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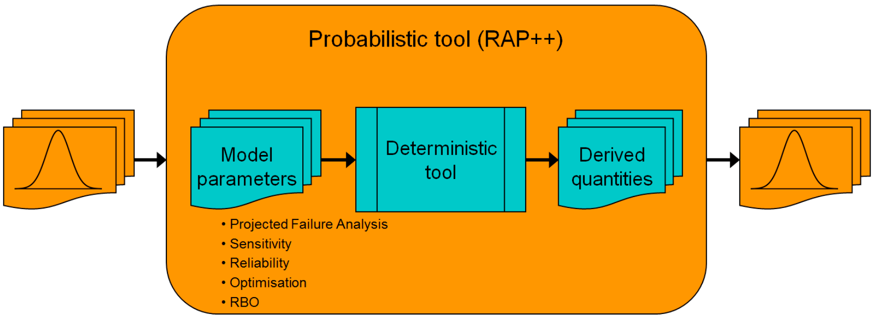
Executive summary

Structural Failure Risk Assessment, Potential and Pitfalls



Problem area
Risk assessments of structures or components have become more common practice. The risk or probability of failure can be computed by means of a probabilistic analysis, in which all the important sources of scatter (uncertainties) in model parameters are taken into account by their corresponding distribution functions. The resulting probabilistic problem can be solved by means of a probabilistic method.

Description of work
An overview is presented of the different steps involved in a risk assessment and the pitfalls that one should be aware of. The main steps are the selection of a proper deterministic model that can represent the failure modes, the characterisation of the most important scatter sources and the selection of a probability threshold. Good probabilistic methods exist, but differ in accuracy, efficiency and robustness.

Results and conclusions
The paper ends with a number of general recommendations and pitfalls that gives guidance in performing a risk assessment.

Applicability
A number of probabilistic tools are available nowadays that functions as a shell on top of any existing deterministic tool. One such tool is the NLR in-house developed reliability Analysis Program RAP++, which is briefly presented. With such a tool a risk assessment can be performed relatively easily, provided the availability of the necessary input.

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Summary

Risk assessments of structures or components have become more common practice. The risk or probability of failure can be computed by means of a probabilistic analysis, in which all the important sources of scatter (uncertainties) in model parameters are taken into account by their corresponding distribution functions. The resulting probabilistic problem can be solved by means of a probabilistic method. Good probabilistic tools are available nowadays that functions as a shell on top of any existing deterministic tool. This paper presents an overview of the different steps involved in a risk assessment and the pitfalls that one should be aware of.

Contents

1 Introduction

Traditionally, engineering structures are designed according to a deterministic approach, in which all the model parameters, as well as the outcome of the analysis, have a unique single value. The uncertainty and scatter inherent in the model parameters are compensated for by introducing scatter and safety factors in the deterministic analysis. For example, a life (damage tolerance) analysis of a structure or component results in a single life estimate which is subsequently divided by a factor 3 or more to account for scatter to obtain the design life. Such an approach often leads to overly conservative designs by assuming a worst-case scenario.

With a probabilistic analysis the uncertainty in model parameters can be taken into account by means of their corresponding distribution functions. Apart from less conservative analysis results, this also yields the reliability level, i.e. probability/risk of failure, and provides directions to improve the design making it more reliable. Furthermore, it can reveal dangerous parameter combinations causing so-called outliers, which show a response of the system away from the majority of the solutions. Understanding these combinations yields a more robust design as well. The risk of failure of a structure can be assessed, and designing a structure in this way prevents its over-design whilst retaining safety/reliability.

In this paper the potential and pitfalls of a risk assessment to determine the reliability level of a structure will be discussed. The following steps can be distinguished in performing a risk assessment (reliability analysis) to determine the reliability or probability of failure:

<u>Step 1 Choice of random model parameters and approximate distribution functions</u>

First, it must be decided which of the model parameters should be treated as random variables. At first, each model parameter that can contain uncertainty can be taken into account. By means of a probabilistic sensitivity analysis it can be determined which of these are really important for the failure mode(s) at hand and thus should be taken into account for the accurate determination of the risk level. One should exclude non-physical parameters, such as tuning parameters necessary for the correct operation of a numerical algorithm.

Based on available data, regression analysis in combination with goodness-of-fit tests or even engineering judgement, distribution functions can be assigned to each of these model parameters. At first these can be (crude) approximate ones for use in the sensitivity analysis. In the case of engineering judgement one assigns a distribution type based on physics, together with a known mean value from the deterministic analysis and an upper bound of the coefficient of variation. In this way a conservative distribution is defined overestimating the true scatter. Section 2 will provide some more details.

Step 2 Choice of failure function(s)

The choice of failure function(s) is determined by the underlying deterministic problem (e.g. design allowables). Usually the same failure criteria as applied in the deterministic analysis are applied making this step straightforward. A continuous failure function in the stochastic domain is preferred by most probabilistic methods. Multiple failure functions (criteria) can be specified. Their interrelationship can be described by means of a failure tree. When the failure functions are dependent, for instance analysing multiple failure functions for the same component that affect one another, the failure tree should be evaluated at each sample point in the probabilistic space to avoid an overestimation of the probability of failure. In case of independent failure modes, the probability of failure for each failure function can be computed separately and the overall probability of failure can be determined according to the rules presented in section 3.

Step 3 Probabilistic sensitivity analysis

By means of a probabilistic sensitivity analysis it can be examined how sensitive the failure functions are for changes in the model parameters. In this way the most important scatter sources, the cause of potential problems, can be identified and modified to make a design more robust. Furthermore, it can reveal dangerous combination of model parameters causing so-called outliers, which show a response of the system away from the majority of the solutions. Understanding these combinations can contribute to an improved design as well.

A sensitivity analysis consists of a limited number of simulations and can be performed with crude approximations of the true distribution functions. Accurate distributions are thus not necessary in this step and conservative estimates suffice if the only goal is to determine the relative sensitivities, i.e. ranking of the random variables according to their importance.

A sensitivity analysis should typically be performed prior to a reliability analysis. In this way the number of random variables can be (considerably) reduced, significantly reducing the analysis time of a reliability analysis, since their efficiency and often stability (apart from Monte-Carlo Simulation) strongly depends on the number of random variables. Failure in most cases is a local phenomenon and will therefore only depend on a limited number of model parameters, e.g. material properties in the region around the failure location. Another advantage of a limited number of random variables is the limited number of distribution functions which have to be determined accurately. In many cases the scatter in a model parameter for which no accurate scatter information is available can be left out in a subsequent reliability analysis, because the model is not sensitive for scatter in this parameter.

Step 4 Selection of the most important random variables and accurate distribution functions

Based on the results of the sensitivity analysis, the most important random variables are identified. If an approximate distribution was assigned, it should be replaced by an accurate one, requiring a considerable amount of data. The accuracy of the computed probability value strongly depends on the accuracy of the tails of the random variable distribution functions, demonstrated in section 6. Good characterisation of these tails therefore is important and requires many data values. A good estimate of the variance requires considerably more data values than the mean. No general rule can be provided for the minimum required number of data values. For the often applied lognormal and Weibull distribution, (Ref. 1) provides some guidance as well as (Ref. 2). A too high variance yields a conservative probability of failure. Small sample sizes often lead to overestimation of the variance (Ref. 3).

Step 5 Solution of the probabilistic problem, requiring a probabilistic method

The original deterministic model is now fed with random variable information, requiring special probabilistic methods for its solution. Various methods exist that differ in their efficiency, robustness and accuracy, see also section 3. Efficiency is important to limit the amount of analysis time, since the probabilistic problem requires the solution of many deterministic problems which can easily become prohibitively expensive. The robustness of a method expresses the capability to solve complex systems of dependent failure modes. This is a prerequisite for application to real life problems which often consists of multiple dependent failure locations and/or multiple failure modes.

Step 6 Interpretation of the results

An important issue in interpretation of the results obtained from a probabilistic analysis is the allowed probability of failure, discussed in some more detail in section 4. The selection of a threshold probability value seems a disadvantage at first, complicating the design process. However, the choice of safety and scatter factors is also a decision, prescribing indirectly the level of safety, and has large implications on the cost of a structure. The only difference is that the values for these factors have historically been defined based on experience and are applied without questioning. The value is normally fixed, not allowing the designer any flexibility and not knowing the real safety level, which is in fact a big disadvantage. This is not the case for a probabilistic (reliability) analysis.

In the next sections a number of the above steps will be discussed in more detail. Section 2 provides some more details about the distribution functions assigned to the model parameters making them random variables. The probabilistic methods to solve the probabilistic problem are more detailed in section 3. The main result is the probability of failure. Section 4 gives some guidance in the selection of the threshold probability of failure. A number of tools exist to perform risk assessments. One such tool, RAP++ (Reliability Analysis Program), is briefly presented in section 5 and a brief example showing a number of pitfalls is discussed in section 6 and summarised in section 7.

2 Distribution functions

Many types of distribution functions exist and an overview can be found in (Ref. 4). Most distribution functions have two or three parameters. An estimate of the distribution parameters can be obtained by fitting a distribution to a set of (experimental) data points applying the method of maximum likelihood estimation (MLE) (Ref. 5) or the method of least-squares (Ref. 6). The method of maximum likelihood is the most robust from a statistical point of view.

The distribution type can be selected based on physical observations, for instance no clear preference for a higher or lower value implies a symmetrical distribution like the normal distribution, or a parameter that expresses some limit often is best modelled by an extreme value distribution. The normal distribution is often applied. A disadvantage is that for very low probabilities (high reliability) the value can become negative, which is physically impossible. A lognormal or Weibull distribution is the most common to represent the variability in fatigue life of a component. From a physical point of view a Weibull distribution, being an extreme value distribution, is more appropriate. Better is to have sufficient data and test various distributions using probability plots and statistical tests as discussed below.

The number of sample points necessary to achieve a proper fit can be quite large: thirty to fifty or more than a hundred, depending on the type of distribution function and the required accuracy of the tails. To accurately characterise the tails of a distribution often a lot of point are required, see also section 6. With a few points, the MLE method can be badly biased. For small sample sizes without heavy censoring (i.e. datasets consisting of failures as well as non-failures) it is better to use a least-squares estimate. For small sample sizes with heavy censoring it is better to use MLE.

For the MLE method confidence bounds can be determined for the distribution parameters, which bound the solution for a given confidence level, normally 95%. The confidence interval will contain the value of the parameter with a reasonable degree of certainty and thus gives an indication of the level of accuracy of the solution. The larger the interval, the less reliable the parameter values are. Confidence bounds can be computed from the Fisher information matrix (Ref. 7), but other methods exist as well like Bayesian, Likelihood ratio, beta binomial.

Most of the distribution functions can be plotted on so-called probability paper, which transforms the cumulative distribution function such that the sigmoidal shape is converted to a straight line, see left picture of figure 1. The advantage of such a plot over the cumulative distribution function plot, right picture of figure 1, is that it is easy to detect whether a sample of data points can be fitted by a specific distribution type. Plotted on corresponding probability paper should reveal a more or less straight line through the data points.

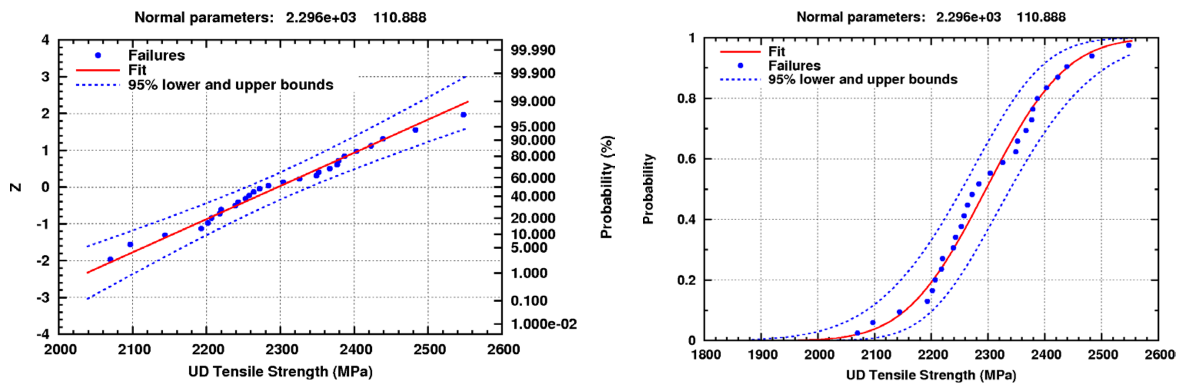

Figure 1 Normal distribution function for the tensile strength, on probability paper (left) and as cumulative distribution function (right)

Another way to check whether the fitted distribution follows the data set is by means of goodness-of-fit tests. These tests do not tell you that you do have a certain type of distribution function they only tell you when the data make it (un)likely that you do. These tests result in a value for some test statistic. This value is compared against a critical value to determine whether the test is passed and the data can be described by this distribution. Well known test are (Ref. 6):

- Anderson-Darling
  This test is more sensitive to deviations in the tails of a distribution than the older Kolmogorov-Smirnov test. A disadvantage is that the critical values depend on the specific distribution.
- Chi-square
  This test can be applied to any discrete or continuous distribution function. A disadvantage is that the sample size should be sufficiently large.
- Hollander-Proschan
  This test compares the theoretical reliability to the Kaplan-Meier estimate and is the only one that is applicable to multiple-censored data.
- Kolmogorov-Smirnov
  This test is based on the maximum distance between the examined and empirical distribution function and is as such applicable to any continuous distribution function. A disadvantage is that it is more sensitive near the centre of the distribution than at the tails.

3 Probabilistic methods

Any probabilistic/reliability problem can be mathematically expressed as:

$$p_f = P\left(G\left(\underline{x}\right) \leq 0\right) = \iint_{G\left(\underline{x}\right)\leq 0} f\left(\underline{x}\right) d\underline{x} \tag{1}$$

where p_f is the probability of failure of interest, $\underline{x}$ a vector containing the deterministic model-parameters which are treated as random variables, $G(\underline{x})$ is the failure/**limit-state function**(s) and $f(\underline{x})$ the joint probability density function (JPDF) being the combination of all the individual distribution functions in the n-dimensional stochastic space. An example is shown in the left picture of figure 2 for the case of two standard normal random variables U_1 and U_2, which for instance represent geometric properties, material properties, or loads, with mean equal to zero and the standard deviation equal to one. Any continuous distribution can be converted to standard normal space by means of a non-linear transformation (Ref. 8).

The limit-state function $G(\underline{x})$ is a simple transformation of a failure function. An often applied limit-state/failure function in fracture mechanics is $K_I\,(a) > K_C$ denoting the situation where the stress intensity factor K_I at a certain crack size a becomes larger than the fracture toughness K_C of the material resulting in instable crack growth. This yields the limit-state function $G(\underline{x}) = K_C - K_I\,(a)$. A negative value of the limit-state function, $G(\underline{x}) < 0$, denotes a combination of parameter values for which the structure is in a failure-state and a positive value, $G(\underline{x}) > 0$, a safe-state. A value of zero, $G(\underline{x}) = 0$, is denoted as the **limit-state** separating the failure and safe domain. The right picture of figure 2 depicts the limit-state function G as function of the same two standard normal random variables. The red-line depicts the limit-state.

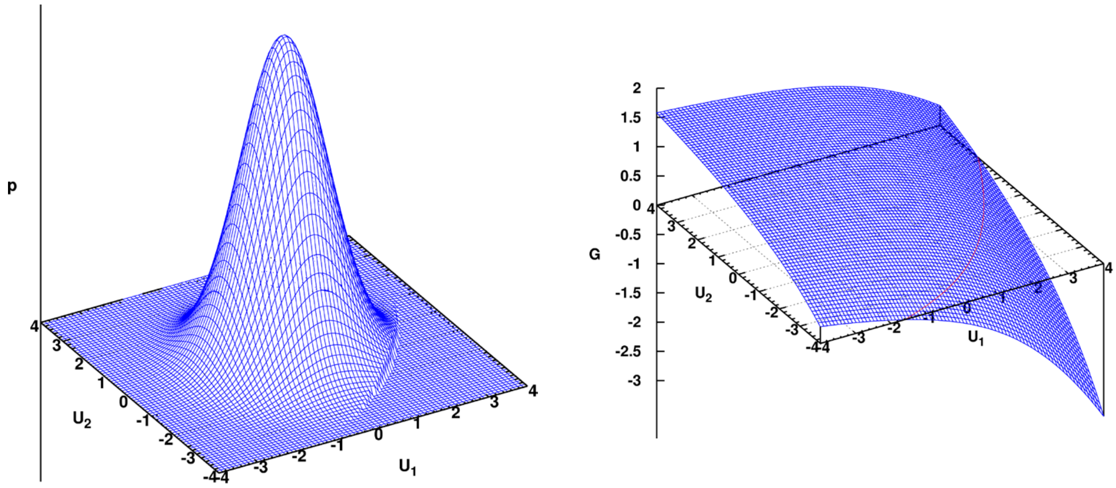

Figure 2 Example joint probability density function f(U) (left) and limit-state (failure) function G(U) for two standard normal variables

The probability of failure of the integral equation (1) basically is the volume integral of the JPDF that lies in the failure domain and is visualised as the volume of the JPDF $f(\underline{x})$ in the left picture of figure 2 located in the failure domain cut-away by the limit-state $G(\underline{x})=0$ in the right picture of figure 2. Since the total volume equals one, the cut-away volume equals the probability of failure.

The objective of a reliability analysis is to obtain a solution of this integral equation. In general, this cannot be done by numerical integration due to the following reasons:

- The integral equation is multi-dimensional, i.e. one integral for each random variable. Numerical integration becomes rapidly more expensive with increasing number of integrals. Furthermore, the accuracy of the solution will decrease rapidly as well, due to the type of functions used.
- In general, the joint probability density function $f(\underline{x})$ will be unknown in explicit form. Only the individual distribution functions of each of subsequent random variables are known.
- In general, the integration boundary (limit-state) is unknown explicitly, but known in implicit form only. For example, to obtain one point on the limit-state a number of deterministic analyses have to be performed. It is noted that the integration bounds in equation (1) are determined by the deterministic model only.

In order to obtain a solution of the integral equation (1) a large number of probabilistic methods have been developed in the past decades, e.g. see Ref. 9, and even recently. The most well-known and simple method is the Monte Carlo method, but it is extremely inefficient especially when dealing with smaller probabilities of failure ($< 10^{-3}$), which is in general the case for engineering structures. Much more efficient methods, such as FORM, SORM and Importance Sampling methods, have been developed in the past decades to improve efficiency. The most well-known probabilistic methods are:

- Monte-Carlo simulation (MCS, Ref. 9)
- Monte-Carlo simulation Importance Sampling (MCS-IS, Ref. 9)
- Latin-Hypercube Sampling (LHS, Ref. 9)
- Directional Sampling (DS, Ref. 10)
- First-Order Reliability Method (FORM, Ref. 9)
- Second-Order Reliability Method (SORM, Ref. 11)

Two NLR in-house developed methods, with improved characteristics are:

- Adaptive Radial Based Importance Sampling (ARBIS, Ref. 12)
- Adaptive Directional Importance Sampling (ADIS, Ref. 13)

Table 1: Quantitative comparison of different probabilistic methods

Method	Accuracy	Efficiency	Robustness
MCS (LHS)	High	Low	High
DS (ARBIS)	High	Medium-low	High
MCS-IS	Medium	Medium	High
ADIS	High-medium	High-medium	High-medium
SORM	Medium-low	High-medium	Low
FORM	Low	High	Low

The strong and weak points of the various methods are summarised in table 1 with respect to their relative accuracy, efficiency and robustness. The efficiency is defined as the number of deterministic analyses (limit-state function evaluations), for example finite element analyses, necessary to reach a sufficiently accurate solution. Robustness is defined as the ability to handle complex limit-state(s), such as: multiple failure points, multiple failure (limit-state) functions, mixed terms, noisy limit state, etc.

Basically, all the probabilistic methods differ from one another in the way they select the evaluation points in the stochastic space, i.e. for which values of the random variables the next deterministic problem is solved. In general, the more complex the algorithm, i.e. the more knowledge about the characteristics of the general problem (1) that are implemented in the algorithm, the fewer function evaluations are required to obtain a reliable solution. On the other hand the more complicated the algorithm the less robust the method will be, in general. What is envisaged is a *robust* probabilistic method which is able to determine small probabilities (*accurate*) with as few function evaluations (*efficient*) as possible.

Another very attractive method is to construct a surrogate model (e.g. response surface) for each of the response variables of the underlying deterministic problem by performing a number of deterministic analyses and fit a response surface through these points. Many methods exist to determine the location of the points in which the responses are evaluated, such as the so-called Design Of Experiments (DOE): factorial designs, Central Composite Designs, Box-Behnken design, Latin Hypercube Samples.

This surrogate model is subsequently used in the probabilistic analysis applying one of the above probabilistic methods. This is very efficient, because evaluating the surrogate model is very cheap and can be done millions of time with little computational effort. Even direct MCS can be applied in this way. However, although this method seems to be very attractive from a computational point of view, it should be applied with great care for the following reasons:

- Construction of the surrogate model becomes (very) expensive with increasing number of random variables (i.e. degrees of freedom, curse of dimensionality) losing the advantages.
- The accuracy of the results heavily depends on the accuracy of the surrogate model, which is not very accurate for many applications. Furthermore, for probabilistic analysis the surrogate model only needs to be accurate at the limit-states! This is not accomplished by the previously mentioned DOE methods. Therefore a new strategy has been developed by NLR in ADIS, which is efficient using a surrogate model approach while guaranteeing accuracy!

A structure can have multiple failure modes, which can best be represented by a failure or fault tree. This is a tree where at the end-points the failure modes are located and the cross points of the branches consists of AND- or OR-gates, e.g. figure 3. OR-gates are applied for failure modes in **series**, for which the total (system) reliability R, or probability of failure P=1-R, for independent failure modes can be obtained from:

$$R = \prod_{i=1}^{n} R_i = \prod_{i=1}^{n} 1 - P_i \qquad (2)$$

in which the entire system will fail if any failure mode is triggered. This is called the product law of reliabilities, expressing that the reliability is decreased by adding another component in the chain.

AND-gates are applied for failure modes in **parallel**. The total probability of failure *P* can be obtained from:

$$P = \prod_{i=1}^{n} P_i = \prod_{i=1}^{n} 1 - R_i \qquad (3)$$

in which the entire system will fail if all failure modes are triggered, for instance a structure that has multiple load paths, where the remainder of the structure can carry the load after one fails. This is called the product law of unreliabilities, expressing that the probability of failure is decreased by adding another component.

Often multiple failure modes can best be modelled by a series system and equation (2) applies. In case the failure modes are not independent, equation (2) can still be applied resulting in a conservative prediction. An accurate result can be obtained by evaluating the failure tree inside the probabilistic method for each sampling point.

A system consisting of failure modes both in series and parallel, which is the general form of the failure tree, e.g. figure 3, can be analysed by first replacing the reliabilities of the parallel modes with an equivalent reliability according to equation (3) and thereafter replacing the resulting series failure modes with the total system reliability *R* according to equation (2). This is similar as the replacement of a system of resistors by its equivalent resistance.

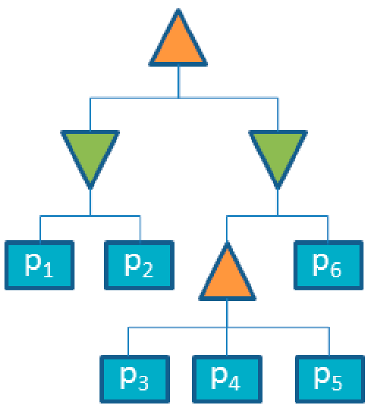

Figure 3 Example failure tree with ▲ = OR gate, ▼ = AND gate, ■ = failure function

4 Probability threshold

In the ISO standard 2394 (Ref. 14) target probability values for lifetimes are mentioned, based on the relative costs of safety measures against the consequences of failure. In general, the target lifetime probability of failure will be smaller than 10^{-3} for engineering structures.

Table 2: Lifetime probability of failure targets according to Ref. 14

Relative costs of safety measures	**Consequences of failure**			
	Small	**Some**	**Moderate**	**Great**
High	1	10^{-1}	10^{-2}	10^{-3}
Moderate	10^{-1}	10^{-2}	10^{-3}	10^{-4}
Low	10^{-2}	10^{-3}	10^{-4}	10^{-5}

The military standards for fighter aircraft, MIL-STD-1530C (Ref. 15) and MIL-STD-882D (Ref. 16), also include proposed probability of failure ranges. An example is depicted in table 3, where the green, orange and red categories are, respectively, acceptable, undesired and unacceptable. Once aircraft component risk shifts into an orange category, fleet management has to take action, for example repair, replacement, change in fleet usage, change of inspection scheme.

Table 3: Lifetime probability of failure targets according to Ref. 16

SEVERITY PROBABILITY	**Catastrophic > $1M**	**Critical > $200K**	**Marginal > $10K**	**Negligible > $2K**
Frequent $> 10^{-1}$				
Probable $> 10^{-2}$				
Occasional $> 10^{-3}$				
Remote $> 10^{-6}$				
Improbable $< 10^{-6}$				

The probability of failure (risk level) can be expressed in terms of:

- Cumulative risk, which is the fraction of the population that is expected to fail over the design lifetime of the component, as presented in the above tables. This does not take into account the change in risk rate over the lifetime. For example for fighter aircraft this is about 10^{-3}, i.e. 1 in 1000 aircraft (Ref. 17). The value can be easily determined from the cumulative failure distribution function.

- Instantaneous risk is defined as the probability of failure per time increment, e.g. per operation hour or per flight. For fighter aircraft this is about 10^{-6} per flight (Ref. 15 and Ref. 17). The instantaneous risk value is determined from the hazard function $h(t)$, which depends on the failure probability density function $f(t)$ and cumulative distribution function $F(t)$ according to:

$$h(t) = \frac{f(t)}{1 - F(t)} \qquad (4)$$

5 Probabilistic tool RAP++

Nowadays, a number of general probabilistic software tools exist that solves the probabilistic problem and act as a shell around any deterministic tool adding the probabilistic layer, preventing the need to modify the source code. Such a probabilistic tool is RAP++ (Reliability Analysis Program, Ref. 18) that has been developed at NLR over the past 15 years. RAP++ is a very versatile probabilistic tool and can be easily applied as shell around any existing deterministic tool (provided it can be run in batch mode) using a simple interface, adding the probabilistic layer to perform a probabilistic analysis, figure 4. Examples of software tools to which RAP++ has been interfaced using a generic interface are: NASTRAN (FEM), ABAQUS (FEM) and NASGRO (crack growth).

The tool has, for example, been applied for several risk assessments of aircraft engine components. These are related to fatigue and fracture phenomena, with respectively, crack initiation and crack growth as the dominant failure mode.

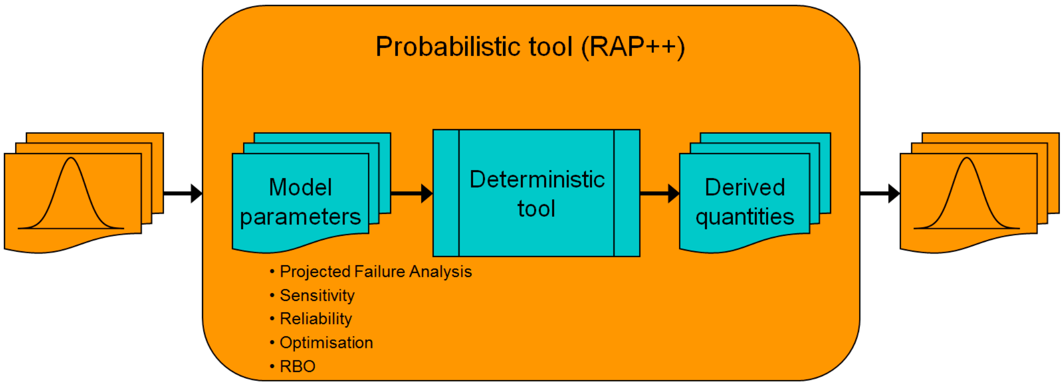

Figure 4 Schematised probabilistic analysis

The source code is written in C++, fully object oriented, resulting in a much more robust and extendible program. Currently supported platforms are: Windows and Linux. The quality of program is guarded via a source code repository to enable a full source code history in which all changes made can be viewed and allowing previous releases to be regenerated. Furthermore, a large database of unit and global test cases exists to check correct performance after modifications have been made. The global test cases are also used to fine tune the program for optimal performance.

The program's main capabilities include 1) Statistical Data Analyses to determine whether data points from different sets can be pooled; if there are any outliers; whether data sets are correlated; and which distribution function applies best to a set of (experimental) data points. The output of these analyses can serve as input for a probabilistic analysis, such as: 2) Projected Failure Analysis to determine any future failures; 3) Probabilistic Sensitivity Analysis to determine how sensitive the design is for scatter in model parameters. Correlation and sensitivity coefficients denote the most important random variables; 4) Reliability Analysis to determine the reliability (probability of failure) of a design; 5) Optimisation and Reliability Based Optimisation to determine a reliable optimal (structural) design given some objective, design variables and constraints.

Other features include a Failure Tree Analysis to model multiple failure modes, which are correctly evaluated for each deterministic analysis performed. All major distributions functions are available. Random variables may be correlated. The program can be run in batch mode and contains a powerful expression editor with which complex functions can be defined for use as limit-state or objective function and constraints. Design of experiments (DOE) is available to construct response surface or Kriging meta models in order to improve efficiency when required. An automatic restart capability exists to restart an analysis at the point where it was terminated. An automatic debug facility exists that generates all required debug information after a crash. An extensive user- and technical manual is available, as well as a Java based Graphical User Interface. A license of the program can be obtained from the author.

6 Brief example of a risk assessment

In this section an example is given of the sensitivity of the probability of failure for deviations in the distribution tails and correlation between random variables. The example concerns a single edge crack tension problem loaded by a displacement field, for which the crack growth life is determined with NASGRO (Ref. 19).

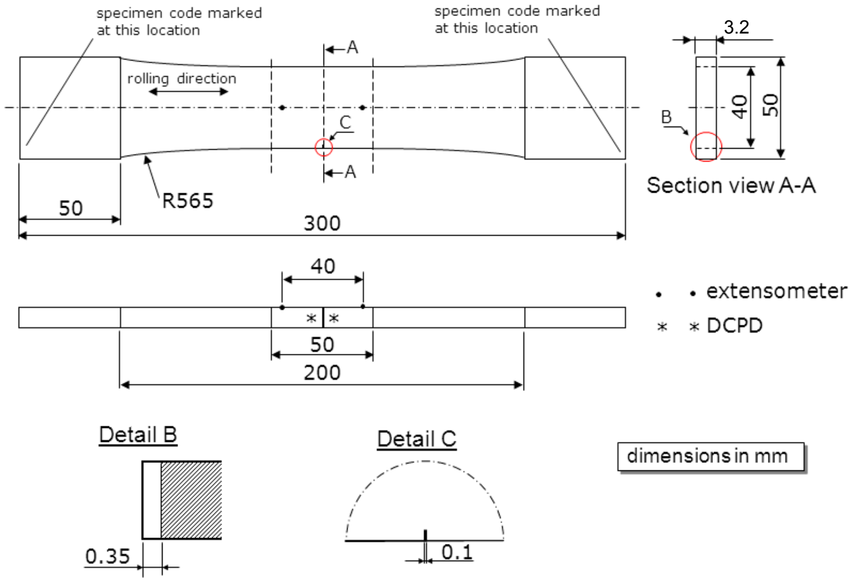

Figure 5 Single edge crack tension (SET) fatigue crack growth specimen

The dominant scatter sources, determined by a probabilistic sensitivity analysis, were the initial crack length a_i, the fracture toughness K_c, the modulus of elasticity E, and the NASGRO crack growth equation parameters C and n (Ref. 20). These two parameters are strongly correlated depicted by the solid-line in figure 6, i.e. any random combination of both parameters lies on or close to this solid-line. This correlation should be taken into account as will be demonstrated underneath.

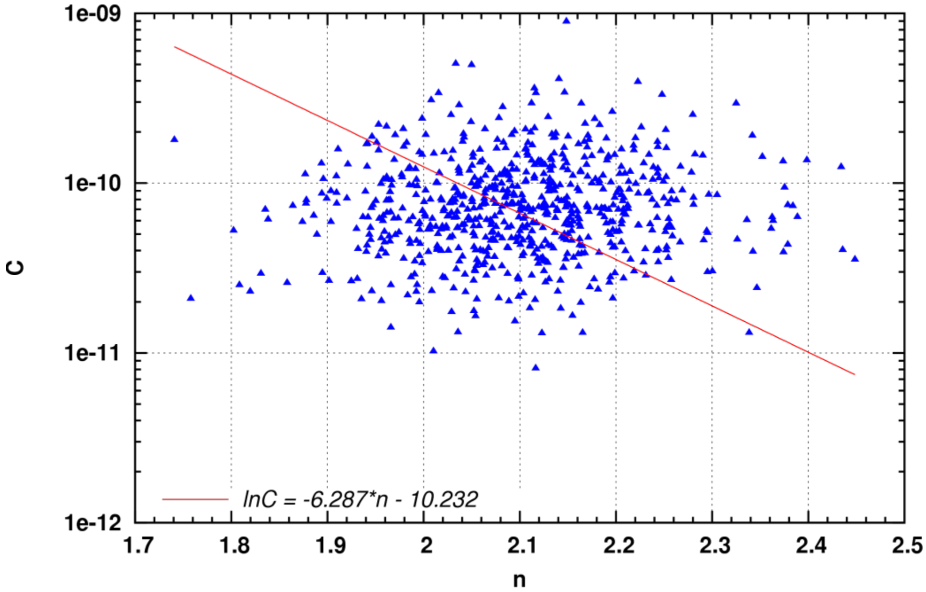

Figure 6 Correlation of the NASGRO crack growth law parameters C and n (solid-line) and uncorrelated random combinations drawn from their corresponding distributions (blue triangles)

A reliability analysis was performed with RAP++ to determine the probability of failure (POF) for a design life of 47,000 cycles, being a factor 3 lower than the deterministic crack growth life. The POF obtained with the various probabilistic methods and their efficiency is denoted in table 4. MCS is very inefficient, although very robust, and was ended after 3000 simulations without reaching an answer. A sufficiently converged answer requires 9 million simulations. All the other methods reached a comparable answer. DS is much more efficient than MCS followed by ARBIS. ADIS, FORM and SORM are of comparable efficiency. This corresponds to the quantitative comparison of table 1. The accuracy of FORM and SORM is unknown, which can be improved for all the other (sampling) methods by increasing the number of simulations. Furthermore, FORM and SORM are not robust in the case of a complex limit state, such as a highly nonlinear failure function, multiple design points or a combination of failure functions (serial and parallel systems).

Table 4: Probability of failure P(Life < 47,000) of different probabilistic methods

Probabilistic method	Probability of failure	Number of deterministic analyses
MCS	-	3000 (9,000,000)
DS	3.70e-5	4348
ARBIS	2.82e-5	1053
ADIS	3.42e-5	76
FORM	3.39e-5	46
SORM	3.54e-5	64

To demonstrate the importance of taking into account the correlation between random variables, the above analysis is repeated without the correlation between C and n. This allows any combination of C and n, depicted in figure 6 by the triangles, instead of the straight solid-line of the correlated case. The resulting probability of failure was **0.125**, which is four orders in magnitude higher than the correlated result of table 4. Hence, the effect of neglecting correlations on the probability of failure can be huge, leading to completely false POF values. Nevertheless, this POF-value is very conservative due to the much larger scatter in the random variables C and n when treated uncorrelated, which is true in general.

Another estimate of the probability of failure can be obtained by fitting a distribution function on the 3000 crack growth lifes obtained in the MCS. A Logistic, Normal and Weibull distribution function all yielded a good fit. The obtained Normal and Logistic distribution are depicted in figure 7 on probability paper. The points are located on or near the straight line representing the fit, indicating a proper fit. This was further supported by test statistics not presented here. Their cumulative distributions are depicted in figure 8, showing a similar cumulative distribution for all three types. The tails of the distribution function, shown in the right plot, however, shows a larger deviation, indicating an error in describing the true behaviour in this region.

For all three distribution functions the probability value can be computed for a life of 47000 cycles, representing an estimate for the POF, presented in table 5.

Table 5: Probability of failure P(Life < 47000) estimated from the fitted distribution

Type	Probability of failure
PoF	3.E-05
Logistic	2.2E-03
Normal	2.4E-04
Weibull	1.7E-08

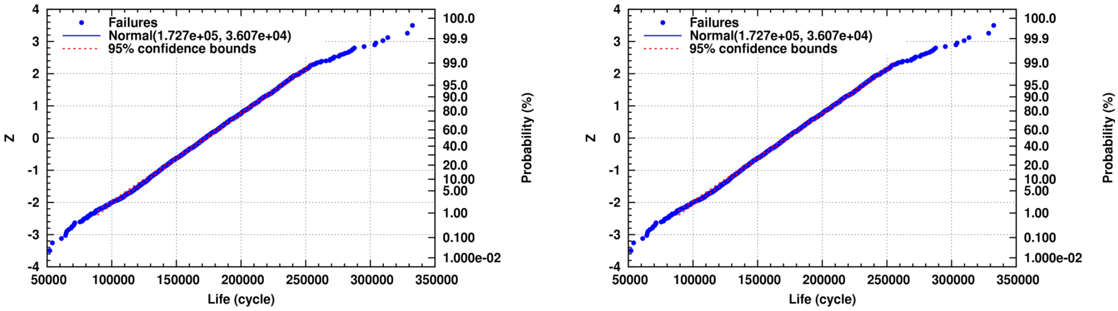

Figure 7 Fit of crack growth life on Normal (left) and Logistic (right) probability paper

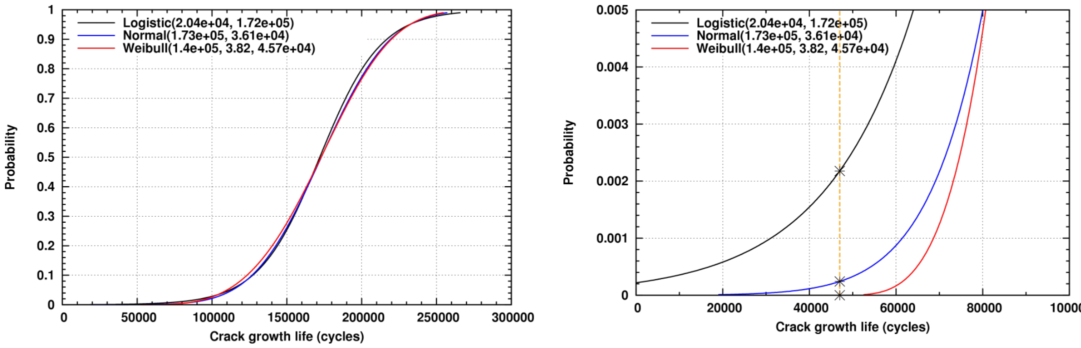

Figure 8 Whole (left) and left-tails only (right) cumulative distribution plot of the fitted Logistic, Normal and Weibull distribution function

The difference in these values is huge and none is of the same order as the results obtained from the reliability analyses presented in table 4. The right picture of figure 8 depicts the tails of the corresponding distributions, where the vertical dotted orange line denotes the component life of 47000 cycles and the stars mark the probability values. It is clear from this plot that the tails are quite different and cause the huge differences in values for the probabilities. Of all three, the Normal distribution best represents the true behaviour in the tail yielding a conservative POF closest to the values obtained with the probabilistic methods.

The importance of the tail is also shown by the crack growth life value computed for a probability of failure of 1E-6 shown in table 6. The logistic distribution then even predicts a negative life.

Table 6: Life estimate for a probability of failure of 1E-6 obtained from the fitted distribution

Type	Life (cycles)
Logistic	-109,836
Normal	1401
Weibull	49462

7 Recommendations and pitfalls

The paper ends with a number of general recommendations and pitfalls that gives guidance in performing a risk assessment.

- The applied underlying deterministic model should be able to accurately represent the “failure” behaviour. Probabilistic methods cannot be used to improve the deterministic model. If the outcome of the deterministic model for all mean parameter values is not close enough to the mean of the experimental results, the conclusion can be drawn that the deterministic model is lacking some physical phenomenon and cannot predict the real behaviour well.
- A minimum of 30 to 50 specimens, ideally from different batches, should be tested to characterise the distribution function parameters of an important scatter source. To accurately characterise the tails of a distribution, needed for an accurate reliability analysis, many more (up to hundreds) tests are required.
- A probabilistic sensitivity analysis should be performed previous to a reliability analysis to identify the most important scatter sources, the cause of potential problems. Furthermore, it can reveal dangerous combinations of model parameters causing so-called outliers, which show a response of the system away from the majority of the solutions. Understanding these combinations can contribute to an improved design as well. Approximate distributions suffice.

- The selected distribution type has limited influence on the outcome of a probabilistic sensitivity analysis to characterise the most important scatter sources. The effect on the reliability can be huge since this value is highly determined by the tails of the distribution function.
- Correlation between random variables should be taken into account and can have a huge effect on the probability of failure.
- An importance sampling method is the preferred probabilistic method to compute the reliability due to its high robustness, accuracy and (reasonable) efficiency. These methods can be applied, at least to some extent, in combination with parallel computing facilities, which can drastically reduce the computation time. Although, application of parallel computing in combination with commercial software tools is often (very) limited due to the required number of licences. Of the applied sampling methods ADIS is by far the most efficient. The probabilistic methods FORM and SORM should be used with care to ensure a correct reliability estimate is obtained.
- The computational effort depends on the number of random variables. For many problems this number will be less than 10. In those cases, a probabilistic sensitivity analysis requires about 100 simulations of the deterministic model. A reliability analysis requires about 100 to a 1000 simulations, depending on the probabilistic method applied.
- Surrogate models that replace the expensive deterministic method should be applied with care, making sure it is accurate and not requires a higher computational effort to generate than direct application of the deterministic method.
- An allowed (threshold) probability of failure must be chosen, to determine whether a structure is safe. Deterministic methods on the other hand pretend to be absolutely safe.

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