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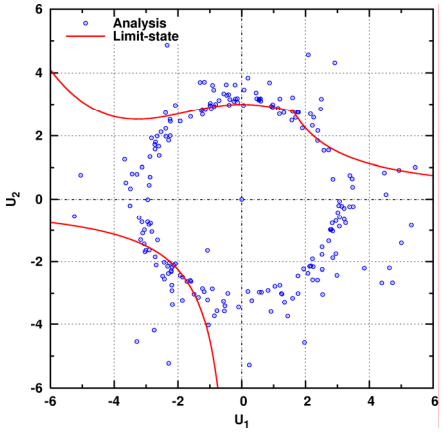
Executive summary

Adaptive Radial-Based Importance Sampling method for structural reliability



Problem area
The evaluation of the failure probability is a basic problem in structural reliability analyses. The failure probability can be formulated in terms of an integral equation that represents the volume of the joint probability density function located in the failure domain. In the past decades many methods have been presented to solve this integral equation, such as sampling methods based on Monte-Carlo simulation and directional simulation and methods based on an analytical solution of the integral equation: first- and second-order reliability method.

Description of work
In this paper an adaptive radial-based importance sampling (ARBIS) method is presented to solve the integral equation. The radial-based importance sampling (RBIS) method, excluding a β-sphere from the sampling domain, is extended with an efficient adaptive scheme to determine the optimal radius β of the sphere. The adaptive scheme is based on directional simulation.

Results and conclusions
Several numerical examples demonstrate the efficiency, accuracy and robustness of the scheme.

Applicability
The ARBIS method can be applied as a black-box and is of particular interest in applications with a low probability of failure, for example in structural reliability, in combination with a small number of stochastic variables.

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Author(s)
F.P. Grooteman

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Adaptive Radial-Based Importance Sampling method for structural reliability

Nationaal Lucht- en Ruimtevaartlaboratorium, National Aerospace Laboratory NLR

Anthony Fokkerweg 2, 1059 CM Amsterdam,
P.O. Box 90502, 1006 BM Amsterdam, The Netherlands
Telephone +31 20 511 31 13, Fax +31 20 511 32 10, Web site: www.nlr.nl

Nationaal Lucht- en Ruimtevaartlaboratorium
National Aerospace Laboratory NLR

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F.P. Grooteman

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Adaptive Radial-Based Importance Sampling method for structural reliability

Frank Grooteman *

National Aerospace Laboratory NLR, Anthony Fokkerweg 2 1059 CM Amsterdam, The Netherlands

Abstract

In this paper an adaptive radial-based importance sampling (ARBIS) method is presented. The radial-based importance sampling (RBIS) method, excluding a β-sphere from the sampling domain, is extended with an efficient adaptive scheme to determine the optimal radius β of the sphere. The adaptive scheme is based on directional simulation. The underlying basic methods are presented briefly. Several numerical examples demonstrate the efficiency, accuracy and robustness of the scheme. As such, the ARBIS method can be applied as a black-box and is of particular interest in applications with a low probability of failure, for example in structural reliability, in combination with a small number of stochastic variables.

Keywords: Importance Sampling; Monte-Carlo Simulation; Structural reliability; Failure probability; Adaptive

1 Introduction

The evaluation of the failure probability is a basic problem in structural reliability analyses. The failure probability can be formulated as:

$$p_f = P\{G(\underline{x}) \le 0\} = \int_{G(\underline{x})\le 0} f(\underline{x})\, d\underline{x} \tag{1}$$

where $\underline{x}$ represents the vector of stochastic variables of the reliability problem and $f(\underline{x})$ the joint probability density function in X-space. $G(\underline{x})$ is the failure or limit-state function, defining a safe state when $G > 0$ and a failure state when $G < 0$. The hyper-surface separating the safe from the failure domain $G = 0$ is called the limit-state. The integral represents the volume of the joint probability density function located in the failure domain.

In the past decades many methods have been presented to solve this integral equation, such as sampling methods based on Monte-Carlo simulation (MCS) and directional simulation (DS) [1-2] and methods based on an analytical solution of the integral equation: first-order reliability method (FORM) and second-order reliability method (SORM) [3].

* Tel.: +31-527-24-8727; fax.: +31-527-24-8210.
E-mail address: grooten@nlr.nl (Frank Grooteman).

FORM and SORM approximate the limit-state with, respectively, a first-order or an incomplete second-order function. Furthermore, the underlying solution method requires the solution of an optimization problem to find the smallest distance to the limit-state. FORM, and to a lesser extent SORM, are often very efficient. However, neither method is robust in the case of a complex limit-state, such as a highly non-linear failure function, multiple failure points or a combination of failure functions (serial and parallel systems). An example of a series system having multiple design points is given in Fig. 1. In general, the accuracy of the solution is unknown, because either narrow confidence bounds cannot be obtained or they require an extra computational effort (e.g. importance sampling).

On the other hand, MCS and DS are very inefficient compared with FORM and SORM, especially for small probability values. Nevertheless, convergence to the exact solution is guaranteed for an increasing number of simulations, and confidence bounds on the solution are available in the case of a finite number of simulations. Furthermore, these methods are very robust in the sense that they can handle complex limit-states.

Various methods have been presented to improve the efficiency of the two basic sample methods (MCS and DS): for example [4-8], referred to as importance sampling techniques. The basic idea is to concentrate sampling near the most important part(s) of the limit-state(s), that is points on $G(\underline{x}) = 0$ located closest to the origin in U-space. A widely applied approach is to shift the sampling centre from the origin to the design point. Often a FORM analysis, having the mentioned disadvantage, is applied first to obtain knowledge about the design point. An alternative strategy is to gather knowledge about the failure domain and thus limit-state(s) during sampling and use this knowledge to guide the sample domain towards the most important regions. This is called an adaptive method, e.g. [9].

An importance sampling method originally proposed by Harbitz [6], referred to as the radial-based importance sampling (RBIS) method, is to exclude an n-dimensional sphere called "β-sphere" from the safe part of the sampling domain. The remaining sampling domain is restricted to values outside the sphere located in the tail part of the joint probability density function. In principle no knowledge about the location of the design point(s) is required. The method converges to the exact solution provided the sphere is located in the safe domain, which can be easily checked during sampling. However, the optimal choice is a sphere that touches the limit-state, maximising the excluded region. Hence the optimal sphere radius β is the smallest distance to the limit-state given by the most probable (design) point (MPP).

In this paper a very efficient, accurate and robust adaptive scheme is presented to determine the optimal sphere radius. Because of these characteristics, the resulting method can be applied as a black-box and is for most structural reliability applications much more efficient than crude Monte-Carlo method.

2 Adaptive Radial-Based Importance Sampling (ARBIS)

A set of dependent non-normal stochastic variables $\underline{x}$ can always be transformed to a set of independent standard normal variables $\underline{u}$, called the U-space, by applying appropriate transformations, [10-12]. The remainder of the paper is therefore restricted to the U-space. Before presenting the adaptive scheme the original idea of Harbitz [6] is briefly presented.

2.1 Radial-based Importance Sampling (RBIS)

The method of Harbitz [6] is based on a simple but effective importance sampling method, excluding a β-sphere from the sample domain, Fig. 1. The sphere has to be located inside the safe domain. The optimal radius β is equal to the distance to the Most Probable Point, which is the point on the failure surface (limit-state) that is closest to the origin.

The probability content of the excluded sphere is given by:

$$p = P\{|U| \le \beta\} = P\{|U|^2 \le \beta^2\} = \chi_n^2(\beta^2) \tag{2}$$

where χ_n^2 is the chi-square distribution function with n degrees of freedom equal to the number of stochastic variables.

The probability integral of Eq. (1) can be rewritten in terms of a conditional probability, yielding:

$$p_f = P\{G \le 0 \,|\, |U| > \beta\} P\{|U| > \beta\} \tag{3}$$

The first term can easily be obtained by Monte-Carlo sampling outside the sphere. The second term is given by Eq. (2), thus yielding:

$$p_f = \frac{N_{fail}}{N_{sim}}\left(1 - \chi_n^2(\beta^2)\right) \tag{4}$$

In a crude MCS most sampling points would be located inside the sphere. Disregarding this part of the domain can save a huge amount of samples. Therefore, this method can be much more efficient than MCS, especially for small probabilities of failure occurring in structural reliability analyses. The method has similar characteristics as MCS, such as convergence to the exact solution (accuracy) and the capability to handle complex limit-state(s) (robustness).

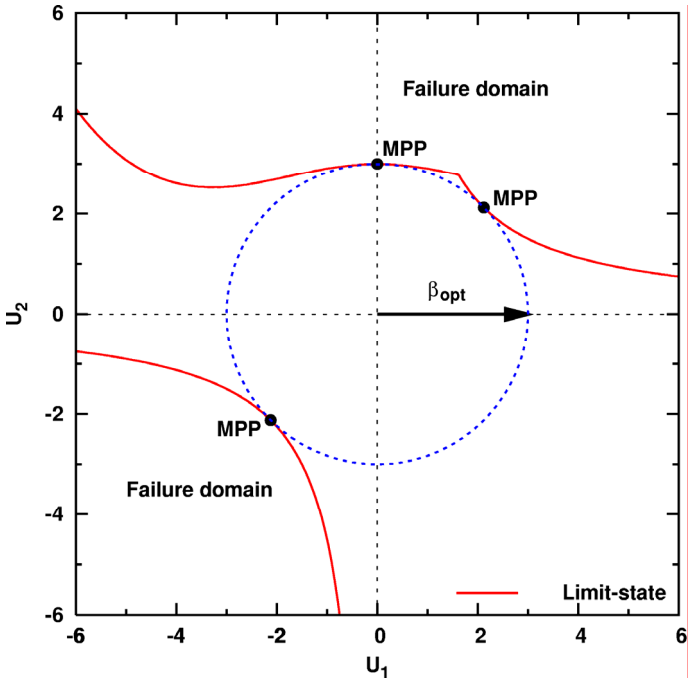
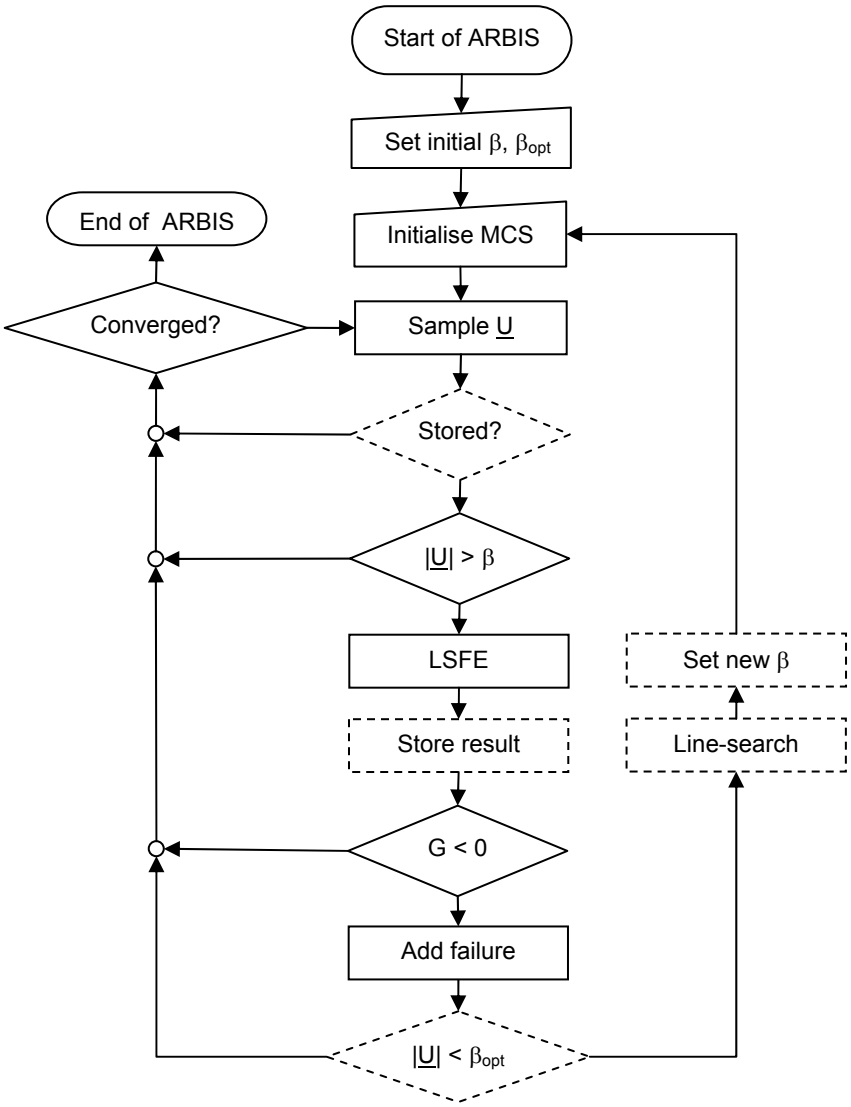


Fig. 1. Optimal Radial Based Importance Sampling, problem 12 in section 3.

2.2 Adaptive scheme

The RBIS method proposed by Harbitz assumes that the MPP or design point is known. At the start of a probabilistic analysis, no information about the limit-state is available. The unknown MPP has to be determined first. For this a FORM analysis can be used, which is efficient but not robust. The latter makes it less suitable for practical applications. An alternative adaptive scheme is presented here that is robust, efficient and guarantees an optimal radius β (accurate). The basic steps are depicted in the flow chart below. The dashed blocks in the flow chart represent the new adaptive part.



An initial value of the radius β (β_0 in Fig. 2) is chosen such that the excluded sphere is located in the failure domain, contrary to the safe domain requirement of the previous section. This can be achieved by selecting an initial β that results in a low probability p_0 of the sample domain outside the sphere. With equation (2) this yields:

$$\beta = \sqrt{\chi_n^{-2}(1-p_0)} \tag{5}$$

The initial value of β_{opt} is set to a very high value representing the unknown MPP. Next, the Monte-Carlo method is initialised by selecting a start seed for the random number generator used to generate the sample points in U-space. For points outside the sphere ($|\underline{U}| > \beta$) the limit-state function is evaluated (LSFE) and the result (failure or safe) is stored. If the sample point is located in the failure domain (dot in Fig. 2), a line-search (see section 2.3) is performed in this direction to determine the point on the limit-state (see Fig. 2), usually requiring two-to-three G-function evaluations. The resulting distance $\tilde{\beta}_{opt}$ is a first approximation of

the distance to the MPP and is used to determine a new radius β (β_1 in Fig. 2) of the sphere. This new radius is selected somewhat smaller than $\tilde{\beta}_{opt}$ according to:

$$p_0 = 1 - \chi_n^2\left(\tilde{\beta}_{opt}^2\right)$$

$$\beta = \sqrt{\chi_n^{-2}\left(1 - \frac{p_0}{p_{step}}\right)} \tag{6}$$

in which 1-p_{step} multiplied by 100 denotes the percentage of samples located in the sample domain between the spheres β and $\tilde{\beta}_{opt}$, indicated by the dashed circles in Fig. 2. Only a new failure point located in this part of the domain triggers a new line-search, ensuring that a limited number (two-to-seven) of line-searches are performed to converge to the MPP. This is important, because all the points evaluated in a line-search cannot be added to the Monte-Carlo set and are therefore extra points reducing efficiency. After the line-search, the Monte-Carlo simulation is restarted using the same seed for the random number generator. In this way, the same set of sample points is regenerated and the information stored for points evaluated in a previous Monte-Carlo cycle is re-used.

The value of p_{step} should be selected close to 1, minimising the sample domain between β and $\tilde{\beta}_{opt}$. This prevents unnecessary sampling after the MPP has been located, since all samples in this domain are redundant. However, a value of p_{step} close to 1 can result in locking of the adaptive part of the algorithm, thereby producing erroneous results. In that case, no sample point is obtained in the failure region between both spheres before convergence (see section 4) has been reached. A value of p_{step} = 0.8 has proven to be a good choice.

The adaptive approach is robust and always converges to the MPP for any initial β-value, even if the initially selected β value is too small, i.e. in the safe domain away from the MPP. If the sphere is far from the MPP, this can result in a large number of simulations before the first failure point is found, because most sample points are located close to the sphere. This reduces the efficiency of the algorithm, because all sample points with a radius less than the final β_{opt} are not part of the final sample set. The efficiency is then vastly improved by performing a first line-search in a direction with negative G-gradient, i.e. the sample point has a lower G-value than the origin, instead of postponing the line-search until a failure point has been found.

A small initial p_0, which results in an initial sphere located far in the failure domain, is the best choice. In general, this quickly results in a first failure point. In structural reliability the probability of failure is usually small ($< 10^{-5}$). Selection of an initial p_0 of 10^{-6}, denoting the probability content outside the sphere including the part in the safe domain, generally suffices.

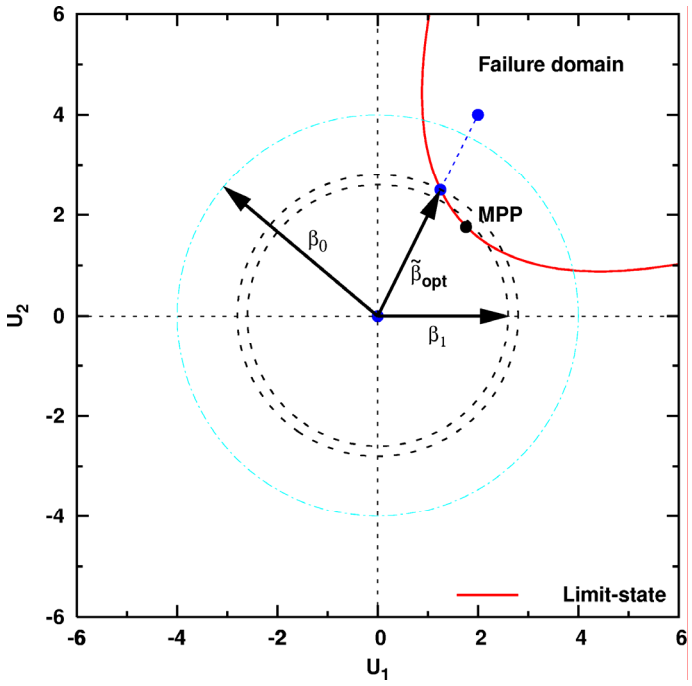


Fig. 2. Adaptive scheme to determine the MPP. β_0 initial radius, $\tilde{\beta}_{opt}$ first estimate of MPP and β_1 new radius.

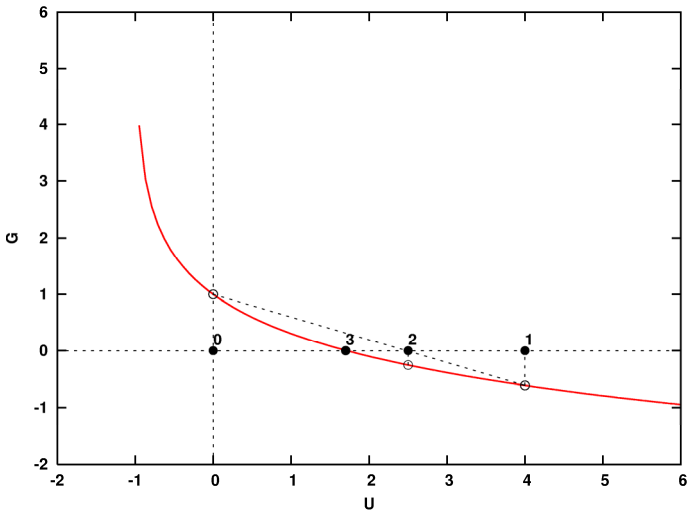


Fig. 3. Line-search procedure.

2.3 Line-search scheme

If a failure point (Fig. 2) is found with a distance to the origin that is less than the current minimum distance $\tilde{\beta}_{opt}$, a line-search is performed in that direction to locate the point on the limit-state that is an improved estimate of the MPP. The procedure is one-dimensional and schematically depicted in Fig. 3. The LSF-value at the origin is determine once at the start of ARBIS (point 0 in Fig. 3) and is used as a scaling value as well. A linear function is fitted through this point and the known failure point (point 1), thereby determining a first

estimate of the limit-state point (point 2). Next, the G-function value is determined in this point and a quadratic fit is made resulting in an improved estimate. This procedure is repeated until the limit-state point is found having an absolute error tolerance of 0.01. A higher accuracy is unnecessary, because Eq. (6) guarantees that the MPP is always located outside the sphere. Usually the process converges in two-to-three iterations. The search is aborted after a maximum of five iterations, to prevent spending an excessive amount of analysis time in rare cases.

3 Numerical Examples

The ARBIS method is here applied to a set of widely used test problems obtained from the literature, representing a broad range of possible limit-states that can occur in practice. The problems are summarized in Table 1, in which the last column gives the corresponding reference. Since these problems are used by various authors, the reference is not necessarily the first one. Because of the simple nature of the limit-state functions they can be evaluated many times, making a near exact evaluation possible by crude Monte-Carlo. This near exact value is given in column 5 of Table 1. Column 6 gives the final (optimal) β value.

The ARBIS method is examined on

- Efficiency

  This is reflected by the number of G-function evaluations necessary to obtain a converged solution. This number is compared with crude Monte-Carlo and RBIS.

- Robustness

  This reflects how the method performs in the case of a complex limit-state function: noisy failure function, highly non-linear failure function, multiple failure points and/or multiple failure functions.

- Accuracy

  The method always converges to the 'exact' solution provided enough samples are taken into account.

Sampling is ended when the maximum relative error in the probability value is below a threshold value. Hence an equal accuracy level is obtained with the Monte-Carlo, ARBIS and RBIS method and therefore their efficiencies can be compared. The maximum relative error is given by:

$$E_{\max}^{rel} = z_{\alpha/2}\, COV_{p_f} = \Phi^{-1}\left(\frac{\gamma+1}{2}\right) COV_{p_f} \tag{7}$$

where γ is the confidence level. For each failure point the current value of the COV_{Pf} is checked against a threshold value, where COV_{Pf} is given by:

$$COV_{Pf} = \sqrt{\frac{1-P_f}{N_{sim}\, P_f}} \tag{8}$$

For all problems the threshold coefficient of variation (COV) for the probability of failure p_f was set to 0.1. This means that with 95 % confidence the relative error in the estimate of the probability of failure p_f is less than:

$$E_{\max}^{p_f} = 1.96\,COV_{p_f} \approx 20\% \tag{9}$$

This accuracy is acceptable for most engineering applications. In general, the real error will be less than 10 %, which is often better than the errors produced in other parts of the analysis (e.g. accuracy of the underlying deterministic model and numerical model). Reducing the COV value reduces the error at the expense of more simulation.

The value obtained for the probability of failure with the ARBIS method, given the above accuracy level, is presented in column 5 of Table 1 between parentheses. The required number of simulations is presented in Table 2, columns 2 to 4, for respectively the Monte-Carlo method, ARBIS and RBIS with optimal radius. The fifth column of Table 2 presents the gain in efficiency of ARBIS over the Monte-Carlo method. RBIS was applied using the final β-value obtained in ARBIS, which is the optimal radius being close to the MPP. The difference in efficiency with ARBIS, presented in column 6, is therefore a measure of the efficiency of the adaptive scheme and reflects the number of G-function analyses spent in the subsequent line-searches to determine the MPP. These values show that the adaptive scheme is very efficient, because only a small number of additional G-function analyses are required compared with the optimal RBIS method.

3.1 Discussion

The various problems serve to demonstrate the efficiency and robustness of the ARBIS method. In all problems the same value for p_0 of 10^{-6} and p_{step} of 0.8 was used, see section 2.2. As explained in the previous section, all results have a similar accuracy level, by selecting a fixed value for the coefficients of variation of p_f. The relative error in p_f was well below the maximum expected error of 20% for all problems.

Robustness is demonstrated by the noisy limit-state of problem 1, multiple failure points of problem 2, highly non-linear limit-states of problems 6 to 8; multiple failure functions of problems 9 to 14, where multiple failure points are present in problems 12 and 14 as well. For all problems the ARBIS method proved to be very robust.

Fig. 4 shows sample plots obtained with ARBIS for the two-dimensional problems, clearly demonstrating the approach. The sample points inside the excluded sphere, see for example problems 5, 6 and 12, including the point in the origin, are related to the line-search method and are left out of the sample set used to calculate the probability value.

As expected, ARBIS is much more efficient for most problems than MCS. The gain in efficiency is less for problems 1 and 3, which have an increased number of variables combined with a high probability of failure value. The efficiency strongly depends on the probability value. The probability value is determined by the location of the MPP (β_{opt}) and shape of the limit-state. A rough estimate of the minimum number of required sample points is given by:

$$N > \left(\frac{z_{\alpha/2}}{\gamma}\right)^2 \frac{1-\chi_n^2\left(\beta_{opt}^2\right)}{p_f} \tag{10}$$

The farther away the location of the MPP, the larger is β_{opt}, reducing the number of simulations. This is reflected by the numerator of the above equation. Problem 2 shown in Fig. 4 is an example, demonstrating an extreme reduction in samples for small probabilities. On the other hand, for a very narrow shaped limit-state the contribution of the failure domain to the whole domain outside the sphere is small, increasing the required number of simulations. This is reflected by the denominator of the above equation. Problem 8 shown in Fig. 4 gives an example of a very narrow limit-state. The probability value for more realistic problems is in general less than 10^{-5}. The gain in efficiency is therefore expected to be (much) higher for more realistic problems than shown for some of the test problems.

Because of its robustness, ARBIS can be applied as a black-box and is of particular interest in situations where a low probability of failure is expected, such as in structural reliability analyses, consisting of a small number of stochastic variables. Like most sampling methods, ARBIS is suitable for application on a parallel computer, which is an ongoing trend [13], compensating for its lower efficiency compared with FORM and SORM by simultaneous analyses. Because of this, the robustness of the stochastic method becomes of increasing importance.

Table 1
Limit-state function descriptions

Case	Limit-state function	Stochastic variables	Description	P_f (P_f^{ARBIS})	β_{opt}	Ref.
1	$g = x_1 + 2x_2 + 2x_3 + x_4 - 5x_5 - 5x_6 + 0.001\sum_{i=1}^{6}\sin(100x_i)$	$x_{1...4}$: LN(120,12) x_5: LN(50,15) x_6: LN(40,12)	Linear LS with noise term	1.22e-02 (1.32e-02)	2.361	[4]
2	$g = x_1x_2 - 146.14$	x_1: N(78064.4, 11709.7) x_2: N(0.0104, 0.00156)	Multiple failure points	1.46e-07 (1.11e-07)	5.443	[4]
3	$g = 2 + 0.015\sum_{i=1}^{9}x_i^2 - x_{10}$	$x_{1...10}$: N(0, 1)	Quadratic LS 10 terms	5.34e-03 (5.6e-03)	2.103	[3]
4	$g = 0.1(x_1 - x_2)^2 - \frac{(x_1 + x_2)}{\sqrt{2}} + 2.5$	x_1: N(0, 1) x_2: N(0, 1)	Quadratic LS with mixed term, convex LS	4.16e-03 (3.71e-03)	2.481	[14]
5	$g = -0.5(x_1 - x_2)^2 - \frac{(x_1 + x_2)}{\sqrt{2}} + 3$	x_1: N(0, 1) x_2: N(0, 1)	Concave LS	1.05e-01 (1.12e-01)	1.625	[14]
6	$g = 2 - x_2 - 0.1x_1^2 + 0.06x_1^3$	x_1: N(0, 1) x_2: N(0, 1)	Non-linear LS with saddle point	3.47e-02 (3.58e-02)	1.996	[3]
7	$g = 2.5 - 0.2357(x_1 - x_2) + 0.00463(x_1 + x_2 - 20)^4$	x_1: N(10, 3) x_2: N(10, 3)	Highly non-linear LS	2.86e-03 (2.60e-03)	2.431	[15]
8	$g = 3 - x_2 + (4x_1)^4$	x_1: N(0, 1) x_2: N(0, 1)	Highly non-linear LS	1.80e-04 (2.03e-04)	2.925	[16]

Case	Limit-state function	Stochastic variables	Description	P_f (P_f^{ARBIS})	β_{opt}	Ref.
9	$g_1 = 2.677 - x_1 - x_2$ $g_2 = 2.500 - x_2 - x_3$ $g_3 = 2.323 - x_3 - x_4$ $g_4 = 2.250 - x_4 - x_5$ $g = \max(g_1, g_2, g_3, g_4)$	$x_{1...5}$: N(0, 1)	Parallel system	2.11e-04 (1.96e-04)	2.738	[16]
10	$g_1 = -x_1 - x_2 - x_3 + 3\sqrt{3}$ $g_2 = -x_3 + 3$ $\min(g_1, g_2)$	x_1: N(0, 1) x_2: N(0, 1) x_3: N(0, 1)	Series system	2.57e-03 (2.81e-03)	2.953	[2]
11	$g_1 = -x_1 - x_2 - x_3 + 3\sqrt{3}$ $g_2 = -x_3 + 3$ $\max(g_1, g_2)$	x_1: N(0, 1) x_2: N(0, 1) x_3: N(0, 1)	Parallel system	1.23e-04 (1.11e-04)	3.434	[2]
12	$g_1 = 2 - x_2 + \exp(-0.1x_1^2) + (0.2x_1)^4$ $g_2 = 4.5 - x_1 x_2$ $\min(g_1, g_2)$	x_1: N(0, 1) x_2: N(0, 1)	Series system Multiple failure points	3.54e-03 (4.51e-03)	2.925	[16]
13	$g_1 = 2 - x_2 + \exp(-0.1x_1^2) + (0.2x_1)^4$ $g_2 = 4.5 - x_1 x_2$ $\max(g_1, g_2)$	x_1: N(0, 1) x_2: N(0, 1)	Parallel system	2.50e-04 (2.03e-04)	3.219	[16]
14	$g_1 = 0.1(x_1 - x_2)^2 - \frac{(x_1 + x_2)}{\sqrt{2}} + 3$ $g_2 = 0.1(x_1 - x_2)^2 + \frac{(x_1 + x_2)}{\sqrt{2}} + 3$ $g_3 = x_1 - x_2 + 3.5\sqrt{2}$ $g_4 = -x_1 + x_2 + 3.5\sqrt{2}$ $g = \min(g_1, g_2, g_3, g_4)$	x_1: N(0, 1) x_2: N(0, 1)	Series system Multiple failure points	2.18e-03 (2.72e-03)	2.925	[14]

Table 2

Number of deterministic analyses required by the different stochastic methods

Case	**MCS**	**ARBIS**	**RBIS opt**	**Δ(MCS - ARBIS)**	**Δ(ARBIS – RBIS)**
1	7655	3520	3498	4157	22
2	$> 10^9$	67	60	$> 10^9$	7
3	17 830	16 674	16 555	1275	119
4	27 096	1215	1142	25 954	73
5	942	155	141	801	14
6	2734	307	281	2453	26
7	36 835	1914	1900	34 935	14
8	354 130	4867	4789	349 341	78
9	361 701	67 427	67 345	294 356	82
10	37 659	1096	1086	36 573	10
11	563 723	4484	4472	559 251	12
12	24 902	216	190	24 712	26
13	351 660	1930	1907	349 753	23
14	41 220	465	413	40 807	52

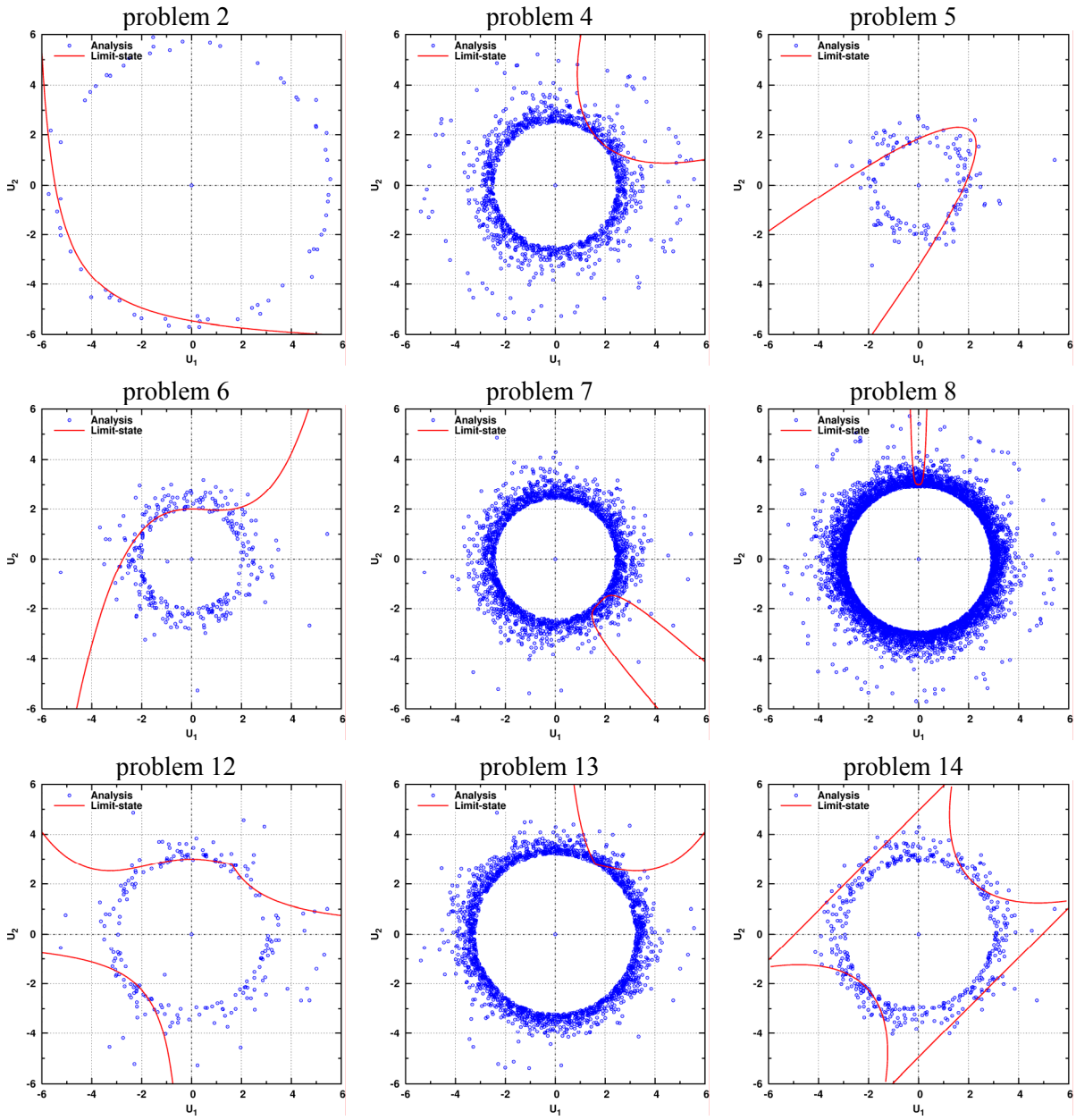


Fig. 4. Sample plots for the two-dimensional test problems.

4 Conclusion

Importance sampling methods are more efficient than Monte-Carlo Simulation and Directional Simulation, but require information about the location of the limit-state(s), especially the part closest to the origin in U-space. Gathering this information can be expensive and can fail to locate all the important parts. In this paper, the Radial-Based Importance Sampling method has been extended with a very efficient and robust adaptive scheme that automatically determines the optimal radius of the excluded sphere. For this reason the

method can be applied as a black-box and is of particular interest in applications with a low probability of failure, such as structural reliability, in combination with a small number of stochastic variables. Furthermore, the method is suitable to be applied on a parallel computer.

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