaged shaft speed of the building points of the speed line is given. The term un-scaled in the figure refers to the fact that the data are not "transformed" to component design point data as described in the section with header GSP.

Finally the operation point data for both media are given in table 2. The shaft speed for CO_2 at equivalent conditions is 12,965 rpm. The shaft speed when the usual scaling on (γ,R) in the gas turbine world (based on total properties) would be 12,860 rpm (a difference of about 0.8%). The required power for the compression of CO_2 is 25% less at a 20% higher mass flow. Main reason is the higher molecular mass of CO_2.

Table 2 Corresponding operating data for compressor with air and CO_2 as medium at $P_{t,ref}$ and $T_{t,ref}$ operating conditions

Parameter	Air	CO_2
N [rpm]	16,450	12,965
W [kg/s]	19.82	23.7
π [-]	7.07	7.07
η [-]	0.85	0.85
P [kW]	5,052	3,750

2.7 Mismatch of scaling rules based on CO_2 example of section 2.6

As described in section 2.2 scaling of turbo components with media of different γ needs special attention since from a theoretical point of view this is not possible without modifying the hardware. A measure for the evolving mismatch applying the scaling rules through the turbo component may be the ratio in exit (virtual) flow areas assuming the entry and exit Mach numbers to be equal. The latter is done for convenience cancelling out a large number of terms in the equations. The virtual exit compressor area ratio is then given by (media 1 and 2 applying the scaling rules to the outlet mass flow equation)

$$\frac{A_2}{A_1} = \sqrt{\frac{(1+\frac{(\pi_2^{\frac{(\gamma_2-1)}{\gamma_2}}-1)}{\eta_2})}{(1+\frac{(\pi_1^{\frac{(\gamma_1-1)}{\gamma_1}}-1)}{\eta_1})}\frac{\pi_1}{\pi_2}} \quad \{39\}$$

and the virtual exit ratio of the turbine by

$$\frac{A_2}{A_1} = \sqrt{\frac{(1-\eta_2(1-\frac{1}{\pi_2^{\frac{(\gamma_2-1)}{\gamma_2}}}))}{(1-\eta_1(1-\frac{1}{\pi_1^{\frac{(\gamma_1-1)}{\gamma_1}}}))}\frac{\pi_2}{\pi_1}} \quad \{40\}.$$

The contribution under the square root follows from the difference in temperatures and the remaining part from the difference in pressure ratios. Ideally the area ratio would be close to 1 if the scaling is valid indicating that the Mach numbers and velocity triangles are nearly constant. In the real world the area is fixed and the Mach number and velocity triangles will vary. Deviation from this value therefore is a measure of the validity of the scaling results and is shown for the CO_2 example as function of the pressure ratio in fig. 6.

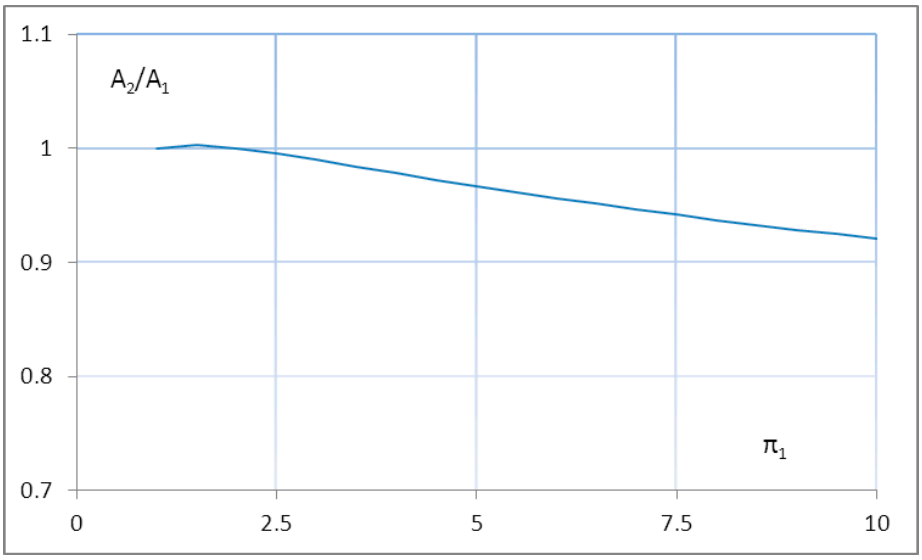

Fig. 6: Virtual exit area ratio of the compressor as function of pressure ratio $\pi_{1,air}$ at equal entry and exit Mach numbers with two media (air and CO_2, η=0.8)

Allowable variation of this parameter is about 3% ($0.97<A_2/A_1<1.03$). For CO_2 scaling this would give a maximum pressure ratio of about 4.5, which would mean that the compressor in section 2.6 should have been scaled part (or stage) wise. The consequence is that the scaling rules only apply to the lower part of the map. Note that for media with equal γ and η the ratio is exactly 1 (as it is for real hardware). The occurrence of η in {39} and {40} also indicate that variation of Mach numbers and velocity triangles may result from differences in component efficiency.

3 Conclusions

Compressor and turbine scaling rules have been investigated. It is argued that scaling rules based on static flow properties are preferred, since these are much more directly related to the flow phenomena and the physics in the turbo components. Main disadvantages by applying this scaling method are that it requires more detailed information than is present in the general used component maps (in the current analysis the flow area of the turbo component or a characteristic Mach number) and that the scale factors are dependent on individual points in the map. Traditionally used scaling factors on total flow parameters generally are constant along the whole map area. The following conclusions can be drawn from the analysis:

1. Map scaling based on static flow properties is highly preferred but requires more detailed information on turbo component characteristics. This is certainly the optimum choice if real hardware with sufficient component data is available in a project.
2. Most obvious scaling parameters for a compressor are the intake axial Mach number and a characteristic circumferential Mach number of the rotor, for a turbine the total Mach number in the throat of the NGV and a characteristic Mach number of the rotor. It is noted that in the scaling rules not both are required since one of them comes into the equations as a ratio, which for the scaling process is assumed to be constant for 2 media.
3. The generally used scaling method on total flow properties with uniform scale factors are a good alternative when detailed information on turbo component data is lacking. This is normal and standard practice since detailed turbo component data are highly proprietary to industry.
4. The essential and most difficult part of the scaling process refers to the turbo component efficiency, which requires detailed loss models.
5. The difference between the scaling on total and static flow parameters is larger for the turbine than for the compressor since the Mach number in the throat area of the NGV is normally higher (choked flow) than the compressor inlet axial Mach number.
6. The scaling rules for media with different γ's need to be handled with care. For small pressure ratios (π<3) the method is allowed. For larger pressure ratios the scaling should be done stage wise. An exit area ratio parameter is derived as validity criterion.
7. Scaling of real hardware is of limited value and cannot replace component testing due to the complexity of the involved flow phenomena.

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