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NLR-TP-2006-686

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This report contains a paper presented at 8th International Conference on Probabilistic Safety Assessment and management (PSAM8), at New Orleans, Louisiana, USA on 14-19 May 2006.

Customer:	National Aerospace Laboratory NLR
Working Plan number:	2005 AT.1.A
Owner:	National Aerospace Laboratory NLR
Division:	Air Transport
Distribution:	Unlimited
Classification title:	Unclassified
	September 2006

Approved by:

Author	Reviewer	Managing department
ME '06	Anonymous peer reviewers	22/11/6

STRUCTURED ASSESSMENT OF BIAS AND UNCERTAINTY IN MONTE CARLO SIMULATED ACCIDENT RISK

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SUMMARY

Monte Carlo simulation of an accident risk model of a complex safety critical operation provides valuable feedback to the decision makers that are responsible for the safety of such operation. By definition, such a Monte Carlo simulation model differs from reality at various points and levels. Hence, the feedback to the decision makers should include an assessment of the combined effect of these differences in terms of bias and uncertainty at the simulated risk level.

In literature the assessment of risk bias and uncertainty due to differences in parameter values has received most attention, e.g. Morgan and Henrion (1990) [1], Kumamoto and Henley (1996) [2]. Obviously, there are many other differences between model and reality than due to parameter value differences only.

The paper presents a structured approach for the assessment of bias and uncertainty in Monte Carlo simulation of accident risk due to differences in parameter values as well as differences that fall beyond the parameter level. For the assessment of differences in parameter values we follow the first-order differential analysis of bias and uncertainty in the accident risk under log-normal assumptions, e.g. [1], and combine bias and uncertainty estimates of parameter values with log-normal risk sensitivities for these parameter variations. Because the number of parameter values may be large, this assessment is performed in two phases. In the first phase an initial bias and uncertainty assessment of parameter values is performed largely using expert knowledge. The second phase focuses on the parameter values that have the largest effect on the risk level; for these, statistical data is collected and sensitivity analysis is performed by running dedicated Monte Carlo simulations.

For the assessment of bias due to other differences than parameter value differences, the paper combines the two structured approaches by Zio and Apostolakis (1996) [3]. One of their approaches assumes alternate hypotheses for the risk case considered, develops an alternate model for each alternate hypothesis, assesses the risk level for each alternate model, and elicits experts on the probability that each alternate model is correct. Their second approach uses an adjustment factor to compensate for differences between model and reality, and elicits experts for the estimation of this adjustment factor. The novelty in this paper is to combine, per non-parameter difference, one alternate hypothesis with one adjustment factor, and to evaluate the bias through the following two estimates for each non-parameter difference:

1. the probability that there is a difference, i.e. the alternate hypothesis is correct; and
2. the conditional risk bias given that the alternate hypothesis is correct, i.e. the conditional adjustment factor.

These estimates per non-parameter difference are evaluated by teams of safety experts and operational experts, and then combined into an overall bias estimate for all non-parameter differences. The estimation of these two factors by experts appears to work quite naturally, especially since the estimation of the conditional risk bias is supported by the risk sensitivity knowledge for each of the model parameters stemming from assessment of the parameter value differences. The novel structured bias and uncertainty assessment approach is illustrated for a Monte Carlo simulation based accident risk assessment for an air traffic operation example.

1 INTRODUCTION

Within the large variety of safety critical industries, air traffic poses exceptional multi-agent communication and coordination challenges to the design of advanced operations. Each aircraft has its own crew, and each crew is communicating with several human operators in different air traffic management and airline operational control centers on the ground in order to timely receive instructions critical to a safe flight. The implication is that safety of air traffic is the result of highly distributed interactions between multiple human operators, procedures, and technical systems.

Accident risk assessment through Monte Carlo simulation of novel air traffic operations provides valuable safety feedback to the designers and decision makers of these operations [4], [5]. Such Monte Carlo simulations are directed to nominal as well as non-nominal situations in air traffic situations and provide the basis for risk evaluations such as the probability of a collision between a pair of aircraft. By definition, a model differs from reality and the resulting accident risk results are uncertain and may be biased. Air traffic operation designers and decision makers are in need of feedback that includes an assessment of the bias and uncertainty of these differences and their combined effect at the level of accident risk.

For assessment of uncertainty in risk assessment various approaches have been proposed (e.g., [1], [3], [6], [7], [8], [9]). One categorization in uncertainty sources is made by distinguishing between aleatory (or stochastic) and epistemic (or state-of-knowledge) uncertainties. Aleatory uncertainty reflects the inherent randomness of processes and is usually represented by probability distributions in the model itself. Epistemic uncertainty reflects restrictions in the state-of-knowledge used for the development of the model. Another categorization is made by distinguishing between uncertainties that arise from phases in computational modeling and simulation. For instance, [3] distinguishes conceptual model uncertainty, mathematical model uncertainty and computer code uncertainty. A more detailed categorization is used in [8], which considers uncertainty due to activities related to conceptual modeling, mathematical modeling, discretization and algorithm selection, computer programming, numerical solution, and solution representation. In spite of the recognition of these wide ranges of types of uncertainty in realistic accident risk assessment problems, until recently the assessment of uncertainty in parameter[1] values has attracted the largest part of academic interest, as is illustrated by [9]. Obviously, there are many other types of differences than those related to parameter values, for example: numerical approximations, model structural differences, hazards that are not incorporated in the model, differences between the assumed and the operational concept in reality. In preparation to assessing the effect of differences between model and reality, all types of differences have to be identified first, and subsequently each difference has to be formulated in terms of an unambiguous model assumption.

For the larger set of differences between accident risk model and reality, Zio and Apostolakis [3] developed two rather unique approaches for a structured assessment of bias and uncertainty. One method assumes alternate hypotheses, develops models for each hypothesis, assesses the risk level for each of these models, and elicits experts on the probability that each model is correct. The second method uses an adjustment factor to compensate for the differences, and elicits experts for the estimation of this adjustment factor. In this paper we combine the adjustment factor and alternate hypotheses approaches of [3] into one method. The key towards this is a decomposition of an adjustment factor for each model structural difference into a product of two factors: 1) how often does the difference not apply, and 2) how severe is the difference when applicable. The estimation of these two factors by experts appears to work quite naturally, especially since the severity estimation is supported by risk sensitivity knowledge for each of the simulation model parameters.

The paper is organized as follows. First, Chapter 2 develops the mathematical framework of bias and uncertainty, based on [10]. Chapter 3 develops the bias and uncertainty assessment process. Chapter 4 illustrates its application to an en-route air traffic scenario. Chapter 5 provides a discussion of results.

2 MATHEMATICS OF BIAS AND UNCERTAINTY ASSESSMENT

2.1 Mathematical problem definition

Accident risk is assessed by first developing a stochastic accident risk model, which includes adoption of assumptions. For the formal bias and uncertainty assessment approach [10] we distinguish assumed parameter values $\overline{v}$ and other model assumptions $\overline{a}$:

[1] In this paper, a parameter is a variable that assumes a Euclidean or integer value, e.g. mean time between failures of a particular system.

- $\overline{v} = \{\overline{v}_1, \ldots, \overline{v}_{n_p}\}$ are the parameter values assumed in the accident risk model, with n_p the number of parameters, and
- $\overline{a} = \{\overline{a}_1, \ldots, \overline{a}_{n_a}\}$ are the other model assumptions (i.e. non-parameter assumptions), with n_a the number of other model assumptions.

In order to capture bias and uncertainty within a mathematical setting, we represent each parameter value assumption and each model assumption as one random variable, and collect all these random variables into two random vectors:

- $V = (V_1, \ldots, V_{n_p})$ is a vector of random variables for the parameter values in the accident risk model,
- $A = (A_1, \ldots, A_{n_a})$ is a vector of random Booleans, where $A_i = 1$ if model assumption $\overline{a}_i$ holds true, and $A_i = 0$ if model assumption $\overline{a}_i$ does not hold true.

We denote the actual risk as $\rho(A,V)$. If all assumptions would have no effect at the risk level, then accident risk equals the conditional expectation of $\rho(A,V)$ given the parameter values are equal to $\overline{v}$ and all other assumptions hold true, i.e., accident risk would be equal to:

$$\rho(\mathbf{1}, \overline{v}) = E\{\rho(A,V) \mid A = \mathbf{1}, V = \overline{v}\}, \tag{1}$$

with $\mathbf{1} \triangleq (1, \ldots, 1)$ a vector with all ones.

It is the aim of the bias and uncertainty assessment to characterize stochastic properties of $\rho(A,V)$, such as expected value and 95% uncertainty interval, in terms of $\rho(\mathbf{1}, \overline{v})$ and the stochastic properties of V and A.

2.2 Mathematics for parameter values

In order to evaluate assumptions due to (random) differences in parameter values, we develop a characterization of $\rho(\mathbf{1}, V)$ in terms of $\rho(\mathbf{1}, \overline{v})$. For this we define for each parameter a *multiplicative bias* $b_i \triangleq E(V_i) / \overline{v}_i$ and a length l_i of the *95% credibility interval* for the value of the *i*th parameter, such that $\Pr\{V_i \in [b_i \overline{v}_i / l_i, b_i \overline{v}_i l_i]\} = 0.95$. Given this multiplicative character of parameter value variations, it is customary to assume that V_i is lognormally distributed [2], [3]. For the evaluation of the effect of variation in a parameter value we define the log-sensitivity of the risk for parameter variation [10]:

$$s_i \triangleq \frac{\partial \ln \rho(\mathbf{1}, v)}{\partial \ln v_i} = \frac{v_i}{\rho(\mathbf{1}, v)} \frac{\partial \rho(\mathbf{1}, v)}{\partial v_i}, \tag{2}$$

which is equal to the normalized sensitivity or elasticity defined in [1], as is shown in the right-hand-side of Eq. (2). The assessment approach of this paper characterizes the bias and uncertainty in the risk in terms of b_i, l_i and s_i. Under a number of conditions it can be shown [10] that the expected value and the 95% credibility interval of $\rho(\mathbf{1}, V)$ are, respectively,

$$E\{\rho(\mathbf{1}, V)\} = \rho(\mathbf{1}, \overline{v}) \times \widehat{B} \times \exp(\tfrac{1}{8} \widehat{U}), \tag{3}$$

$$\Pr\{\rho(\mathbf{1}, V) \in \left[\rho(\mathbf{1}, \overline{v}) \times \widehat{B} \times \exp(-\sqrt{\widehat{U}}), \rho(\mathbf{1}, \overline{v}) \times \widehat{B} \times \exp(\sqrt{\widehat{U}})\right]\} = 0.95, \tag{4}$$

where

$$\widehat{B} \triangleq \prod_{i=1}^{n_p} b_i^{s_i} \tag{5}$$

is the total bias due to all bias contributions of the parameter value assumptions, and

$$\widehat{U} \triangleq \sum_{i=1}^{n_p} (\ln \ell_i^{|s_i|})^2 \tag{6}$$

is due to all uncertainty contributions of the parameter value assumptions. It should be noticed that in this approach all parameter variations are treated as being independent of each other.

2.3 Mathematics for bias due to other differences

In this paper we assume that model assumptions due to other differences (i.e. those that are not differences in parameter values) impose bias on the expected value of the risk, but do not have effect on the size of the 95% credibility interval of the risk. Hence, these other differences are assumptions to have a bias imposed factor Ψ in risk only:

$$\Psi \triangleq E\{\rho(A,V)\} / E\{\rho(1,V)\}. \tag{7}$$

For this factor Ψ, we adopt the following factorization:

$$\Psi = \prod_{i=1}^{n_a} \Psi_i, \tag{8}$$

$$\Psi_i = \frac{E\{\rho(A,V) \mid A_1 = 1, \ldots, A_{i-1} = 1\}}{E\{\rho(A,V) \mid A_1 = 1, \ldots, A_i = 1\}}. \tag{9}$$

Now we define:

$$p_i^{\mathrm{f}} \triangleq \Pr\{A_i = 0 \mid A_1 = 1, \ldots, A_{i-1} = 1\}, \tag{10}$$

$$q_i^{\mathrm{f}} \triangleq \frac{E\{\rho(A,V) \mid A_1 = 1, \ldots, A_{i-1} = 1, A_i = 0\}}{E\{\rho(A,V) \mid A_1 = 1, \ldots, A_i = 1\}}. \tag{11}$$

Within the context of the alternate hypotheses and adjustment factor approaches of [3], the probability p_i^{f} that assumption $\overline{a}_i$ is false can be interpreted as the probability of the alternate hypothesis that assumption $\overline{a}_i$ is false, and the conditional risk bias q_i^{f} given that assumption $\overline{a}_i$ is false can be interpreted as an adjustment factor. With the above definitions of p_i^{f} and q_i^{f}, (9) can be shown to satisfy:

$$\Psi_i = p_i^{\mathrm{f}} q_i^{\mathrm{f}} + (1 - p_i^{\mathrm{f}}). \tag{12}$$

Substituting (12) in (8) and evaluation yields:

$$\Psi = \prod_{i=1}^{n_a} \left[1 + p_i^{\mathrm{f}} (q_i^{\mathrm{f}} - 1)\right]. \tag{13}$$

Combining the above results, the expected risk due to the bias and uncertainty of the parameter value assumptions and the bias of non-parameter assumptions yields

$$E\{\rho(A,V)\} = \Psi \times E\{\rho(1,V)\} = \Psi \times \rho(1,\overline{v}) \times \widehat{B} \times \exp(\tfrac{1}{8}\widehat{U}), \tag{14}$$

with Ψ, $\widehat{B}$ and $\widehat{U}$ satisfying equations (13), (5) and (6), respectively. Moreover, the 95% credibility interval due to the bias and uncertainty in parameter values and the bias of non-parameter assumptions satisfies:

$$\Pr\{\rho(A,V) \in \left[\Psi \times \rho(1,\overline{v}) \times \widehat{B} \times \exp(-\sqrt{\widehat{U}}), \Psi \times \rho(1,\overline{v}) \times \widehat{B} \times \exp(\sqrt{\widehat{U}})\right]\} = 0.95. \tag{15}$$

In words, (15) means that with 95% probability, the risk lies in a credibility interval that ranges from a minimum level $\Psi \times \rho(1,\overline{v}) \times \widehat{B} \times \exp(-\sqrt{\widehat{U}})$ to a maximum level $\Psi \times \rho(1,\overline{v}) \times \widehat{B} \times \exp(\sqrt{\widehat{U}})$.

3. BIAS AND UNCERTAINTY ASSESSMENT PROCESS

As explained in Chapter 1, by definition, a Monte Carlo simulation model differs from reality at various points and levels. In order to get a hold on these differences, the first step is to identify them and to formulate them as model assumptions. Next, the assessment of bias and uncertainty due to these assumptions is done in the following three phases:

1. initial evaluation of parameter value assumptions,
2. simulation-supported evaluation of parameter value assumptions, and
3. evaluation of other (i.e. non-parameter) assumptions.

3.1 *Initial evaluation of parameter value differences*

A Monte Carlo simulation model of accident risk for a complex safety-critical operation typically includes a large number of parameter values. It may not be practically feasible to evaluate the risk sensitivity for all these parameter values by Monte Carlo simulations. Therefore, a first step in the bias and uncertainty assessment is an initial evaluation of the parameter value assumptions based on expert knowledge and available statistical data. This evaluation uses classes of the size of uncertainty and bias of parameter values, and the risk sensitivity for parameter variation. The classes shown in Table 1 have been developed to support this initial evaluation.

Based on the classes in Table 1, initial judgments are acquired of the following items:

- the direction (larger/smaller) and size of the bias b_i of each parameter,
- the size of the 95% credibility interval l_i of each parameter,
- the sign and size of the risk sensitivity s_i of each biased parameter,
- the size of the risk sensitivity s_i of each unbiased parameter.

This judgment is usually based on expert knowledge, or may be based on experimental or statistical data. The risk sensitivity of each parameter depends on the risk for which the evaluation is done.

Table 1: Terminology for size of uncertainty, bias and risk sensitivity.

Qualitative term	Uncertainty l_i, bias b_i		Risk sensitivity $\lvert s_i \rvert$	
	Modal value	**Interval**	**Modal value**	**Interval**
Major	10	[6.83, ∞]	4	[2.67, ∞]
Considerable	5	[3.15, 6.83]	2	[1.33, 2.67]
Significant	2.25	[1.75, 3.15]	1	[0.67, 1.33]
Minor	1.5	[1.30, 1.75]	0.5	[0.33, 0.67]
Small	1.2	[1.13, 1.30]	0.25	[0.17, 0.33]
Negligible	1.1	[1, 1.13]	0.125	[0, 0.17]

Based on the qualitative evaluations of the bias, the 95% credibility interval and the risk sensitivity of each parameter value, the risk uncertainty $l_i^{|s_i|}$ for each parameter is determined using Table 2, and the risk bias $b_i^{s_i}$ for each biased parameter is determined via a table for risk bias similar to Table 2.

Table 2: Risk uncertainty as result of parameter uncertainty and associated risk sensitivity.

Risk uncertainty $l_i^{\lvert s_i \rvert}$		Uncertainty l_i					
		Major	*Considerable*	*Significant*	*Minor*	*Small*	*Negligible*
Risk sensitivity $\lvert s_i \rvert$	*Major*	*Major*	*Major*	*Major*	*Considerable*	*Significant*	*Minor*
	Considerable	*Major*	*Major*	*Considerable*	*Significant*	*Minor*	*Small*
	Significant	*Major*	*Considerable*	*Significant*	*Minor*	*Small*	*Negligible*
	Minor	*Considerable*	*Significant*	*Minor*	*Small*	*Negligible*	*Negligible*
	Small	*Significant*	*Minor*	*Small*	*Negligible*	*Negligible*	*Negligible*
	Negligible	*Minor*	*Small*	*Negligible*	*Negligible*	*Negligible*	*Negligible*

3.2 *Simulation-supported evaluation of parameter value differences*

Parameters values that were judged by experts to have a more than *Negligible* effect on the risk uncertainty or risk bias, are now evaluated using Monte Carlo simulation of the accident risk model. This is done in the following steps.

- Make a list of all parameters that have a more than *Negligible* effect on the risk uncertainty and determine for each of these parameters a quantitative estimate of the length of the 95% credibility interval l_i.

- Make a list of all parameters that have a more than *Negligible* effect on the risk bias and determine for each of these parameters a quantitative estimate of the bias b_i.
- Perform Monte Carlo simulations to evaluate the log-sensitivity s_i for all parameters identified in steps 1 and 2. The log-sensitivity is determined via first order estimation of the derivative of Eq. (2).
- Determine the risk uncertainty $l_i^{|s_i|}$ for each parameter assumption and combine these in Eq. (6) with the results of the expert-based evaluation of parameter assumptions (using the modal value for *Negligible* risk in Table 1).
- Determine the risk bias $b_i^{s_i}$ for each biased parameter assumption and combine these in Eq. (5) with the results of the expert-based evaluation of parameter assumptions (using the modal value for *Negligible* risk in Table 1).

3.3 Expert-based evaluation of other differences

The evaluation of other differences (i.e. non-parameter assumptions) is based on expert elicitation and is supported by the insights gained from the sensitivity analysis and evaluation of the parameter value assumptions in the previous phases. The expert elicitation is supported by using classes for risk bias such as shown in Table 1 and classes for the probability that an assumption does not apply, as shown in Table 3.

Table 3: Qualitative terminology for the probability that an assumption does not apply.

Qualitative term	**Probability p_i^{f}**	
	Modal value	**Interval**
Typical	0.8	[0.64. 1]
Regular	0.4	[0.25, 0.64]
Frequent	0.15	[0.09, 0.25]
Less Frequent	0.06	[0.04, 0.09]
Infrequent	0.028	[0.02, 0.04]
Unlikely	0.01	[0, 0.02]

The evaluation of the non-parameter assumptions is done by the following steps:

- Order these assumptions such that efficient evaluation is supported. In accident risk assessment we considered, the usual order of classes of these assumptions is: a) numerical approximation assumptions, b) model structural assumptions, c) hazard coverage assumptions, d) operational concept assumptions. Within these classes a suitable ordering is also chosen.
- Evaluate the probability that an assumption does not apply using the classes in Table 3, taking into consideration previous evaluations of dependent assumptions.
- Evaluate the risk bias due to non-applicability of an assumption using the classes in Table 1.
- Evaluate the combined effect of items 2 and 3 on the risk bias due to a non-parameter assumption via Table 4.
- Evaluate the total bias due to the non-parameter assumptions by Eq. (13).

Table 4: Risk bias due to probability and effect of the case that a non-parameter assumption does not apply

Risk bias $1+p_i^{\mathrm{f}}(q_i^{\mathrm{f}}-1)$		**Probability that assumption does not apply p_i^{f}**					
		Typical	*Regular*	*Frequent*	*Less frequent*	*Infrequent*	*Unlikely*
Bias due to non-applicability q_i^{f}	*Major*	*Major*	*Considerable*	*Significant*	*Minor*	*Small*	*Negligible*
	Considerable	*Considerable*	*Significant*	*Minor*	*Small*	*Negligible*	*Negligible*
	Significant	*Significant*	*Minor*	*Small*	*Negligible*	*Negligible*	*Negligible*
	Minor	*Minor*	*Small*	*Negligible*	*Negligible*	*Negligible*	*Negligible*
	Small	*Small*	*Negligible*	*Negligible*	*Negligible*	*Negligible*	*Negligible*
	Negligible	*Negligible*	*Negligible*	*Negligible*	*Negligible*	*Negligible*	*Negligible*

4 EXAMPLE OF BIAS AND UNCERTAINTY ASSESSMENT OF AN AIR TRAFFIC OPERATION

4.1 Accident risk model

We consider an hypothetical air traffic example within an en-route sector that consists of two streams of air traffic, flying in opposite direction, at a single flight level. This example has been developed with the aim to understand how air traffic control (ATC) influences accident risk, and how far the nominal spacing *S* between opposite traffic streams can safely be reduced.

For the hypothetical air traffic control example an accident risk Monte Carlo simulation model was developed [11] using Dynamically Coloured Petri Nets [4], [12]. It includes dynamic stochastic representation of aspects such as pilot performance, controller performance, aircraft dynamics, navigation, surveillance, radar, ATC flight plan and aircraft flight plan. During the development of the model, the following numbers of assumptions were adopted: 89 parameter value assumptions, 24 numerical approximation assumptions, 23 model structural assumptions, 21 non-covered hazards assumptions, and 5 operational concept assumptions.

On the basis of the developed Petri Net model, Monte Carlo simulations were performed to evaluate probability distributions of the trajectories of the aircraft for a range of values for the lane spacing *S*. These Monte Carlo simulations addressed several combinations of nominal and non-nominal events regarding, e.g., communication systems, navigation systems, surveillance systems and modes of aircraft dynamics. Speed-up of the Monte Carlo simulation was achieved by decomposing the simulations in a sequence of conditional Monte Carlo simulations, and then combining the results of these conditional simulations [13]. Further processing of the data from the Monte Carlo simulations by a collision risk model gives the accident risk curve presented in Figure 1. The first part of the curve (up to about 10 km) is mostly determined by encounters between aircraft flying nominally along their lanes as expected by the air traffic controller. The second part of the curve (from about 10 km) is mostly determined by encounters between aircraft of which one makes an unexpected sharp turn. The model-based accident curve crosses the ICAO (International Civil Aviation Organization) defined target level of safety at a spacing of about $S = 25$ km.

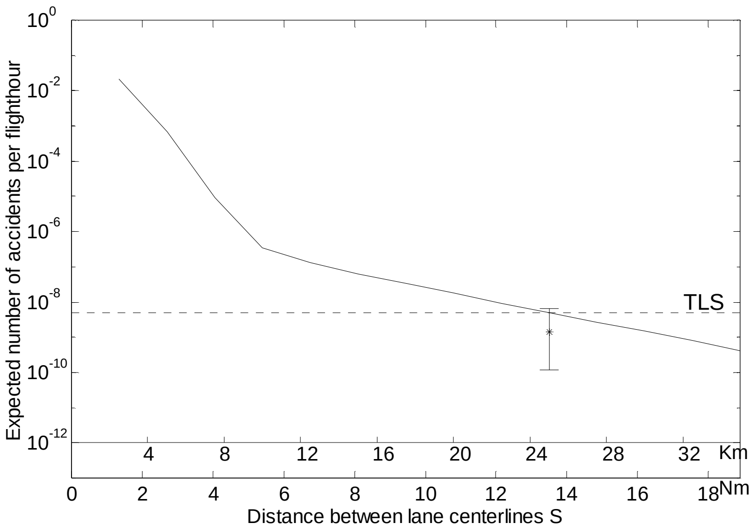

Figure 1 ATC routine monitoring model-based accident risk as function of lane spacing *S* (continuous line). Results of the bias and uncertainty assessment for *S*=25 km: expected accident risk (denoted by *) and 95% credibility interval (bar).

4.2 Bias and uncertainty assessment

First the differences between model and reality of conventional en-route air traffic were identified, and formulated in terms of model assumptions. Next they were evaluated using the bias and uncertainty assessment steps of chapter 3. This particular evaluation was done by safety analysts and supported by expert interviews. The first step considered expert-based evaluation of parameter assumptions. About 80% of the parameter value assumptions were evaluated to have a *Negligible* risk uncertainty and 4% of the parameter values were considered biased, with only one parameter having a more than *Negligible* bias effect. Parameter value differences that potentially have a more than *Negligible* bias or uncertainty effect at the level of risk, were further evaluated using dedicated Monte Carlo simulations. Some examples of the evaluation of the uncertainty in parameter values are given in Table 5; here, the quantitative values have been translated back to qualitative values according to Table 1.

Using the insights gained by the bias and uncertainty assessment of the parameter value assumptions including the sensitivity analysis, the other differences between Monte Carlo simulation model and reality were evaluated. Table 6 shows examples of the assessed bias for all other types of differences between simulation model and reality.

The combined effect of all differences in terms of bias and uncertainty at the accident risk level is depicted in Figure 1. At S = 25 km the actual risk is expected to be 3.5 times smaller than the model simulated risk level. The 95% credibility interval has been assessed to range from a factor 4.5 higher to a factor 12.2 lower than the expected risk. Comparing the expected risk with the target level of safety, it follows that the safe spacing for the operation considered may be about 4 km less than concluded on basis of the model-simulated risk level.

Table 5 Examples of uncertainties in the model risk due to uncertainty in parameter values.

Parameter	Uncertainty l_i	Risk sensitivity $\lvert s_i \rvert$	Risk uncertainty $l_i^{\lvert s_i \rvert}$
Number of aircraft entering each lane per hour	*Significant*	*Significant*	*Significant*
Probability of wrong clearance by controller in opportunistic control mode	*Major*	*Significant*	*Significant*
Standard deviation of vertical position of aircraft	*Minor*	*Significant*	*Minor*
Maximum course deviation during turn of aircraft	*Significant*	*Minor*	*Minor*
Mean duration of communication by controller	*Significant*	*Minor*	*Minor*
Lateral acceleration in turn	*Minor*	*Minor*	*Small*
Standard deviation of transversal position in non-nominal mode	*Significant*	*Negligible*	*Negligible*

Table 6 Examples of bias assessment (+ or - indicate higher/lower risk of the actual operation with respect to the modeled operation).

Other model assumption	Type	Prob. p_i^{f}	Bias q_i^{f}	Risk bias (Table 4)
No semi-circular use of route structure	Concept	*Typical*	*Major -*	*Major -*
There is no Short Term Conflict Alert system	Concept	*Typical*	*Significant -*	*Significant -*
Aircraft do not run out of fuel	Hazard coverage	*Unlikely*	*Significant +*	*Negligible +*
Pilot does not disconnect the autopilot deliberately	Hazard coverage	*Unlikely*	*Minor +*	*Negligible +*
Ground aircraft tracking uses alpha-beta filter and single radar coverage only is considered	Model structure	*Typical*	*Minor -*	*Minor -*
Pilot performance mode is independent of modes of technical systems or air traffic controller	Model structure	*Regular*	*Significant +*	*Minor +*
There is zero probability that the aircraft pair collides after time T_H	Numerical	*Unlikely*	*Significant +*	*Negligible +*

5 DISCUSSION

This paper has developed a novel structured approach to assess bias and uncertainty at accident risk level that are caused by differences between Monte Carlo simulations and reality. In order to enable such a structured approach we developed a novel mathematical model that captures various types of differences and analyses them in terms of bias and uncertainty at the risk level. A crucial preparatory step is to identify all these differences and to formulate each of them unambiguously in terms of a model assumption.

Evaluation of parameter value assumptions is supported by a sensitivity analysis of the model-based risk for variation of single parameter values, while all other parameters have their nominal values. This is then combined with estimation of the bias and 95% credibility interval of the assumed parameter values to yield an estimate of bias and uncertainty, in terms of expected value and 95% credibility interval, at the accident risk level of the Monte Carlo simulation model.

The bias assessment for other differences than those of parameter values is achieved by a combination of the alternate hypotheses and adjustment factor approaches of [3]. In particular, the difference in the expected risk due to each non-parameter difference is evaluated via two variables:

1. the probability that the assumption is false (the alternate hypothesis), and
2. the conditional risk bias given that the assumption is false (the adjustment factor).

These variables are evaluated by teams of safety experts and operational experts, taking into account dependencies between their evaluations. The estimation of these two factors by experts appears to work quite naturally, especially since the estimation of the conditional risk bias is supported by the risk sensitivity knowledge for each of the model parameters stemming from assessment of the parameter assumptions. It has been illustrated how this structured approach can be effectively applied to an en-route air traffic operation, including a wide range of types of differences.

Further refinement of the mathematical background is ongoing. One direction [14] is to extend the mathematical background such that bias and uncertainty assessment can be done for a whole range of operational conditions, rather than for one specific working condition (e.g. for all S values in Figure 1, rather than for one S value). Another direction is to extend the bias and uncertainty mathematical model such that conditions regarding log-linearity and conditional independency between differences in parameter values are relaxed [15]. Two other complementary directions are:

i) to extend our expert based bias assessment of non-parameter differences to conditions under which the mathematical bias and uncertainty model holds true; and

ii) to incorporate estimation of uncertainty in risk level due to differences other than parameter values.

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