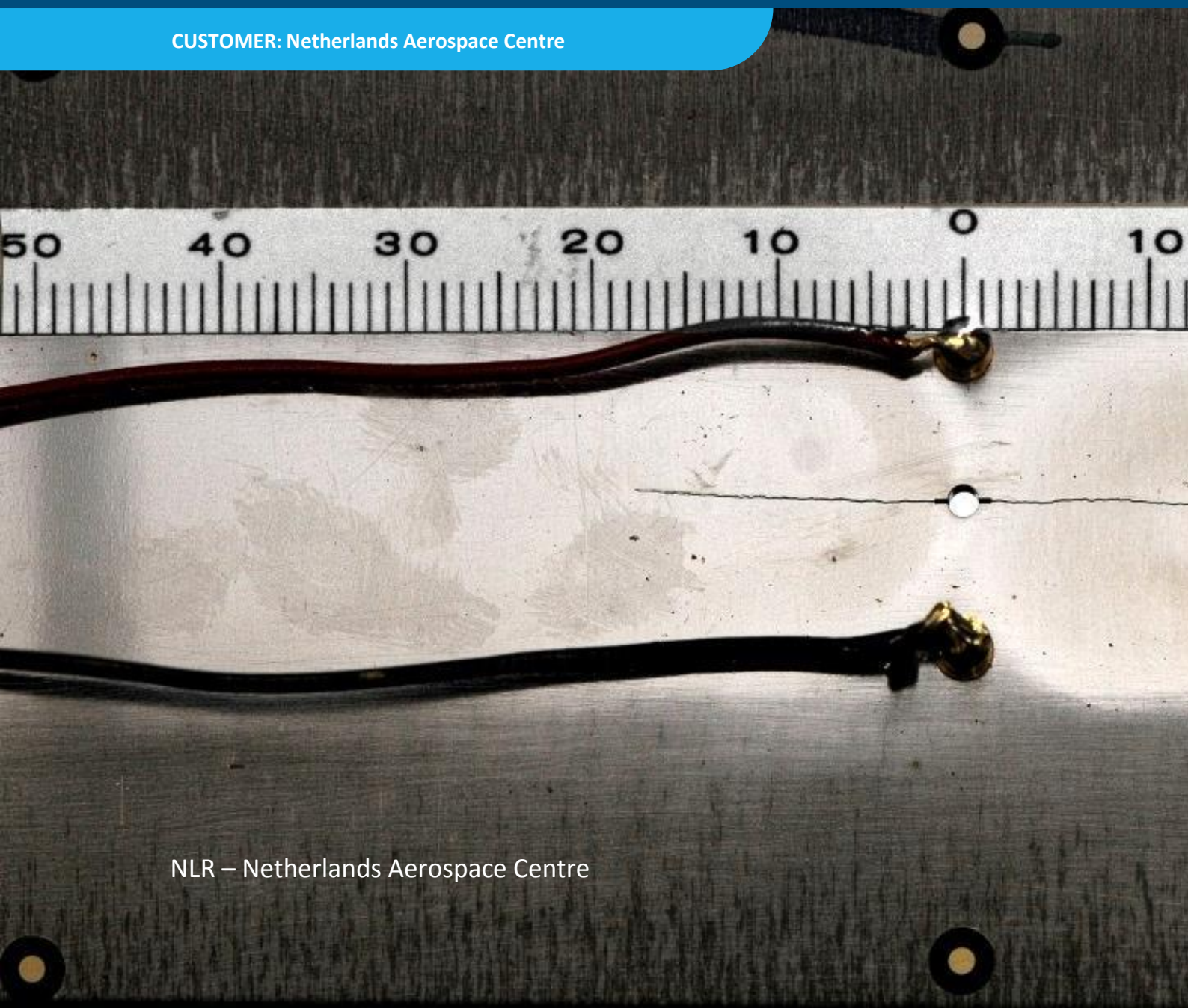


On the validity of the Paris equation

CUSTOMER: Netherlands Aerospace Centre



Netherlands Aerospace Centre

NLR is a leading international research centre for aerospace. Bolstered by its multidisciplinary expertise and unrivalled research facilities, NLR provides innovative and integral solutions for the complex challenges in the aerospace sector.

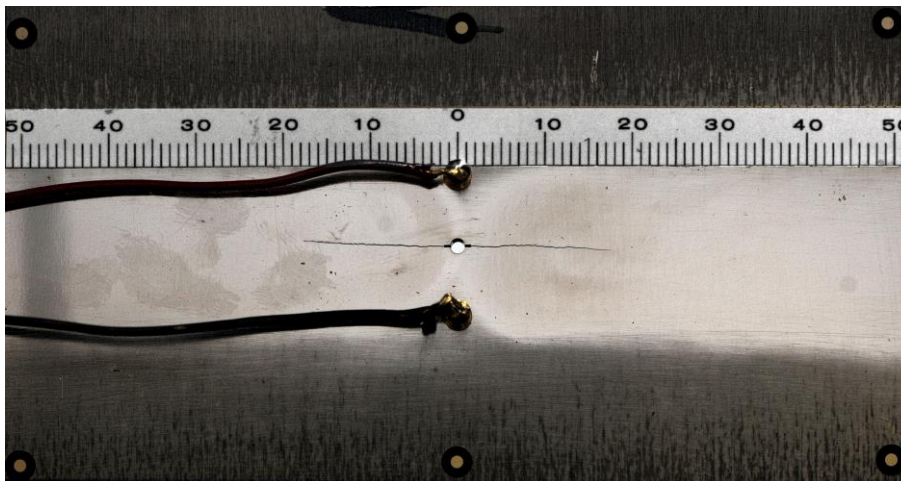
NLR's activities span the full spectrum of Research Development Test & Evaluation (RDT & E). Given NLR's specialist knowledge and facilities, companies turn to NLR for validation, verification, qualification, simulation and evaluation. NLR thereby bridges the gap between research and practical applications, while working for both government and industry at home and abroad.

NLR stands for practical and innovative solutions, technical expertise and a long-term design vision. This allows NLR's cutting edge technology to find its way into successful aerospace programs of OEMs, including Airbus, Embraer and Pilatus. NLR contributes to (military) programs, such as ESA's IXV re-entry vehicle, the F-35, the Apache helicopter, and European programs, including SESAR and Clean Sky 2.

Founded in 1919, and employing some 650 people, NLR achieved a turnover of 71 million euros in 2016, of which three-quarters derived from contract research, and the remaining from government funds.

For more information visit: www.nlr.nl

On the validity of the Paris equation



Problem area

Paris et al. were the first to show that the fatigue crack growth rate in metals shows a power law relationship with the stress intensity factor range. However, the empirical Paris equation is dimensionally correct only when the dimensions of the constant in the equation are changed with the power law exponent. It is generally accepted that the exponent is material-dependent. In the paper we show that the exponent also changes between specimens and crack lengths.

Description of work

Constant amplitude crack growth tests were performed on middle tension specimens of 6.35 mm thick aluminium alloy AA7075-T7351. The crack length was measured by direct current potential drop. The resulting data have been modelled using a dimensionally correct power law equation with different exponents for different crack growth regimes.

REPORT NUMBER

NLR-TP-2017-148

AUTHOR(S)

E. Amsterdam

REPORT CLASSIFICATION

UNCLASSIFIED

DATE

May 2017

KNOWLEDGE AREA(S)

Health Monitoring and Maintenance of Aircraft
Aircraft Material and Damage Research

DESCRIPTOR(S)

engineering alloys
fatigue crack growth rate
pivot point
power law
stress state

Results and conclusions

The mathematical concept of a pivot point allowed 1) to model crack growth with two different exponents using a dimensionally correct equation and 2) to model the crack growth variations in all specimens by varying only one parameter; the power law exponent for the plane strain condition.

It is possible to extend the power law behaviour to smaller and larger crack lengths and to exactly model the fatigue crack growth from an EDM starter notch to final failure with multiple power law exponents at different crack length ranges using a dimensionally correct equation. This is in contrast with the smooth FCGR curves observed in literature or obtained by fitting NASGRO equations to da/dN data that is obtained by incremental polynomial fitting of the a - N curve. The transition points between the different power law regions correspond to the original transition points traditionally observed for aluminium alloys. One of the transition points is associated with a change in the plastic zone volume due to the transition from plane strain to plane stress. It is expected that the change in power law exponents is due to changes in the plasticity around the crack tip. Similar trends between the plasticity in materials and the Paris exponent are observed for materials in general.

Applicability

The multiple power law behaviour allows fitting of the a - N curve to obtain the fatigue crack growth rate for a specimen without scatter. The lack of scatter allows determining the effect of factors with small influences on the FCGR (for example: thickness, frequency or humidity). Since the pivot points are independent of the exponents, it should be possible to determine the influence of stress range on the FCGR with high accuracy. These measurements would support in building a physics based model of fatigue crack growth. Knowledge of the value of the lowest pivot point allows for accurate extrapolation to smaller crack lengths.

GENERAL NOTE

This report is based on a presentation held at the 17th Australian International Aerospace Congress (AIAC17), Melbourne, February 27-28, 2017.

NLR

Anthony Fokkerweg 2

1059 CM Amsterdam

p) +31 88 511 3113 f) +31 88 511 3210

e) info@nlr.nl i) www.nlr.nl



Dedicated to innovation in aerospace

NLR-TP-2017-148 | May 2017

On the validity of the Paris equation

CUSTOMER: Netherlands Aerospace Centre

AUTHOR(S):

E. Amsterdam

NLR

This report is based on a presentation held at the 17th Australian International Aerospace Congress (AIAC17), Melbourne, February 27-28, 2017.

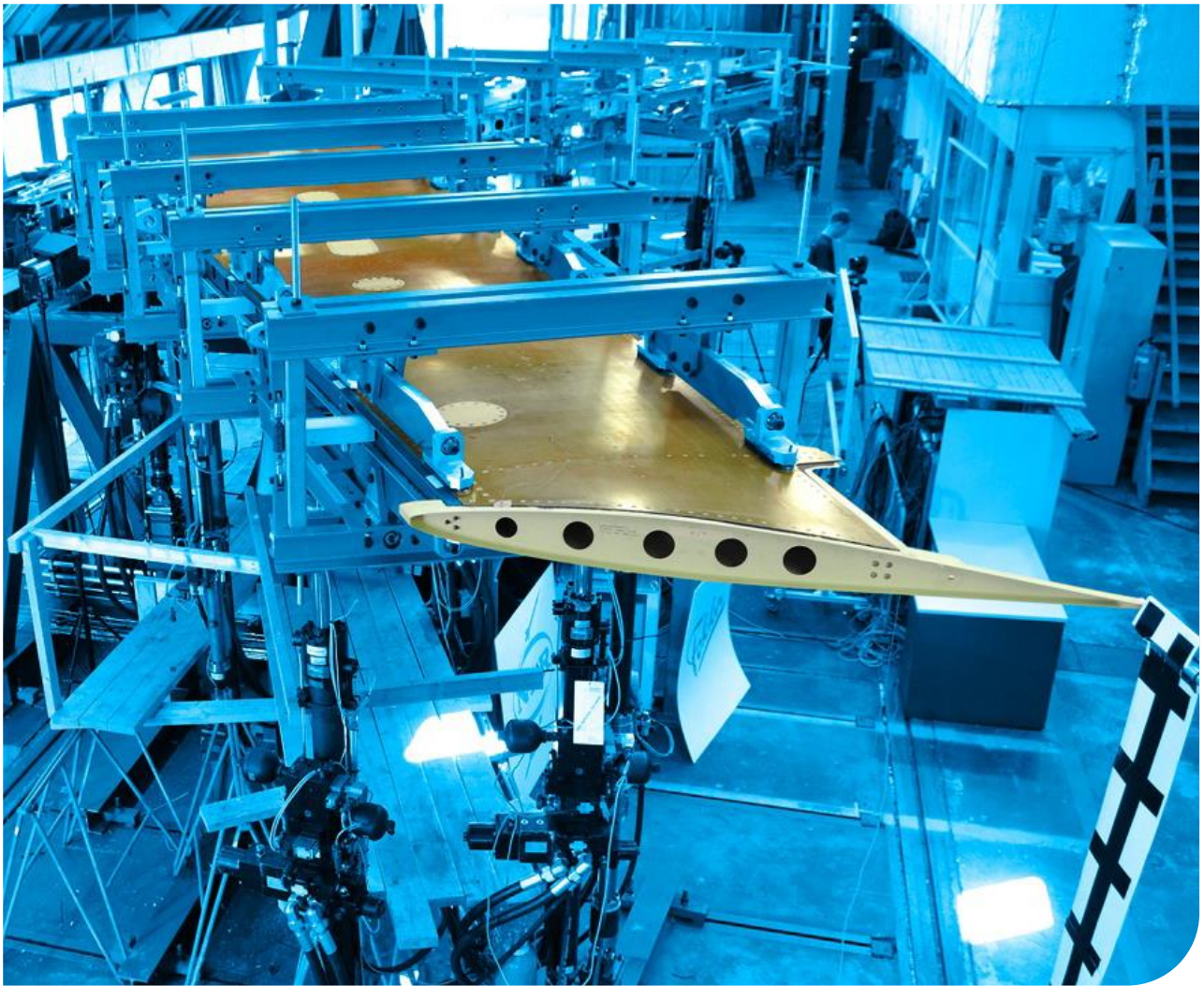
*The contents of this report may be cited on condition that full credit is given to NLR and the authors.
This publication has been refereed by the Advisory Committee AEROSPACE VEHICLES.*

CUSTOMER	Netherlands Aerospace Centre
CONTRACT NUMBER	- - -
OWNER	NLR
DIVISION NLR	Aerospace Vehicles
DISTRIBUTION	Unlimited
CLASSIFICATION OF TITLE	UNCLASSIFIED

APPROVED BY :		
AUTHOR	REVIEWER	MANAGING DEPARTMENT
E. Amsterdam	M. Bos	O.S. Zoeteweij
DATE	DATE	DATE

Summary

Paris *et al.* were the first to show that the fatigue crack growth rate in metals shows a power law relationship with the stress intensity factor range. However, the empirical Paris equation is dimensionally correct only when the dimensions of the constant in the equation are changed with the power law exponent. It is generally accepted that the exponent is material-dependent. In the paper we show that the exponent also changes between specimens and crack lengths. It is possible to exactly model the fatigue crack growth from an electro discharge machining starter notch to final failure with multiple power law exponents at different crack length ranges using a dimensionally correct equation. It is expected that the change in power law exponents is due to changes in the plasticity around the crack tip. Similar trends between the plasticity in materials and the Paris exponent are observed for materials in general.



Contents

Abbreviations	6
1 Introduction	7
2 Experimental set-up and results	8
3 Discussion	10
4 Conclusions	13
References	14

Abbreviations

ACRONYM	DESCRIPTION
ΔK	stress intensity factor range
a	crack length
AA	Aluminium Alloy
C	constant
da/dN	fatigue crack growth rate
DCPD	Direct Current Potential Drop
EDM	Electrical Discharge Machined
Eqn	Equation
FCGR	Fatigue Crack Growth Rate
N	number of cycles
NLR	Netherlands Aerospace Centre
PZS	Plastic Zone Size
S	stress
TP	Transition Point
β	geometry correction factor

1 Introduction

Paris *et al.* were the first to show that the fatigue crack growth rate (FCGR) has a power law relationship with the stress intensity factor range, ΔK [1]:

$$\frac{da}{dN} = C_R \cdot \Delta K^n \quad (1)$$

where da/dN is the FCGR, C_R is a constant and n is the power law exponent. C_R and n are both regarded as material parameters. The stress intensity factor range is defined as:

$$\Delta K = \beta(a)(S_{\max} - S_{\min})\sqrt{\pi a} \quad (2)$$

where S is the applied stress and a is the crack length. The geometry correction factor $\beta(a)$ depends on the geometry and the crack length. From 1963, many adaptations of Eqn 1 (the Paris equation) have been proposed to incorporate the experimental results at low ΔK , near the supposed threshold, and at high ΔK , near the fracture toughness regime [2]. Fracture mechanics has not been able to deduce the Paris equation from basic principles or a physical model, and so it is still an empirical equation. Another major issue is the fact that this equation is dimensionally correct only when the dimensions of the constant C_R are changed with a change in the exponent, n . It is generally accepted that the exponent is material-dependent. Zuidema *et al.* also observed a decrease in the exponent for aluminium alloy AA 2024 when shear lips develop [3]. Usually, fatigue cracks start flat and grow in mode I (tensile mode), until at a certain ΔK the specimen (or component) starts to develop shear lips. The shear lip development in aluminium alloys AA 2024-T3 and AA 7075-T6 starts at a critical growth rate of about $1 \cdot 10^{-7}$ m/cycle [3]. In fact, this correlation has already been observed in the 1960's. Wilhem showed that the point of decrease in the exponent of the da/dN vs. ΔK curve corresponded to the onset of the transition to plane stress conditions for 7075-T6 and 2024-T3 [4]. The transition onset occurred at a certain effective stress intensity factor range and corresponding crack growth rate [4]. Zuidema *et al.* indicate that the shear lips are not responsible for a decrease in n , but that they, like n , are an effect from an underlying phenomenon [3].

Pivot point

When crack growth rate data are equation-fitted, then a change in n also gives rise to a change in the constant C . Many people have reported a linear relationship between $\log C$ and the exponent, n [5]. If there is a linear relationship between $\log C$ and n , this results in a relative crack growth rate equation that is dimensionally correct [5]:

$$\frac{da}{dN} = \frac{da}{dN_b} \left(\frac{\Delta K}{\Delta K_z} \right)^n \quad (3)$$

When ΔK_z is equal to 1, Eqn 3 is equal to Eqn 1. The linear relationship between $\log C$ and the exponent, n , introduces a so-called pivot point in the da/dN vs. ΔK plot. The pivot point is located at the coordinates $\Delta K_z, da/dN_b$ and allows describing the variation between specimens tested under similar conditions by only a variation in n .

A number of papers have been written about the pivot point. However, there is much doubt about there being a physical basis for a pivot point [5]. It concludes that the pivot point is material-dependent and may be associated with a transition mechanism in the FCGR curve [6]. In the present paper the mathematical concept of a pivot point will be used to model crack growth with multiple exponents using a dimensionally correct equation.

2 Experimental set-up and results

Constant amplitude crack growth tests were performed on 29 identical middle tension specimens (dimensions: 500 x 160 mm²) that were obtained from a single plate of 6.35 mm thick aluminium alloy AA7075-T7351. The rolling direction was in the length of the specimens. Fatigue crack growth starter notches were central holes (1.6 mm diameter) with 0.7 mm deep electrical discharge machined (EDM) slots on either side of the hole (total starter notch length of 3 mm). Holes were drilled at 8 mm above and below the start notch hole for copper pins. These were used for automated crack length measurements by direct current potential drop (DCPD). The current was introduced to the specimen at the specimen clamping. The area next to the starter notch was polished for optical crack growth measurements on both front and rear sides of the specimens. The optical measurements (4 for each crack length measurement) were averaged and used to check the DCPD measurements. A constant amplitude 13.5 Hz sinusoidal load was introduced by an electrohydraulic test machine with a 200 kN load cell. The maximum and minimum stresses during the load cycles were 80 MPa and 8 MPa, corresponding to a stress ratio $R = 0.1$. The specimens were pre-cracked to a total crack length $2a$ of 4 mm. Failure was defined as occurring at a half crack length a of 40.8 mm even though actual failure occurred at larger crack lengths. The failure criterion of 40.8 mm was used because all specimens gave crack length data points up to at least 40.8 mm. Fig. 1 shows examples of the obtained fatigue crack growth curves: specimen 1 had the lowest number of cycles to failure and specimen 26 one of the highest number of cycles to failure.

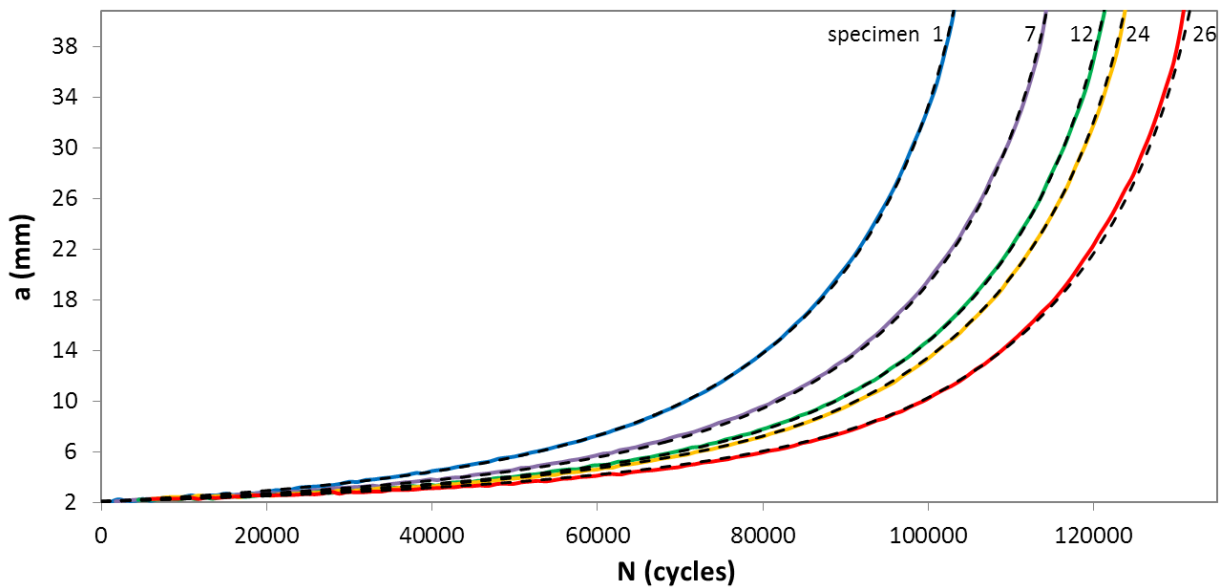


Fig. 1: Crack length vs. cycles for five of the 29 specimens (colored lines). The dotted black lines represent the model (Eqn 3), where the differences between specimens are captured by differences in the exponent, n , for crack lengths smaller than 9.69 mm

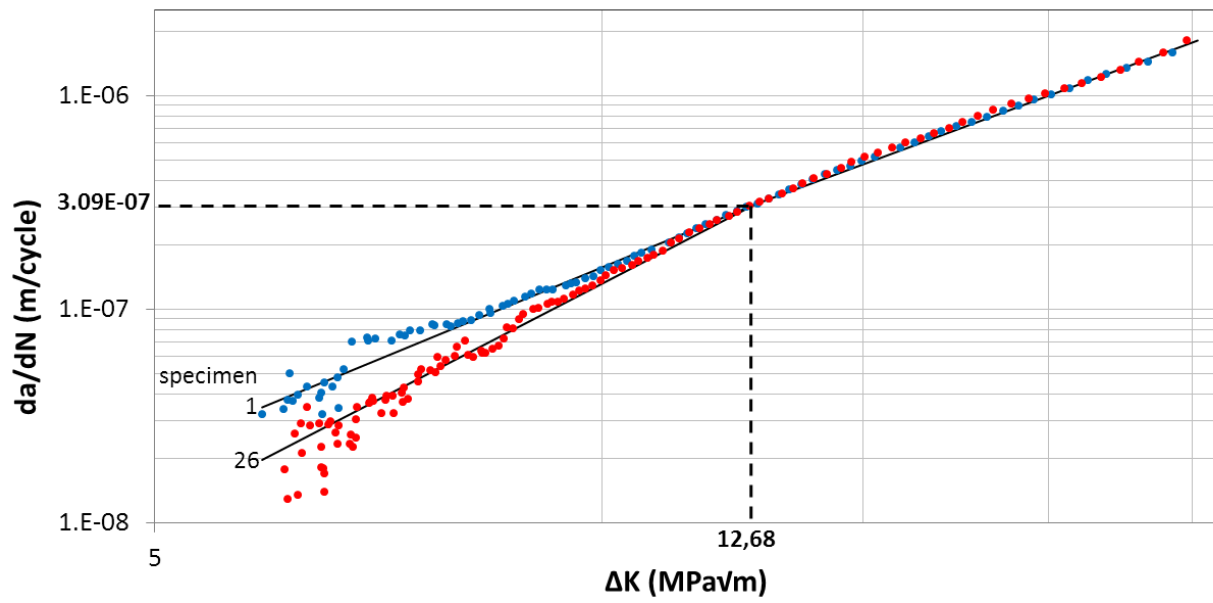


Fig. 2: da/dN vs. ΔK for specimen 1 and 26. The black lines represent the fits of the $a-N$ curve before and after the pivot point crack length using Eqn 3

3 Discussion

It was noticed that the number of cycles to grow the crack from about 10 mm to 40.8 mm was approximately the same for all specimens and that the shapes of the curves were similar (see Fig. 1). This indicates that the constant, da/dN_b , and exponent, n , from Eqn 3 were both the same for this range of crack growth. The differences in the number of cycles to failure were caused by the differences in crack growth up to about 10 mm. Fig. 2 shows $da/dN - \Delta K$ for specimens 1 and 26. Visual best fit lines of the initial parts of the crack growth rates were used to determine the pivot point. These lines intersect at $\Delta K=12.68 \text{ MPa}\sqrt{\text{m}}$ ($a=9.69 \text{ mm}$). It is apparent that the $da/dN - \Delta K$ for specimens 1 and 26 up to $12.68 \text{ MPa}\sqrt{\text{m}}$ cannot be modelled correctly using different constants, but can only be modelled correctly by varying the exponent n in Eqn 3. On the other hand, as mentioned above, for crack lengths $>10 \text{ mm}$ the constant, da/dN_b , and exponent, n , were the same for all specimens. The crack length vs. cycles curves are continuous, therefore the fatigue crack growth rate has to be the same at about 10 mm for all the curves. For that reason, the mathematics of a pivot point have been used to model the crack growth in the specimens (Eqn 3), however with two exceptions: (i) different exponents are used before and after the pivot point, and (ii) a single exponent is used for crack lengths greater than about 10 mm, while the crack growth curve variations at shorter crack lengths have been captured by varying the exponent. The entire FCGR curve can be described for all specimens by Eqn 3, where n varies between 2.86 and 3.62 for crack lengths smaller than 9.69 mm and $n=2.57$ for crack length greater than 9.69 mm. Fig. 1 shows the measured crack lengths and the model for 5 specimens. The $a-N$ curve for each specimen can be perfectly modelled by changing only one parameter that is characteristic for each specimen, i.e. the exponent, n , for crack lengths smaller than 9.69 mm. The value for n is determined from the crack length vs. cycles ($a-N$) curve of each specimen. Fitting the $a-N$ curve is preferable to using the FCGR curve, which derives from the $a-N$ curve and results in increased scatter. At the pivot point the crack growth rate is $3.09 \cdot 10^{-7} \text{ m/cycle}$ and is independent of the exponent n . Therefore, all model curves in Fig. 1 are continuous. Why the exponent is different for each specimen in the plane strain region is presently unknown.

A specimen from a different plate and batch was tested to investigate the possibility to extend the power law behaviour to smaller and larger crack lengths. The DCPD crack lengths were stored at a higher rate compared to the 29 specimens from the other plate to better distinguish possible transitions in FCGR. Fig. 3 shows $da/dN - \Delta K$ for this specimen by classical incremental polynomial fitting of the $a-N$ curve. Four sections and three different transition points can be clearly observed (TP1 – TP3). TP2 represents the transition from flat to slanted crack growth (and therefore from plane strain to plane stress conditions) and the transition occurs at a higher ΔK value compared to the 29 specimens ($16.06 \text{ MPa}\sqrt{\text{m}}$ vs. $12.68 \text{ MPa}\sqrt{\text{m}}$). Fig. 4 shows the $da/dN - \Delta K$ obtained by fitting the $a-N$ curve between the transition points (crack lengths) using Eqn 3. Fitting the $a-N$ curve with multiple power laws using Eqn 3 results in an excellent fit over the entire $a-N$ curve (see Fig. 5). When the obtained results of the fits of the $a-N$ curve are plotted in the Fig. 3, the results are in excellent agreement with the FCGR obtained by classical fitting of the $a-N$ curve. Since the FCGR is exactly described by the transition point and the exponent, it is easy to extrapolate the FCGR to smaller values of ΔK . When this data is used to model crack growth from the EDM starter notch, the crack length after 45.000 cycles (number of pre-cracking cycles) matches the crack length at the start of the test (see the insert in Fig. 5). The stress concentration factor of the EDM notch is 8.7 and the maximum stress at the notch tip is much higher than the ultimate tensile strength of the material. Therefore, it is valid to assume that crack growth occurs from the first cycle.

Since the Paris equation is dimensionally not correct and does not allow modelling of the fatigue crack growth rate with different exponents (as in Fig. 2) or with multiple exponents (as in Fig. 4), the validity of the Paris equation is limited.

For the tested specimens there was a gradual transition from flat to slanted crack growth (and therefore from plane strain to plane stress conditions) between 7-11 mm and 22-25 mm crack lengths [5]. The average onset of the transition at the front and back of specimen 1 was at $a = 9.7$ mm. This corresponds excellently with the 9.69 mm crack length for the pivot point. From linear elastic fracture mechanics it is known that there is a sharp increase in the crack resistance curve when the plane stress condition starts to evolve [7]. The crack resistance is mostly determined by plasticity, it is therefore expected that a change in the plastic zone volume results in a change in crack growth rate. Fleck *et al.* both have shown the general trend that the power law exponent decreases if the plasticity in a material increases [8]. Serdyuk *et al.* measured the plastic zone size (PZS) and FCGR of magnesium MA12 alloy in vacuum at room- and low temperature. The temperature reduction leads to a decrease in PZS and an increase in the exponent [9]. The decrease in PZS was partially caused by the change in yield stress due to the temperature drop. Therefore, a change in exponent is also expected if the volume of plasticity increases within a specimen of the same material. Therefore, it is expected that the change in PZS due to the transition from plane strain to plane stress is responsible for the change in the power law exponent.

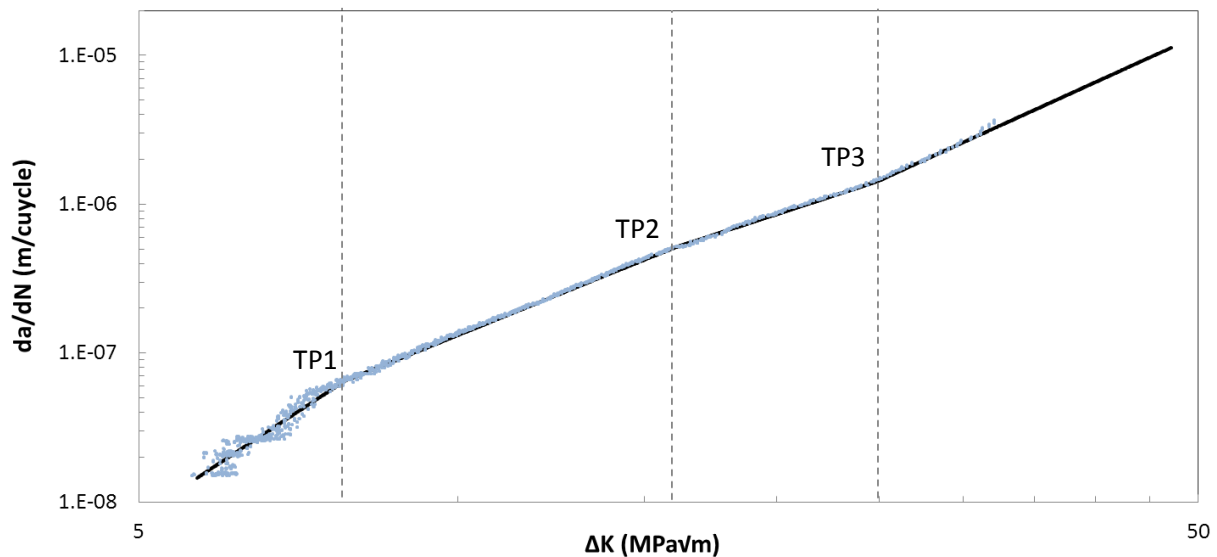


Fig. 3: da/dN vs. ΔK by classical incremental polynomial fitting of the a - N curve for the specimen from a different plate and batch (blue dots). The black line represents the fits of the a - N curve between the transition points crack lengths using Eqn 3 (see also Fig. 4). The vertical dotted lines indicate the transition points

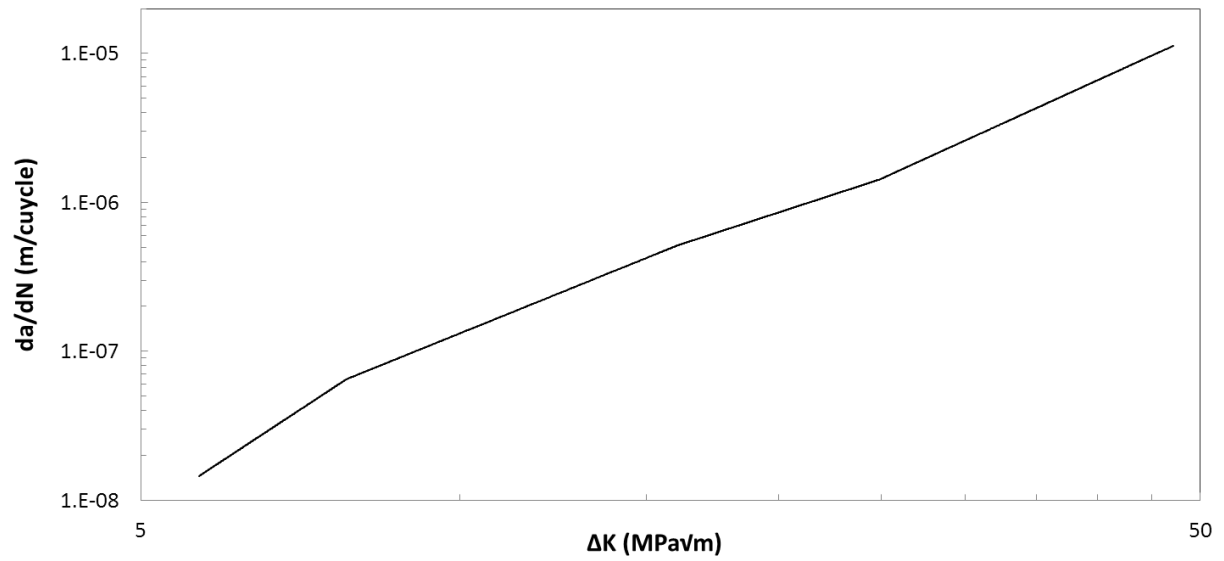


Fig. 4: da/dN vs. ΔK by fitting the a - N curve with multi power laws for the specimen from a different plate and batch

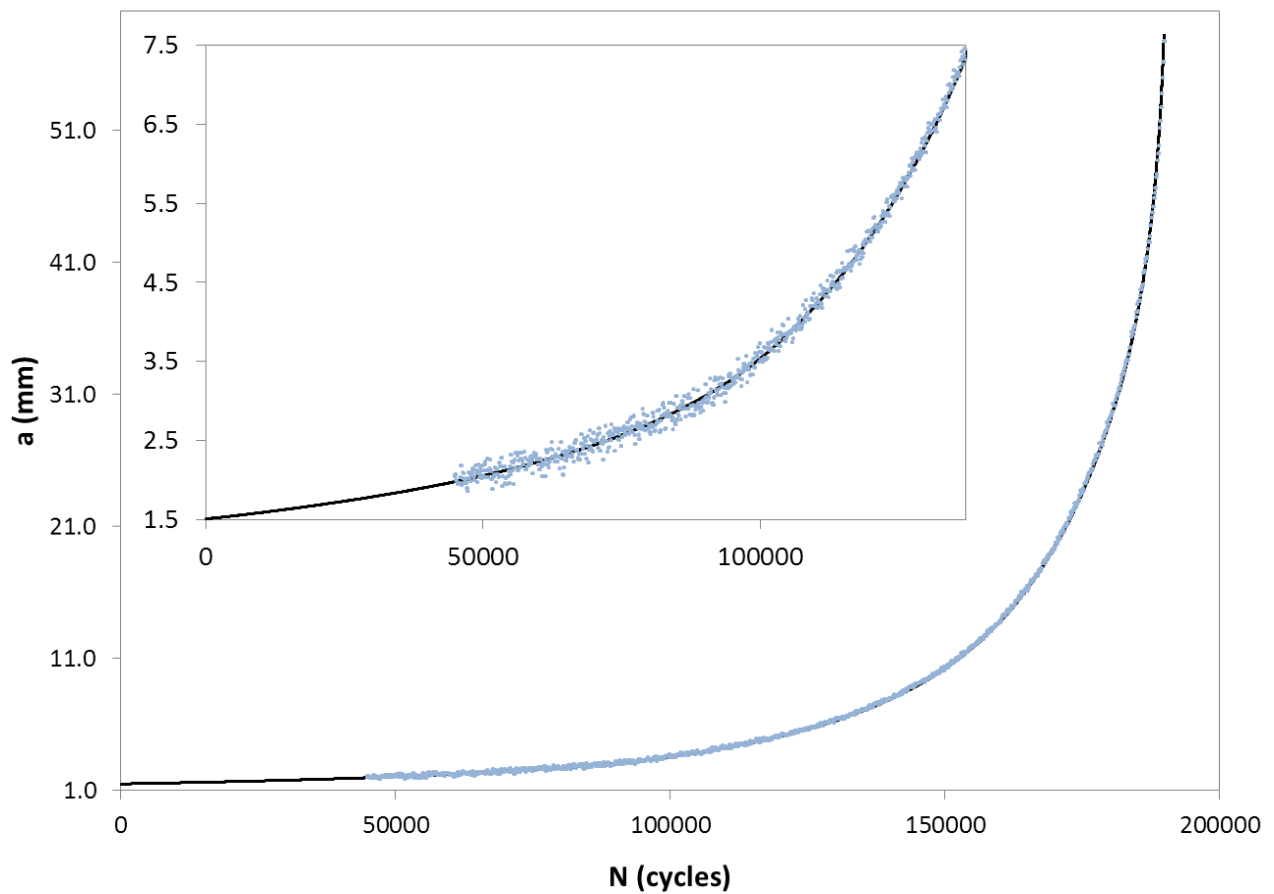


Fig. 5: Crack length vs. cycles for the specimen from a different plate and batch. The black line represents the fits of the a - N curve between the transition points crack lengths using Eqn 3. The insert shows crack length vs. cycles from the starter notch (1.5 mm) to 7.5 mm. The data points start after of 45,000 cycles (number of pre-cracking cycles)

4 Conclusions

The mathematical concept of a pivot point allowed 1) to model crack growth with two different exponents using a dimensionally correct equation and 2) to model the crack growth variations in all specimens by varying only one parameter; the power law exponent for the plane strain condition. It is possible to extend the power law behaviour to smaller and larger crack lengths and to exactly model the fatigue crack growth from an EDM starter notch to final failure with multiple power law exponents at different crack length ranges using a dimensionally correct equation. This is in contrast with the smooth FCGR curves observed in literature or obtained by fitting NASGRO equations to da/dN data that is obtained by incremental polynomial fitting of the $a-N$ curve. The transition points between the different power law regions correspond to the original transition points traditionally observed for aluminium alloys. One of the transition points is associated with a change in the plastic zone volume due to the transition from plane strain to plane stress. It is expected that the change in power law exponents is due to changes in the plasticity around the crack tip. Similar trends between the plasticity in materials and the Paris exponent are observed for materials in general.

References

1. Paris, P. and Erdogan, F., A critical analysis of crack propagation laws, *Transactions of the ASME*, 1963, pp. 528-533.
2. Hartman, A. and Schijve, J., The effects of environment and load frequency on the crack propagation law for macrofatigue crack growth in aluminium alloys. *Engineering Fracture Mechanics* 1(4), 1970, pp. 615-631.
3. Zuidema, J., Veer, F. and Van Kranenburg, C., Shear lips on fatigue fracture surfaces of aluminium alloys, *Fatigue & Fracture of Engineering Materials & Structures* 28, 2005, pp. 159-167.
4. Wilhem, D.P., Investigation of cyclic crack growth transitional behavior. Fatigue Crack Propagation, *ASTM* 415, Am Soc Testing Mats, 1967, pp. 363-383.
5. Amsterdam, E. and Grooteman, F., The influence of stress state on the exponent in the power law equation of fatigue crack growth, *International Journal of Fatigue* 82, 2016, pp. 572-578.
6. Iost, A., Temperature dependence of stage II fatigue crack growth rate. *Engineering Fracture Mechanics* 45, 1993, pp. 741-750.
7. Janssen, M., Zuidema, J. and Wanhill, R.J.H., *Fracture Mechanics*. 2nd Ed. VVSD, Delft, 2002.
8. Fleck, N.A., Kang, K.J. and Ashby, M.F., The cyclic properties of engineering materials. *Acta Metallurgica et Materialia* 42(2), 1994, pp. 365-381.
9. Serdyuk, V.A. and Grinberg, N.M., The plastic zone and growth of fatigue cracks in magnesium MA12 alloy at room and low temperatures. *International Journal of Fatigue* 5(2), 1983, pp. 79-85.

NLR

Anthony Fokkerweg 2

1059 CM Amsterdam, The Netherlands

p) +31 88 511 3113 f) +31 88 511 3210

e) info@nlr.nl i) www.nlr.nl