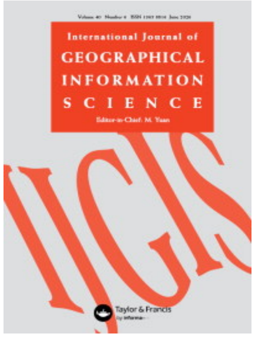

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RESEARCH ARTICLE

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Uncertainty in visibility: a scoping review of the probable and fuzzy viewshed for observer location optimization

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ABSTRACT

Although the probable and fuzzy viewshed have been recognized as critical in visibility analysis, they remain underutilized in practical applications such as surveillance drone positioning, telecommunications tower placement, and helicopter battle-position selection. Traditional approaches often assume a binary (boolean) notion of visibility, overlooking real-world factors like uncertainty in terrain data, partial occlusion from vegetation, or the effect on visibility by light sources, atmospheric haze, and target size. This scoping review systematically maps research on non-boolean visibility models and identifies several key gaps. First, there is a lack of methods that integrate both probabilistic and fuzzy approaches for observer placement. Second, while research has addressed DEM uncertainty and vegetation, few studies combine multiple factors or apply their methods to multi-observer or path-planning problems. Finally, research into 3D applications remains sparse, even though such work is critical for tasks like military helicopter missions or surveillance drone flights. Consequently, we highlight the need for more robust modeling of combined visibility factors and clearer strategies for incorporating both probable and fuzzy criteria in real-world operational settings. Bridging these gaps will enable more accurate and reliable visibility analyses across diverse domains, from city planning to helicopter mission planning.

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Probability of visibility; fuzzy viewshed; observer location optimization; literature review; visibility analysis

1. Introduction

Visibility analysis, a subfield of Geographic Information Science (GISc), focuses on analyzing what can be seen from a certain observer point. Determining whether something is in-view often involves stepping through a raster or ray-tracing between points to check for obstructing terrain or objects. If none are present, the points are considered visible. A viewshed calculation extends this concept in all directions around an observer location, mapping visible areas. For instance, in archaeology, a viewshed can

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be used to model the visual control exerted from an Iron Age settlement. This helps researchers assess the site's strategic advantages in monitoring movement and mobility through the surrounding landscape (Fábrega-Álvarez and Parcero-Oubiña 2019).

Beyond these methods, some approaches incorporate voxel-based or volumetric representations to handle complex 3D environments (Chmielewski and Tompalski 2017). Meanwhile, an isovist model (Tandy 1967), more common in architecture, describes the 2D region visible from a specific vantage point (often ignoring terrain elevation), although these methods have been updated to work in 3D environments (Wassim *et al.* 2014).

Another subset of research within visibility analysis focuses on determining optimal observer locations when they are not predefined. One of the first to define this problem was Victor Klee (Honsberger 1976) with the Art Gallery Problem. The goal of this problem is to find the minimum number of guards, or viewpoints, needed to see every point within a simple polygon. This fundamental concept extends to other applications, such as maximizing the view over an area represented by a digital elevation model (DEM) with a minimum number of observers, as done by Cervilla *et al.* (2015). Some applications for this include telecommunications (Kashyap *et al.*2014), firefighting (Mistick *et al.* 2023), surveillance sensor placement (Akbarzadeh *et al.*2013), counter-terrorism sniper hazard assessment (VanHorn and Mosurinjohn 2010), drone operations (Zorbas *et al.*2016), or helicopter battle positions (Milton and Williams 2002). In the majority of publications in this area, boolean visibility is assumed, which means that the subject or area is completely visible or not at all. In reality, a complex phenomenon like visibility is influenced by weather conditions, such as sun glare, fog, and rain, or by environmental conditions, such as partial occlusion from trees. These factors can influence the probability of visibility of an object or the ability to distinguish an object, even if the object is in line-of-sight (LOS).

Moreover, uncertainties in terrain data can lead to inaccuracies in viewshed calculation, where even small errors can lead to significant over- or underestimations (Huss and Pumar 1997). Yet, publications that incorporate such uncertainties remain limited. For example, methods such as the probable viewshed, which calculates the likelihood a location is visible, or the fuzzy viewshed, which quantifies the clarity of an object (Fisher 1994), remain an understudied research field. A review by Wheatley and Gillings (2000), discussed factors like DEM accuracy, vegetation, seasonal cycles, climate and weather, and background clarity. Although it referenced literature on how these factors influence visibility, relatively few researchers have addressed these topics since then.

Consider a helicopter mission aiming to plan a route while avoiding enemy air defense. Stiles (2000) stresses the importance of accounting for the uncertainty in the DEM data for such a helicopter mission, where a small error in terrain data can lead to large errors in radar visibility calculations. During such helicopter missions, helicopters also need to plan a location from which they hover to support or execute a mission, which is called a Battle Position (BP) (Army 2016). In planning such positions, experienced pilots seek a location where it is difficult to see the helicopter, for example, by ensuring the sun or moon is behind the observer (i.e. the helicopter) from the target's (e.g. a threat) perspective; by staying within an area covered by sun shadow, moon shadow, or artificially produced shadow; or by blending in with land as the background, as opposed to the sky (Milton and Williams 2002). Although factors like the probability of an unobstructed LOS between observer and target, or a target's clarity from the observer's point of view, are considered critical in military missions, the contribution of these factors to visibility uncertainty is not well-researched within the military context. This is an expected consequence of the confidential nature of military studies. Surprisingly, there is also little research on these factors outside the military context. Returning to the example of determining a BP, which can be regarded as an observer location optimization problem that incorporates non-boolean visibility factors, the goal of this review is to target literature (both military and non-military) examining the causes of uncertainty in whether an object is visible, and how such uncertainty is managed when optimizing observer locations.

1.1. Objectives of this literature review

This scoping review aims to identify and examine relevant literature, establishing a foundation for our investigation. By assessing existing work, highlighting the research gaps to lay the groundwork for future research. Accordingly, this review addresses the following research question:

> What is an effective way to model uncertainty in visibility arising from uncertain terrain and vegetation data, environmental conditions, and target characteristics?

To answer this research question, a scoping review is conducted to systematically map the research done and to identify the potential research gap in this area. The objective of this scoping review is to find what factors contribute to the uncertainty in visibility and investigate how those factors are modeled in literature. This is with the overall goal to investigate how non-boolean visibility can be integrated into observer location optimization.

The remainder of this review is organized as follows. Section 2 describes the methodology that is used to conduct this review. Section 3 presents the results of the review where each of the visibility criteria is grouped to the probable or fuzzy viewshed, how these criteria can be combined and publications that discuss some form of observer location optimization. Section 4 provides a discussion of the literature reviewed. Section 5 gives a short overview of the key research gaps.

2. Review method

This scoping review is conducted largely based on the guidelines for PRISMA scoping reviews by Tricco *et al.* (2018). The aim of the literature search in this review is to find publications that focus on the non-boolean viewshed, mainly, but not limited to, applications in aviation and defense. To find publications in this area, the main keywords were identified as: 'probability of visibility', 'probabilistic visibility', 'probable viewshed', 'fuzzy viewshed'. In addition, terms as 'battle position', 'firing position', and 'observer location optimization', are also considered primary keywords due to their relevance in determining observer positions in a military context, where uncertainty in visibility is a critical factor (Stiles 2000). The full keyword search can be found in section A.

2.1. Eligibility criteria

For inclusion in this review, publications must focus on quantifying uncertainty in visibility within the GISc domain, mainly focusing on publications using LOS or viewshed calculations. They are grouped into three categories: theory introduction, viewshed calculation application, and observer location optimization.

- Theory introduction includes publications that propose new methods for calculating visibility uncertainty for specific factors.
- Viewshed calculation application covers publications that use such methods to compute the viewshed.
- Observer location optimization involves cases where the observer's location is undetermined and must be identified—such as finding optimal sites for cell towers, surveillance cameras, drones, or military helicopters. Objectives can range from maximizing coverage to maintaining visibility on a target while staying outside a threat's line of sight.

In each category, only those addressing non-boolean visibility are considered, with the exception of publications in the observer location optimization category, where no relevant non-boolean visibility publications were found. Nevertheless, this category remains central, given the critical role of non-boolean visibility factors in planning drone surveillance sites, sniper positions, or helicopter battle positions—each considered an observer location optimization problem. A dedicated subsection on this category is therefore included in the results, alongside a separate section exploring the combination of non-boolean visibility factors.

Papers focused on visibility not in a GISc domain—for example, visibility of sensors in robotics for collision avoidance—are excluded. Furthermore, only papers published in English were considered, and the publications were required to provide an explanation of the mathematical model used to some degree. Although many military research publications address other parts of the electromagnetic spectrum (e.g. radar or radio waves), this review focuses exclusively on visible light. As noted by Milton and Williams (2002), determining observer locations—such as in helicopter BP planning—largely hinges on visibility. Furthermore, visibility is highly susceptible to uncertainty, influenced by a multitude of factors. Once a model is developed that effectively manages these uncertainties, it can likely be adapted to radio waves or radar as well. We chose not to automatically filter these publications by using the NOT operator, however, these publications are filtered out manually.

2.2. Information sources

To keep the list of publications in the computational and/or aviation/defense domain, specific databases and journals have been chosen in this regard. The databases chosen were as follows:

- NPS
- Web of Science
- IEEE

- AIAA

To decide the most relevant journals a Scopus search was done with the keywords *'viewshed'* AND *'intervisibility'*. These two keywords were chosen as papers containing the keyword viewshed often describe the application of a visibility algorithm and intervisibility as this is the term often used in military publications when describing visibility. The five most relevant journals for this literature review are found to be International Journal of Geographical Information Science (IJGIS), Landscape and Urban Planning, Journal of Archaeological Science, Land, and Computers and Geosciences. All of these journals are present in the Web of Science database, so they do not have to be added separately.

With the given keyword search, all records obtained from these databases were screened. This was done with the help of ASReview (Van De Schoot *et al.*2021), to filter out papers obtained that do not address visibility in the GISc domain. ASReview was able to help automatically filter out these publications.

After the screening process, records were excluded by the eligibility criteria, and finally, the remaining records included in this study were all read and excluded if they were not deemed relevant after all. Citation search was also performed on relevant records to increase the number of studies included in this review.

3. Results

The keywords search in the four databases and two journals resulted in a set of 617 publications out of which 37 were duplicates. The final number of publications included in this study is 48. The final result of the literature study performed is shown in Figure 1.

In section 3.1, the included studies are discussed and categorized according to which part of the probable or fuzzy viewshed they belong. Furthermore, in section 3.2, methods for combining probable or fuzzy viewsheds are discussed and in section 3.3 publications concerning observer location optimization are presented.

3.1. Probable and fuzzy viewshed

The term fuzzy viewshed was first introduced by Fisher (1992), where he initially defined it as an enhanced visibility model that accounts for errors in digital elevation models (DEMs) by estimating the probability that each point is visible from a specific location, often using methods like Monte Carlo simulations to generate multiple DEM realizations and averaging the resulting viewsheds. This approach acknowledges uncertainties in elevation data and provides a probabilistic assessment of visibility rather than a simple yes/no determination. Later, Fisher (1994) corrected himself and named this the probable viewshed instead. Here, he instead defined the fuzzy viewshed as a visibility model that represents varying degrees of clarity with which objects are seen from a specific location, accounting for factors like distance, atmospheric conditions, and individual perception. Instead of a simple boolean outcome, it uses a decay function to depict visibility as a gradual transition from clear to obscured,

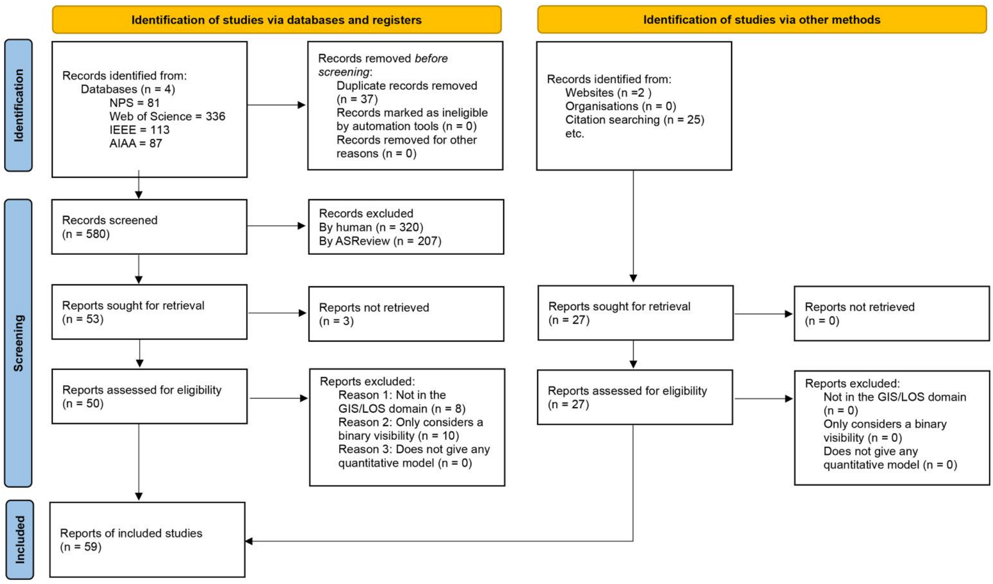

Figure 1. Flow diagram (Tricco *et al.* 2018) for updated systematic reviews which included searches of databases, registers, and other sources.

which can be characterized with terms such as ‘usually visible’, ‘sometimes visible’, and ‘visible only under very favorable conditions’. In short, the probable viewshed is defined as the likelihood that LOS beams from the object to the observer will not be blocked, and the fuzzy viewshed describes how clearly the object is visible.

Fisher provided a few examples of a fuzzy viewshed, such as distance decay that increases with fog or mist. A more complex example is where solar glare is introduced; the observer may only see a limited distance into the sun but a larger distance away from it. The rest of this section will focus on these and other identified probable or fuzzy types of visibility and ways of combining them.

In summary, probable viewshed defines the probability that there is an LOS between an object or certain area and an observer, which can be influenced by DEM errors, but also probabilistic placement of layers of vegetation, where it is often infeasible to model every leaf and branch. Fuzzy viewshed then determines the clarity of that object or area which can result from factors like vegetation, distance decay, size, light conditions or sources, weather conditions, or background. In section 3.1.1 through section 3.1.7 will discuss these factors and give examples of how these are modeled in literature.

3.1.1. DEM accuracy

Publications focusing on DEM accuracy are mostly found with this review’s search strategy. The effect was first discussed by Fisher (1990), where later he and other publications focused on the effect errors in DEM data have on practical applications concerning visibility. Fisher (1993) extends the calculation of the probable viewshed by taking into account additional sources of errors, such as interpolation between grid points in DEM data. In a later publication, Fisher (1995) shows how different applications of the probable viewshed would be calculated and how their desired values would differ for e.g.

military observation or hiker viewpoints. This is also done for multiple observers. He also suggests that interpreting a probable viewshed can reflect the effort a viewer might have to make to see a location. Thus, a location with a small probability of being seen will gain a higher probability if the viewer increases their elevation by standing on tiptoe, on the roof of a vehicle, or on a viewing platform. Nackaerts *et al.* (1999) apply the method suggested for calculating probable viewsheds by Fisher (1992), however, in this study, the visibility of individual cells is also assessed.

Wheatley and Gillings (2000) stress the importance of the accuracy in the modeling of terrain features, especially those close to the observer, which influences the visibility more than features far away from the observer. Moreover, uncertainty in hilltop or crest height can have a much larger effect on the uncertainty in visibility than, for example, valley bottoms or sides of hills.

Kyriakidis *et al.* (1999) discusses an interesting method to combine DEMs of different qualities, which can, for example, assist in finding targeting areas where more accurate elevation measurements are required.

An application of the probable viewshed is presented by Murphy *et al.* (2018) for a Roman communication network of towers and forts in Scotland. The paper uses this method to account for potential errors in the DEM and to confirm if the observed intervisibility between sites is genuine or a result of data uncertainty. The probable viewshed analysis supported the results of the initial boolean viewshed. It showed that a primary group of towers had high probability values of being intervisible, while a second group of towers had a low probability of being seen from other sites. This confirmed that the visibility patterns were based on actual intervisibility and not simply caused by DEM errors.

Finally, Loots *et al.* (1999) analyze the probable viewshed for a Hellenistic City Defence System, but wrongly calls it a fuzzy viewshed, as they are directly referring to the uncertainty in visibility due to DEM error, which should be named the probable viewshed according to Fisher (1994) definitions. The error made was also later confirmed by Ogburn (2006).

Multiple publications also stress the importance of taking the uncertainty in a DEM into consideration for military applications. Henrico *et al.* (2020) compare two DEMs of different accuracies for calculating whether specific target areas were visible or not. When using a lower accuracy, it is advised to take the uncertainty into account and not use such a low-accuracy DEM for the determination of a helicopter landing zone. Similarly, Stiles (2000) stresses the importance of taking DEM quality into account for helicopter path planning. Small errors in the height of a hill can lead to high errors in the LOS calculations of a radar, making paths that appear safe according to the DEM in reality unsafe. It is also mentioned that since vegetation and micro-terrain are usually not captured, this can lead to errors that reduce the visibility of an enemy radar. Xin *et al.* (2005) extend upon this by calculating a route for a helicopter that takes into account the uncertainty in the DEM data as well as errors in the locations of the target and observers. The optimization of the route assumes a terrain-following navigation system that is trying to stay below the LOS horizon of a hostile sensor. It also considers that being spotted a few times for a short duration is preferable to being spotted once for a long duration by the hostile sensor. Huss and Pumar (1997) investigate the probability of visibility due to DEM quality and add the effect of terrain roughness. They found that DEM databases investigated are good for predicting masking, but less reliable for predicting visibility of observers. Finally, Baer *et al.* (2005) introduce an algorithm to calculate LOS through uncertain terrain, accounting for vegetation. The terrain is divided into small bins; at each step, the probability of a ray passing through a bin is computed based on terrain elevation, its probabilistic error, and vegetation density. The full equation for the calculation and combination of these probabilities is found in Section B.1. These probabilities then combine to yield the overall likelihood that the LOS remains unobstructed.

3.1.2. Vegetation

Instead of considering vegetation as an 'error', as done by some publications in the previous section on DEM quality, one can use a digital surface model (DSM)[1], where the vegetation is also modeled. This, however, raises the problem of how to model the transparency of vegetation. Most publications assign some form of probable visibility due to vegetation, although some focus on a fuzzy viewshed from vegetation. One could see this as the probability an LOS is blocked by a leaf or branch, for example (probable viewshed), or instead how well an object is still distinguishable when it is behind vegetation (fuzzy viewshed).

One of the first to discuss the problem of visibility through vegetation is Llobera (2007), based on initial research by Dean (1997). In this publication, the probability of visibility is calculated as a function of the density of vegetation an LOS beam passes through, based on the Beer-Lambert law, which is one of the methods to model visibility in three-dimensional cluttered scenes. The Beer-Lambert law is an equation that is normally used for the permeability of a photon traveling through a macroscopically homogeneous medium. Llobera (2007) derived from the standard Beer–Lambert law an equation that can be used for the permeability of vegetation instead (Equation (1)).

$$p(x) = e^{-k(x) \cdot x} (k \geq 0, x > 0) \tag{1}$$

Here, x represents the distance from the observer along the LOS to the target, with $x = 0$ at the origin of the coordinate system. Then, $k(x)$ is a function based on the density of vegetation along the LOS. The definition of $k(x)$ is not explicitly found and is derived by the authors of this literature review in Equation (2). The full derivation can be found in Section B.2. $p(x)$ describes the probability of the LOS not having encountered an obstacle by distance x.

$$k(x) = \rho(x)\beta(x, z) \tag{2}$$

Here, $\rho(x)$ is the density of vegetation (number of plants/trees per unit of volume) at a distance x from the observer. $\beta(x, z)$ is the effective cross-sectional area of the vegetation encountered at distance x and relative height z (compared to the observer's location) that the LOS goes through. This cross section accounts for the geometry of leaves, branches, or trunks that block the LOS. In other words, $\rho(x)$ and $\beta(x, z)$ together describe the 'obstacle density' along the path; larger values of either quantity lead to higher attenuation. Langer and Mannan (2012) takes a similar approach to modeling visibility, in which they also arrive at a Beer-Lambert equation for the probability of visibility. However, their derivation is based on Poisson-distributed disk center in 3D space. They arrive at Equation (3).

$$p(x) = \lambda e^{-\lambda(x-x_0)} \tag{3}$$

Here, $\lambda = \eta\pi R^2$ and η represents the density of the Poisson process. They also take into account scenarios with varying object shape and distribution. Their work highlights how variations in object geometry—and not just density—can influence visibility probabilities in cluttered scenes.

Overall, the key insight is that while the Beer–Lambert law was first introduced for homogeneous optical media, it can be generalized to spatially varying media, including vegetation. By defining a space-dependent attenuation coefficient $k(x)$ that reflects local vegetation density and geometry, one obtains a tractable model for the probability of seeing through layers of vegetation along a given line of sight.

There are also various implementations where visibility through vegetation is modeled. Govindaraju *et al.* (2014) applies the Beer-Lambert law for the path planning of small surveillance drones. It is argued that since these drones fly low and relatively slow, modeling transparency through vegetation is essential to find strategic waypoints that ensure optimal search of the target regions. The authors argue that since the observer is now a drone flying over a forest, the crown of a tree are now the layers of vegetation the drone's camera has to look through. These thin layers or slices can all have a different density (δ_i). The Beer-Lambert law for probability of visibility for a drone flying over a forested region is shown in Equation (4).

$$P_v(r) = \exp^{-\omega \sum_{i=1}^{N} \delta_i \Delta x_i} \tag{4}$$

Here, $\sum \delta_i$ is the sum of the average tree crown cover densities along each section of vegetation through which of the LOS beam travels, while ω is a constant to convert the crown cover density into decay constant. Then, Δx is a small distance increment along the LOS, similar to Δx in the previously mentioned publication by Llobera (2007). r is the horizontal distance to the target from the surveillance drone.

Some publications do not directly use the Beer-Lambert law, but assume a transparency value, sometimes named a vegetation index, to model vegetation. For example, Bang *et al.* (2010) discuss infiltration-route analysis, a military application of GISc. In this publication, an algorithm is created where thermal observation devices (TODs) are randomly placed. The infiltrator is assigned a certain concealment probability to the TODs based on the type of vegetation found in the area the infiltrator is passing through. The precise calculation of the values for each vegetation index is not explained. Interestingly, the observers (TODs) use thermal vision instead of visual light for detection. Also Maaiveld *et al.* (2023) assume a value for vegetation transparency, whose calculation is not discussed. In this publication, they consider an unmanned ground vehicle in a military context, tasked with finding vantage points to observe a location of interest. A concealment value is assigned to vegetation in a 2D raster environment. For these vantage points, a route is also calculated, and a multi-objective optimization is performed to determine a trade-off between the ease of travel of those routes and their vantage value due to vegetation or buildings. Guth (2009) also uses a vegetation index but argues that existing land classification or vegetation indices do not provide an ideal estimate of tree heights for visibility calculations. It suggests to either use percent canopy coverage or integrating Light Detection and Ranging (LiDAR) data into a point cloud instead.

Percent canopy coverage can be determined from aerial LiDAR data, as demonstrated by Vatandaslar *et al.* (2024). While this data is not typically used in standard viewshed calculations, it can serve as an input for probabilistic models. For example, the probability of visibility could be reduced in areas with a high canopy cover, which would help create a more realistic, non-binary assessment of what can be seen through sparse forests.

The limitations of using a 2.5D DSM are further explained by Rášová (2018), who presents three methods of adding vegetation to a DSM in 2.5D. The first method is adding vegetation as a solid obstacle with a known spatial extent, the second method is giving that obstacle a partially transparent value, and the third method is adding vegetation as a solid obstacle with unknown spatial extent and using the probable viewshed to calculate visibility. However, Rášová (2018) also acknowledges that a 3D point cloud model would be needed to accurately model various parts of the tree, different tree types, the effect of the distance of the observer from the vegetation, and the viewing angle.

One study that applies a 3D point cloud method is Murgoitio *et al.* (2013), who construct a model based on tree trunk obstructions derived using LiDAR data. The modeled trunk obstructions were found to have a reference visibility model derived from digital photography and perform better than a bare Earth visibility model or a mean vegetation height visibility model.[2]

Several other methods of modeling visibility through vegetation exist, for example, Magoc *et al.* (2010), which will be discussed later in section 3.2, defines the influence of vegetation density as a fuzzy measure and discusses how it interacts with other fuzzy criteria influencing the fuzzy viewshed.

Bartie *et al.* (2011) models trees as 2.5D objects with a permeability that is a function of the distance from the observer to the tree and decreases closer to the trunk of the tree. The values for the permeability change per season, which is one of the only publications that does this. The permeability is defined as the area that a cross-section of vegetation covers, similar to defined by Equation (1).

Finally, Chmielewski and Tompalski (2017) use a voxel-based approach to model visibility through vegetation, leveraging airborne laser scanning (ALS) data to create 3D voxel grids that represent vegetation and other obstacles. The study examines the influence of voxel size on visibility analysis accuracy, testing various voxel resolutions and point cloud thinning methods to determine optimal parameters. The study classifies the visibility through vegetation as a fuzzy variable, where it describes objects (outdoor advertising boards in this case) with a level of visibility parameter between zero and nine. It uses ground truth visibility points and computer vision to determine the visibility level.

3.1.3. Target location

Besides errors in terrain data, there can also be uncertainty in the target location that affects visibility calculations. Consider again a helicopter trying to stay out of a radar's

LOS, as depicted by Stiles (2000). If not only the DEM but also the radar's exact location is uncertain, it could alter which paths are deemed safe. Similarly, if that same helicopter is searching for a certain target whose exact location is unknown, it will influence which vantage points offer a clear LOS to the target. So there can be uncertainty in both the observer's location or the target's location. Huss and Pumar (1997) point out that even a slight shift of the observer's (an enemy threat in this case) position can dramatically change visibility. They point out that if an observer is stationed in a certain area, they may move around to find the best vantage point. However, when placing observers (e.g. enemy personnel or radars) in Geographical Information System (GIS) software, this potential movement is often not accounted for, even though it could significantly affect LOS calculations. As mentioned before, Xin *et al.* (2005) also point this out for the application of a helicopter mission and calculates the influence of uncertainty in the observer's location, which is an enemy radar in this case.

Although uncertain threat location seems a relevant topic in 2D drone path planning, such as in Dogan and Zengin (2006) and Haraguchi *et al.* (2007), most research focuses on maintaining a control link with the drone rather than ensuring a visual LOS. Saboor *et al.* (2024) created a probabilistic line-of-sight (PLOS) model for urban environments. The model does not use a direct 3D map of a specific city; instead, it uses statistical parameters to generate a representative urban layout. It then predicts the LOS probability based on the drone's altitude and its position relative to the ground user, factoring in these statistical parameters.

Stucky (1998) provides an example of boolean 3D visibility applied to route-planning for scenic, strategic, hidden, and withdrawn paths, with the latter three intended for military operations. However, it does not incorporate any form of probable visibility. It would be interesting to examine how DEM errors, vegetation, or target location uncertainty might affect these optimal paths.

3.1.4. Distance and target size

Fuzzy viewshed as result of distance and target size was first introduced in an example by Fisher (1994). Consider a DEM (raster based), where each cell's value represents elevation and is identified by row *i* and column *j*. The equation to calculate the fuzzy viewshed, or clarity, is shown in Equation (5), where $\mu(x_{ij})$ is the fuzzy membership at the cell at the cell in row *i*, column *j*, d_{vp-ij} is the distance from the viewpoint to that cell.

$$\mu(x_{ij}) = \begin{cases} 1 & \text{for } d_{vp-ij} \leq b_1 \\ \dfrac{1}{1 + \left(\frac{d_{vp-ij} - b_1}{b_2}\right)^2} & \text{for } d_{vp-ij} > b_1 \end{cases} \tag{5}$$

The idea is that a target is perfectly visibility until a distance b_1, after which the clarity decays with distance. The distance b_1 depends on the size of the target, where b_1 decreases with a smaller target. A second distance, b_2, is introduced so that the fuzzy membership drops to 0.5 at $b_1 + b_2$. This point is called the 'crossover point', where clarity visibly diminishes, and some observers may struggle to distinguish the object from its background.

Fisher's method for distance decay was later improved by Ogburn (2006). Fisher used values $b_1 = 1km, b_2 = 3km$, but did not suggest how to modify them based on target size. The modified fuzzy viewshed accounts for the decline in visibility with distance by incorporating the size of the target and human visual acuity thresholds into the decay function. By adjusting Fisher's original formula using the trigonometric relationship between object size and distance (at specific visual arcs), the method produces a visibility model where the fuzzy membership values decrease gradually with distance, reflecting how an object's clarity diminishes based on its size and the observer's ability to perceive it. This is done through Equation (6).

$$a = \frac{d_{target}}{s_{target}} = \frac{1}{2\tan(\beta/2)} \tag{6}$$

Here, d_{target} is the distance to the target, s_{target} is the width of the target, and β is defined as the visual arc, which depends on the viewers limit of recognition acuity, where a value of $1'$ $(\frac{1}{60}^{\circ})$ is taken for normal 20/20 unaided vision. The resulting a is the distance multiplier, which can be substituted for the crossover point, $b_1 + b_2$ in Equation (5). When keeping b_1 constant to a value of $1\,km$, the decrease in clarity is much more gradual than the suggested values by Fisher. As example, a human with 20/20 vision would be able to distinguish an object of one meter wide from a distance of $6880\,m$. This approach results in a target size-sensitive fuzzy viewshed that more accurately represents real-world visibility conditions.

Fraser (1983) described the Higuchi viewshed, which is seen as an important fundamental work on fuzzy viewshed that takes distance into account as well. The Higuchi viewshed can be described as a distinction between features nearby (foreground), at a medium distance (middle ground), or long distance (background). Nearby features directly shape the observer's immediate experience, while medium-distance features provide context and reference points. Faraway features may still be visible, but they are less distinct and typically less significant for immediate needs. Later, Wheatley and Gillings (2000) described this method in more detail and explained how the Higuchi viewshed could be created in GIS. The three visibility zones (foreground, middle ground and background), were not based on fixed distances, but on other analytical criteria chosen by the researcher, such as the perceived size of an object or its cultural significance.

Llobera (2007) later extended this concept by determining which of the three visibility zones an object is in by determining the visual angle that an object occupies in an observer's field of view. This directly models how prominent an object appears to a person. For example, it defines the foreground as the area where an object occupies more than 15° of the visual field and the 'background' as the area where it occupies between 0.1° and 0.5°.

Fábrega-Álvarez and Parcero-Oubiña (2019) performed field experiments to establish empirical distance thresholds for different levels of 'visual control' over an individual in a landscape. Instead of using generic zones, the paper defines a hierarchy of visual resolution based on what can be perceived. Using a person as the visual target, the study found the following: detection of the individual required a distance of $2550 - 2100m$; human recognition required $1250 - 975m$; and basic recognition, where distinct parts of clothing and limb movements become clear, occurred at 600 *m*. Furthermore, detailed individual recognition, such as discerning the shape of a hat, was possible at 225 *m*, and full identification based on details like accessories was possible at 60 *m*. The paper proposes using these experimentally-derived distances to create a more meaningful fuzzy viewshed, which they term a Fuzzy Individual Distance Viewshed. In this model, the different levels of recognition are assigned corresponding fuzzy membership values, combining the gradual decay of a fuzzy model with values that have real-world meaning.

Another relevant term for this section is visualscape, introduced by Llobera (2003), which he defined as the spatial representation of any visual property generated by, or associated with, a spatial configuration. Part of the visualscape is the visual exposure, which deals with how much of a feature or a terrain is visible at each location, rather than finding out whether a location is visible or not. Using the visual exposure allows for some interesting analysis. For example, by taking the local gradient of visual exposure at each location, one could find the quickest way to get out of sight from a certain feature.

An example of an application of the calculation of the fuzzy viewshed due to distance decay is using Ogburn's methods shown by Murphy *et al.* (2018), again for the Roman tower communication system. This method provides a more realistic model by including a decay function that demonstrates the breakdown in visibility over distance. The analysis confirmed that while the main communication towers had excellent visibility to and from the central site, the direct intervisibility between other towers further away was significantly lower. This finding suggests that the central site could have acted as a crucial relay station, passing signals between towers that had poor direct visibility with each other due to the large distances involved.

Other authors took the sensing capabilities of a camera into account, instead of that of the human eye. Akbarzadeh *et al.* (2013) used probabilistic membership functions for the sensing range and sensing angle, which represents the sensing capabilities of a camera, taking into account critical environmental factors such as terrain topography. The distance membership function is assumed to be a sigmoid function. The viewing angle of the camera is also taken into account; however, the target size is not taken into account in this publication.

Bishop (2021) tested visibility decay functions on camera pictures taken of wind turbines. The contrast ratio of the wind turbines with the background is taken as the level of visibility and is compared versus distance in a linear, inverse linear, and logarithmic manner. It was found that a linear decay is sufficient for distances up to 11 km. Building on this finding, the study calculated the non-boolean, cumulative visual impact by modeling each of the 56 turbines with a linear distance decay function in a GIS environment. These individual impact maps were then summed together, resulting in a final graded impact score for each location rather than a boolean viewshed.

Hognogi *et al.* (2022) is an example of a study where the distance decay function given in Equation (5) is applied for territorial planning for a landscape in Romania and concludes this viewshed analysis can be used for determining the most optimal lookout points.

3.1.5. Light conditions or sources

Fisher (1994) also explains how solar glare can make it more difficult for an observer to see a certain target and incorporates this in his fuzzy viewshed calculation. The solar glare is modeled as simple trigonometric functions that modify the b_1 and b_2 distances from Equation (5) to b_1' and b_2', for which the equations are shown in Equation (7).

$$\begin{aligned} b_1' &= b_{1_{\min}} + \left((b_1 - b_{1_{\min}}) \times c\right) \\ b_2' &= b_{2_{\min}} + \left((b_2 - b_{2_{\min}}) \times c\right) \end{aligned} \tag{7}$$

Here, c is defined as in Equation (8).

$$c = \begin{cases} \sin\psi & \text{for } \left|\sin\left(\frac{\theta}{2}\right)\right| < \sin\psi, \\ \left|\sin\left(\frac{\theta}{2}\right)\right| & \text{for } \left|\sin\left(\frac{\theta}{2}\right)\right| \geq \sin\psi. \end{cases} \tag{8}$$

Here, ψ represents the elevation angle of the sun, which measures how high the sun is above the horizon. θ denotes the azimuth angle of the sun, indicating the compass direction from which the sunlight is coming. Both angles are defined relative to the observer's orientation as they look toward the target. Specifically, when the sun is positioned directly behind the target from the observer's viewpoint, both the elevation angle (ψ) and the azimuth angle (θ) are zero degrees.

Surprisingly, there have not been any other mentions found of the influence of solar glare on the fuzzy viewshed. Solar glare is, however, a concept extensively researched in driving safety. For example, Guo *et al.* (2023) defines minimum and maximum angles from the observer to the position of the sun that can result in a disability, disturbing, or acceptable glare. Fuzzy logic in visibility is not mentioned in any of the publications focusing on driving safety.

3.1.6. Weather conditions

Fisher (1994) also suggests how fog or mist affects the fuzzy viewshed. He achieves this by introducing a sudden drop at the crossover point and slightly limiting the visibility in the foreground. The modifications for b_1 and b_2 given in Equation (5) are now shown in Equation (9).

$$\mu(x_{ij}) = \begin{cases} 1 & \text{for } d_{vp-ij} \leq b_1, \\ 0 & \text{for } d_{vp-ij} > b_1 + 2 \times b_2, \\ \sin\left(\left(\frac{d_{vp-ij} - b_1}{2 \times b_2}\right) \times 90^\circ\right) & \text{for } b_1 + 2 \times b_2 \geq d_{vp-ij} > b_1. \end{cases} \tag{9}$$

Fisher also mentions that Equation (9) is not suitable for atmospheric haze, as haze commonly stays close to the ground. Thus, if the LOS passes near the ground surface, it will experience a reduction in clarity. Conversely, if it remains farther from the ground, clarity can be preserved, allowing objects in the background to appear sharper than those in the foreground. In this scenario, clarity improves as the LOS moves higher above the ground. Fisher suggests this can be modeled with Equation (10)

$$\mu(x_{ij}) = \begin{cases} 0 & \text{if } ij \text{ is out-of-view,} \\ \min[\mu(h_1), \mu(h_2), \ldots, \mu(h_n)] & \text{if } ij \text{ is in-view.} \end{cases} \tag{10}$$

Here, $\mu(x_{ij})$ is defined as the fuzzy membership of a target viewed through haze, determined by the minimum membership value from n locations along an LOS. The membership at each location, $\mu(h)$, is a function of the LOS height above the terrain, h, and is calculated with Equation (11).

$$\mu(h) = 1 - \frac{d_{haze}}{(d_{haze} + h)} \tag{11}$$

Here, d_{haze} is the height of significant haze. This is the altitude where the haze density is such that the membership function $\mu(h) = 0.5$. It represents a critical point where haze starts to significantly impact visibility.

Although these equations give some indication on how clarity can decay with weather conditions, they do not seem to be based on any physics, but rather on what would seem a plausible decay rate. Although not directly related to the probable or fuzzy viewshed, Schwartz *et al.* (2021) discuss how rain, snow, and fog affect viewing distance. The viewing distance is generally calculated with Equation (12).

$$V_d = \frac{-ln(C_R)}{\sigma_c} \tag{12}$$

Here, σ is the attenuation coefficient, which is specific to the weather condition. The publication explains how rain, snow and fog affect the attenuation coefficient. C_R is defined as the contrast ratio, typically set at 2% because it represents the threshold at which human vision can reliably distinguish an object from its background under standard viewing conditions. However, this value has been questioned in recent literature, such as Lee and Shang (2016), as the 2% threshold assumes an ideal scenario of a black object against a white background. In reality, object and background colors often vary, making this assumption less applicable to many real-world conditions. Consequently, recent studies have advocated for higher contrast ratio thresholds, arguing that a 2% threshold may underestimate the difficulty of distinguishing objects under less-than-ideal conditions. Although the concept of the fuzzy viewshed is not explicitly discussed in these papers, the principles they explore align closely with the ideas of uncertainty and gradation inherent to the fuzzy viewshed.

3.1.7. Background

Concealing an object is easier when it is set against a landscape background rather than outlined against the sky. Fisher (1996) terms this the 'horizon viewshed'. He categorizes the horizon viewshed into three categories: 1) the location is simply in-view, 2) local horizon, which could be the top a landscape feature such as a hill, which has more land surface as background, 3) global horizon, where the land surface is seen to meet the sky.

Fisher's horizon viewshed has been applied by Fontani (2017), who has created an algorithm applying Fisher's Horizon Viewshed for power transmission line placement, to study the visual impact of such a placement. With the model created, power or radio tower placement can be optimized for the least visual pollution as seen from a nearby town. The visual impact of wind turbines also depends on their background, as argued by Bishop (2019), where he argues white wind turbines cause less visual pollution when having a cloudy background. This is due to the decreased contrast ratio of the turbine and background, similar to how it is explained in section 3.1.6 for the influence of viewing distance by fog, haze or rain. How much light reaches the wind turbines themselves, due to the time of the day or the weather, also influences the contrast ratio and thus visibility.

To measure how much an object stands out against its background, it is necessary to analyze its camouflage. Camouflage in the visual and infrared spectrum aims to minimize the contrast of a target by matching its surface properties to its background as stated by Toet and Hogervorst (2020). This publication assesses different methods of camouflage, which can be grouped into three types: subjective approaches through experiments with human observers, objective computational approaches using image analysis (such as saliency models), and objective approaches through physical measurements. While there are many methods to assess camouflage, there are no standardized and internationally accepted procedures, and the quality of these methods depends on the scenarios and the environments in which they are used.

3.1.8. Classification of factors

Taking into account the definitions of the probable and fuzzy viewshed, DEM accuracy can be clearly classified under the probable viewshed, consistent with the concept introduced by Fisher (1994). Vegetation, however, is more complex, as it has been attributed to both the probable and fuzzy viewshed in the literature, although it is mostly treated as probable visibility. Both interpretations have merit, yet in a realistic GIS setting—where not every branch or leaf is modeled exactly—it is more logical to treat vegetation as a probable factor, reflecting the likelihood that a particular vegetation layer will obstruct the line of sight.

Publications explicitly addressing target or observer location uncertainty in GISc are scarce. However, whenever its potential importance is noted, it appears in the context of probable visibility, and is therefore classified as such.

The criteria of distance, target size, light conditions or sources, weather conditions, and background clearly belong to the fuzzy viewshed. These factors do not affect the probability of the line of sight, but rather the clarity of the target. Among them, weather conditions have received slightly more attention, often focusing on how weather impacts viewing range. We argue that relaxing or altering assumptions about this viewing range essentially aligns with the concept of the fuzzy viewshed.

A final table showing whether a factor is grouped under probable or fuzzy viewshed in GISc context is shown in Table 1.

3.2. Combining visibility

Most of the previously mentioned publications focus on quantifying the probable or fuzzy viewshed for only one of the factors mentioned; there are very few publications discussing how multiple factors can be combined. There are publications, however,

Table 1. Classification of factors according to the probable and fuzzy viewsheds.

Category	Factor
Probable viewshed	DEM accuracy
	Vegetation
	Target location
Fuzzy viewshed	Distance and target size
	Light conditions and sources
	Weather conditions
	Background

that combine the visibility from multiple observers instead. Fisher (1994) illustrates how probable viewsheds from multiple observers can be combined via union or intersection. For two observers a and b, determining the probability p that an object at row i and column j is visible to at least one of them involves using Equation (13), which is standard probability rule for non-independent events.

$$p(x_{A\cup B})_{ij} = p(x_A)_{ij} + p(x_B)_{ij} - \left(p(x_A)_{ij} \times p(x_B)_{ij}\right) \tag{13}$$

Similarly, if one wants to determine if something is visible to both observers, one could use Equation (14), which is the standard probability rule for independent events.

$$(p_{x_{A\cap B}})_{ij} = p(x_A)_{ij} \times p(x_B)_{ij} \tag{14}$$

Wheatley (1995) takes this a step further and introduces a method called the cumulative viewshed analysis, where many viewsheds can be combined, which could theoretically be applied to probable viewsheds as well.

Although not explicitly mentioned by Fisher (1994), one could use Equation (14) for combining probable viewsheds from different criteria of the probable viewshed as well, such as DEM accuracy and vegetation. This is how Baer *et al.* (2005) handles the combination of DEM error and vegetation shown in Section B.1.

Combining visibility factors for fuzzy measures can be more complicated than combining factors for probable viewsheds. One study that does combine fuzzy measures in the viewshed is Magoc *et al.* (2010), where a set of fuzzy criteria is chosen to calculate a single value for the visibility of a target. The set is chosen to be distance, elevation, geographic location, time of day, sun position in the sky, weather conditions, and vegetation density. The publication considers how various criteria interact, for example, under rain or fog conditions; the sun's position could be less influential on visibility. To accommodate such interactions, visibility can be computed by the Choquet integral, which is defined in Equation (15).

$$(C)\int_I f\ d\mu_m = \sum_{i=2}^{n} \left(f(\sigma(i)) - f(\sigma(i-1))\right)\mu_m(A(i)) \tag{15}$$

Here the total visibility is captured in f, where the set of fuzzy criteria are accounted for in a non-additive way via μ_m. μ_m is a non-additive measure defined over a subset of criteria. When f is a (known) function mapping each criterion i in I to a real number, we can define a permutation σ on the set $I = \{1, 2, \ldots, n\}$ such that $f(\sigma(1)) \leq f(\sigma(2)) \leq \ldots \leq f(\sigma(n))$. $A(i)$ is a subset of indices that includes the i $^{\text{th}}$ criterion and all criteria that follow it in the sorted order. Because μ_m must assign importances to all subsets of criteria, its direct construction can be expensive. In practice, these values are often elicited from experts (domain specialists who judge the relevance of each subset). Alternatively, one can try to quantify each criterion's individual and combinatorial contribution when values for total visibility are available from experiments or simulations. Using regression analysis (such as least absolute deviation) one could then obtain the approximate weights of the Choquet integral. To avoid exponential complexity, the measure μ_m can be restricted to a 2-additive fuzzy measure. In that case, only first- and second-order interactions need to be specified, reducing the computational burden to $O(n^2)$ instead of $O(2^n)$ while still reflecting key synergies or redundancies among criteria. An example of the calculation of this integral with explanation of each step can be found in Section B.3.

Fuzzy viewsheds due to different criteria can also be combined by calculating the influence of each criterion on the b_1 and b_2 distances in Equation (5), (7) and (10).

When instead combining fuzzy viewsheds from multiple observers, the overall fuzzy viewshed can be obtained by taking the worst clarity (i.e. the minimum membership value) for each grid cell, as shown in Equation (16):

$$\mu_{(A\cap B)}(x) = \min[\mu_A(x), \mu_B(x)]. \tag{16}$$

Another challenge lies in finding a way to integrate probable and fuzzy viewsheds. Fisher (2000) explains the distinction between the two, noting that fuzzy viewsheds rely on fuzzy set memberships, which represent a possibility distribution, whereas probable viewsheds are based on a probability distribution. One approach to combine them is to first establish a threshold for the fuzzy viewshed that determines whether the viewer can distinguish an object from its background, effectively converting the fuzzy viewshed into a boolean viewshed. This boolean viewshed can then be integrated with the probable viewshed. Interestingly, this process resembles the method used for the contrast ratio when predicting viewing distances under various weather conditions, as previously discussed by Schwartz *et al.* (2021).

Another method is to transform a fuzzy viewshed into a probable viewshed by evaluating the probability of the fuzzy criteria affecting visibility. For instance, when considering a viewshed influenced by fog, the probability of fog occurring in a given area can be used as the basis. By applying a threshold after calculating the fuzzy viewshed, the probable viewshed can then be derived.

3.3. Observer location optimization

The term observer location optimization, introduced in this literature review, is considered the overarching category of what is often called the maximum coverage problem, the Art Gallery Problem, filtering candidate viewpoints, the minimum observer problem, or siting observers. It deals with determining observer positions when they are not predefined. Typically, this requires multiple viewshed calculations to identify one or more optimal viewpoints. The main objective is often to maximize the cumulative viewshed, which is the total visible area across the landscape (Wheatley 1995).

One of the first to introduce a problem of this type was Victor Klee in 1973, where the goal was to observe every point of an art gallery by positioning a minimum number of guards. Recently, Abrahamsen *et al.* (2022) has shown that the Art Gallery Problem is $\exists\mathbb{R}$-complete, meaning its difficulty is equivalent to solving systems of polynomial equations over real numbers. This result justifies the lack of simple combinatorial algorithms for the problem. Consequently, this finding rules out many algorithmic approaches that rely on constructing a finite set of rational candidate points for the guards.

Rana (2005) illustrates a similar concept in the context of surveillance cameras, where observers are placed to minimize overlap using a greedy-search algorithm called Rank and Overlap Elimination. VanHorn and Mosurinjohn (2010) used multiple viewshed calculations to identify potential positions from which sniper terrorism threats could target specific sites, notably factoring in weapon capabilities such as rifle range. Another application is the placement of cell phone towers as done by Kashyap *et al.* (2014). To narrow down suitable locations, only higher elevations (such as hilltops) are considered. In populated areas, effective coverage radius decreases, and this is factored into the optimization process along with tower height, aiming to balance sufficient coverage with minimum cost.

Akbarzadeh *et al.* (2013) is one of the few publications that employs a probabilistic model for visibility in the context of observer location optimization. They define a sensor coverage model whose probability of visibility decreases with distance and relative angle, then use an evolutionary algorithm to determine the best placement and orientation of sensors on a university campus. Their objective function prioritizes coverage of pedestrian walkways over streets, and excludes building rooftops altogether.

Cervilla *et al.* (2015) and Heyns (2020) introduce an efficient algorithm for placing the minimum number of observers to achieve maximum coverage of a masked area. Yu *et al.* (2016) partition the region into multiple subareas to maximize coverage, outperforming commonly used algorithms in both accuracy and speed. Wang and Dou (2020) extend this approach by considering overlapping coverage among viewpoints, creating more accurate results while requiring fewer observers. Most recently, Wang *et al.* (2024) improve performance in mountainous terrains and increase stability through globally oriented data processing, making it the most robust approach to date.

Finally, although not a direct application of observer location optimization, the concept of a Complete Intervisibility Database (CID) is a relevant application, as described by Dowers and Mineter (2005). The CID is a pre-computed database that stores the visibility from every point in a landscape to every other point. This pre-computation enables rapid analyses, such as calculating the cumulative visibility to identify locations with high visual exposure or areas that offer concealment, which is often used for military path planning or potential observation location planning. Caldwell *et al.* (2003) discuss Tactical Decision Aids (TDAs) that the CID can be used for. One such TDA is used to analyze the percentage of a target that is visible from an observation point to determine how much of that target or area will be visible. This analysis can be used to identify the number and location of optimal observer sites for viewing a target feature. Another TDA allows for the determination of the least or most visible path. This analysis can also be integrated with other inputs, such as vegetation or slope, to inform path or observation point planning.

Table 2. Classification of the reviewed papers.

		Theory introduction	Viewshed calculation application	Observer location optimization
Boolean viewshed		*not included*	*not included*	Cervilla *et al.* (2015); Akbarzadeh *et al.* (2013); Kashyap *et al.* (2014); VanHorn and Mosurinjohn (2010); Rana (2005); Wang and Dou (2020); Wang *et al.* (2024); Heyns (2020); Abrahamsen *et al.* (2022); Caldwell *et al.* (2003);
Probable viewshed	DEM accuracy	Fisher (1990); Fisher (1992); Fisher (1993); Kyriakidis *et al.* (1999); Baer *et al.* (2005);	Fisher (1995); Henrico *et al.* (2020); Huss and Pumar (1997); Nackaerts *et al.* (1999); Loots *et al.* (1999); Murphy *et al.* (2018)	
	Vegetation	Dean (1997); Llobera (2007); (Langer and Mannan 2012); Murgoitio *et al.* (2013); Bartie *et al.* (2011); Baer *et al.* (2005); Chmielewski and Tompalski (2017); Rášová (2018)	Guth (2009)	Maaiveld *et al.* (2023);
	Location	Xin *et al.* (2005); Saboor *et al.* (2024)		
Fuzzy viewshed	Distance and target size	Fisher (1994); Ogburn (2006); Fraser (1983); Llobera (2003); Llobera (2007); Fábrega-Álvarez and Parcero-Oubiña (2019);	Hognogi *et al.* (2022); Murphy *et al.* (2018); Bishop (2021);	Akbarzadeh *et al.* (2013)
	Light conditions or sources	Fisher (1994)		
	Weather conditions	Fisher (1994); Schwartz *et al.* (2021); Lee and Shang (2016)		
	Background	Fisher (1996); Toet and Hogervorst (2020);	Fontani (2017); Bishop (2019);	

3.4. Summary of results

The publications are divided in theory introduction, viewshed calculation application, and observer location optimization When a publication both introduces a new method and calculates a viewshed as example, it is categorized under 'method introduction'. As mentioned before in section 2, for the observer location optimization category, boolean visibility publications are added to in the literature search. The final result of the papers that could be placed into one or more of these categories are shown in Table 2 and plotted in Figure 2. The total number of publications that could be categorized is 35. The publication years of papers and their category is plotted in Figure 3.

It can been seen from the figure that publications in the observer location optimization are only done using boolean visibility, publications on location uncertainty

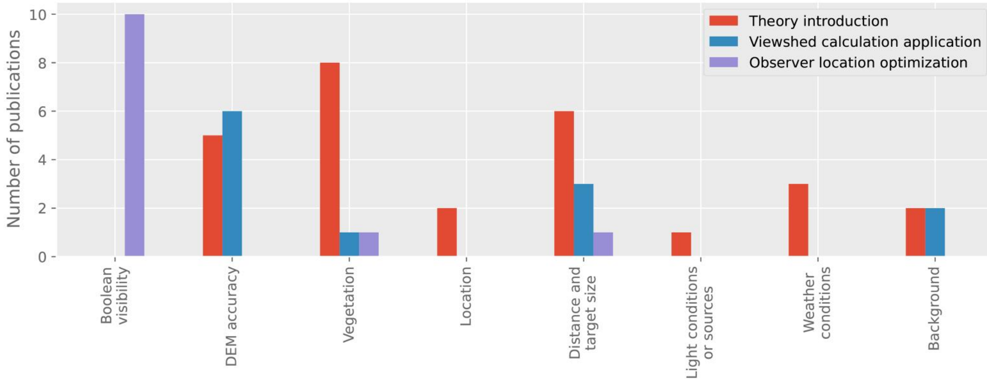

Figure 2. Number of publications found focusing on boolean, fuzzy and probable visibility per category.

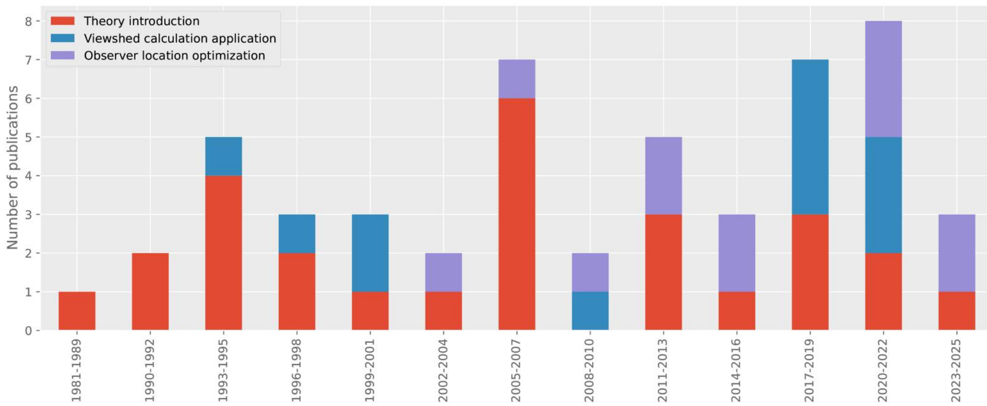

Figure 3. Number of the included studies published each year for each category.

are scarce and there are not many publications on the fuzzy viewshed altogether since its introduction. Furthermore, most observer location optimization publications are more recent than those in other categories as seen in Figure 3, suggesting that interest in this area has grown in recent years.

4. Discussion

The main distinction between fuzzy and probable viewshed is that the latter describes the probability of a location being visible, while the fuzzy viewshed portrays the degree to which objects can be distinguished (Fisher 1999). The accuracy of a DEM is attributed to the probable viewshed where, for example, the uncertainty in the height of a hill can lead to a probability of an LOS reaching a target. Vegetation, is usually attributed to a probable viewshed or a transparency/permeability value between zero and one. In most publications that use permeability, the permeability is equated to the probability of visibility.

For both DEM accuracy and vegetation individually there seem to be numerous theories and applications available. However, only one study (Baer *et al.*2005) combines both factors to compute probable visibility. Although observer location optimization has received growing attention—and various authors (Stiles 2000, Xin *et al.*2005) have stressed the significance of accounting for probable visibility—very few publications integrate probable visibility into observer location optimization, as Wang and Dou (2020), Rana (2005), Cervilla *et al.* (2015) did for the boolean viewshed. Maaiveld *et al.* (2023) is the only publication that addresses vantage point determination for an autonomous military ground vehicle seeking a location behind see-through vegetation to launch an attack.

Another factor that may be considered part of the probable viewshed is uncertainty in the target's location or observer's own location, which can lead to significant changes in visibility. In the existing 3D viewshed literature, this issue appears largely unexplored, highlighting a research gap in this area.

Furthermore, although Fisher (1994) explains how to combine multiple probable viewsheds, this method has not been applied in any publications on the viewshed calculation or observer location optimization to the best of the authors' knowledge, also highlighting a research gap in this area.

The existence of a LOS does not necessarily guarantee that an object can be seen or, more importantly, distinguished. Factors influencing visual clarity include distance, target size, lighting conditions or sources (such as glare caused by the sun or moon), weather conditions, and the background against which an object is viewed. These factors collectively define the fuzzy viewshed, though it is not limited to these alone, as other factors may also influence the fuzzy viewshed. Since its introduction by Fisher (1994), there has been limited research expanding on the theory of fuzzy viewsheds and the calculation of these influencing criteria. While the impact of weather on visibility range is relatively well-studied, solar glare has primarily been investigated in the context of driving safety, with little focus on its effects on object detection. Similarly, research on target size and distance is sparse and largely limited to studies of human vision. Furthermore, the influence of background has been minimally addressed in the context of fuzzy viewsheds, with more in-depth studies found in the field of camouflage techniques. No publications have been identified that incorporate a fuzzy viewshed into observer location optimization, highlighting a significant research gap in this area.

Returning to the earlier example provided by Milton and Williams (2002)—planning a helicopter battle position (BP)—multiple criteria that come into play during this planning can be classified as fuzzy or probable visibility factors, effectively treating this as an observer location optimization problem. Despite the importance of these criteria being underscored by Milton and Williams (2002), no existing research integrates them into a single unified framework for observer location optimization. This lack of integration represents a key gap in the literature.

5. Conclusions and future work

This literature review was written in order to answer the research question '*What is an effective way to model uncertainty in visibility arising from uncertain terrain and vegetation data, environmental conditions, and target characteristics?*'. It has systematically mapped the landscape of research on uncertainty in visibility, specifically in the context of probable and fuzzy viewsheds and their implications for observer location optimization. Our study has identified key factors contributing to non-boolean visibility and provided insights in how these factors have been modeled, including their limitations. The review highlights the gaps in the integration of the probable and fuzzy viewshed into GISc-based analysis. The key research gaps identified are:

- **Applying probable visibility to observer location optimization** (also referred to as the minimum observer problem, maximum coverage, siting observers, or candidate viewpoint determination), the importance of which has been stressed by multiple authors.
- **Considering uncertainty in a target's position**, which can drastically influence the observer or the target visibility.
- **Further quantifying the role of fuzzy criteria on the fuzzy viewshed.** Since its introduction, limited research has addressed how these criteria affect visibility. No studies have integrated the fuzzy viewshed into observer location optimization, likely because there has been insufficient work on quantifying and combining fuzzy criteria for observer location optimization.

Existing research has partially tackled certain aspects of these gaps, such as incorporating DEM accuracy and probabilistic vegetation modeling. However, a framework combining both probable and fuzzy viewshed factors remains mostly unexplored. Literature reviewed here provides a starting point indicating that while the groundwork for handling non-boolean visibility exists, there is still progress to be made in more accurately modeling and combining these factors.

For practitioners in GIS, the findings of this review aim to highlight the limitations of boolean visibility models and suggest that more nuanced approaches could improve real-world applications. By incorporating fuzzy and probabilistic methods, decision-makers can better place observers with a more realistic assessment of visibility conditions. This leads to better informed planning in critical applications such as battle position selection, or surveillance locations. For researchers, this review provides a road map for future research into integrating uncertainty-aware models into GISc.

While this review covers the current state of the art in uncertainty modeling in visibility, limitations remain:

- This review focuses on literature related to the visible light spectrum. Although visibility is an important aspect of GISc, uncertainty should be taken into account for more environment variables, such as other parts of the electromagnetic spectrum.
- A limitation of the methods that are presented here for the modeling and combination of factors is a lack of empirical validation. This would require simulations or controlled experiments or field tests, which could be difficult to conduct.
- Integration of non-boolean visibility is underexplored; future research should explore multi-criteria modeling of visibility.

Concluding, this review lays the groundwork for advancing the integration of uncertainty in visibility modeling within GIScience.

Notes

1. DEM is a 'bare Earth' elevation model which is a superset of a digital terrain model (DTM) and DSM, with a DTM being a DEM augmented with features like breaklines and ridges while a DSM being a DTM that includes the natural and human-made features on the Earth's surface as explained by Llames (2024).
2. The bare Earth visibility model or a mean vegetation height visibility model are vegetation correction models applied to a DEM.

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Data and codes availability statement

The data and codes that support the findings of this study are available at the link: https://doi.org/10.6084/m9.figshare.30406360.

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Appendix A. Full keyword search

NPS: *'probability of visibility' OR 'probabilistic visibility' OR 'probable viewshed' OR 'fuzzy viewshed' OR 'battle position' OR 'candidate viewpoints'*

IEEE: *(surveillance OR target OR defense OR military) AND (intervisibility OR viewshed OR 'visibility model' OR 'observation point') OR 'probability of visibility' OR 'probabilistic visibility' OR 'probable viewshed' OR 'fuzzy viewshed' OR 'battle position' OR 'firing position' OR 'overwatch position' OR 'candidate viewpoints'*

AIAA: *(surveillance OR target OR defense OR military) AND (viewshed OR 'visibility model' OR 'observation point' OR 'candidate viewpoints')*

WoS: *(surveillance OR target OR defense OR military) AND (intervisibility OR viewshed OR 'visibility model' OR 'observation point') OR 'probability of visibility' OR 'probabilistic visibility' OR 'probable viewshed' OR 'fuzzy viewshed' OR 'battle position' OR 'firing position' OR 'overwatch position' OR 'candidate viewpoints'*

IJGIS: *(surveillance OR target OR defense OR military) AND (intervisibility OR viewshed OR 'visibility model' OR 'observation point') OR 'probability of visibility' OR 'probabilistic visibility' OR 'probable viewshed' OR 'fuzzy viewshed' OR 'battle position' OR 'firing position' OR 'overwatch position' OR 'candidate viewpoints' OR 'maximum coverage' OR 'siting problem'*

JDMS: *(surveillance OR target OR defense OR military) AND (intervisibility OR viewshed OR 'visibility model' OR 'observation point') OR 'probability of visibility' OR 'probabilistic visibility' OR 'probable viewshed' OR 'fuzzy viewshed' OR 'battle position' OR 'firing position' OR 'overwatch position' 'candidate viewpoints' OR 'siting problem'*

Appendix B. Equations: explanations, derivations and examples

B.1. Baer LOS uncertainty calculation

$$\begin{aligned}&\mathrm{Losp}(s) = \mathrm{Losp}(\lfloor s \cdot \Delta s/\Delta t \rfloor)\cdot \\ &\textstyle\int_{-\infty}^{+\infty} P(z(t), z_t(t), \sigma_t(t)) \cdot 2 \cdot \sum \frac{\Delta S}{\cos(\mathrm{el})} \cdot S_{\frac{1}{2}}(e(s), n(s), z_r(s) - e) de\end{aligned} \tag{B1}$$

The equation computes the LOS probability ($\mathrm{Losp}(s)$) along a distance s from viewer to target by considering terrain errors ($e = z - z_t$), step sizes (Δs), and terrain correlation cell properties ($t, \Delta t$). It accounts for elevation ($z_t(t)$), elevation error ($\sigma_t(t)$), and cosine of the elevation angle ($\cos(\mathrm{el})$). The function $S_{\frac{1}{2}}(e(s), n(s), z_r(s))$ represents the half-penetration distance based on easting ($e(s)$), northing ($n(s)$), and ray altitude ($z_r(s)$). The core idea behind this distance is rather than treating vegetation as an on/off obstruction, the algorithm uses ($S_{\frac{1}{2}}$ as the distance a LOS can travel into a vegetation layer before it has a 50% chance of being blocked. How this value could be determined is not discussed by the authors.

B.2. Beer-Lambert law for vegetation

Consider an arbitrary number of LOS N_0 shot randomly at an area α. The ratio of area blocked by vegetation to the total area α is defined as ϕ. The number of initial LOS that would pass through n layers of vegetation (surfaces α) can be written as: $N = N_0 \cdot (1n\phi)^{n(x)}$, assuming all ϕ's are identical. This assumption is taken, because Llobera (2007) is considering the same type of trees the LOS beam is passing through. The fraction $\frac{N}{N_0}$ is defined by Llobera (2007) as the probability $p(x)$ that one of the original N_0 will pass.

Δx is a slide of vegetation, x is the distance along the LOS.

With Taylor expansion:

$$\begin{aligned}
p(x) &= \frac{N}{N_0} = (1-\phi)^n, \quad \phi(x) = \Delta x \cdot \rho(x) \cdot \beta(x,z), \quad n = \frac{x}{\Delta x} \\
p(x) &= e^{ln(1-\phi(x))^{\frac{x}{\Delta x}}} \\
p(x) &= e^{\frac{x}{\Delta x} ln(1-\phi(x))}, \quad ln(1-\phi(x)) = -\phi(x) + \frac{\phi(x)^2}{2} - \frac{\phi(x)^3}{3} + \ldots \\
ln(1-\phi(x)) &\approx -\phi(x), \quad \text{when} \quad \Delta x \to 0 \\
p(x) &= e^{-\rho(x)\cdot\beta(x,z)\cdot x}
\end{aligned}$$

Without Taylor expansion:

$$\begin{aligned}
p(x) &= \frac{N}{N_0} = (1-\phi)^n, \quad \phi(x) = \Delta x \cdot \rho(x) \cdot \beta(x,z), \quad n = \frac{x}{\Delta x} \\
p(x) &= e^{ln(1-\phi(x))^{\frac{x}{\Delta x}}}, \quad b = 1-\phi(x), \quad a = \frac{1}{b} \\
p(x) &= e^{ln(\frac{1}{a})^{\frac{x}{\Delta x}}} \\
p(x) &= e^{\frac{1}{\Delta x} ln(a)^{-x}} \\
p(x) &= e^{-x\frac{1}{\Delta x} ln(\frac{1}{1-\phi(x)})} \\
p(x) &= e^{-k(x)x}, \quad \text{with} \quad k(x) = \frac{1}{\Delta x} ln(\frac{1}{1-\phi(x)})
\end{aligned}$$

B.3. Choquet integral example

Criteria I_1, I_2, I_3, which are the object distance, vegetation density and weather condition respectively with criteria example values:

$$f(I_1)_{d=3km} = \frac{10-3}{10} = 0.7, \quad f(I_2)_{forest} = 0.5, \quad f(I_3)_{clearsky} = 0.9$$

These values are taken from Magoc *et al.* (2010), where, for example, the utility function for distance is taken to be linearly decreasing from 1 for a viewing distance of 10 *km* to 0 at 0 *km*.

Fuzzy relationships, which could be obtained by expert input, or from calculating the individual and combined importance of criteria when values for the total visibility are available. Values could be, for example:

$$\begin{aligned}&\mu(\{I_1\}) = 0.2, \quad \mu(\{I_2\}) = 0.3, \quad \mu(\{I_3\}) = 0.4,\\ &\mu(\{I_1, I_2\}) = 0.5, \quad \mu(\{I_1, I_3\}) = 0.6, \quad \mu(\{I_2, I_3\}) = 0.7,\\ &\mu(\{I_1, I_2, I_3\}) = 1.\end{aligned}$$

Which are values taken from the paper (Magoc *et al.* 2010).

Sort from low to high:

$$f(I_2) = 0.5 \ \leq \ f(I_1) = 0.7 \ \leq \ f(I_3) = 0.9$$

So the permutation σ is:

$$\sigma(1) = I_2, \quad \sigma(2) = I_1, \quad \sigma(3) = I_3$$

Choquet integral:

$$(C)\int f \ d\mu_m = \sum_{i=2}^{n} (f(\sigma(i)) - f(\sigma(i-1))) \ \mu_m\big(A_{\sigma(i)}\big)$$

Filling in values:

$$\begin{aligned}&(f(\sigma(1)) - f(\sigma(0))) \ \mu_m(\{I_2, I_1, I_3\}) = (0.5 - 0) \cdot 1 = 0.5\\ &(f(\sigma(2)) - f(\sigma(1))) \ \mu_m(\{I_1, I_3\}) = (0.7 - 0.5) \cdot 0.6 = 0.2 \cdot 0.6 = 0.12\\ &(f(\sigma(3)) - f(\sigma(2))) \ \mu_m(\{I_3\}) = (0.9 - 0.7) \cdot 0.4 = 0.2 \cdot 0.4 = 0.08\end{aligned}$$

The total visibility is then:

$$(C)\int f \ d\mu_m = 0.5 + 0.12 + 0.08 = 0.7$$