# Crosstalk modelling of unshielded wire pairs

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***Abstract* — Low-frequency analysis on multi-conductor transmission line equations results in closed-form leading order expressions for near-end crosstalk. Two wiring configurations are introduced for which leading order dependency on all model parameters can be derived. An interesting result is a 24 dB/octave decrease of crosstalk when distance is increased between two wire pairs, oriented parallel to an infinite, perfectly conducting ground plane.**

***Index Terms—Near-end crosstalk; low-frequency analysis; multi-conductor transmission line***

## I. Introduction

The installation of a large number of electric and electronic systems in cars, aircraft and other transport vehicles requires routing of many wires and cables. Electromagnetic interference between these conductors might result in malfunction of the connected systems. The electromagnetic coupling between wires has been studied since many years by using multi-conductor transmission line (MTL) equations. The solution of these equations has been considered by White [1], Paul [2] and many others. The unintentional coupling between wires is usually referred to as crosstalk which needs to be reduced as much as required. When wires and cables are put closer to each other crosstalk might increase. Simultaneously the increase of electric and electronic systems in transport vehicles requires the installation of more cables and wires while the available space remains limited. Therefore, there is a need to reconsider guidelines for routing and harnessing of cables. Knowledge about dependencies of crosstalk on geometric parameters (such as distance between wires and height above a ground plane), frequency and impedances is a prerequisite for such guidelines.

In the present paper we discuss the derivation of closed-form expressions for near-end crosstalk between wire pairs. To verify the modelling MTL simulations and measurements are performed. By numerical solution of the MTL equations it appears that the differential mode crosstalk decreases by 24 dB/octave when the distance between the wire pairs increases. So far this amount of decrease could not be explained by analytical expressions available in literature. White [1] presented only expressions for crosstalk between two single wires above a ground plane. These expressions predict the well-known decrease of 12 dB/octave with respect to the separation distance between wires. In this paper we present a mathematical approach where we derive expressions for near-end crosstalk (NEXT) of multi-conductor transmission lines which contain all model parameters. This rigorously proves the decrease of differential mode NEXT by 24 dB/octave for wire pairs oriented parallel and close to a ground plane. Similar analysis shows this is a special case, since other wire pair configurations show the familiar decrease of 12 dB/octave. Besides separation distance, the calculated closed-form expression clearly shows the dependency of crosstalk on all other geometric parameters and terminal loads. Starting point for the approach is the matrix system of MTL equations for finite transmission lines as presented in Paul [2]. Mathematical analysis is used to obtain relations between near-end crosstalk and model parameters. By applying a low-frequency analysis approach to this system of equations it is possible to find closed-form expressions for crosstalk for several cable configurations. In this paper we derive such an analytic formula for two wire configurations. The first one regards two single, unshielded wires above an infinite, perfectly conducting ground plane as was analyzed by White [1]. We show that our closed-form expressions correspond with those of White. Next, we apply our approach to crosstalk between two unshielded and untwisted wire pairs above a ground plane. To this end, the logarithmic expressions in the inductance matrix are carefully expanded in series of the parameter $\alpha$ which is defined as the ratio between intra-pair separation distance and the distance from the centre of one pair to that of another pair. Knowledge about modelling of untwisted pairs acts as a basis for analysis of twisted pairs. In a forthcoming paper we will apply our mathematical approach also to shielded wire configurations.

## II. Methodology

Consider a configuration of n perfect conductors of length $\ell$ situated parallel to and in close proximity of an infinite, perfectly conducting ground plane. We denote by $\mathbf{V}_0$ the vector of which entry $k$ represents the voltage difference between conductor $k$ and the reference plane. Its related vector of currents will be given by $\mathbf{I}_0$.

Each pair of conductors can be observed as a transmission line. Electromagnetic fields might induce coupling between each pair of transmission lines causing crosstalk. Near-end crosstalk between transmission lines is defined as the ratio of the voltage difference between conductors in one transmission line, over that same voltage difference of another transmission line. This is made explicit with the following equation:

$$\gamma_{NE} = \frac{\mathbf{U}_2^T \mathbf{V}_0}{\mathbf{U}_1^T \mathbf{V}_0}, \tag{1}$$

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in which the vectors $\mathbf{U}_1$ and $\mathbf{U}_2$ are used to select the right voltages from $\mathbf{V}_0$. Based on different configurations with a possible varying number of conductors, different entries of $\mathbf{V}_0$ need to be added or subtracted. This is managed by different $\mathbf{U}_1$ and $\mathbf{U}_2$.

Since we are interested in the precise dependencies of crosstalk on model parameters, a first objective is to find an explicit expression for the vector of voltages. Analytical formulas were presented before for two-conductor transmission lines [1]. In its book on MTL equations Paul introduces a method to analyse configurations containing *n* conductors [2]. It contains matrix formulas that can be solved to find the currents in all these conductors. We use low-frequency approximations for the chain matrices which are described by formula (4.48) of Paul [2]. Furthermore, we assume that no voltage source is contained in the termination network at the far-end ( $\mathbf{V}_\mathbf{L} = 0$ ). Then, matrix equation (4.90) of [2] becomes:

$$\left[2\mathbf{Z} + j\omega\ell(\mathbf{L} + \mathbf{ZCZ})\right]\mathbf{I}_0 = \left[\mathbf{I}_n + j\omega\ell\mathbf{ZC}\right]\mathbf{V}_\mathbf{S}. \quad (2)$$

Here $\mathbf{I}_n$ is the n-dimensional identity matrix, ω is the angular frequency of the signal travelling down the transmission line and $\ell$ the length of the line. The impedance matrix containing the termination impedances of the transmission line is denoted by $\mathbf{Z}$ and we assume equal loads on both ends of the transmission lines. The inductance and capacitance matrix are given by $\mathbf{L}$ and $\mathbf{C}$. All possible voltage sources form the entries of $\mathbf{V}_\mathbf{S}$, which in our case of one such source is given by:

$$\mathbf{V}_\mathbf{S} = V_S \mathbf{U}_1, \quad (3)$$

in which $V_S$ is the magnitude of the voltage source. The vector of voltages can be obtained from corresponding currents by:

$$\mathbf{V}_0 = \mathbf{V}_\mathbf{S} - \mathbf{Z}\mathbf{I}_0. \quad (4)$$

### *A. Low-frequency approximation*

We use a low-frequency approximation in order to Taylor expand the inverse of the left hand side matrix of (2). When we assume $\omega\ell\left\|\mathbf{Z}^{-1}\mathbf{L} + \mathbf{CZ}\right\| \ll 1$ we can obtain:

$$\mathbf{V}_0 = \frac{V_s}{2}\left[\mathbf{I}_n + \frac{1}{2} j\omega\ell(\mathbf{L}\mathbf{Z}^{-1} - \mathbf{ZC})\right]\mathbf{U}_1 + O(\omega^2\ell^2). \quad (5)$$

This expression provides us with an explicit formula for the voltage difference of all conductors compared to the reference plane, based on all model parameters such as impedance, capacitance and inductance. Next expression (5) is substituted into (1). For the derivation of closed-form crosstalk expressions we need to specify the vectors $\mathbf{U}_1$ and $\mathbf{U}_2$ in (1) and the matrices $\mathbf{Z}$, $\mathbf{L}$ and $\mathbf{C}$ in (2) for specific wire configurations.

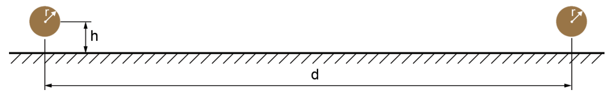


Figure 1. White's case with two wires above an infinite, perfectly conducting ground plane

## III. CROSSTALK ANALYSIS

Norton equivalent representations of the electrical schemes corresponding to our wire configurations enable us to find impedance matrices. The per-unit-length inductance and capacitance matrices depend on the geometrical properties like mutual distances and height above ground plane. In the following section at first we consider White's case with the aim to verify our mathematical methodology as described in chapter II. Subsequently we apply our methodology to a situation with two wire pairs close to a ground plane.

### *A. White*

Consider two single wires which are positioned parallel and close to an infinite, perfectly conducting ground plane (see Fig. 1). It has been extensively described in for instance [1]. The matrix equations given in (2) require numbering of the wires. Here, culprit is number one, victim is number two. The wires have equal height to ground plane *h* and radius *r*. The distance between the two wires is defined as *d*. We presume a nonzero impedance between wires and ground plane, which yields as impedance matrix:

$$\mathbf{Z} = \begin{bmatrix} Z_c & 0 \\ 0 & Z_v \end{bmatrix}, \quad \text{with} \quad Z_v = R_{c,v}, Z_c = R_{c,c}. \quad (6)$$

Here, $R_{c,c}$ and $R_{c,v}$ are common-mode impedances between culprit respectively victim wire and ground. An admittance matrix $\mathbf{Y}$ is defined as the inverse of the impedance matrix. Since in this configuration a source is added between culprit wire and ground, this implies that $\mathbf{U}_1 = (1,0)^T$ in (3).

#### *1) Inductance and capacitance matrices*

The solution of MTL equations requires the specification of per-unit-length parameters corresponding to the configuration. Here it is assumed that the wires are situated in a homogeneous and lossless medium. By formula (3.66) of Paul [2], we have:

$$\mathbf{L} = \beta\begin{bmatrix} \ln(2h/r) & \ln\sqrt{1+x} \\ \ln\sqrt{1+x} & \ln(2h/r) \end{bmatrix}, \quad (7)$$

in which $\beta = \mu_0/2\pi$ and $x = 4h^2/d^2$. By inverting this matrix, the capacitance matrix becomes:

$$\begin{aligned} \mathbf{C} &= \mu_0\varepsilon_0\mathbf{L}^{-1} \\ &= \frac{\mu_0\varepsilon_0\beta}{\det(\mathbf{L})}\begin{bmatrix} \ln(2h/r) & -\ln\sqrt{1+x} \\ -\ln\sqrt{1+x} & \ln(2h/r) \end{bmatrix}. \end{aligned} \quad (8)$$

Here $\varepsilon_0$ and $\mu_0$ are the permittivity and permeability of free space. Other media are modelled with $\varepsilon = \varepsilon_r \varepsilon_0$, $\mu = \mu_r \mu_0$.

*2) Near-end crosstalk*

To obtain a closed-form expression for NEXT the numerator and denominator of (1) need to be calculated. Since the crosstalk in this case is defined as the ratio of the voltage of the victim wire and the voltage of the culprit wire, we define $\mathbf{U}_1 = (1,0)^T$ and $\mathbf{U}_2 = (0,1)^T$. Consequently:

$$\begin{aligned} \mathbf{U}_1^T \mathbf{V}_0 &= \frac{V_S}{2} \mathbf{U}_1^T \mathbf{U}_1 + O(\omega \ell) \\ &= \frac{V_S}{2} + O(\omega \ell) \\ \mathbf{U}_2^T \mathbf{V}_0 &= \frac{V_S}{4} j\omega\ell \mathbf{U}_2^T \left( \mathbf{L}\mathbf{Z}^{-1} - \mathbf{Z}\mathbf{C} \right) \mathbf{U}_1 + O(\omega^2 \ell^2) \\ &= \frac{V_S}{4} j\omega\ell \left( Z_c^{-1} l_{21} - Z_v c_{21} \right) + O(\omega^2 \ell^2). \end{aligned} \tag{9}$$

Here, $l_{ij}$ and $c_{ij}$ are entries of $\mathbf{L}$ and $\mathbf{C}$. The above shows a clear distinction between capacitive and inductive coupling. This also holds for our final crosstalk formula, which follows from substitution of (9) into (1). The inductive near-end crosstalk becomes:

$$\gamma_{NE,ind} \approx j\omega\ell \frac{\mu_0}{8\pi R_{c,c}} \ln\left(1 + 4h^2 / d^2\right), \tag{10}$$

and the capacitive near-end crosstalk reads:

$$\gamma_{NE,cap} \approx \frac{1}{2} j\omega\ell \frac{\varepsilon_0 \pi R_{c,v} \ln\left(1 + 4h^2 / d^2\right)}{\ln^2\left(2h / r\right) - \ln^2 \sqrt{1 + 4h^2 / d^2}}. \tag{11}$$

The total NEXT follows from addition of the inductive and capacitive expressions:

$$\gamma_{NE} \approx \gamma_{NE,ind} + \gamma_{NE,cap}. \tag{12}$$

The equation for capacitive coupling corresponds exactly with the corresponding formula in White [1]. For inductive coupling White obtains twice the value of crosstalk given by (10). This originates from a different value obtained for mutual inductive coupling *M*. White states that it should equal:

$$M = \frac{\mu_0}{2\pi} \ln\left(1 + \frac{4h^2}{d^2}\right),$$

which was obtained by a mirroring technique that replaces the ground plane by two new wires. Paul gives half this value for mutual inductance, i.e. $l_{12}$ in equation (7). Walker states that this mirroring technique leads to twice as many field lines with the same amount of current as present in the actual configuration with ground plane [3]. This explains why White predicted too much crosstalk and why the actual mutual inductance is half of *M*.

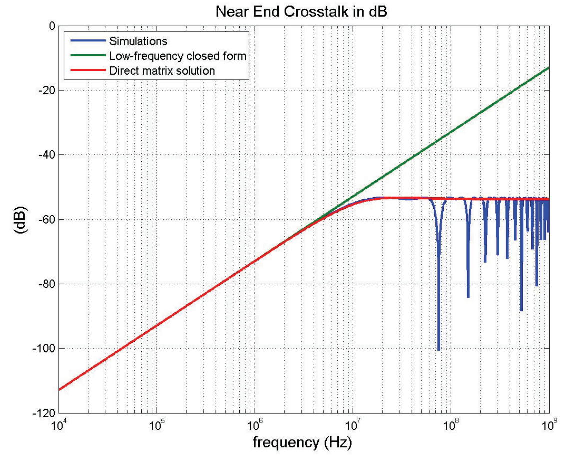


Figure 2. MTL simulations (blue) of NEXT for two unshielded wires above a ground plane. The green line shows the low frequency approximation obtained by summation of (10) and (11). The red line is the direct solution found from Paul's matrix equation, given by (2).

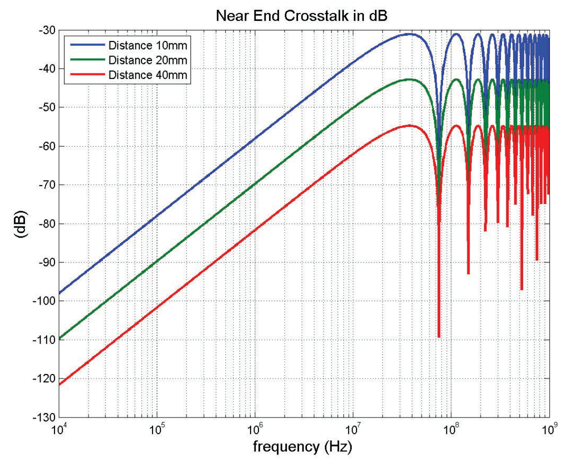


Figure 3. MTL simulations of NEXT with different separation distances for White's case with two unshielded wires above a ground plane.

In order to verify the crosstalk expressions in (10) and (11) we have compared the outcome with results of MTL simulations of wires with finite length $l = 2$ m, wire radii $r = 0.5625$ mm, height above ground plane $h = 1.67$ mm and distance between wires $d = 20$ mm. Furthermore, the common mode resistances are $R_{c,v} = R_{c,c} = 100$ Ohm. The results are displayed in Fig. 2. The result of an MTL simulation is displayed in blue. The green line shows the outcome of the sum of (10) and (11). Indeed for the linear part of crosstalk it is equal to MTL simulations. The red line represents the crosstalk that follows from the direct solution $\mathbf{I}_0$ in (2) and its subsequent substitution in (4) respectively (1). This result excludes resonance phenomena due to finite transmission line effects, but it provides an accurate upper plateau. Actually,

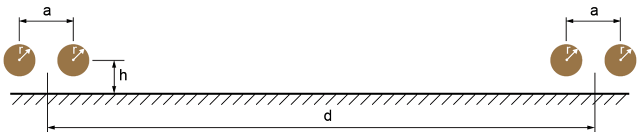


Figure 4. Two wire pairs oriented parallel above an infinite, perfectly conducting ground plane

an approximation of the plateau value can be calculated from the analytical solution of (2) to be:

$$\gamma_{plateau} \approx \frac{\left(Z^{-1} l_{12} - Z c_{12}\right)}{\left(Z c_{11} + 3 Z^{-1} l_{11}\right)}, \tag{13}$$

in which $Z = R_{c,v} = R_{c,c}$. The intersection of this plateau with the outcome of (12) yields the transition frequency where the plateau begins:

$$f_T \approx \frac{1}{\pi l \left(Z c_{11} + 3 Z^{-1} l_{11}\right)}. \tag{14}$$

This transition frequency yields an accurate value for the characteristic dimensionless length $l / \lambda$, where resonance phenomena due to finite length of the wires start. This value is much more accurate than empirical values like $l / \lambda = 0.1$ which can be found in literature [1].

As follows from (10) and (11), the inductive part decreases with increasing culprit impedance, whereas the capacitive coupling is directly proportional to victim impedance. Furthermore, assume $h << d$. Then, $\ln\left(1 + 4h^2 / d^2\right)$ may be approximated by $4h^2 / d^2$. Thus, inductive crosstalk increases with the square of height and decreases with the square of the distance between the conductors. For the capacitive crosstalk a similar behavior can be expected, except that the logarithms in the numerator of (11) might be of influence depending on the actual values of $h$, $d$ and $r$. This crosstalk dependence on the distance between the wires is verified by MTL simulations shown in Fig. 3. Inspection of this figure clearly reveals a decrease of 12 dB when the distance is doubled.

## B. *Wire pairs close to a ground plane*

In this section we consider a wire configuration in which two wire pairs are situated parallel to a ground plane (see Fig. 4) and a differential voltage source is included between both wires of the culprit pair. All four conductors again have the same height to ground plane *h* and wire radius *r*. The intra-pair separation distance is *a* and the distance from the centre of one pair to the centre of another pair is *d*. We number the wires one to four from left to right, defining wires one and two to be the culprit pair. The matrix equation given in (2) still holds. Since we have four wires and a ground plane the dimensions of impedance, inductance and capacitance matrices are four.

### 1) *Impedance matrix*

The termination networks are modelled as in Fig 5. By Norton equivalent representation techniques, we obtain the following impedance matrix:

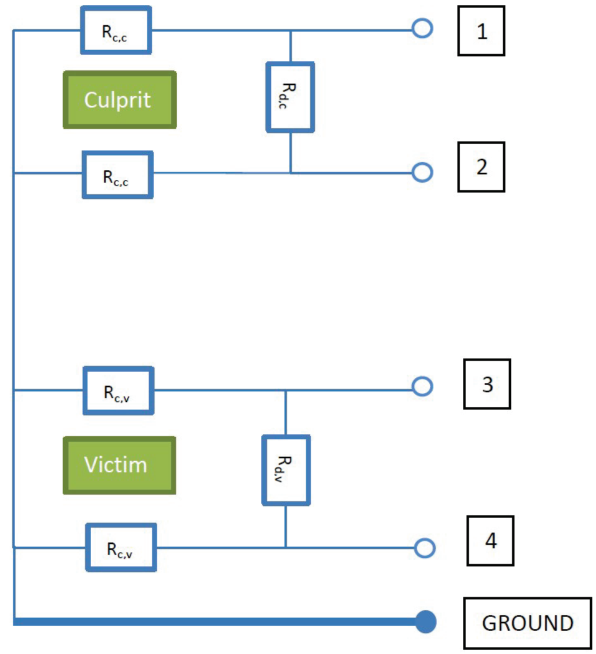


Figure 5. Termination network for two wire pairs above a ground plane

$$\mathbf{Z} = \begin{bmatrix} \mathbf{Z}_c^* & \varnothing \\ \varnothing & \mathbf{Z}_v^* \end{bmatrix}, \tag{15}$$

in which:

$$\mathbf{Z}_m^* = R_{d,m} \begin{bmatrix} c_m + \kappa_m & c_m - \kappa_m \\ c_m - \kappa_m & c_m + \kappa_m \end{bmatrix}. \tag{16}$$

Here $m \in \{v, c\}$ represents victim or culprit. Moreover $\kappa_m = \left[2\left(R_{d,m} / R_{c,m} + 2\right)\right]^{-1}$, $c_m = \left[R_{d,m}\left(R_{d,m} / R_{c,m} + 2\right) / R_{c,m}\right]^{-1} + \kappa_m$ · The variable $R_{d,m}$ is the differential-mode and $R_{c,m}$ the common-mode impedance of either culprit or victim wire pair. Since a differential source was added between the culprit wires we define $\mathbf{U}_1 = \left(-1, 1, 0, 0\right)^T$. By (3) this implies that a voltage difference of $2V_S$ is enforced.

### 2) *Inductance and capacitance*

We apply formula (3.66) of Paul to obtain the inductance matrix for this configuration in a homogeneous medium:

$$\mathbf{L} = \begin{bmatrix} \mathbf{L}_{000} & \mathbf{L}_{12} \\ \mathbf{L}_{12}^T & \mathbf{L}_{000} \end{bmatrix}, \tag{17}$$

with $\mathbf{L}_{000}$ the inductance matrix between two wires of a single pair. This matrix follows from the right hand side of (7) with $x$ replaced by $y = 4h^2 / a^2$. Furthermore:

$$\mathbf{L}_{12} = \frac{\beta}{2}\begin{bmatrix} \ln(1+x) & \ln\left(1+\frac{x}{(1+\alpha)^2}\right) \\ \ln\left(1+\frac{x}{(1-\alpha)^2}\right) & \ln(1+x) \end{bmatrix}.$$

Here $x = 4h^2/d^2$ and $\alpha = a/d$. The capacitance matrix is obtained by inverting the matrix $\mathbf{L}$.

The intention of this crosstalk analysis is to obtain a closed-form expression for the leading order behaviour of NEXT that contains all model parameters. To this end we derive a Taylor expansion of the matrix $\mathbf{L}_{12}$ in terms of $\alpha$. Hence we assume $a \ll d$ and obtain:

$$\hat{\mathbf{L}}_{12} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} b + \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \frac{\beta x}{x+1} \cdot \alpha + \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \frac{\beta}{2} \cdot \frac{3x+x^2}{(x+1)^2} \cdot \alpha^2, \tag{18}$$

in which $b = \beta/2 \cdot \ln(1+x)$. Substituting (18) into (17) yields a Taylor approximation for $\mathbf{L}$:

$$\hat{\mathbf{L}} = \mathbf{L}_{00} + \mathbf{L}_{01} \cdot b + \mathbf{L}_1 \alpha + \mathbf{L}_2 \alpha^2, \tag{19}$$

in which:

$$\mathbf{L}_{00} = \begin{bmatrix} \mathbf{L}_{000} & \varnothing \\ \varnothing & \mathbf{L}_{000} \end{bmatrix}.$$

Evidently, we approximated the inductance matrix by a second order polynomial in $\alpha$.

To obtain the capacitance matrix the inverse of the inductance matrix is required. Let $\mathbf{L}_0$ be given by the first and second term of (19), $\mathbf{L}_0 = \mathbf{L}_{00} + \mathbf{L}_{01} \cdot b$. By the assumption $a \ll d$ we obtain:

$$\begin{aligned} \hat{\mathbf{C}} &= \mu_0 \varepsilon_0 \hat{\mathbf{L}}^{-1} \\ &= \mu_0 \varepsilon_0 [\mathbf{I}_4 - \mathbf{L}_0^{-1}\mathbf{L}_1 \alpha - (\mathbf{L}_0^{-1}\mathbf{L}_2 - \mathbf{L}_0^{-1}\mathbf{L}_1\mathbf{L}_0^{-1}\mathbf{L}_1)\alpha^2]\mathbf{L}_0^{-1} \\ &= \mathbf{C}_0 + \mathbf{C}_1 \alpha + \mathbf{C}_2 \alpha^2, \end{aligned}$$

in which the Taylor approximation for the inverse of a quadratic polynomial was used. It remains to determine the inverse of $\mathbf{L}_0$. We assume that $x \ll 1$ by which $b \ll 1$. This only holds for wire pairs close enough to the ground plane ($h < d/2\sqrt{2}$), for which one can obtain:

$$\hat{\mathbf{L}}_0^{-1} = [\mathbf{I}_4 - \mathbf{L}_{00}^{-1}\mathbf{L}_{01} \cdot b]\mathbf{L}_{00}^{-1}.$$

*3) Near-end crosstalk*

To determine the desired closed-form expression we define $\mathbf{U}_1 = (-1,1,0,0)^T$ and $\mathbf{U}_2 = (0,0,-1,1)^T$. Next we calculate the numerator and denominator of equation (1):

$$\begin{aligned} \mathbf{U}_1^T\mathbf{V}_0 &= \frac{V_S}{2}\mathbf{U}_1^T\mathbf{U}_1 + O(\omega\ell) \\ &= V_S + O(\omega\ell). \\ \mathbf{U}_2^T\mathbf{V}_0 &= \frac{V_S}{4} j\omega\ell \mathbf{U}_2^T\left[\mathbf{L}\mathbf{Z}^{-1} - \mathbf{Z}\mathbf{C}\right]\mathbf{U}_1 + O(\omega^2\ell^2) \\ &= \frac{1}{8} j\omega\ell \kappa_c^{-1} R_{d,c}^{-1}\left(l_{13} + l_{24} - (l_{14} + l_{23})\right) \\ &\quad -\frac{1}{2} j\omega\ell \kappa_v R_{d,v}\left(c_{13} + c_{24} - (c_{14} + c_{23})\right) \\ &\quad + O(\omega^2\ell^2). \end{aligned}$$

The nice structure of both inductance and capacitance matrix enables us to obtain the following expression for the near-end capacitive crosstalk of two wire pairs above a ground plane:

$$\gamma_{NE,cap} \approx \left[\lambda_1 \cdot \frac{x^2\ln(1+x)}{(x+1)^2} + \zeta_1 \cdot \frac{3x+x^2}{(x+1)^2}\right] \cdot \alpha^2, \tag{20}$$

in which $\lambda_1 = j\omega\ell\mu_0\varepsilon_0\kappa_v Y_{34}^{-1} a_2 (a_1)^2 \beta^3$ and $\zeta_1 = -j\omega\ell\mu_0\varepsilon_0\kappa_v Y_{34}^{-1}(a_1)^2 \beta/2$. In these expressions $a_1 = \mathbf{L}_{00}(1,1) - \mathbf{L}_{00}(1,2)$ and $a_2 = \left[-\mathbf{L}_{00}^{-1}\mathbf{L}_{01}\mathbf{L}_{00}^{-1}\right](1,4)$. The inductive coupling is given by:

$$\gamma_{NE,ind} \approx \chi_1 \cdot \frac{3x+x^2}{(x+1)^2} \cdot \alpha^2, \tag{21}$$

in which $\chi_1 = -j\omega\ell Y_{12}\beta/8\kappa_c$. The total NEXT equals the sum of (20) and (21).

## IV. LEADING ORDER CLOSED-FORM EXPRESSION

In the previous analysis we made two essential assumptions, namely $a/d \ll 1$ and $4h^2/d^2 \ll 1$. The latter can be used once more to expand (20) and (21) around $x = 0$. This way we obtain the leading order terms of (20) and (21). If moreover all original model parameters are substituted back into the more simplified notations, we obtain a closed-form expression for the NEXT of this wire configuration. The leading order inductive crosstalk is given by:

$$\gamma_{NE,ind} \approx j\omega l \frac{3\mu_0}{2\pi R_{d,c}}\left(R_{d,c}/R_{c,c} + 2\right) a^2 h^2 d^{-4}. \tag{22}$$

The corresponding capacitive crosstalk becomes:

$$\gamma_{NE,cap} \approx j\omega l \frac{24\pi\varepsilon_0 R_{d,v}}{\left(\frac{R_{d,v}}{R_{c,v}} + 2\right)} \frac{a^2h^2}{\ln^2\left(r^2\left(\frac{1}{4h^2} + \frac{1}{a^2}\right)\right)} d^{-4}. \tag{23}$$

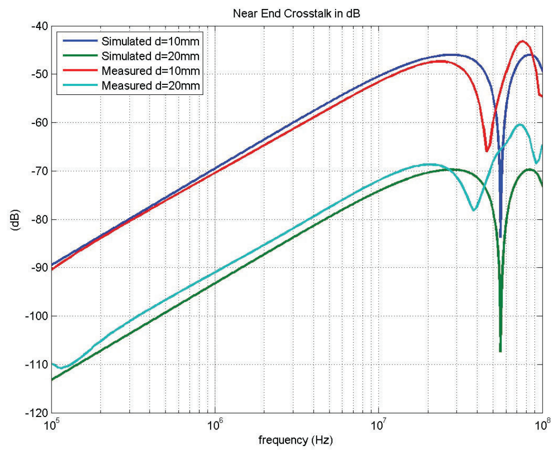


Figure 6. MTL simulations and measurements of NEXT for two different separation distances for the wire configuration given in Fig. 4.

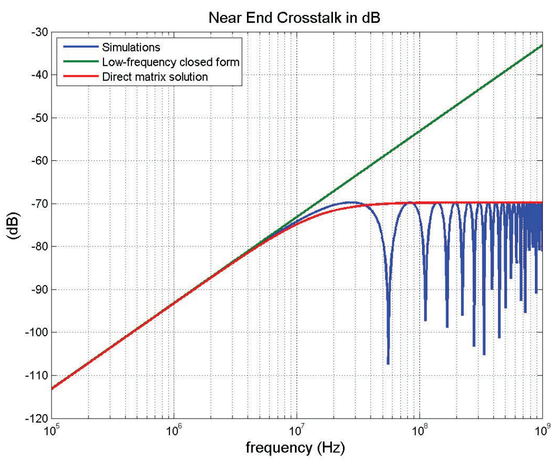


Figure 7. MTL simulations (blue) of NEXT for the wire configuration given in Fig. 4. The green line shows the low frequency approximation obtained by summation of (22) and (23). The red line is the direct solution found from Paul's matrix equation, given by (2).

To verify the modelling of unshielded wire pairs we performed both MTL simulations and NEXT measurements. Results are shown in Fig.6 for two different separation distances. For simulations the geometric values corresponded to $\varepsilon_r = 2$, $l = 1.9$ m, $r = 0.49$ mm, $a = 2.5$ mm and $h = 1.5$ mm. Load values were equal to $R_{d,v} = R_{d,c} = 112.5$ Ohm and $R_{c,v} = R_{c,c} = 450$ Ohm.

The measured crosstalk values correspond well to the outcome of MTL simulations. Fig. 6 confirms the predicted fourth order decrease of crosstalk with respect to wire-pair separation distance. The difference between the two simulated values shows a 24 dB/octave decrease. Between the measurements this difference is nearly the same. This suggests that this wiring configuration is a special case, because of the significant difference between this decrease, and the 12 dB/octave observed in White's case and other configurations.

In Fig. 7 we compare results of MTL simulations with the sum of our closed-form near-end crosstalk expressions in (22) and (23). Values of all parameters were as given above and a fixed distance between the wire pairs of $d = 20$ mm was chosen. The blue line shows the MTL simulations, which in the linear region coincide with our approximations (given by the green line). Finally, as before the red line represents the direct solution of Paul's matrix equation given by (2). The corresponding voltages are obtained by (4) after which the displayed NEXT was calculated with (1). This shows that again Paul's approximation excludes the finite transmission line effects, but provides an accurate upper boundary.

## V. CONCLUSIONS

We have described a mathematical approach to calculate closed-form expressions (22) and (23) for near-end crosstalk of unshielded wire pairs, oriented parallel to a ground plane. These expressions only contain logarithmic functions, geometrical data of wire pairs and culprit and victim impedances. The correctness of the mathematical approach has been verified for the classical configuration of two single wires above a ground plane, in which the approach provides expressions which correspond with formulas in literature.

For two wire pairs close a ground plane the closed-form expressions prove that the differential mode crosstalk decreases by 24 dB/octave by increasing distance between the wire pairs. This has been verified by means of measurements and MTL simulations. Furthermore, the expressions show that capacitive coupling is directly proportional to victim differential-mode impedance, while inductive crosstalk is inversely proportional to culprit differential-mode impedance. Here it assumed that differential-mode impedances in (22) and (23) are small in comparison to common-mode impedances (in our measurements the differential-mode impedances were 112.5 Ω and the common-mode impedances were 450 kΩ). Finally, expressions (22) and (23) also show the influence of the surrounding medium. The permittivity only affects the capacitive crosstalk, whereas the permeability influences the inductive coupling.

## REFERENCES


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