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Probabilistic Approaches toward Conflict Prediction

G.J. Bakker, H.J. Kremer and H.A.P. Blom

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Probabilistic Approaches Toward Conflict Prediction

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Abstract

In this paper four conflict prediction approaches are considered: a classical geometric approach, two variations of a probabilistic approach developed by Paielli & Erzberger, based on conflict probability and overlap probability, and a novel probabilistic approach. The objective of all conflict prediction approaches is to evaluate a set of planned or predicted trajectories on their conflict potential and to supply other Air Traffic Management (ATM) subsystems with the conflict information. The classical geometric approach and approaches based on conflict probability and overlap probability are briefly reviewed. The novel probabilistic approach is described and explained in more detail. Simulation results for ATM examples are provided and compared for the four approaches on flexibility of usage and imposed restrictions on aircraft behaviour.

I. Introduction

In this paper, the conflict prediction part of conflict probing will be considered. We will consider four approaches concerning conflict prediction. The first approach is the classical geometrical approach; the second approach is the probabilistic approach described in Refs. 1 and 2; the third approach is a variation of the second approach, and the fourth approach is a novel probabilistic approach, which is based on collision risk formulae ([3]).

The objective of all conflict prediction approaches is to evaluate a set of planned or predicted trajectories for their conflict potential and to supply other air traffic management (ATM) subsystems with the conflict information. In this paper the focus will be on the detection of conflicts between predicted aircraft trajectories in ATM.

When predicting aircraft trajectories, the prediction uncertainty increases with the prediction period. This is caused by the fact that prediction errors accumulate over time. It is assumed that the trajectories that predict the future aircraft behavior are four-dimensional trajectories. A four-dimensional trajectory is defined by predicted three-dimensional positions and corresponding predicted times that are given for all points on that trajectory. These four-dimensional trajectory predictions are evaluated for their conflict potential.

This paper will compare the mentioned conflict prediction approaches with which conflict potential is evaluated with pairs of predicted four-dimensional trajectories. It is a continuation of Ref. 4. The paper is organized as follows.

First, the classical geometric conflict prediction approach will be considered. Some limitations will be highlighted that create the reason why we will study probabilistic conflict prediction approaches. The first probabilistic conflict prediction approach that will be considered is based on conflict probability ([1], [2]). This approach will be reviewed briefly. The second probabilistic approach is based on overlap probability and is introduced as a variation of the first probabilistic approach. The third probabilistic approach that will be considered is based on collision risk formulae ([3]). This approach will be explained briefly.

Issues like flexibility of usage and restrictions on aircraft behavior of the four approaches will be discussed, and conclusions will be drawn.

II. Conflict Prediction Approaches

A. Geometric Conflict Prediction Approaches

The classical geometric conflict prediction approach that is performed with a pair of predicted four-dimensional trajectories will be considered. Input for the geometric conflict prediction is the predicted four-dimensional trajectory. The uncertainty of the predicted four-dimensional trajectory is translated into areas around the predicted trajectory. Let us refer to these areas as protection zones. The protection zones are such that at any time in the future, the probability that an aircraft is inside its protection zone is larger than some threshold. The size and shape of the protection zones may vary with time. The protection zones for the horizontal plane and for the vertical plane are defined independently. Horizontal and vertical distances between protection zones should be such that they are safe. Two aircraft are said to be in geometric conflict when the distance between the protection zones of those aircraft becomes smaller than the minimum allowed distance between them [e.g. defined by the International Civil Aviation Organization (ICAO)]. Information like the duration (e.g. time interval in which two aircraft are in geometric conflict) and minimum distance between the protection zones can be generated (e.g. Refs. 5 and 6).

B. Limitations of Geometric Approaches

Let us start by considering various causes that result in aircraft deviating from their predicted four-dimensional trajectories. These causes exist in all parts of ATM, some examples are 1) wind modeling and prediction errors, and 2) tracking, navigation and control errors.

Large wind modeling and prediction errors can result in aircraft that deviate from their predicted trajectory. The same result applies for large tracking, navigation and control errors. Conflict prediction methods predict aircraft deviations from their predicted trajectory, and on the basis of this prediction, conflict potential is evaluated. Geometric conflict prediction approaches translate the mentioned prediction uncertainties in areas around the predicted aircraft positions (protection zones). The main limitation of this geometric approach to conflict prediction is its tendency to be overly conservative in handling uncertainties in aircraft behavior. For example, climbing or descending aircraft are given a lot of moving space. To improve conflict prediction, uncertainties should be handled less conservatively than geometric approaches handle them. However, uncertainties should still be handled conservatively enough to keep the sky safe.

The key attribution of this paper is that the mentioned limitation of geometric approaches towards conflict prediction can be overcome by an appropriate probabilistic approach. Furthermore, using probabilistic conflict prediction, more information about conflicts or encounters can be provided (e.g., probabilities, collision risks), which can be exploited for an improved quality of the decision whether there is a conflict or not. (Thus one might expect the number of false and missed conflicts to be reduced). Therefore, there is a clear reason to study probabilistic conflict prediction approaches.

In this paper three probabilistic approaches are discussed. The first probabilistic approach is the conflict probability approach ([1], [2]). The second probabilistic approach is a variation of the first probabilistic approach and is based on overlap probability (also based on the method described in Refs. 1 and 2). The third probabilistic approach is based on collision risk formulae ([3]). There are some basic differences between the probabilistic approaches. These differences will become clear when the approaches are described.

C. Conflict Probability Approach

The authors of Refs. 1 and 2 have developed a method to evaluate conflict probabilities. The approach is initially developed to predict conflicts in the horizontal plane only. In their approach a conflict is defined as a situation in which the separation between aircraft falls below a certain separation threshold. Evaluation of conflict potential is done based on the evaluated conflict probabilities.

In Refs. 1 and 2 the conflict prediction is focused on free flight. The future deviations of the aircraft from the expected four-dimensional trajectories are predicted by probability density functions. They realized that in free flight the further you predict a trajectory in the future, the less certain these predictions are. Note that this does not need to be the case in the four-dimensional ATM philosophy, in which aircraft are kept within some boundaries around their planned four-dimensional trajectory.

In the case of free flight, the decision whether aircraft will approach each other too closely is seen as a tradeoff between efficiency and certainty. To optimize this tradeoff, the authors of Refs. 1 and 2 developed a method to describe the certainty. The approach aims to predict the probability that the separation between two aircraft falls below a certain separation threshold (e.g., ICAO separation standards). This probability is called conflict probability. The goal is to keep the conflict probability below some acceptable level. In order to evaluate the conflict probability, they assume that it is realistic to model the deviations of the aircraft from their expected trajectories by Gaussian density functions. Using the direction of the relative velocity at time of minimum predicted separation, i.e., as the minimal horizontal distance between the expected trajectories, the probability density function of the relative position at that time is obtained. An analytical expression is obtained to estimate the conflict probability. For a more extended treatment the reader is referred to Refs. 1 or 2.

D. Overlap Probability Approach

So far, the approach described in Refs. 1 and 2 is used to predict the probability that the separation between two aircraft falls below a threshold that is determined by (e.g. ICAO) separation standards; this probability is called conflict probability. If, however, for this threshold a value like the size of an aircraft is used, then the same approach yields the overlap probability. Thus, overlap probability follows from a variation of the approach developed by Refs. 1 and 2; with the threshold reduced to the size of an aircraft, the overlap probability reflects the probability that the aircraft physical volumes overlap.

E. Collision Risk Approach

In our novel probabilistic approach, the conflict potential is evaluated through collision risk formulae ([3]), which are a generalized version of Reich's collision risk approach ([7]) adopted by ICAO. The generalizations have been developed because the Reich model applies under rather restrictive assumptions only.

The resulting collision risk equals the probability of collision between two aircraft. The steps that have to be taken in the novel approach are as follows. First, the joint probability density functions of the positions and velocities of individual aircraft are predicted, and then the joint probability density function of the relative position and velocity of an aircraft pair is evaluated. Then the collision risk for the aircraft pair is evaluated using the generalized Reich collision risk equations. This novel collision risk approach will be briefly elaborated next.

III. Collision Risk Modeling

A. Generalized Reich Collision Risk Model

In this section we briefly discuss the generalized Reich collision risk model without going too much into the mathematical details. For a detailed description we refer to Refs. 3 and 8.

Let the stochastic process $\{ s_t^i \}$ represent the position of the center of aircraft *i*, and let $\{ v_t^i \}$ represent its velocity.

Next, with s_t^i and s_t^j representing the positions of the centers of aircraft pair (*i,j*), the relative position is represented by the process $s_t \stackrel{\Delta}{=} s_t^i - s_t^j$, and the relative velocity is represented by the process $v_t \stackrel{\Delta}{=} v_t^i - v_t^j$.

Now we define an in-crossing of a certain area *D* around the origin as follows. The relative position s_t enters *D* at time *t*, if

$$s_{t-\Delta} \in D^c \text{ and } s_t \in D \quad \text{for } \Delta \downarrow 0$$

where D^c is an open set in R^3 and equals the complement of *D*. Each entering of *D* by the relative position s_t is called an in-crossing.

The in-crossing rate is defined as the expected number of in-crossings at time *t* per unit time and is denoted by $\varphi(t)$. In Ref. 3, the in-crossing rate is defined as

$$\varphi(t) \overset{\Delta}{=} \lim_{\Delta \downarrow 0} \frac{P\left\{s_t \in D \ , s_{t-\Delta} \in D^C\right\}}{\Delta} \tag{1}$$

We can express the collision risk between aircraft (probability of an in-crossing) for a time period $[t_1,t_2]$, denoted by $P_{ic}(t_1,t_2)$, as follows ([3]):

$$P_{ic}(t_1,t_2) = \int_{t_1}^{t_2} \varphi(t)\, dt \tag{2}$$

In Ref. 3, a characterization of the in-crossing rate $\varphi(t)$ has been derived under very general conditions. This model is called the generalized Reich collision risk model, in which it is assumed that the process $\{s_t,v_t\}$ admits a density function $p_{s_t,v_t}(.)$. For numerical evaluation of $\varphi(t)$, there is a need to characterize the probability density function $p_{s_t,v_t}(s,v)$ for the relative position s_t and the relative velocity v_t. Characterizing this probability density thus is an important part of the collision risk prediction problem.

B. Gaussian Case

To be able to compare the collision risk approach with the other approaches, we will assume that the position and velocity of each individual aircraft is Gaussian distributed with some mean and covariance. Using the well-known fact that a linear combination of Gaussian variables is also Gaussian, it is clear that the relative position and velocity are also Gaussian distributed.

Using the Gaussian probability density function of relative position and velocity, the in-crossing rate (1) can then be evaluated.

Next, the collision risk approach, the conflict probability approach, the overlap probability approach and the classical geometrical approach will be compared by applying them to Gaussian ATM examples.

IV. Comparison of Approaches

First of all, it should be noted that the collision risk approach deals with the problem of conflict (collision) prediction in a three-dimensional sense: horizontal and vertical movements are incorporated, also when they are not independent of each other. This implies a significant improvement over geometric approaches where the horizontal and vertical distances between protection zones are monitored independently and the conflict prediction approach of Refs. 1 and 2, which tends to define the probability of a horizontal conflict or overlap independently from the probability of a vertical conflict or overlap.

Next, the conflict prediction approaches are compared with each other by applying them to a two-dimensional example that was already described in Ref. 1. In the described ATM example, aircraft move in the horizontal plane only.

A. Situation and Modelled Uncertainties

In the examples, some parameters that define the situation can be distinguished. Which parameters and how they were taken is explained in the following. The exact values of the appropriate parameters are given in the sections in which the examples are discussed.

The probability density functions of the positions of the aircraft at a certain time are characterized by the predicted positions and their uncertainties in the across-track and along-track direction (the uncertainties are assumed to be Gaussian distributions, and so they are characterized by the standard deviations). The deviations in along-track and across-track direction are assumed independent of each other.

The positions of both aircraft are predicted in time. The expected magnitudes of the ground speeds are assumed constant for both aircraft. The predicted across-track uncertainty in position (standard deviation) is constant for both aircraft. The predicted along-track uncertainty in position (standard deviation) is 0 for both aircraft at current time and increases linearly in time (given by a growth rate). The routes that can be formed by connecting the predicted aircraft positions are straight lines in the horizontal planes that cross each other, except for a path angle of 0 deg, in which case the aircraft are predicted to fly on parallel routes.

The situation described above is visualized in Fig. 1. All conflict prediction approaches will be applied to the above situation. In the simulations we have evaluated 1) the predicted minimum distance between protection zones around the aircraft (classical geometric approach, e.g., Refs 5 and 6); 2) the conflict probability (see Refs. 1 and 2); 3) the overlap probability (version of Refs. 1 and 2 with threshold at 50 m); and 4) the collision risk following our novel approach.

The threshold on which the geometric approach and the conflict probability approach defined in Refs. 1 and 2 are based is taken 5 n miles (5 n miles is the currently used ICAO separation standard for en-route airspace). The threshold used for evaluating the overlap probability is set to 50 m. The novel probabilistic approach needs extra input parameters, the across-track standard deviation of the velocity, the along-track standard deviation of the velocity and the size of the boxes which represent the aircraft. For these parameters, some reasonable values were used: standard deviation of the velocity is 2% of the ground speed in either direction and independent of each other. The length and width of the box enclosing one aircraft are 50 m.

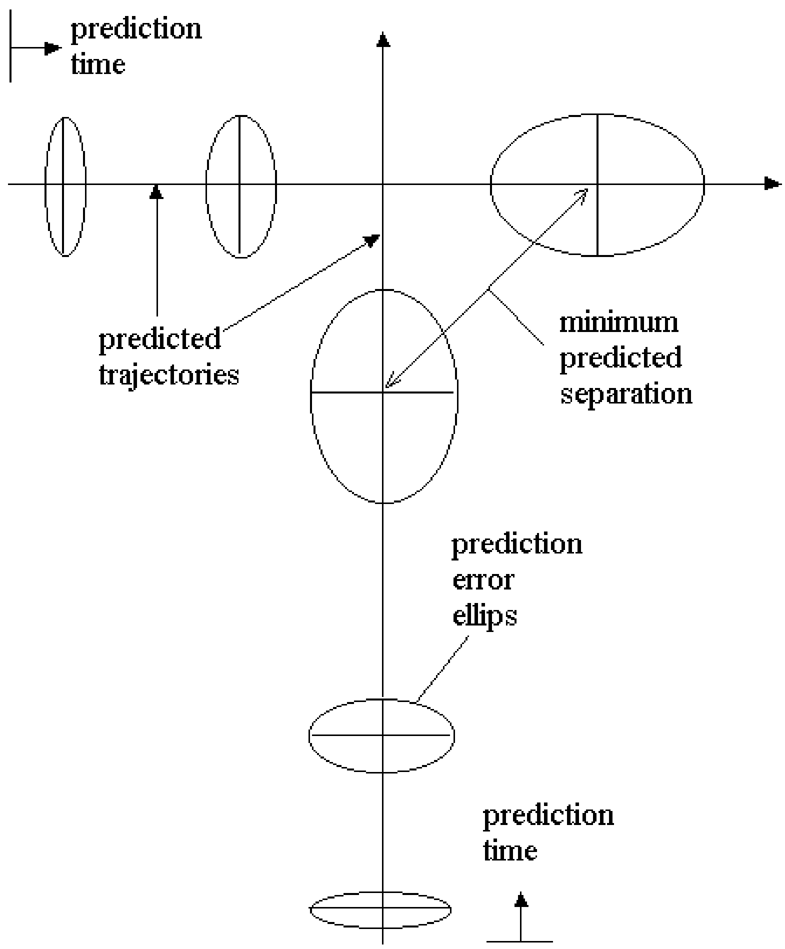

Figure 1: ATM example in the horizontal plane.

The collision risk is evaluated for the time interval which starts 5 minutes before the aircraft reach their minimum predicted separation until 5 minutes after they have reached their minimum predicted separation. In the geometric approach, the size of the protection zone is defined as a box whose length is equal to along-track standard deviation of position and whose width is equal to the across-track standard deviation of position. The length of the box lies in the predicted velocity direction.

The evaluation of the minimum predicted distances between the protection zones, the conflict probability, the overlap probability and the collision risk can be done for various sets of simulation parameters. Performance of the conflict prediction approaches in various situations are compared by varying the following simulation parameters: 1) minimum predicted separation, 2) path crossing angle, 3) predicted ground speed of the aircraft, 4) time before minimum predicted separation, 5) growth-rate of the along-track standard deviation of position, and 6) across-track standard deviation of position.

The results of some examples will be shown.

B. Example 1

In this example, from Ref. 1, the minimum predicted separation between the aircraft is 6 n miles. The path angle between the predicted aircraft routes is 90 deg. The predicted ground speed magnitude of both aircraft is 480 kn. The time before minimum predicted separation is varied from 40 min to 1 min. The growth rate of the along-track standard deviation of position is 15 kn for both aircraft, and the across-track standard deviation of position is 1 n mile and constant for both aircraft.

The result of varying the time before minimum predicted separation is that the along-track standard deviation of position at time of minimum predicted separation is varied from 10 n miles to 0.25 n mile.

In the geometric approach, the minimum predicted distances between the protection zones are evaluated. If a 'geometric' conflict is detected, the probability of a conflict is 1; otherwise it is 0. The geometric approach was used with a 1-sigma value for the assumed area of aircraft; the length and width of the area is equal to the along-track and across-track standard deviation, respectively. Figure 2 shows the results of the geometric and conflict probability approach. Figure 3 shows the results of the conflict probability and the collision risk approach. In Fig. 3, all curves are normalized (in order to fit within a linear scale figure).

To make the difference between the probabilistic approaches more clear, we use a logarithmic scale to plot the results of the example see Fig.4 for all three probabilistic approaches.

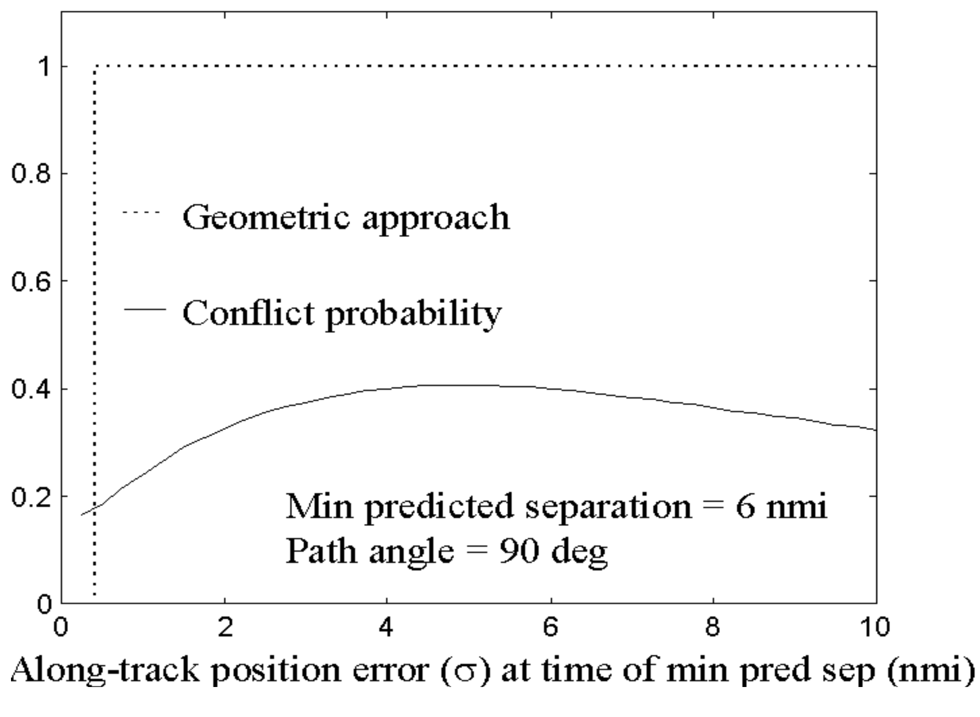

Figure 2: Geometric approach and conflict probability in example 1.

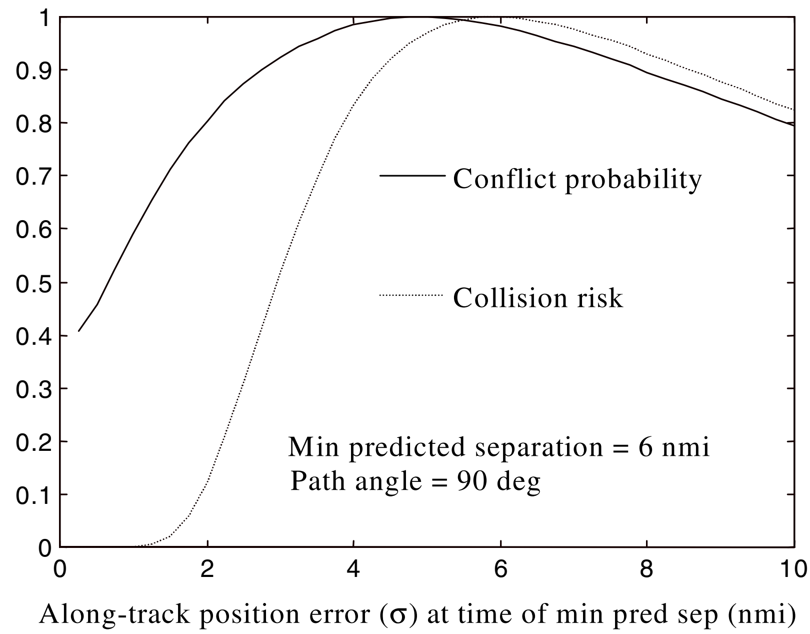

Figure 3: Conflict probability and collision risk in example 1 (normalized).

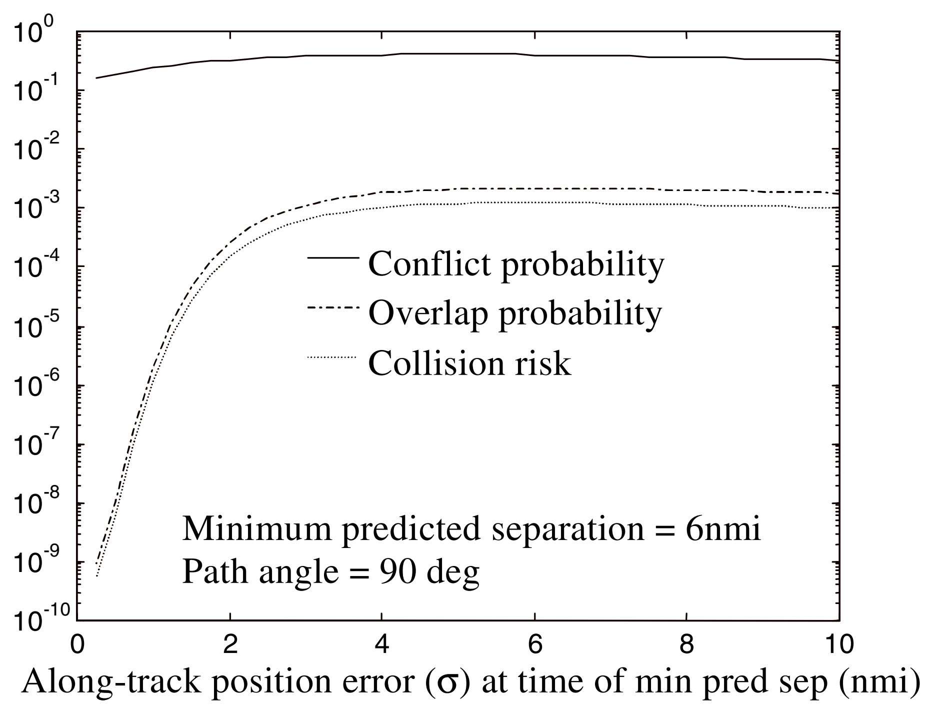

Figure 4: Conflict probability, overlap probability (with threshold reduced to 50 m) and collision risk in example 1 (log scale).

C. Example 2

In this example, also from Ref. 1, we change the minimum predicted separation between the aircraft from 6 n miles to 4 n miles in the set of simulation parameters for example 1. In Fig. 5, conflict probability, overlap probability and collision risk are plotted using a log scale.

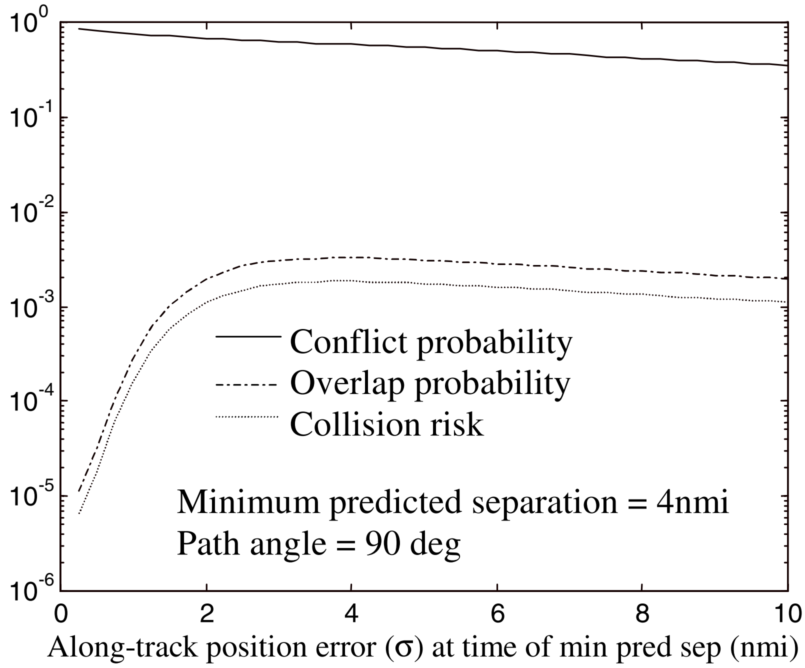

Figure 5: Conflict probability, overlap probability (with threshold reduced to 50 m) and collision risk in example 2 (log scale).

D. Example 3

In this example, we further compare collision risk and overlap probability. The minimum predicted separation is 6 n miles. The path angle is varied from 0 to 360 deg. The ground speed of one aircraft is 420 kn and the ground speed of the other aircraft is 480 kn. In all situations, the faster aircraft crosses behind the slower aircraft (except for path angle 0 when the routes are parallel). The time before minimum predicted separation is varied from 20 min to 1 min. The growth rate of the along-track standard deviation of position is 10 kn for both aircraft. The across-track standard deviation of position is 1 n mile and constant for both aircraft. The overlap probability (threshold reduced to 50 m) and collision risk are evaluated.

The results are given in Figs. 6-9. In the three-dimensional figures, the horizontal axes represent the time to minimum predicted separation (minutes) and the path angle. The position of one aircraft at time of minimum predicted separation is translated to (0,0) in the horizontal plane. All points on a circle in the horizontal plane represent the same time that this aircraft needs to fly from its current position to its position at time of minimum predicted separation (0,0). So each point in the horizontal plane represents a possible position where one aircraft currently is. The heading of the other aircraft (conflicting aircraft) is given in the figures. The vertical axis represents the overlap probability respectively the collision risk. In the two-dimensional figures, the axes are the same as the horizontal axes of the three-dimensional figures. Possible current positions of one aircraft relative to its position at time of minimum predicted separation are coloured according to the value of the overlap probability respectively collision risk (the colouring- scale is shown in the figures).

Figures 6 and 8 do not give a very clear view of the differences between overlap probability and collision risk. Figures 7 and 9, however, do show a clear difference between overlap probability and collision risk, especially when the aircraft are close to the position where the predicted separation reaches its minimum. Therefore, in Fig. 10 the overlap probability and collision risk are evaluated for situations where the aircraft are 4 min before they reach their minimum predicted separation. The path angles are varied from 0 to 360 deg. This figure shows a significant difference between overlap probability and collision risk.

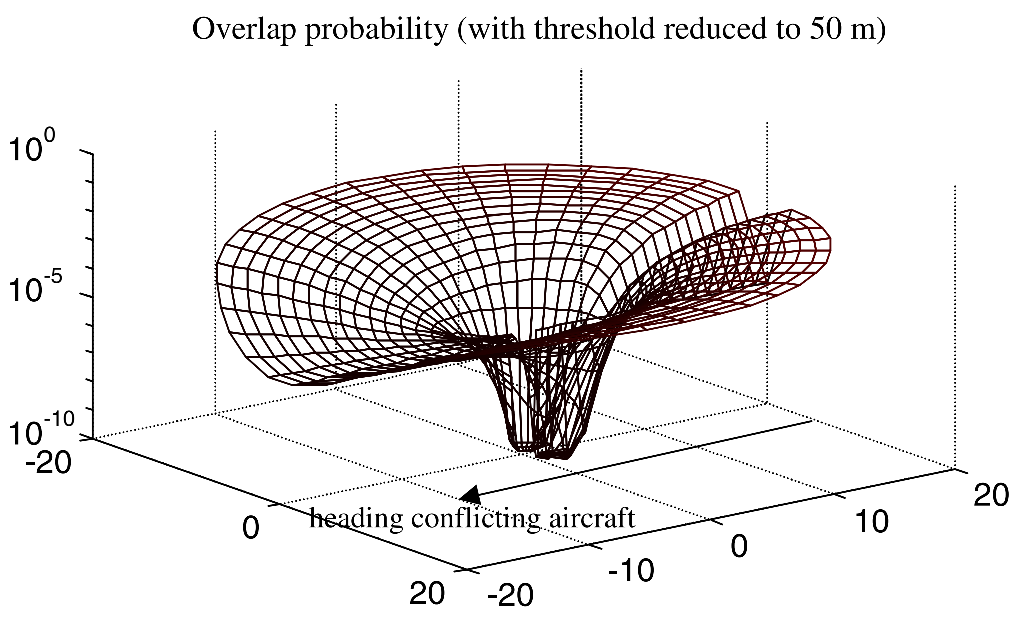

Figure 6: Overlap probability (z-axis) with threshold reduced to 50 m is represented for various path angles and times to minimum predicted separation (horizontal axis).

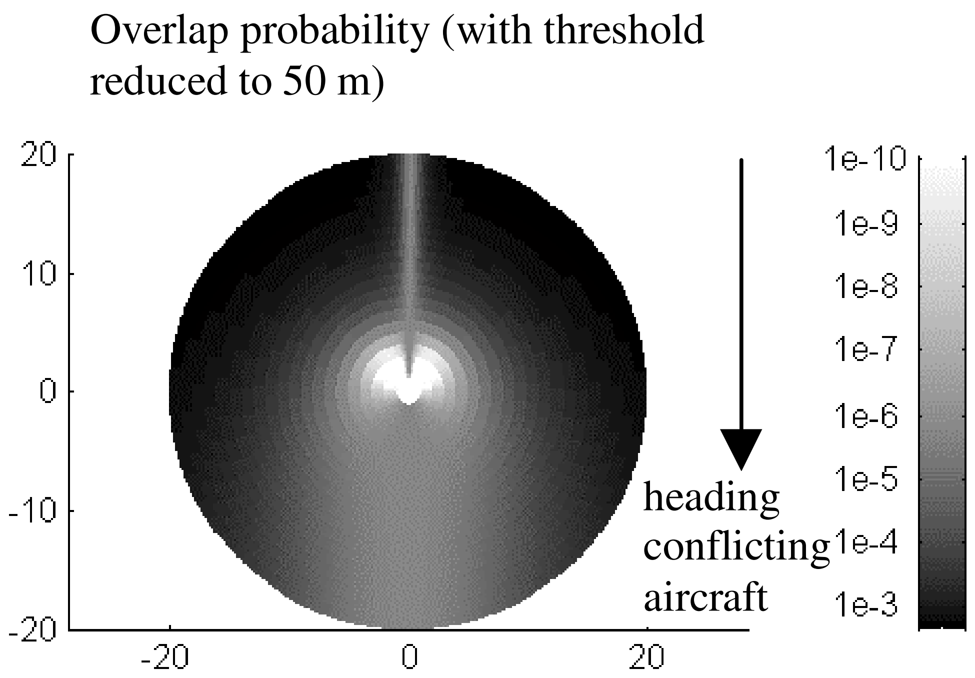

Figure 7: Overlap probability (with threshold reduced to 50 m) is represented by colours for various path angles and times to minimum predicted separation.

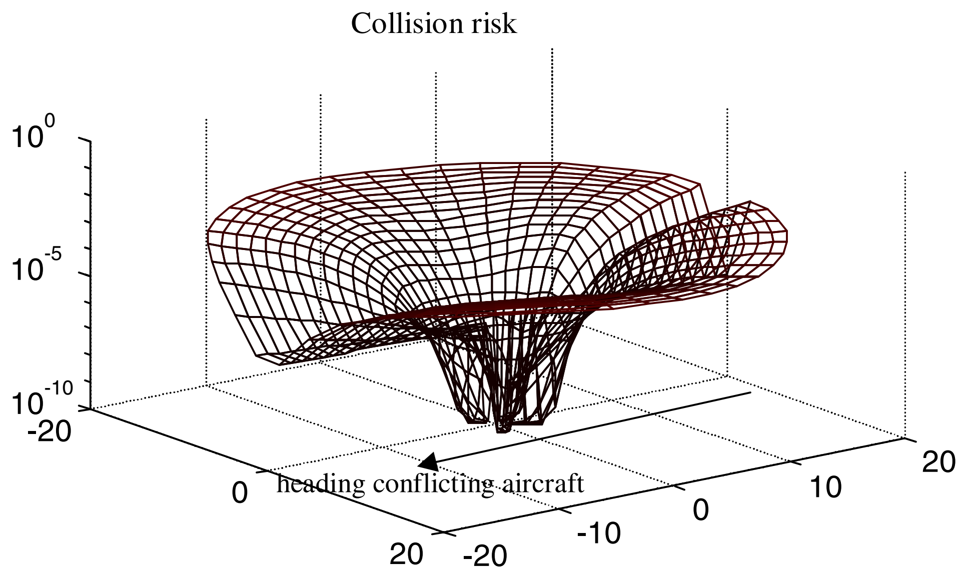

Figure 8: Collision risk (z-axis) is represented for various path angles and times to minimum predicted separation (horizontal axis).

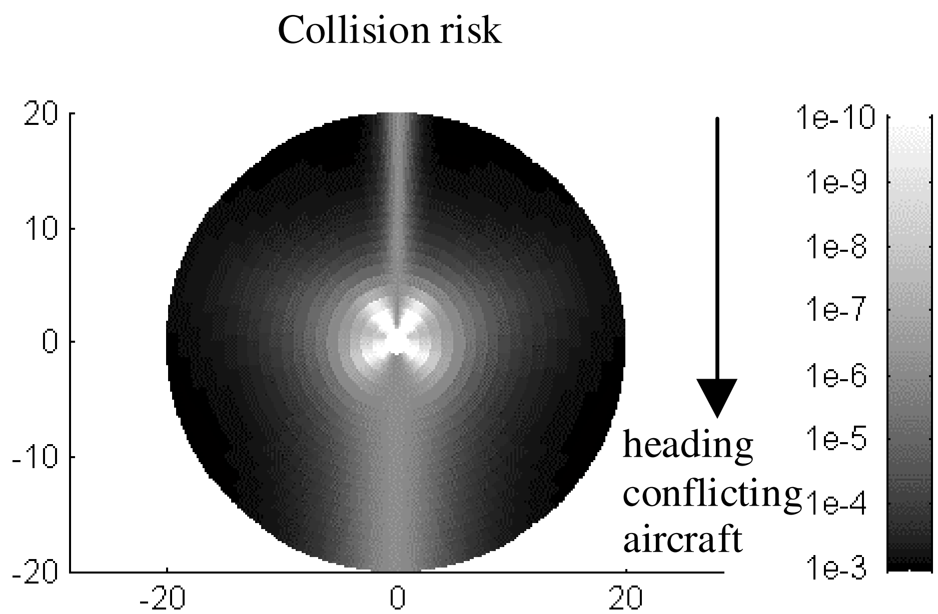

Figure 9: Collision risk is represented by colours for various path angles and times to minimum predicted separation.

E. Example 4

The situation simulated in this example is the same as was simulated for example 3, except for the fact that the faster aircraft now crosses before the slower aircraft instead of behind the slower aircraft.

The overlap probability (with threshold reduced to 50 m) and collision risk are evaluated for path angles between 0 and 180 deg and the time before minimum predicted separation is 4 min. Figure 11 shows the overlap probabilities and the collision risk results. The overlap probabilities are the same for the situations in example 3 and example 4. The collision risk results for examples 3 and 4 differ.

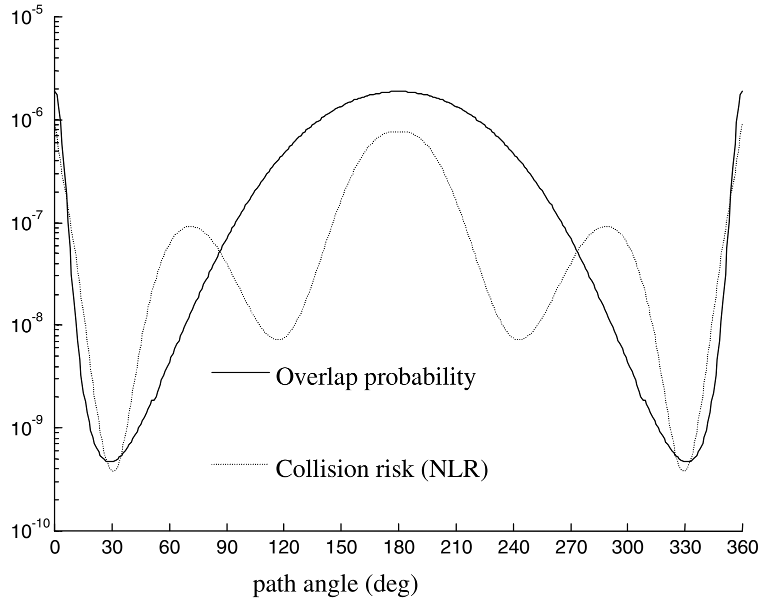

Figure 10: Overlap probability (with threshold reduced to 50 m) and collision risk for various path angles and 4 minutes before time of minimum predicted separation of 6 n miles.

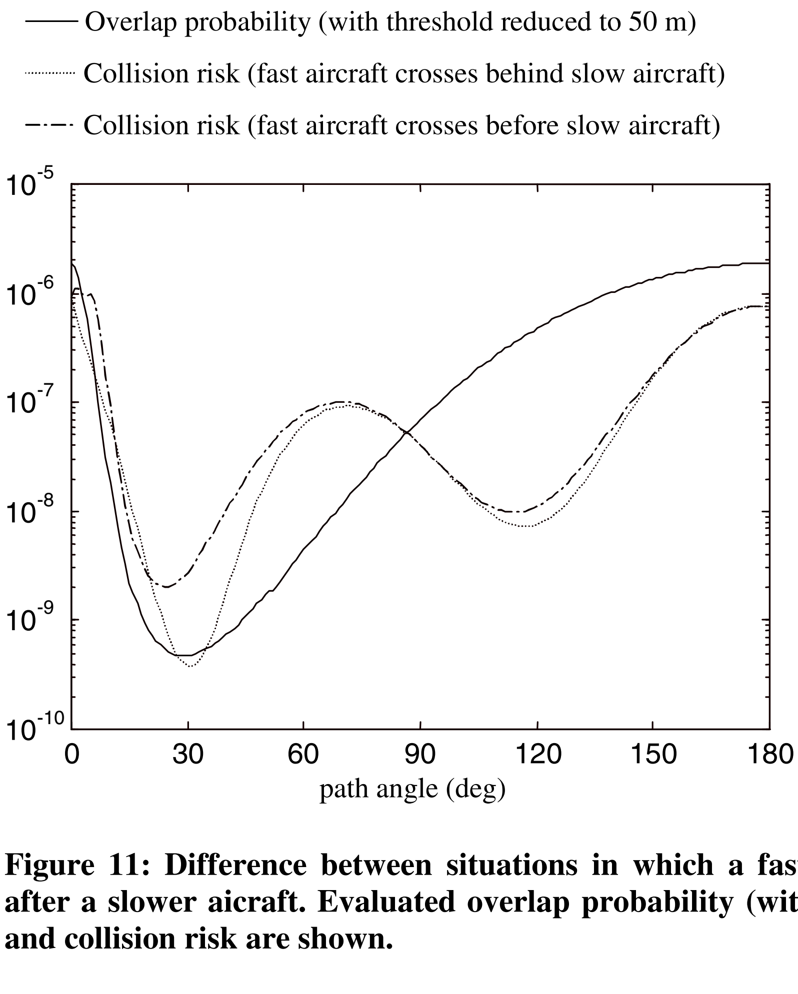

Figure 11: Difference between situations in which a faster aircraft crosses before or after a slower aicraft. Evaluated overlap probability (with threshold reduced to 50 m) and collision risk are shown.

V. Discussion of the results

A. Flexibility of Usage

From the results of example 1, we can conclude that even with a protection zone that is represented by an uncertainty area of 1-sigma only (70 % containment), the geometric approach does not show flexibility in its usage. It can be seen from Figs. 2 and 3 that the probabilistic approaches show more flexibility to changes in along-track standard deviation of position. Thus lets us take a closer look at the probabilistic approaches.

In Fig. 4, it can be seen that for large uncertainties in the along-track position the conflict probability, overlap probability, and the collision risk are approximately equally sensitive to changes in the uncertainty. Thus for tools that concentrate on situations in which large uncertainties are common, all probabilistic approaches can be used. A good example where large uncertainties are common, is a flow management tool. However, if the uncertainties in along-track position become smaller, collision risk and overlap probability are much more sensitive to changes in along-track uncertainty than conflict probability. When the uncertainty in the along-track position decreases in Fig. 4, the collision risk and overlap probability values decrease very fast to very small values, whereas conflict probability values decrease very slow. Thus in this example, when the aircraft get closer to the point of minimum predicted separation, the more flexible collision risk and overlap probability become than conflict probability. For small uncertainties, it is easier to separate safe situations from unsafe situations by using collision risk or overlap probability than conflict probability.

In Refs. 1 and 2, the authors already concluded that conflict probabilities for minimum predicted separations below 5 n miles have a different shape than for minimum predicted separations above 5 n miles. If the minimum predicted separation is larger than 5 n miles, the shape of the conflict probabilities is as plotted in Fig. 3; first with increasing along-track position uncertainty from 0, the conflict probability increases from 0 to a maximum and then it decreases again. If the minimum predicted separation is smaller than 5 n miles, with increasing along-track position uncertainty from 0, the conflict probability decreases from its maximum monotonically towards 0. For collision risk and overlap probability, such a distinction is not necessary, as can be seen from Figs. 4 and 5. For a minimum predicted separation of 4 n miles and 6 n miles, with decreasing along-track position uncertainties, the collision risk and overlap probability slowly increase to a certain maximum and then decrease to very small values. As a result, we conclude that conflict probabilities can give no information with regard to the possible modifications of ICAO separation standards, where collision risk and overlap probability can.

B. Tradeoff Between Velocity Magnitude and Period of Encounter

Figures 4 and 5 show that in the simulated situations the overlap probability and collision risk have similar shapes. Thus in these situations they show similar flexibility in their usage. This can be explained as follows.

The overlap probability (and conflict probability) is evaluated based on a random indication of relative position and velocity. The magnitude of the relative velocity does not have any effect on the overlap probability (and conflict probability), only the direction of the relative velocity does. The implication of using a random indication for evaluation of the conflict and overlap probability is that the period of the encounter or possible conflict is not taken into account and thus has no effect on the result. Uncertainty in relative velocity is also not taken into account.

Collision risk is evaluated from in-crossing rates integrated over time. On any moment in time, the magnitude, direction, and uncertainty of the relative velocity are used for evaluation of the in-crossing rate at that time. So the magnitude, direction, and uncertainty of the relative velocity are all incorporated in the collision risk. Implication of integrating the in-crossing rates over time is also that the period of encounter or possible conflict is incorporated in the collision risk.

In general, the magnitude of the velocity and the period of encounter or potential conflict will have opposite effects on the collision risk and no effect on overlap probability (and conflict probability).

This can be explained as follows. The faster the aircraft fly, the shorter the encounter or period of potential conflict will be. The larger magnitude of the relative velocity will enlarge the in-crossing rates during the period of encounter or possible conflict. The consequence of the smaller time period is a potential reduction of collision risk, whereas the larger in-crossing rates create a potential increase of collision risk. Collision risk will show a tradeoff between these effects; overlap probability (and conflict probability) will not.

From the results given in Figs 4 and 5, it can be concluded that in these situations, the above tradeoff is such that the two consequences balance each other out. This, however, is not always the case.

From the results of Example 3 (Figs 6-10), it is straightforward that the overlap probability and collision risk are symmetric with respect to 0 path angle, i.e., overlap probability / collision risk in case path angle is β, is the same as in case path angle is $-\beta$. To obtain a complete picture of the situation, the results for all path angles between 0 and 360 degrees are given in the figures.

The overlap probability with threshold reduced to 50 m and collision risk are evaluated for various path angles and times to minimum predicted separation. The shapes of the light/dark areas in Figs. 7 and 9 show a difference. From this difference, it can be concluded that the opposite effects of the magnitude of the velocity and the period of encounter or potential conflict are not always in balance. This is further explained by considering the overlap probability and collision risk on the 4-min circle (all positions on this circle represent a possible position of one aircraft 4 min before its position at time of minimum predicted separation).

The results are shown in Fig. 10. In this figure, the focus is on the possible situations 4 min before time of minimum predicted separation. Overlap probabilities show that the worst situations are represented by 0 (or 360) and 180 deg path angle. The best situation is represented by 30 deg path angle. Based on collision risk, the worst situations are reached for path angles of 0 (or 360) and 180 deg. The best situation is represented by 30 (or 330) deg path angle. The path angles for which the collision risk really differs from the overlap probability (up to a factor 15) are the path angles between 30 and 180 deg (or 330 and 180 deg). Around 70 deg path angle a local maximum in collision risk (local minimum in safety) is achieved and around 120 deg a local minimum in collision risk appears. The above results can be explained by a tradeoff between the magnitude of the relative velocity and the period of encounter or potential conflict.

The overlap probability is not capable to take the above described opposite effects into account. Collision risk however does take these effects into account. Example 4 shows the tradeoff when a faster aircraft crosses before a slower aircraft instead of crossing behind the slower aircraft, which was simulated in example 3. Figure 11 shows that overlap probability is the same for both cases, but collision risk differs significantly for path angles around 30 deg. So collision risk can distinguish between the simulated situations and overlap probability cannot.

C. Imposed Restrictions on Aircraft Behavior

Most conflict prediction approaches assume some restrictions on aircraft behavior. In the geometric approach, the more dynamic the aircraft behavior, the more difficult it is to define an appropriate deterministic protection zone around the aircraft and the more difficult it is to evaluate distances between protection zones. Therefore, geometric conflict prediction approaches tend to be complex in case of dynamic aircraft behavior. To reduce complexity, most geometric conflict prediction approaches assume that aircraft fly in straight lines.

The probabilistic approach in Refs. 1 and 2 yields a search for the moment of minimum predicted separation. It is assumed that the aircraft velocities and prediction errors are constant during the encounter or period of potential conflict. The conflict probability and overlap probability are derived from a random indication of the aircraft positions and velocities together with their uncertainty corresponding to the moment of minimum predicted separation. Therefore, dynamic aircraft behavior may cause incorrect conflict or overlap probabilities.

The novel probabilistic conflict prediction approach is based on collision risk. Collision risk is evaluated from in-crossing rates integrated over time. At any moment in time, predicted aircraft positions and velocities together with their uncertainties are used for evaluation of the in-crossing rate at that time. The in-crossing rates are evaluated for the whole encounter or period of potential conflict. Collision risk is derived from these in-crossing rates, thus incorporating all dynamics.

D. Advanced Application: Dynamic Spacing

The conflict prediction approaches were compared by considering restrictions on aircraft behavior, flexibility of usage, and conservatism. Now an advanced application of the conflict prediction approaches will be discussed: dynamic spacing.

If the meteorological conditions change, the ATM system should be able to absorb this information and to translate it into use. If we focus on conflict prediction, in bad weather it may be needed to increase aircraft separations (spacing). One possible way to realize this is to change the separation threshold to a value that everybody agrees on and use conflict prediction approaches that make use of this separation threshold ([1], [2]).

Let us refer to methods that dynamically change the separation threshold according to changes in (meteorological) conditions, as dynamic spacing methods. A procedure could be that the right people judge (meteorological) conditions and select a certain separation threshold, based on their experience.

If dynamic spacing methods are developed and used in line with geometric or the conflict probability approach in Refs. 1 and 2, (meteorological) conditions should be translated into separation thresholds that apply for all aircraft. This way an ATM system can be created where the capabilities of highly equipped (expensive) aircraft are not fully used.

The overlap probability approach (variation of the method of Paielli & Erzberger) and the novel probabilistic approach use a probabilistic separation threshold. If the probability density function of position and velocity are given dependent on the (meteorological) conditions, conflict potentials will be predicted dependent on the (meteorological) conditions. In this approach, the dynamic spacing method yields that (meteorological) conditions are translated into probability density functions of position and velocity.

If dynamic spacing methods are developed in line with the novel probabilistic conflict prediction approach (or the overlap probability approach), every aircraft will be judged on its capability to navigate in current conditions. Using this probabilistic approach, spacing between two highly equipped aircraft may be smaller than the spacing between aircraft with less equipment on board. Thus making full use of all aircraft capabilities.

VI. Conclusions

In this paper, an overview is given of four conflict prediction approaches. The classical geometric approach, the conflict probability approach ([1], [2]), the overlap probability approach (a variant of the approach in Refs. 1 and 2), and a novel probabilistic approach. The objective of all conflict prediction approaches is to evaluate a set of planned or predicted trajectories on their conflict potential and to supply other ATM subsystems with the conflict information.

The reason for studying probabilistic conflict prediction approaches is that the classical geometric approach tends to be overly conservative in handling uncertainties in aircraft behavior. In the probabilistic conflict prediction models, modeling of the trajectory uncertainties causes the predictions to be less conservative. The conservatism that is seen as a limitation in the geometric approach can be overcome by an appropriate probabilistic approach.

The first two approaches are briefly reviewed and the overlap probability approach is introduced. Overlap probabilities are evaluated with the threshold reduced to the size of an aircraft. The novel probabilistic approach is explained in more detail. The approaches are compared on various qualities. The results of the comparisons are summarized in the following.

In the studied examples, only two-dimensional straight predicted flight paths were simulated. The reason for simulating straight flight paths lies in the imposed restrictions on aircraft behavior. Dynamic aircraft behavior would cause large complexity in the classical geometric conflict prediction approach and may cause incorrect predictions in the conflict probability and overlap probability evaluated according to the approach of Paielli & Erzberger ([1], [2]). However, the novel probabilistic conflict prediction approach, based on collision risk formulae, incorporates all aircraft behavior.

The conflict prediction approaches were compared on flexibility of usage. Flexibility was judged on the amount of impact the input trajectory uncertainties have on the output of the prediction. The more sensitive the output of the prediction is with respect to changes in the input, the better the approach can distinguish safe from unsafe situations. In this respect, the classical geometric approach showed the worst flexibility in its usage. The conflict probabilities proved to be much less sensitive to changes in the probability density functions than overlap probability and collision risk. Overlap probability and collision risk showed a lot of sensitivity to changes in the probability density functions, especially for small uncertainties in position. The latter makes overlap probability and collision risk extremely valuable in environments where small uncertainties in position are common (e.g. four-dimensional ATM, short term conflict prediction).

In some situations, the overlap probability and collision risk showed similar flexibility. However, the flexibility of the overlap probability and collision risk was not similar in all situations. This was explained by the tradeoff between the period of encounter or potential conflict and the magnitude of the relative velocity. The overlap probability evaluated according to the method in Refs. 1 and 2 does not take the magnitude of the velocity and the period of encounter or possible conflict into account. The evaluated collision risk incorporates all aircraft behavior, and so magnitude of velocity and period of encounter or potential conflict are also taken into account (and the tradeoff between them). An ATM example was simulated where this tradeoff made a difference. Evaluated collision risks indicated that for some path angles it would be safer for a fast aircraft to cross behind a slower aircraft than crossing before the slower aircraft. Overlap probabilities could not distinguish between these situations.

For an ATM system to make full use of (meteorological) conditions information, dynamic spacing methods are necessary. Briefly, this means that known (meteorological) conditions are translated in an amount of space that is necessary to separate aircraft so that they are safe. If the classical geometric approach or the conflict probability approach in Refs.1 and 2 is used, dynamic spacing methods need to be developed that translate (meteorological) conditions in separation thresholds. This means that all aircraft are treated equally, which induces no full use of (expensive) aircraft equipment. If the overlap probability or the novel probabilistic approach is used, dynamic spacing means that models need to be developed that represent aircraft behavior in all (meteorological) conditions. These approaches have the option of taking the quality of the equipment of individual aircraft into account, thus making full use of the aircraft equipment.

In the qualities described, the novel probabilistic approach proves to be the most promising and enables other advanced applications such as the incorporation of the probability density functions for all possible (meteorological) conditions, and the incorporation of collision risk prediction capability into the ATM design.

References

[1] Paielli, R.A., and Erzberger, H., "Conflict Probability Estimation for Free Flight," *Journal of Guidance, Control and Dynamics*, Vol. 20, No. 3, May-June, 1997, pp. 588-596.

[2] Erzberger, H., Paielli, R.A., Isaacson, D.R., and Eshow, M.M., "Conflict Detection and Resolution in the Presence of Prediction Error," *1st USA/Europe Air Traffic Management R&D Seminar*, FAA/EUROCONTROL, 1997.

[3] Bakker, G.J., and Blom, H.A.P., "Air Traffic Collision Risk Modelling," *32nd IEEE Conference on Decision and Control*, Vol. 2, Institute of Electrical and Electronics Engineers, New York, 1993, pp. 1464-1469.

[4] Kremer, H.J., Bakker, G.J., and Blom, H.A.P., "Geometric Versus Probabilistic Conflict Probing," *1st U.S.A./Europe Air Traffic Management R&D Seminar*, FAA/EUROCONTROL, 1997.

[5] Kremer, H.J., Vertegaal, W.C., and Jansen, R.B.H.J., "PHARE Advanced Tools Conflict Probe", National Aerospace Laboratory NLR, NLR Rept. DOC 98-70-18 (volume 3 of 10), Amsterdam, 1998.

[6] Vink, A., de Jong, C.J.M., and Beers, J.N.P., "Medium Term Conflict Detection," *1st USA/Europe Air Traffic Management R&D Seminar*, FAA/EUROCONTROL, 1997.

[7] Reich, P.G., "A Theory of Safe Separation Standards for Air Traffic Control," Royal Aircraft Establishment, 64041, Malvern, UK, 1964.

[8] Bakker, G.J., Kremer, H.J., and Blom, H.A.P., "Geometric and Probabilistic Approaches Towards Conflict Prediction in ATM," (submitted for publication).