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Executive summary

Flight Dynamics Modeling For A Small-Scale Flybarless Helicopter UAV

Problem area

We present a UAV helicopter flight dynamics nonlinear model for a flybarless articulated Pitch-Lag-Flap (P-L-F) main rotor with rigid Blades.

Description of work

The model includes the main rotor, tail rotors and the fuselage. Additionally the paper reviews all assumptions made in deriving the model, i.e. structural, aerodynamics, and dynamical simplifications.

Results and conclusions

The model has been compared with an equivalent FLIGHTLAB nonlinear model. Simulation results show that the match between this model and FLIGHTLAB is very good for static (trim) conditions, is good to very good for dynamic conditions from hover to medium speed flight u = 5 m/sec, is fair to good for dynamic conditions at high speed u = 10 m/sec, and except for the yaw channel is also good in the VRS.

Applicability

The model is applicable for high bandwidth control specifications, for both Clock-Wise (CW) and Counter-ClockWise (CCW) main rotor rotation, and valid for a range of flight conditions including autorotation and the Vortex-Ring-State (VRS). Hence this model could potentially be used to simulate and investigate the flight dynamics of a flybarless small-scale UAV helicopter, including in autorotation and VRS conditions. Future work will focus on the development of linear and nonlinear control schemes, based on an adapted version of this model, to obtain optimal helicopter flight trajectories.

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Flight Dynamics Modeling For A Small-Scale Flybarless Helicopter UAV

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We present a UAV helicopter flight dynamics nonlinear model for a flybarless articulated Pitch-Lag-Flap (P-L-F) main rotor with rigid blades, applicable for high bandwidth control specifications, for both ClockWise (CW) and Counter-ClockWise (CCW) main rotor rotation, and valid for a range of flight conditions including autorotation and the Vortex-Ring-State (VRS). The model includes the main rotor, tail rotor, and the fuselage. Additionally, the paper reviews all assumptions made in deriving the model, i.e. structural, aerodynamics, and dynamical simplifications. Simulation results show that the match between this model and an equivalent nonlinear FLIGHTLAB® model is very good for static (trim) conditions, is good for dynamic conditions from hover to medium speed flight, and is fair to good for dynamic conditions at high speed. Hence, this model could potentially be used to simulate and investigate the flight dynamics of a flybarless UAV helicopter, including in autorotation and VRS conditions.

Nomenclature

ϕ	Bank angle (roll angle)
θ	Inclination angle (pitch angle, or elevation)
ψ	Azimuth angle (yaw angle, heading)
$\mathbf{V}_{k,G}$	Kinematic velocity of the vehicle center of mass
$u_k^b = u$	x component of $\mathbf{V}_{k,G}$ on body frame F_b
$v_k^b = v$	y component of $\mathbf{V}_{k,G}$ on body frame F_b
$w_k^b = w$	z component of $\mathbf{V}_{k,G}$ on body frame F_b (positive down)
$p_k^b = p$	Roll velocity (roll rate) of the vehicle relative to the earth
$q_k^b = q$	Pitch velocity (pitch rate) of the vehicle relative to the earth
$r_k^b = r$	Yaw velocity (yaw rate) of the vehicle relative to the earth

I. Introduction

In the past twenty years, scientific progress related to sensors technology and computational hardware has allowed for sustained research in the field of robotics. In particular when considering flying robots, the availability of increasingly reliable, high performance, and miniaturized sensors, combined with advances in computing power on miniaturized hardware, has yielded impressive developments in the area of Unmanned Aerial Vehicles (UAVs)[a]. These unmanned vehicles have been developed for both civilian and military missions[b], with their *raison d'être* stemming from the need for (real-time) information[c]. Further, UAV

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[a]Although industry and the regulators have now adopted Unmanned Aerial System (UAS) as the preferred term for Unmanned Aircraft, as UAS encompasses all aspects of deploying these vehicles and not just the platform itself.

[b]UAVs have typically been associated with the so-called *DDD* tasks:[1] Dull e.g. long duration, Dirty e.g. sampling for hazardous materials, and Dangerous e.g. extreme exposure to hostile action.

[c]Spanning a broad spectrum, i.e. visual, electromagnetic, physical, nuclear, biological, chemical, or meteorological information.

deployment and recovery from unprepared or confined sites may often be necessary, such as when operating from or above urban and natural canyons, forests, or naval ships. Hence, for those situations a helicopter UAV, capable of flying in and out of such restricted areas, would represent a particularly attractive solution.

I.A. Background

A helicopter is a complex system, and understanding helicopter flight has been a continuous endeavor. Certainly helicopter nonlinear flight dynamics modeling has seen considerable development over the past forty years. We refer here to some of the foundational contributions[d] of the 1970s in Ref. 2–5, of the 1980s in Ref. 6–16, of the 1990s in Ref. 17–23, and for the last decade in Ref. 24–27. For a single main rotor, and briefly summarized, helicopter flight dynamics includes the rigid-body responses combined with higher-frequency modes.[28] These higher-frequency modes are generated by the main rotor system and its interaction with the fuselage and other vehicle components. For flight mechanics and control development purposes, the three most important aspects of these higher order rotor dynamics are blade flapping which allows the blade to move in a plane containing the blade and the shaft, blade lead-lag which allows the blade to move in the plane of rotation, and rotor inflow which is the flow field induced by the rotor at the rotor disk. On these subjects, an extensive discussion covering the various levels of required model complexity may be found in Ref. 15, 22, 29.

In Ref. 15 a general definition of helicopter model sophistication was formulated, to conveniently describe helicopter model complexity. This definition, slightly adapted here for R/C helicopters, is given hereunder

- **Dynamics**. The level of detail in representing the dynamics of the helicopter. This factor determines the validity of the model in terms of the frequency range of applicability, and in the sequel is divided into two sub-categories: (i) low/medium model bandwidths are such that the blade flap/lag dynamics and inflow dynamics are either omitted or elementary modeled, and (ii) high model bandwidths referring to models which do account, in a relatively detailed way, for (most of) those effects.[30, 31]
- **Validity**. The level of sophistication in calculating the helicopter forces, moments, and inflow. This factor determines the domain of validity in the flight envelope and is also divided into two sub-categories: (i) conventional flight as in hover and low speed maneuvers, and (ii) aerobatic/aggressive maneuvers including steep descent flight conditions.

I.B. Small-Scale Helicopter Dynamics: Review of Previous Work

In the past fifteen to twenty years, there has been considerable worldwide activity in research[e] related to automatic flight of small-scale helicopter UAVs. For example, for low to medium bandwidth systems, the usual robustness-performance trade-off has undeniably allowed for quick and successful demonstration (or simulation) of automatic helicopter flight for hover and low speed conditions, see Ref. 32–42. Further, for high bandwidth system specifications, at still conventional flight conditions, model-based automatic flight results can be found in Ref. 43–56, and non-model[f]-based examples have been documented in Ref. 57–60, while vision based systems have been reported in Ref. 61–66. Additionally, high bandwidth system specifications for aggressive/aerobatic flight conditions have also been successfully demonstrated, with the model-based approaches described in Ref. 67–69, and non-model-based ones given in Ref. 70–72. This said, and to the best of our knowledge, none of the previously mentioned high bandwidth model-based approaches are valid for high sink rates or steep descent flight conditions, such as the VRS or autorotation.[73]

I.C. General-Purpose Helicopter Simulation Codes

Several general-purpose helicopter simulation codes exist, often based on a multi-body dynamics approach. These codes have been extensively used, by industry and academia worldwide, such as GenHel,[7] CAMRAD,[74]

[d]Without considering aspects related to Inverse Simulation, Higher Harmonic Control (HHC) or Individual Blade Control (IBC).

[e]In the sequel, due to time and space constraints, we only review contributions in the field of helicopter UAV modeling for control synthesis, excluding thus system identification, navigation, and control aspects.

[f]We refer here to models which are generally not derived from first principles, such as in the areas of machine learning, evolutionary, and genetic algorithms.

FLIGHTLAB,[26] and HOST[24] to name a few. These simulation codes, with a proven track record stretching back two or three decades, are indeed very reliable and highly accurate. They represent excellent tools for among others helicopter flight simulation purposes, operational analysis, crew training, flying qualities investigations, auto-pilot design, load prediction, and vibrations analysis. For all that, these simulation codes have some disadvantages. First, these codes may be seen as third-party black-box models, since often one does not have complete access to their detailed analytical expressions, or corresponding software algorithms and implementations. Second, even when analytical expressions may be available, the multi-body model structure adds a huge amount of detail, resulting in very high order dynamical systems, effectively inhibiting any further manipulation of the analytical expressions. Third, for the calculation of blade and main rotor forces, the integration of the elemental lift and drag forces is often not solvable analytically, due to the complex nature of the expressions, but rather through numerical algorithms, which in turn precludes any use of closed-form expressions based models. Hence, all these reasons restrict the range of control techniques that could be used, when the end goal is to design a helicopter auto-pilot. Here we did not mention the use of lookup tables as a potential issue. Typically, tabular data provide a representation of vehicle propulsion, aerodynamic, or mass properties. Often these lookup tables are obtained as the result of experimental tests. Sometimes, this tabular data formulation is necessary simply because there was not enough knowledge to permit an analytical representation of the laws of physics. Nonetheless, these tables may be approximated by closed form expressions, through careful selection of either high-order polynomial expansions,[75] or (cubic) B-Splines.[76]

Now either white-box[g] or black-box models can be used for auto-pilot design, through judicious model linearizations, provided linear control techniques are employed. For example, these latter include classical methods,[77,78] optimal methods,[79–82] robust methods,[83,84] or receding horizon methods[85–87] methods, that all may be applied in a gain scheduling[88] or (quasi-) Linear-Parameter-Varying (LPV) framework.[89] On the other hand, black-box models cannot be used for auto-pilot design when nonlinear control techniques, that explicitly require closed-form modeling, are sought. Hence, the availability of our model and its corresponding analytical expressions, will allow us to research and evaluate several nonlinear control techniques. Notably, control methods that include (i) feedback linearization,[90,91] (ii) Lyapunov based methods such as sliding mode control,[92–94] or backstepping,[95,96] (iii) differential flatness,[97–100] (iv) State-Dependent Riccati Equation (SDRE),[101–105] or (v) the $\theta - D$ approach[106] may all be investigated in the future, through the use of a modified version of our helicopter nonlinear model.

I.D. Our Research Model

The purpose of our work is to present a small-scale helicopter UAV flight dynamics model for a flybarless, i.e. without a Bell-Hiller stabilizing bar, articulated Pitch-Lag-Flap (P-L-F) main rotor with rigid blades, applicable for high bandwidth control specifications, for both ClockWise (CW) and Counter-ClockWise (CCW) main rotor rotation, and valid for a range of flight conditions including the Vortex-Ring-State (VRS). Now due to space constraints, we present here the model from a common qualitative approach. The complete helicopter analytical expressions can be found in Ref. 107–109.

The nonlinear dynamic model includes the twelve-states rigid body equations of motion, the four-states/blade flap/lag angles and flap/lag rotational velocities, the three-states dynamic inflow, and the single-state main rotor Revolutions Per Minute (RPM). Thus, for a two-bladed helicopter main rotor, the full model includes twenty-four-states, while for a three-bladed helicopter main rotor, the full model includes twenty-eight-states. Besides, the model accommodates for an off-axis response correction factor, for flight in the VRS, and for deterministic[h] wind linear velocity inputs. Static ground effect has been accounted for by a correction factor applied to the non-dimensional total velocity at the rotor disk center. Further, computation of main rotor forces is done numerically through Gaussian quadrature integration, using a low order Legendre polynomial scheme. Additionally, the fuselage model is based upon aerodynamic lift and drag coefficients, which are tabulated as a function of airflow angle of attack and sideslip angles. These lookup tables are derived from a scaled-down full-size helicopter fuselage aerodynamic model. The horizontal and vertical tails have currently not been included, but they will be added at a later stage. For the tail rotor, this

[g] A model where all necessary information is available, e.g. based on available first principles expressions.

[h] Stochastic atmospheric turbulence will be added at a later stage.

latter has been modeled as a Bailey type rotor. Finally the paper reviews all assumptions made in deriving the model, i.e. structural, aerodynamics, and dynamical simplifications, which are valid for stability and control investigations of helicopters up to an advance ratio limit[i] of about 0.3.[5,110,111]

The remainder of the paper is organized as follows. In Section II, an introduction on helicopter main rotor (aero)dynamics is presented. In Section III, the rigid body equations of motion are reviewed. In Sections IV and V, the main and tail rotor models are discussed. In Section VI, simulation results are analyzed. Finally, conclusions and future directions are presented in Section VII.

II. The Helicopter Main Rotor

Since the early 1950s, it is known that including flapping dynamics in a helicopter flight model could produce limitations in rate and attitude feedback gains.[112] In fact blade flapping motion has three natural modes, i.e. coning, advancing, and regressing. The regressing flapping mode is the most relevant when considering the effect of rotor dynamics on handling characteristics, it is the lowest frequency mode of the three, and it has a tendency to couple into the fuselage modes.[111,113–118] Additionally, for helicopter directional axis control, blade lead-lag dynamics ought to be considered for control system design.[119] In particular it is well known that blade lead-lag produces increased phase lag at high frequency, in the same frequency range where flapping effects occur,[116] and that control rate gains are primarily limited by lead-lag-body coupling.[116,120]

Regarding the induced rotor flow, this latter contributes to the local blade incidence and dynamic pressure. This induced flow plays a key role in destabilizing the flapping mode; this may for example result in a large initial overshoot in the vertical acceleration response to an abrupt input in the collective pitch.[116,121] In fact for full-size helicopters, frequencies of inflow dynamics are of the same order of magnitude as those of rotor blade flapping and lead-lag modes. Hence inflow dynamics can have a significant influence on the performance of a main rotor system.[116,121] Inflow models can be divided into two categories, static and dynamic models. For low-bandwidth maneuvering applications, such as trim calculations or flying-qualities investigations, the dynamic effects of the interaction of the airmass with the airframe and rotor may be expected to be negligible, therefore static inflow models may be acceptable.[122] But for high bandwidth applications, dynamic interactions between the inflow dynamics and the blade motion must be considered. Conjointly, dynamic inflow models can be divided into two[j] unsteady categories: the Pitt-Peters dynamic inflow,[126–129] and the Peters-He finite-state wake model.[130–132] The finite-state wake model is a more comprehensive theory than dynamic inflow, not limited in harmonics and allowing to account for non linear radial inflow distributions, while the dynamic inflow model can be thought of as a special case of the finite-state wake model, with only three inflow expansion terms.[131,132] The sophisticated finite-state model is attractive when rotor vibration and aeroelasticity need to be analyzed.[133]

On the subject of wake distortion which is the primary source of the so-called off-axis response problem, observed in maneuvering flight at hover and low speed,[134,135] several modeling approaches have been researched over the years. For the interested reader, we refer here to the aerodynamic interaction between helicopter rotor and body in Ref. 136, the inclusion of a virtual inertia effect associated with the swirl in the rotor wake in Ref. 137, the introduction of an aerodynamic phase lag in flapping and dynamic inflow equations, with the use of system identification techniques in Ref. 138–142, the extended momentum model approach in Ref. 134, 143, 144, the free wake modeling in Ref. 145–147, the dynamic vortex ring modeling in Ref. 148–150, the augmented Pitt-Peters dynamic inflow model in Ref. 135, 151–155, and the augmented Peters-He finite state inflow model in Ref. 154–157.

Concerning ground effects, these can be divided into three domains, namely static ground effect, dynamic ground effect, and the ground vortex. Static ground effect, i.e. when the ground surface is not subject to movements, can be accounted for by a correction factor applied to the non-dimensional total velocity at the rotor disk center.[158] Dynamic ground effect takes place when for instance a helicopter is hovering above a

[i] The flight envelope of small-scale helicopters is well within this limit.
[j] Albeit recent advances in computing power and methodology have made it foreseeable to add a third category, namely that of detailed free-wake models that may be run in real-time for flight dynamics applications.[123–125]

heaving, rolling or pitching surface (such as a ship deck), resulting in unsteady aerodynamics effects at the rotor.[159,160] Additional complications may arise in the eventuality of having a helicopter either (i) hover above an inclined ground plane or ship deck, in which case not only the magnitude but also the distribution of rotor induced velocity changes,[161,162] or (ii) partially hover above a building top or a ship deck.[163] On the subject of ground vortex, this phenomenon is produced by the interaction of the rotor downwash, the ground, and the velocity of translation.[164] The earliest identification of the horseshoe ground vortex occurred in Ref. 165, followed by theoretical investigations in Ref. 166–168, and experimental results in Ref. 169–171. The consequences of such a ground vortex is that characteristics as trim control requirements, effectiveness of the tail rotor, and engine performance[k] may all be markedly changed by the proximity of a ground vortex.[170]

We conclude this section by briefly addressing the issue of atmospheric disturbances, since the main rotor is sensitive to these effects. Indeed wind and wind gusts, induced by atmospheric variations, or by local terrain or man-made structures, will change the aerodynamic conditions at the rotor, hence impact rotor blade lift and drag.[68,172] Atmospheric disturbances may be added as additive perturbations, and these come in three different forms, i.e. constant linear wind velocity, stochastic linear turbulence velocity, and stochastic rotational turbulence velocity. Linear disturbances will affect the vehicle aerodynamic velocity, while rotational turbulence velocities will affect vehicle body roll, pitch, and yaw rates. A low altitude turbulence survey with a huge amount of data can be found in Ref. 173, while Ref. 174 presents an early description of nonstationary[l] low altitude atmospheric turbulence. Additionally, two atmospheric models have extensively been used in the aerospace community. The first one is the *von Karman* model,[175] where an isotropic[m] turbulence model was assumed, which was validated by experimental measurements of low altitude in Ref. 176. The second one is the *Dryden* atmospheric model.[177] The difference between the two lies basically in a small variation of the high frequency content.[178] The *Dryden* model is most frequently used, owing to its greater simplicity of implementation, i.e. by passing white noise through linear filters.[179] On the other hand, FLIGHTLAB users will be more familiar with the *von Karman* spectrum, since this model with extensions from Ref. 180, 181 generates the additive vertical turbulence velocity. Further, the literature on transient and steady-state turbulence (gusts) modeling, and corresponding aircraft and helicopter response, is extensive to say the least. For the interested reader, we refer here to fixed-wing design and response to gusts in Ref. 182–187, while for the rotary-wing case an excellent *tour d'horizon* is provided in Ref. 188–190. Furthermore, the effect of deterministic gusts can be found in Ref. 191–193, analysis of flapping response to gusts in Ref. 194, coupled flap-torsion dynamics to stochastic vertical turbulence in Ref. 195–197, flap-lag dynamics to stochastic vertical turbulence in hover in Ref. 198,199, flap-lag dynamics to stochastic horizontal turbulence in hover in Ref. 200, and in forward flight in Ref. 201, and blade-fixed[n] atmospheric turbulence in Ref. 181, 202–206.

III. Rigid Body Equations of Motion

III.A. Assumptions

- The vehicle has a longitudinal plane of symmetry, and has constant mass, inertia, and Center of Gravity (CG) position, hence fuel consumption and/or payload pickup/release are neglected. The vehicle is also a rigid system, i.e. it does not contain any flexible structures, hence the time derivative of the inertia matrix is zero. Further variations of helicopter CG locations due to main rotor blades position are neglected.

- The vehicle height above ground is very small compared to the earth radius, implying a gravitation independent of height and thus constant. Additionally the center of mass and CG are identical for a constant gravity field.

- The earth is assumed fixed and flat. There is then no longer a distinction between the directions of

[k] In case of engine inlet ingestion of its own exhaust.[169]

[l] A stochastic process whose probability distribution changes when shifted in time or space.

[m] Statistical properties invariant with respect to direction.

[n] Body-fixed atmospheric turbulence refers to the turbulence experienced by a point fixed on a non-rotating vehicle component such as the vehicles CG, while blade-fixed atmospheric turbulence refers to the turbulence experienced by a component of a rotating rotor blade.[202] Indeed the atmospheric turbulence velocities seen by non-rotating vehicle components and rotating blades may be substantially different.

gravitational force and the force of gravity, hence the external force becomes the force of gravity[o]. Gravity is also a function of latitude, for all practical purpose we will consider the medium latitudes of 52°.

- Finally, we neglect the effect of buoyancy or Armichedes force, which is negligible with respect to all other forces.

III.B. Modeling

Classical Newtonian mechanics and the fundamental relationship of kinematics provide us with the standard twelve-state rigid body equations of motion. The model is detailed in Ref. 107, following notations of Ref. 207.

IV. Main Rotor Modeling

IV.A. Assumptions

Structural Simplifications

- Rotor shaft forward and lateral tilt-angles are zero. The blade has zero twist, constant chord, zero sweep, constant thickness ratio, and a uniform mass distribution.
- Rigid rotor blade in bending. Neglecting higher modes (harmonics), since higher modes are only pronounced at high speed.[22,209] Further, blade torsion is neglected since small-scale helicopter blades are generally relatively stiff.
- Rotor inertia inboard of the flap hinge is assumed small and thus neglected.

Aerodynamics Simplifications

- Vehicle flies at a low altitude, hence neglecting air density and temperature variations. Blade element theory[p] is used to compute rotor lift and drag forces. Radial flow along blade span is ignored. Pitch, lag, and flap angles are assumed to be small.
- Momentum theory[q] is used to compute the uniform inflow component.
- Compressibility effects are disregarded, which is a reasonable assumption considering small-scale helicopter flight characteristics. Viscous flow effects are also disregarded, which is a valid assumption for low angle of attacks and un-separated flow.[212,213]
- Aerodynamic interference effects between the main rotor and other helicopter modules, e.g. fuselage or tail rotor, are neglected.

Dynamical Simplifications

- Dynamic twist[r] is neglected. Hence blade CG is assumed to be located on the blade section quarter chord line.
- Unsteady (frequency dependent) effect for time-dependent development of blade lift and pitching moment, due to changes in local incidence are ignored. For example dynamic stall, due to rapid pitch changes, is ignored.
- A balanced rotor is assumed. In general most of the inertial terms, contributing to main rotor moments, vanish[s] when integrated around 2π azimuth.

[o] For further details on the geoid earth and gravity see Ref. 207, 208.

[p] Calculates the forces on the blade due to its motion through the air. It is assumed that each blade section acts as a 2-D airfoil to produce aerodynamic forces, with the influence of the wake contained in an induced angle of attack at the blade section.[210]

[q] States that the total force acting on a control volume is equal to the rate of change of momentum, i.e. mass flow entering and leaving this control volume.[210,211]

[r] Any offset in blade chordwise CG or aerodynamic center position will result in a coupling of the flap and torsion Degrees Of Freedom (DOF) in blade elastic modes.[22]

[s] These terms should be retained when evaluating rotor out-of-balance loads.[4]

IV.B. Modeling

The model uses the rigid blade coupled flap-lag equations of motion for a Pitch-Lag-Flap (P-L-F) hinges sequence. This hinge arrangement is indeed much more useful for modeling the rotor dynamics of small-scale flybarless R/C helicopters. The equations have been obtained through the Lagrangian method,[214] and are reported in Ref. 109. They are valid for a single articulated[t] rotor, with hinge springs and viscous dampers. Further all three hinges are physically separated and the model allows for both CW and CCW rotating main rotors. Although the flap-lag equations of motion are valid for small flap, lag, and pitch angles, the exact tangential and perpendicular blade velocity expressions have been retained, hence full coupling between vehicle and blade dynamics is modeled.

For the rotor forces, the procedure consists in integrating the elementary lift and drag forces over the blade span, then average (integrate) the result over one revolution, and finally multiply by the total number of blades.[107] The integrations are done numerically, through Gaussian quadrature integration, using a fifth order Legendre polynomial scheme.[215,216] For the rotor moments, they include contributions from four different sources: aerodynamics, inertial loads, flap hinge stiffness, and lag hinge damping.[107]

Regarding rotor inflow modeling for flight dynamics, we assumed that it was sufficient to consider the normal component of inflow at the rotor, i.e. the rotor induced downwash.[22] For flight dynamics applications, it was reported in Ref. 133 that the Peters-He model was not remarkably better than the Pitt-Peters formulation. Since our primary interest is flight dynamics, we have thus chosen to implement the more straightforward Pitt-Peters model,[126,128] with a correction for flight in the VRS from Ref. 217, and a pseudo-harmonic term to model thrust fluctuations in the VRS from Ref. 218.

Concerning wake bending during maneuvering flight, which may significantly change the inflow distribution over the rotor resulting in a sign reversal in the off-axis response, we chose to use the extended momentum constant coefficients model of Ref. 134, 143, 144 as it is simple to implement.[24,47,219]

For the aspect of ground effect, only a static ground effect has been accounted for, by a correction factor applied to the non-dimensional total velocity at the rotor disk center.[158]

Finally for the atmospheric disturbance, currently only deterministic wind linear velocity inputs are available. Furthermore, for the case of our helicopter UAV and due to its small scale compared to the large-scale turbulence model, we will assume that the entire rotor disk experiences a spatially uniform turbulence velocity, identical to that at the rotor center, hence spatial gradients effects will be deemed insignificant.

V. Tail Rotor Modeling

V.A. Assumptions

Structural simplifications

- The blade has zero twist, constant chord, zero sweep, and has constant thickness ratio.
- The blade is rigid, hence torsion is neglected.

Aerodynamics simplifications

- Linear lift with constant lift curve slope, and uniform induced flow over the rotor.
- Aerodynamic interference effects from main rotor are neglected.
- Compressibility, blade stall and viscous flow effects are disregarded.

Dynamical simplifications

- No blade dynamics and simplified inflow dynamics.
- Unsteady effects neglected.

[t] Although this model, with a proper combination of hinge offset and springs about the hinge, could also be used to model a hingeless rotor.

V.B. Modeling

The tail rotor is a powerful design solution for torque balance, directional stability and control of single main rotor helicopters. The theory we apply here is based on the work done by Bailey in Ref. 220. The chosen model, reported in detail,[107] is the standard approach towards tail rotor modeling, as implemented among others in Ref. 4, 221, 222.

VI. Simulation Results

Simulation plots and visual comparisons of our model, implemented in a MATLAB® environment,[223] with an equivalent helicopter FLIGHTLAB[26] model are described next, for a R/C flybarless two-bladed main rotor helicopter UAV, which physical characteristics are documented in Appendix B. For the FLIGHTLAB model, the following options have been selected.

- Articulated rotor, and blade element model. Quasi-steady airloads, based on the Peters-He three-state inflow model, with no stall delay effects.
- Bailey tail rotor, and ideal engine.

The model's simulation plots, presented in the sequel, are based on an adapted version of our baseline model. Specifically, the static expressions of the Pitt-Peters inflow model have been retained in lieu of the dynamic ones, since the former ones provide a better match with FLIGHTLAB. This unexpected observed behavior is a subject of ongoing research.

VI.A. Trim Results

A trim condition is equivalent to an equilibrium point, also called an operating point of a nonlinear system, which can be thought of as a specific flight condition.[52] Further, trim settings are a prerequisite for stability analysis, vibration studies, and control systems synthesis. Indeed, any flight vehicle should be able to maintain equilibrium during steady flight conditions, this means that the resultant forces and moments on the vehicle are equal to zero.[224] For helicopters however, the concept of trim is more complicated than of fixed-wing aircrafts.[225] A helicopter has components that rotate with respect to each other and with respect to the air mass. Hence, periodic forces and moments enter the dynamic equations, and we cannot simply eliminate them by averaging.[225]

Our trim module is structured as a constrained optimization problem. At equilibrium the resultant forces and moments on the vehicle should be equal to zero, hence the objective of the trim module is to minimize the three vehicle linear accelerations and the three rotational accelerations. The variables that the algorithm is allowed to manipulate include the four control inputs, and the vehicle roll and pitch states, since these latter two influence the projection of the gravity vector on the body frame. Additionally, constraints are specified, i.e. by assigning fixed values to the three vehicle linear velocities, the three vehicle rotational velocities, and by setting to zero the three dynamic inflow linear accelerations. Now regarding the periodic states, i.e. blade flap and lag positions and velocities, these states are handled by time-marching the nonlinear helicopter model long enough until the transients have decayed. Finally, the remaining four states which include the three vehicle Cartesian position and the vehicle heading are left free, since the position of the helicopter does not influence[u] its dynamic behavior or stability. The optimization is further based on a Newton iteration scheme, similar to that of Ref. 222, which is simple to implement and has been widely used.[226] The Newton method guarantees quadratic convergence, but only guarantees local convergence, and is also sensitive to initial starting values. Even with good starting values, the method can exhibit erratic divergence due to for example numerical corruption.[226] Hence over the years, several other approaches have been researched, for a review of helicopter trim strategies see among others Ref. 22, 224–230.

[u] Although strictly speaking this is not true in vertical flight, due to the ground effect when trimming near the ground, and due to changes in air density when trimming with a non-zero vertical velocity; however for the case of air density variations, these may be neglected when considering small-scale UAV applications, since the flight altitude is generally within 200-300m above ground.

Comparisons of model trim results with FLIGHTLAB are discussed next. In figure 1, figure 4, and figure 7 the roll and pitch angles are plotted as a function of body linear velocities $(u, v, w)^{\mathrm{v}}$. We see that the maximum absolute deviations do not exceed 0.4°, refer also to table 1. In figure 2, figure 5, and figure 8 the main and tail rotors collective inputs are visualized, together with the main rotor power. For the main and tail rotor collective the match with FLIGHTLAB is good to very good, while the agreement on main rotor power is only good up until a velocity[w] around $u = 20$m/s. Further the top figure of figure 2 also gives us the minimum power speed, also called the *bucket speed*, predicted to be around $u = [15 - 18]$ m/s by the model and FLIGHTLAB respectively. Additionally and as expected, we see from the top figure of figure 5 that it takes more power for vehicle starboard flight (i.e. to the right) than for port-side (i.e. left) flight. This is due to the fact that the main rotor turns CW which implies that the tail rotor thrust is oriented towards port-side to counter the main rotor torque. For the longitudinal and lateral cyclic inputs, these are given in figure 3, figure 6, and figure 9. Overall, as given from table 1, the fit can be considered as being good to very good.

We conclude this trim section by quickly addressing the issue of steady autorotative flight.[73] figure 10 gives the main rotor power as a function of vehicle velocity V, for several flight path angles FP, ranging between -15°and -45°. Now in autorotation the clutch[x] is disengaged, and if we neglect the power losses due to the transmission/gearbox and any power losses from the tail rotor[y], then the zero-torque values as given by points A and B in figure 10 ought to give us the required vehicle velocity and flight path angle to achieve a steady autorotative flight, at the nominal main rotor RPM. For the case of our helicopter, we see that both the model and FLIGHTLAB predict these values to be at $V = 10.5$ m/s for $FP = -45°$ and $V = 13.3$ m/s for $FP = -30°$. Finally figure 11 has been added to visualize the effect of vehicle mass variation on the autorotative velocity. We first note the paradox, as reported in Ref. 231, that the autorotative rate of descent increases as the vehicle mass decreases, see points C and D in figure 11. Second, and for the case of our small-scale helicopter, this effect may be qualified as week, since a 25% variation in vehicle mass results only in a 4% variation of vehicle autorotative velocity, and hence rate of descent.

Name	$\lvert\Delta_{max}\rvert$ (in °)		
	along u	along v	along w
Roll	0.35	0.25	0.4
Pitch	0.4	0.25	0.1
MR Collective	0.6	0.6	0.7
TR Collective	0.75	1	0.7
MR Longitudinal Cyclic	0.25	0.6	0.05
MR Lateral Cyclic	0.65	0.25	0.02

Table 1. Maximum absolute deviations in trim between model and FLIGHTLAB

VI.B. Dynamic Results

For the validation of a model dynamic responses, we may consider two approaches. The first one consists in obtaining a linearized model which describes the small perturbation motion about a trimmed equilibrium position. The validation is then carried out by comparing the frequency response predicted by the linearized model and the frequency response obtained from either an equivalent linear FLIGHTLAB model, or from a linear model identified from flight test data. The second approach consists in comparing the time histories of the (nonlinear) model and those obtained from again either FLIGHTLAB, or flight test data. In this paper, we only provide visual comparisons of time histories data with FLIGHTLAB for roll/pitch/yaw angles

[v] With w positive down.

[w] The Bell UH-1H top speed is 60.28m/s, thus based on Froude scaling[68] with $N = 7.75$ the top speed of our R/C helicopter would then be $60.28/\sqrt{N} = 21.65$m/s. Hence for our helicopter a speed of $u = 20$m/s may be considered as a top speed. In fact we do not intend to operate the vehicle beyond a speed range of [10 - 15] m/s.

[x] All helicopters are equipped with an overrunning clutch between the transmission and the engine, so that the rotor does not have to drive a dead engine during autorotation.

[y] In case the tail rotor is still driven by the main rotor even though the clutch is disengaged. Note that this is not the case for all R/C helicopters.

(ϕ, θ, ψ), linear velocities (u, v, w), and rotational velocities (p, q, r). Since a helicopter is also a perfect example of a Multiple-Input-Multiple-Output (MIMO) system, table 2 has been provided to better understand the impact of each input channel on the vehicle response.

		Response			
		Pitch	Roll	Yaw	Climb/Descent
Input Axis	Long stick	Prime	Due to lat flapping	Negligible	Desired in fwd flight
	Lat stick	Due to long flapping	Prime	Undesired in hover, desired in fwd flight	Descent with bank angle
	Rudder	Negligible	Roll due to TR thrust & sideslip	Prime (hover)	Undesired, due to power changes in hover
	Collective	Due to transient & steady long flapping	Due to transient & steady lat flapping & sideslip	Power change varies requirement for TR thrust	Prime

Table 2. Single-rotor helicopter coupling sources (short version, from Ref. 232). *Long* stands for Longitudinal, *Lat* for Lateral

The tests are set to evaluate the open-loop response of this highly unstable model, at a constant main rotor RPM. First, the rotor is allowed to reach a steady-state condition during a time period of 0.5s. Then, for the following 3s we simultaneously apply on the four input channels a sinusoid of $1°$ in amplitude, at a frequency of 2Hz[z].

The first test is run from a hover trim condition, see figure 12, where it can be seen that the overall fit with FLIGHTLAB is good to very good. The second test is carried out to evaluate the medium speed characteristics at $u = 5$m/s, see figure 13, where we can see that the match with FLIGHTLAB is good to very good for $(\phi, \theta, \psi, w, p, q, r)$, while the fit for (u, v) is good for the first 2s, after which the quality of the fit starts to decrease. The third trial is run to check the high speed flight at $u = 10$m/s, see figure 14, where we can see that the match with FLIGHTLAB is acceptable for q, is good to very good for (ϕ, ψ, v, p, r), and the fit for (θ, u, w) is good for the first 2s, after which the quality of the fit starts to decrease. Finally the fourth test is run to check the response in the VRS region, see figure 15, at a value $w = 5$m/s corresponding to approximately one time the induced velocity in hover. Here it can be seen that states $(\phi, \theta, u, v, p, q)$ exhibit a good to very good match with FLIGHTLAB, that the fit for w is good for the first 1.5s - 2s, while the yaw axis (ψ, r) fit deteriorates after 1s.

Regarding the observed discrepancies between our model and FLIGHTLAB, especially those seen at high speed or on the yaw channel in the VRS, these may very probably be attributed to the following five items: (i) validity of the flap-lag equations of motion up to about $u = 10 - 15$m/s, see Ref. 109, (ii) a somewhat distinct implementation of the Bailey type tail rotor, (iii) a distinct implementation of the induced rotor flow, i.e. FLIGHTLAB uses the Peters-He finite-state wake model,[130–132] while our model applies the static

[z]Corresponding to the maximum anticipated closed-loop system bandwidth for autonomous flight.

version of the Pitt-Peters model,[126, 128] (iv) a distinct implementation of the induced rotor flow in the VRS, i.e. FLIGHTLAB uses the method presented in Ref. 233, while our model utilizes a slightly adapted version of Ref. 217, and finally (v) any side-effects due to the model simplifications as presented in Section IV. This said, we believe that most of the observed differences may primarily be attributed to the first three items, namely distinct models and hence behavior of the main rotor blade flap-lag, tail rotor inflow, and main rotor inflow.

VII. Conclusion

We have presented a UAV helicopter flight dynamics nonlinear model for a flybarless articulated Pitch-Lag-Flap (P-L-F) main rotor, with rigid blades, and applicable for high bandwidth control specifications. The model allows for both ClockWise and Counter-ClockWise main rotor rotation, and is valid for a range of flight conditions including autorotation and the Vortex-Ring-State (VRS). Further, this model has been compared with an equivalent FLIGHTLAB nonlinear model. Simulation results show that the match between the model and FLIGHTLAB is very good for static (trim) conditions, is good to very good for dynamic conditions from hover to medium speed flight $u = 5$m/s, is fair to good for dynamic conditions at high speed $u = 10$ m/s, and except for the yaw channel is also good in the VRS. While keeping in mind the model's accuracy reduction at high speed, this model could potentially be used to simulate and investigate the flight dynamics of a flybarless small-scale UAV helicopter, including in autorotation and VRS conditions, as well as provide a basis for model-based control design. Indeed, future work will focus on the development of nonlinear and linear control schemes. In particular, we have currently used an adapted version of this model, based on closed-form expressions, to obtain optimal helicopter flight trajectories, by solving constrained nonlinear optimal control problems. This topic will be elaborated upon in future publications.

Appendix A: Simulation Results

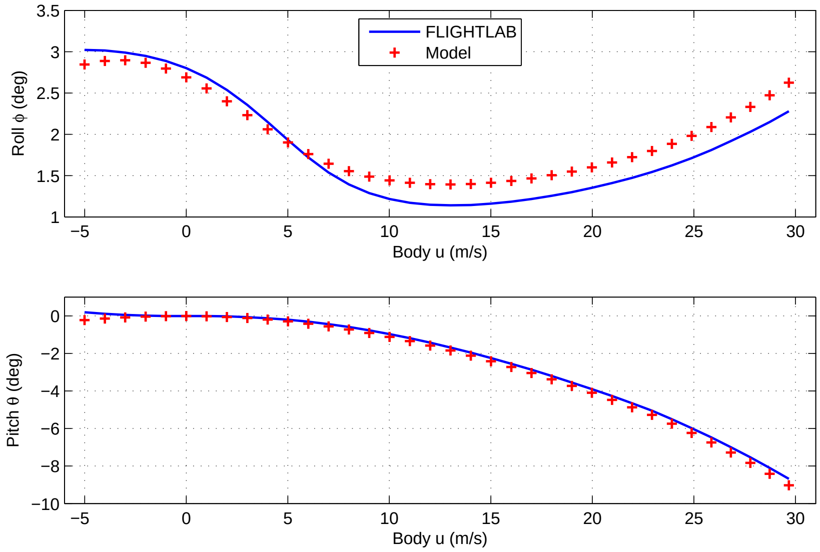

Figure 1. Trim roll and pitch angles as a function of body longitudinal velocity u

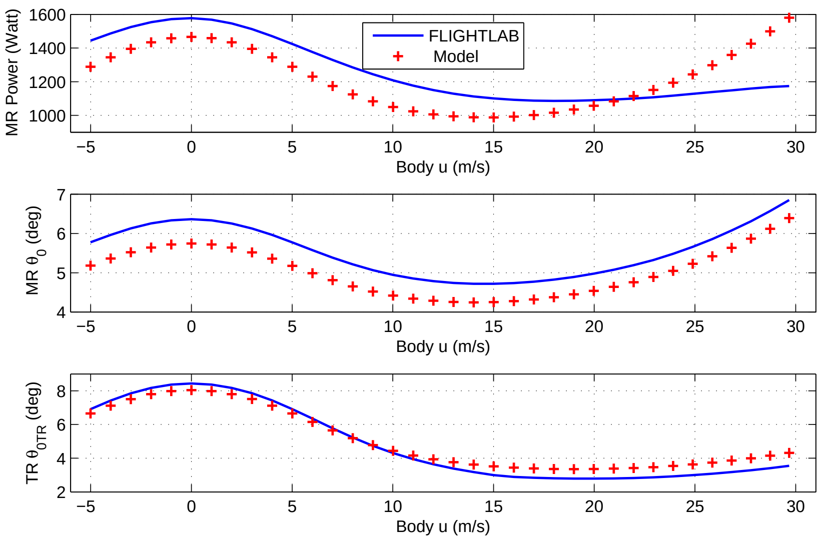

Figure 2. Trim main rotor power and main/tail rotor collective pitch angles as a function of body longitudinal velocity u

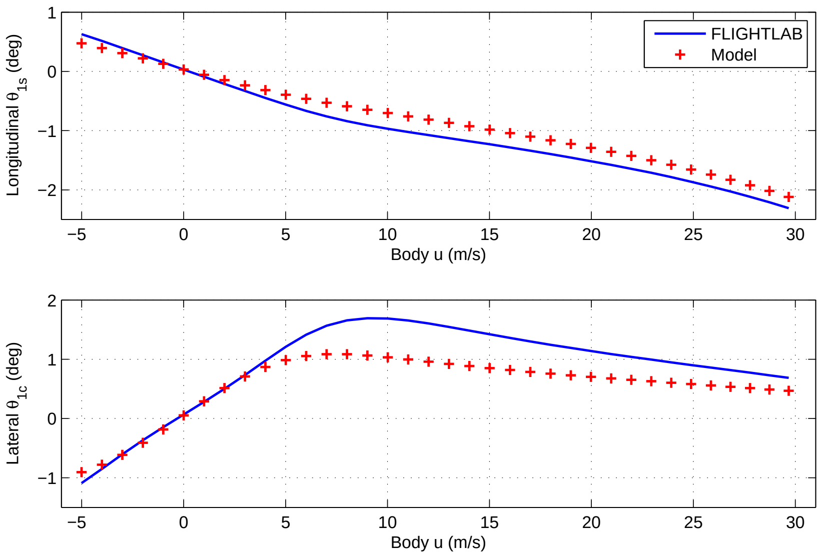

Figure 3. Trim main rotor longitudinal and lateral cyclic pitch angles as a function of body longitudinal velocity u

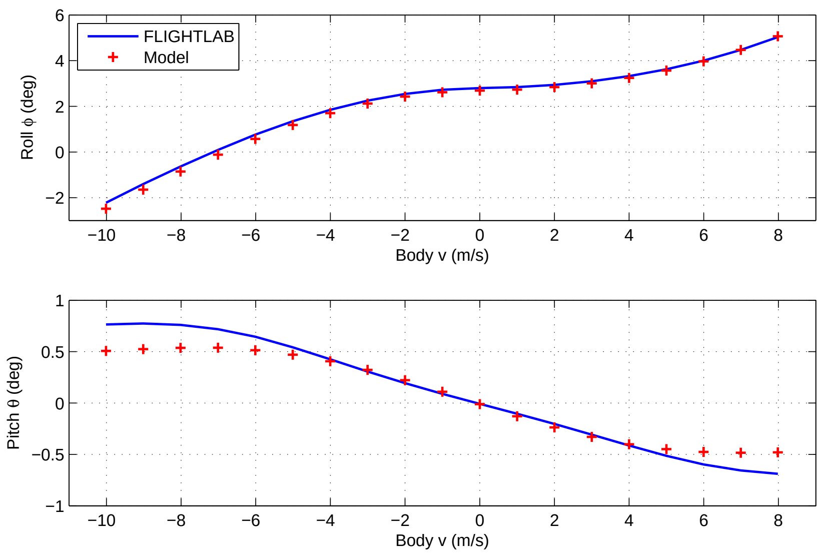

Figure 4. Trim roll and pitch angles as a function of body lateral velocity v

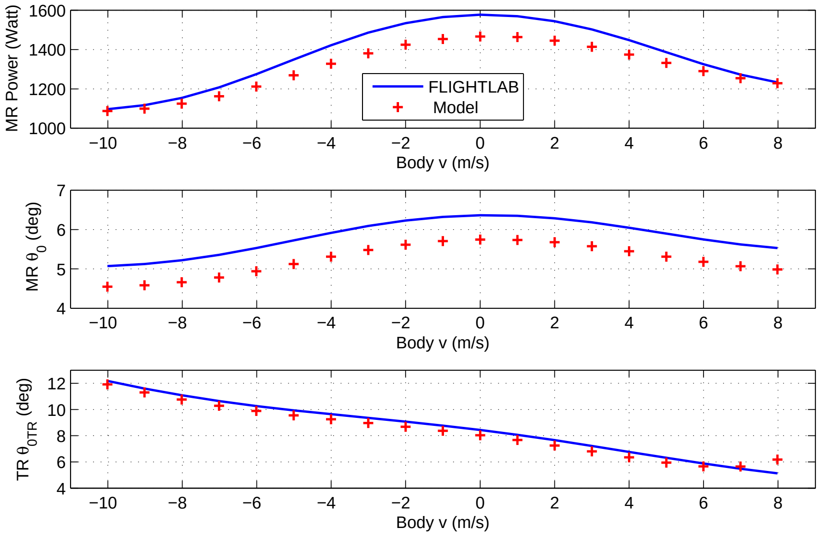

Figure 5. Trim main rotor power and main/tail rotor collective pitch angles as a function of body lateral velocity v

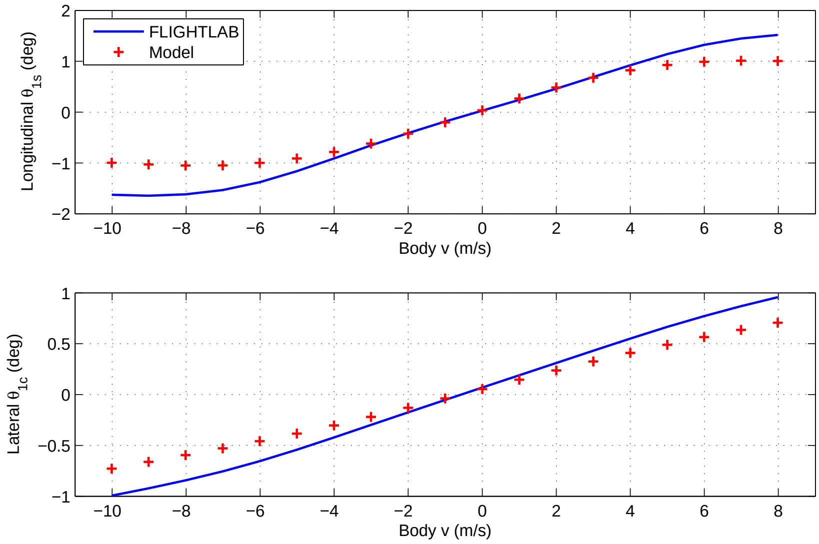

Figure 6. Trim main rotor longitudinal and lateral cyclic pitch angles as a function of body lateral velocity v

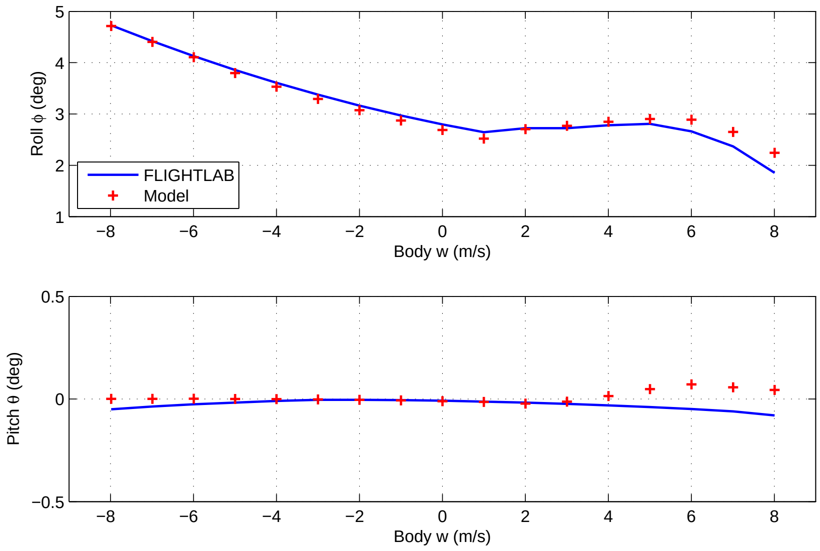

Figure 7. Trim roll and pitch angles as a function of body vertical velocity w

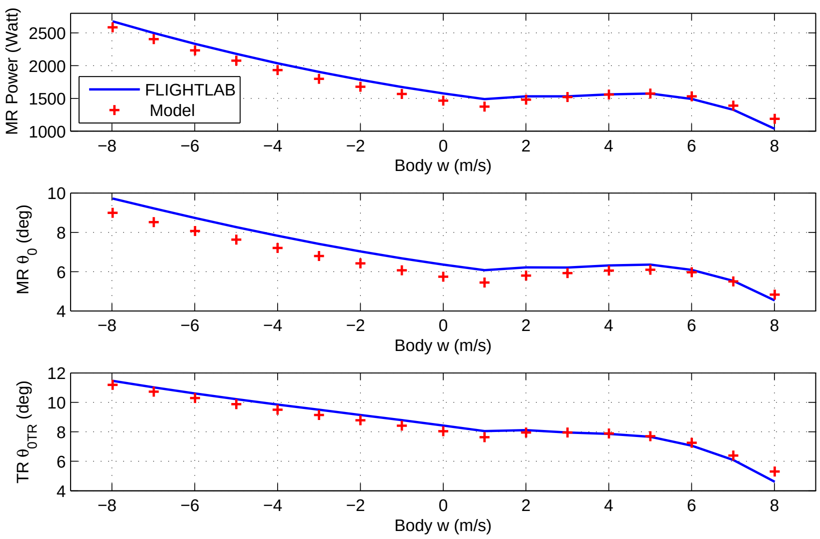

Figure 8. Trim main rotor power and main/tail rotor collective pitch angles as a function of body vertical velocity w

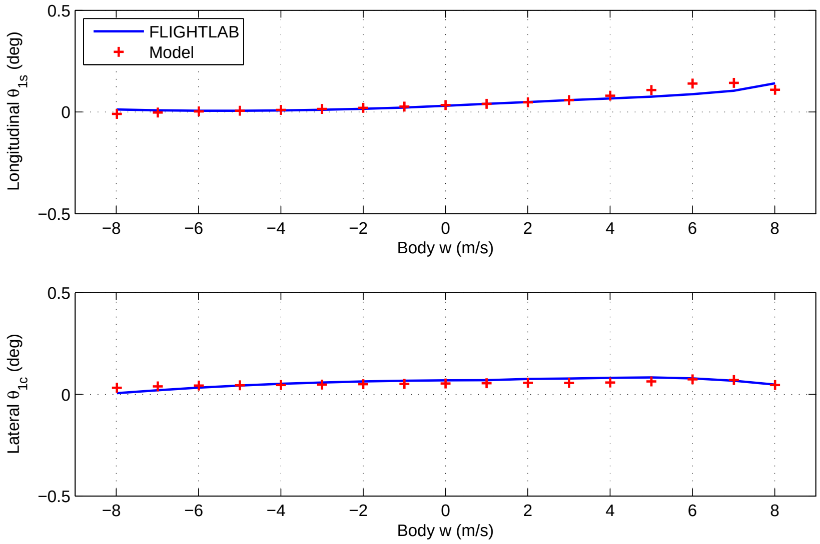

Figure 9. Trim main rotor longitudinal and lateral cyclic pitch angles as a function of body vertical velocity w

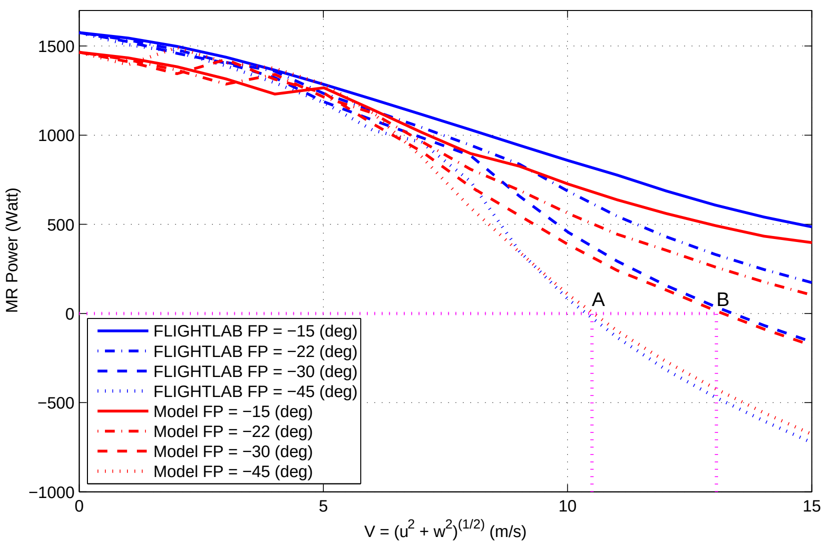

Figure 10. Trim main rotor power as a function of vehicle velocity V and flight path angle FP

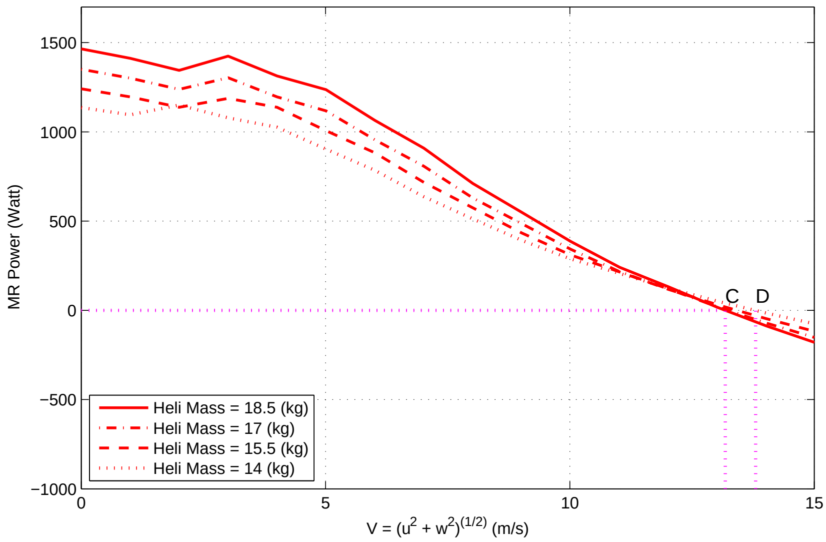

Figure 11. Trim main rotor power as a function of vehicle velocity V for several vehicle mass, at a flight path angle $FP = -30°$

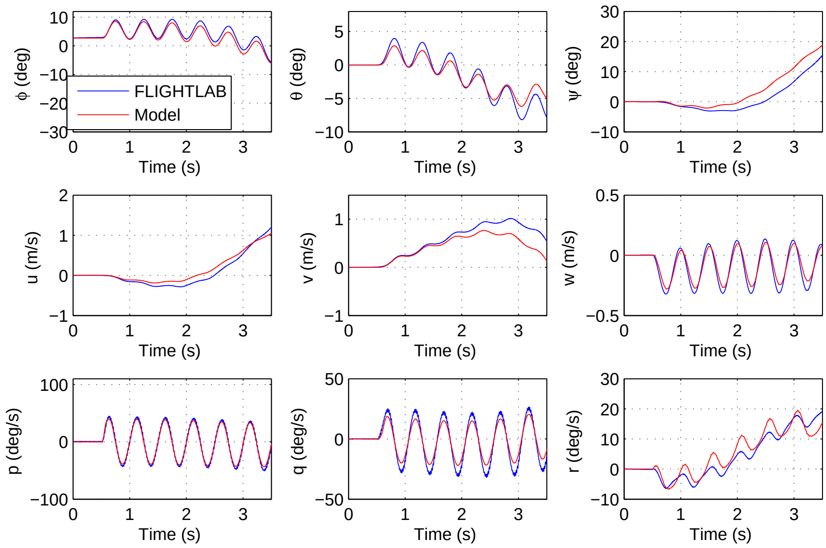

Figure 12. Vehicle response to sinusoidal inputs (at hover)

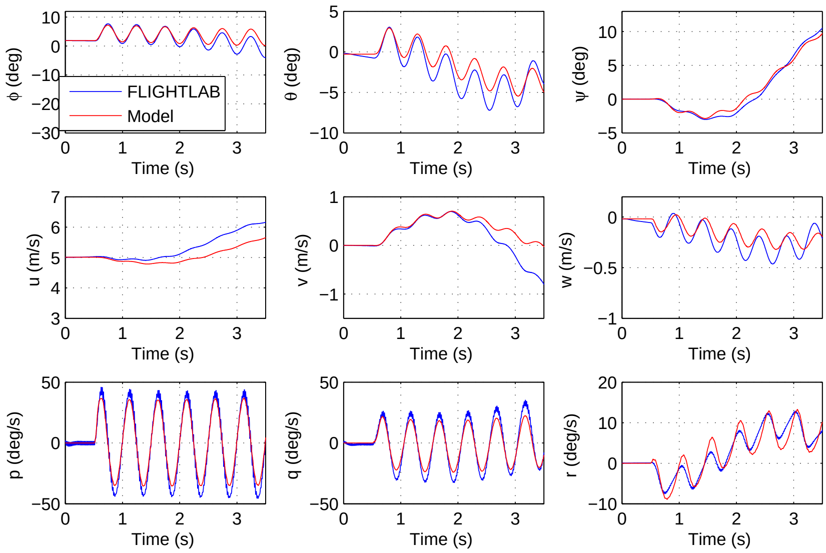

Figure 13. Vehicle motion: response to sinusoidal inputs (at $u = 5$ m/s)

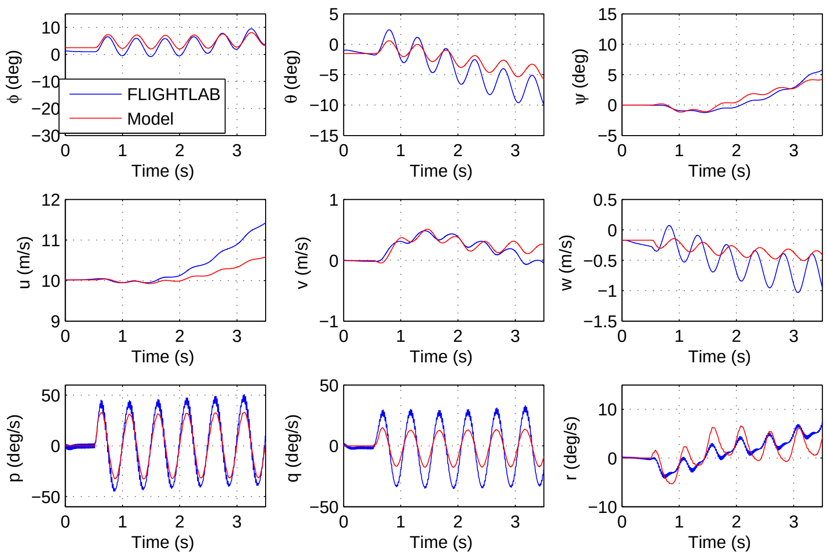

Figure 14. Vehicle response to sinusoidal inputs (at $u = 10$ m/s)

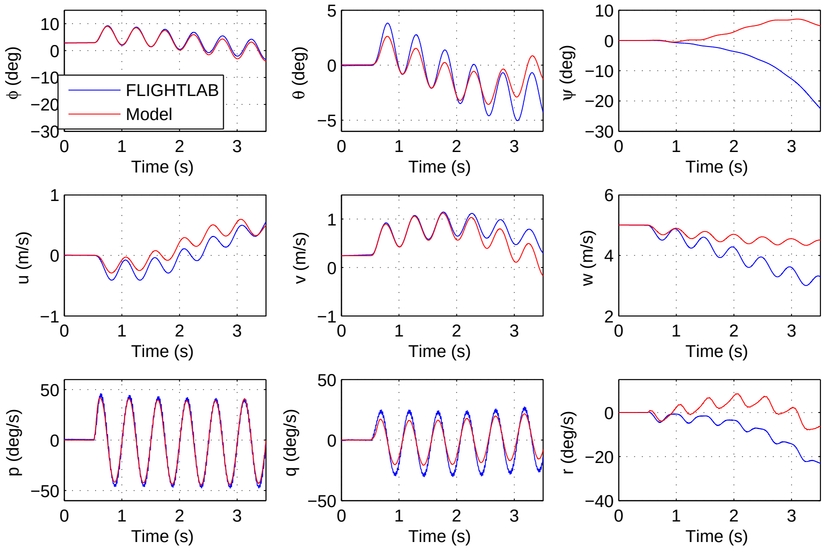

Figure 15. Vehicle response to sinusoidal inputs (at $w = 5$ m/s)

Appendix B: Physical Parameters

	Name	Parameter	Value	Unit
Environment	Air density	ρ	1.2367	kg/m^3
	Static temperature	T	273.15 + 15	K
	Specific heat ratio (air)	γ	1.4	
	Gas constant (air)	R	287.05	$J/kg.K$
	Gravity constant	g	9.812	m/s^2
Vehicle	Total mass	m_{Zerof}	18.5	kg
	Fuselage mass	m_{Fus}	17.94	kg
	Fuselage inertia moment wrt x_b	A	0.3	$kg.m^2$
	Fuselage inertia moment wrt y_b	B	0.76	$kg.m^2$
	Fuselage inertia moment wrt z_b	C	0.86	$kg.m^2$
	Fuselage inertia product wrt x_b	D	0	$kg.m^2$
	Fuselage inertia product wrt y_b	E	0	$kg.m^2$
	Fuselage inertia product wrt z_b	F	0	$kg.m^2$
	X-pos. of fus. CG wrt total CG	x_H	0	m
	Y-pos. of fus. CG wrt total CG	y_H	0	m
	Z-pos. of fus. CG wrt total CG	z_H	0.015	m
	X-pos. of MR hub wrt total CG	x_H	0	m
	Y-pos. of MR hub wrt total CG	y_H	0	m
	Z-pos. of MR hub wrt total CG	z_H	-0.36	m
	X-pos. of TR hub wrt total CG	x_{TR}	-1.150	m
	Y-pos. of TR hub wrt total CG	y_{TR}	0.040	m
	Z-pos. of TR hub wrt total CG	z_{TR}	-0.070	m
	Direction of rotation	Γ	-1 (CW)	
	Number of blades	N_b	2	
	Nominal angular velocity	$\Omega_{MR_{100\%}}$	151.843	rad/s

Main Rotor MR	Rotor radius from hub	R_{rot}	0.944	m
	Swashplate phase angle	ψ_{PA}	0	rad
	Precone angle	β_P	0	rad
	Pitch-flap coupling ratio	$K_{(\theta\beta)}$	0	
	Pitch-lag coupling ratio	$K_{(\theta\zeta)}$	0	
	Spring restraint coef. due to flap	K_{S_β}	271.1635	$N.m/rad$
	Spring damping coef. due to flap	K_{D_β}	0	$N.m.s/rad$
	Spring restraint coef. due to lag	K_{S_ζ}	0	$N.m/rad$
	Spring damping coef. due to lag	K_{D_ζ}	24.4047	$N.m.s/rad$
	Off-axis roll coef.	K_p	0	
	Off-axis pitch coef.	K_q	0	
	Offset distance	e_P	0.035	m
	Offset distance	e_L	0.049	m
	Offset distance	e_F	0.010	m
	Blade mass	M_{bl}	0.277	kg
	Blade twist at tip	θ_{wash}	0	rad
	Blade chord	c_{bl}	0.076	m
	Hub arm chord	c_{hub}	0.015	m
	Root cutout from flap hinge	r_c	0.006	m
	Y-pos. blade CG wrt flap hinge	$y_{G_{bl}}$	0.8932	m
	Tip loss factor	B	0.97	
	Airfoil lift coef.	$c_{l_{bl}}$	NACA0012	
	Airfoil drag coef.	$c_{d_{bl}}$	NACA0012	
	Airfoil pitching moment coef.	c_M	NACA0012	
	Lift deficiency factor	K_{defic}	0.89	
Transmission	Gearbox transmission ratio	GB	4.67	
Tail Rotor TR	Number of blades	$N_{b_{TR}}$	2	
	Rotor radius from hub	$R_{rot_{TR}}$	0.18	m
	Pitch-flap coupling	$\delta_{3_{TR}}$	0	rad
	Preset collective pitch bias	$\theta_{bias_{TR}}$	0	rad
	Partial coning angle wrt thrust	$\beta_{0_{TR}}$	0	rad/N
	Tail blockage constant	b_{t_1}	1	
	Transition velocity	v_{bl}	0	m/s
	Blockage due to vertical fin	k_{bl}	1	
	Correction factor	$K_{TR_{corr}}$	1/1.270	
	Blade chord	c_{TR}	0.035	m
	Tip loss factor	B_{TR}	0.92	
	Airfoil lift curve slope	$c_{l_{(0,TR)}}$	5.73	rad^{-1}
	Blade drag coef.	$c_{d_{TR}}$	0.035	
Actuators	MR collective range	θ_0	[-3,10].π/180	rad
	MR lateral cyclic range	θ_{1c}	[-7,7].π/180	rad
	MR longitudinal cyclic range	θ_{1s}	[-7,7].π/180	rad
	TR collective range	$\theta_{0_{TR}}$	[-12,18].π/180	rad

Table 3: Physical Parameters

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