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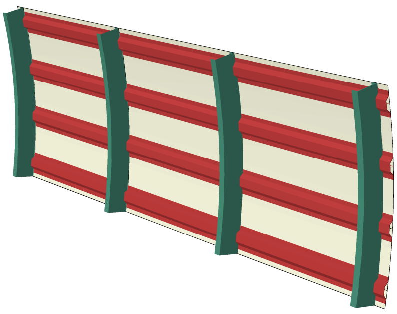
Executive summary

New reliability-based optimisation scheme applied to a composite fuselage panel



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Problem area
To minimise the weight and maximise the performance of aircraft, aircraft structures are optimised more and more in terms of geometry and applied materials. This results in structural designs with a reduced redundancy and as such a lower reliability. Since aerospace structures operate in harsh environments with great uncertainty in operating conditions, and failure can be fatal and costly, a high reliability level is required as well. Moreover, composite materials show a much higher scatter than the traditional aluminium alloys, further decreasing the reliability. The obtained reliability cannot be determined by means of the traditional deterministic approach, but requires a probabilistic approach.

Description of work
Within the framework of the EU-project MAAXIMUS the NLR in-house Reliability Analysis Program (RAP++) has been extended with the capability to optimise a structural design while obtaining a predefined reliability level, called reliability-based optimisation (RBO). Most RBO schemes are based on the FORM probabilistic method, which consists of an optimisation problem as well. The new scheme is based on an equally efficient probabilistic sampling method ADIS, which is much more accurate and robust than FORM.

Results and conclusions
The RBO scheme was demonstrated on a fuselage panel with a composite skin and compared against the deterministic optimum. A number of design variables are at the same time random variables and are taken into account as random design variables.

This report is based on a presentation held at the ECCOMAS Conference on the Mechanical Response of Composites 2011, Hannover, Germany, 21-23 September 2011.

New reliability-based optimisation scheme applied to a composite fuselage panel

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Summary

A new reliability-based optimisation scheme has been implemented in the NLR in-house probabilistic tool RAP++. Most RBO schemes are based on the FORM probabilistic method, which consists of an optimisation problem as well. The new scheme is based on an equally efficient probabilistic sampling method ADIS, which is much more accurate and robust than FORM. The RBO scheme will be demonstrated on a fuselage panel with a composite skin and compared against the deterministic optimum. A number of design variables are at the same time random variables and are taken into account as random design variables.

Contents

1 Introduction

In order to minimise the weight and maximise the performance of aircraft, aircraft structures are optimised more and more in terms of geometry and applied materials. This will result in structural designs with a reduced redundancy and as such a lower reliability. Since aerospace structures operate in harsh environments with great uncertainty in operating conditions and failure can be fatal and costly, a high reliability level is required as well. Moreover, composite materials show a much higher scatter than the traditional aluminium alloys, further decreasing the reliability. The obtained reliability cannot be determined by means of the traditional deterministic approach, but requires a probabilistic approach.

Within the framework of the EU-project MAAXIMUS the NLR in-house Reliability Analysis Program (RAP++) has been extended with the capability to optimise a structural design while obtaining a predefined reliability level, called reliability-based optimisation (RBO). This is an extension of the traditional deterministic optimisation in which a structure is optimised for a given objective function subjected to a number of constraints. In case of an RBO analysis an additional reliability constraint is added.
In this paper an RBO scheme will be described based on probabilistic sampling methods to obtain a more robust and accurate optimisation scheme. Furthermore, a new class of variables, random design variables, will be introduced that can handle model parameters which are a design variable in the optimisation analysis and at the same time a random variable in the reliability analysis. An example, is the composite skin thickness which is an important design variable, but can show considerable scatter that should be taken in to account in the reliability analysis. Hence, the assigned distribution function to model the scatter changes during the optimisation analysis.

2 RBO approach

For each constraint evaluation during the optimisation process, the reliability needs to be evaluated requiring a certain number of deterministic analyses. The probabilistic method applied therefore needs to be as efficient as possible. In table 1 a global overview is provided of the probabilistic methods available in RAP++. The most efficient methods are FORM, SORM and ADIS. Since FORM is based on an analytical expression, the accuracy is only guaranteed for a (almost) linear limit-state. The FORM method is not very robust since it consists of an optimisation analysis. Moreover, it cannot handle complex limit-states, e.g. having multiple failure points or parallel systems of failure functions. For the latter, analytical expressions [1] are available, but these only provide a crude approximation not suitable for reliability based optimisation.

Table 1: Quantitative comparison of different probabilistic methods

Method	Accuracy	Efficiency	Robustness
MCS (LHS)	High	Low	High
DS (ARBIS)	High	Medium-low	High
ADIS	High-medium	High-medium	High-medium
SORM	Medium-low	High-medium	Low
FORM	Low	High	Low

In case of multiple limit-states, the efficiency of FORM becomes less, because the problem needs to be solved for each limit-state function separately. The constraints applied in the optimisation part in general are also the failure functions used in the reliability part.
Hence, for a realistic problem often more than one limit-state function will be active. Not every constraint (failure function) will be equally important though in the reliability analysis. Therefore, some form of constraint screening (for instance by an initial FORM analysis) can be applied to remain with only the most important ones. For an inactive failure function, having no or a far away failure point, the number of function evaluations can become considerable as well, because the optimiser starts in an infeasible point and first has to locate a feasible point which might not exist or is far away in the probabilistic domain. Also, a failure function may become (in)active during the optimisation process due to a change in design variables. Another potential problem is the situation where the origin of the stochastic space lies in the failure domain yielding a probability value close to one. This might happen during the RBO process especially in the first few maxi-cycles starting from an infeasible point or because the Hessian matrix applied in the SQP is still too inaccurate to represent the second-order behaviour. A multiple limit-state problem then only can be solved by inversion of the reliability problem, such that the origin is located in the safe domain again, which is automatically done by RAP++. Due to this, a series system will change in a parallel system with the afore mentioned accuracy problems in case of FORM and SORM. Probabilistic sampling methods on the other hand do not suffer from this issue.
The SORM method, with respect to FORM, can only improve the accuracy for each individual limit-state requiring additional simulations to determine the curvature of the limit-state at the most probable point. The same reasoning as for FORM applies for SORM.

The most common RBO approach in literature, however, uses the FORM method, applying a gradient based optimiser like SQP for both the optimisation and FORM. Basically, two schemes are frequently applied [2]-[4]: a nested (two-loop) scheme and a single-loop scheme. In the

nested approach two separate optimisation problems are solved, one at the design level and one at the reliability level. The latter is performed inside the first each time the constraints are evaluated. In the single-loop approach both optimisations are performed simultaneously, satisfying the probability constraint only at the optimum. To this, the problem is reformulated such that a single optimisation problem results [3]. This approach is more efficient than the nested-loop approach. The accuracy strongly depends on the first-order assumption of the limit-states.
Summarising, a FORM or SORM based RBO scheme can be efficient, but is also not very robust and not able to handle complex sets of limit-state functions, and may also become inefficient. Hence, this approach is currently not applied in RAP++.

Another approach is to apply sampling probabilistic methods instead of the analytically based FORM or SORM. The efficiency of most sampling methods is far too low to be used directly in an RBO scheme, requiring too many function evaluations for each reliability analysis. Hence, these methods only work in combination with a (global) meta (surrogate) model. This surrogate model can be in terms of the random variables only (excluding the design variables), but then requires the generation of a new meta model for each change in design variables. This can be favourable when the number of design variables is much larger than the number of random variables. In other cases a combined design and random variable meta model is the best option. A sufficiently accurate global meta model however requires in general many response points (deterministic analyses) and quickly becomes (very) inefficient for increasing numbers of (design and random) variables.
A nested two-loop approach has been implemented in RAP++ using SQP as the optimiser and a sampling method for the reliability part. For efficiency reasons it is recommended to use DS over ARBIS and not to apply MCS and LHS. The latter are much more computational intensive to reach a similar accuracy. This especially is a drawback in situations where no failure points can be found (POF=0.), which might occur during the optimisation using a global response surface model, due to accuracy limits of the meta model in the outer regions.

A third approach is to apply an efficient sampling method like ADIS without a global meta model, which is equally efficient as FORM demonstrated by various examples [5], but more accurate and much more robust. At each reliability constraint evaluation a local response surface is generated that only has to approximate the most important region(s) of each limit-state. The required number of function evaluations is very limited, guaranteeing accuracy. The response points are automatically generated by the ADIS algorithm and mainly consist of points lying on or near the most important part(s) of the limit-state. This makes the algorithm very efficient and accurate. Contrary to FORM/SORM, the number of required function evaluations is independent of the number of limit-state functions. Apart from this, ADIS is capable to solve very complex limit-state problems [5]. This approach is also available in RAP++ and is the recommended one and will be applied in the fuselage panel example problem.

A model parameter can be a design variable and a random variable at the same time. To facilitate this a random design variable (RDV) is introduced. For instance an RDV is defined for the skin thickness of a plate being a random variable with a Normal distribution function and a design variable with a lower (minimum) and upper (maximum) bound. Besides the specification of the design variable bounds and distribution function, the user has to specify how the distribution is affected by a change in design variable, which effectively shifts the mean of the distribution function. The user has to specify how this affects the scatter (deviation). Currently, three options are implemented: 1) the distribution is shifted using a constant deviation, 2) the distribution is shifted using a constant deviation/mean, i.e. coefficient of variation, 3) the distribution is shifted using a constant variance/mean. The choice in particular depends on the type of scatter modelled and applied in the specification of the distribution.

Another required input quantity is the reliability constraint. This constraint is in terms of the reliability index β, which is related to the probability of failure Pf by the following non-linear expression or in tabular form:

Table 2: Probability of failure versus reliability index

P_f	β	P_f	β
0.1	1.28	1.00E-06	4.75
0.01	2.33	1.00E-07	5.20
0.001	3.09	1.00E-08	5.61
0.0001	3.72	1.00E-09	6.00
1.00E-05	4.26	1.00E-10	6.36

The optimisation problem can be formulated in terms of the probability of failure as well, but this is a highly non-linear function having strong gradients and as such much less suitable to act as a constraint. The reliability is calculated with an accuracy of COV-β = 0.01 (maximum error of 2%) to prevent an excessive amount of function calls.

Since an RBO analysis is computationally intensive, it is recommended to apply the following procedure:

1. Run an optimisation analysis to determine the deterministic optimum. Furthermore, the optimisation process can be examined for problematic issues and the deterministic optimum can be used as starting point for the RBO analysis.
2. Run a probabilistic sensitivity analysis of the initial design assigning a distribution to all relevant model parameters. The distribution functions may be based on a conservative estimate. This will determine the model parameters for which scatter is important, i.e. the ones that have to be treated as random variables. Only for these variables an accurate distribution function has to be supplied. When insufficient data is available to determine an accurate distribution function it is recommended to use an upper bound for the standard deviation. This will decrease the reliability, i.e. increase the probability of failure, being conservative.
3. Run a probabilistic sensitivity analysis of the optimal design, similarly as in the previous step. In the optimum the sensitivities have changed with respect to the initial design. This will assure that scatter in other model parameters is not important.
4. Run a reliability analysis of the initial design using ADIS and FORM/SORM. The probability of failure will indicate whether the initial design is feasible or not. If not, a new initial design can be chosen which will facilitate the RBO analysis. The results obtained with FORM, SORM and ADIS should in general be comparable. If this is not the case and the response surface is accurate, it is an indication of a complex limit-state problem, e.g. multiple failure points or highly non-linearity. The FORM analysis can be used to check the importance of the various limit-state functions.
5. Run a reliability analysis of the optimal design with ADIS and FORM/SORM. Again to determine the reliability and to check the accuracy of the response surface in case of ADIS.
6. Run the reliability based optimisation. The RBO analysis can start at the initial design. A more efficient approach, reducing the number of deterministic analyses, is to start at the deterministic optimum, because the RBO optimum is expected to be close to the deterministic one. The deterministic optimum on the other hand will be an infeasible point in general (too low probability), which is allowed. However, this can be avoided by selecting a new optimum by scaling the design variables which is more efficient. The value of this factor can easily be determined and checked by means of a (few) reliability analysis.

Table 3: Fuselage panel initial model parameters

Model parameter	Value	Unit	Model parameter	Value	Unit
Angle 0	0	degree	G23 Composite	4200	MPa
Angle 45	45	degree	Nu12 Composite	0.35	
Angle 90	90	degree	Radius	2012	mm
E1 Composite	157000	MPa	Skin Thickness	2	mm
E2 Composite	8500	MPa	Sp0degree	0.25	%/100
E Aluminium	72000	MPa	Sp90degree	0.25	%/100
Frame Height	83	mm	Stringer Angle	60	degree
Frame Lower Flange	22	mm	Stringer Height	25	mm
Frame Pitch	656	mm	Stringer Lower Flange Width	27	mm
Frame Thickness	2	mm	Stringer Pitch	150	mm
Frame Upper Flange Width	25	mm	Stringer Thickness	2	mm
G12 Composite	4200	MPa	Stringer Upper Flange Width	25	mm
G13 Composite	4200	MPa			

3 Composite fuselage panel example

In this example a typical fuselage panel being a cut-out of the fuselage of an aircraft, will be analysed. The panel, depicted in figure 1, consists of a thin composite skin (in white) and 4 frames (in green) in the circumferential direction and 4 stringers (in red) in the longitudinal direction to stiffen the structure. Both frames and stringers, are made of aluminium. The frames are U-shaped and the stringers are so-called hat-stiffeners. A fully parameterised model of the fuselage panel was created in Abaqus as a python script. The 27 model parameters and their initial values are provided in table 3. The skin is modelled by linear shell elements and the frames and stringer are modelled by linear beam elements with a section description corresponding to their shape. The beam elements simplifies the model and reduces the number of degrees of freedom. The approximation mainly lies in the increased effective size of the bay area in between two frames and stringers, which is normally smaller due to the added thickness of the frame and stringer feet. This will affect the buckling behaviour of the panel and yield somewhat lower buckling modes (conservative) being the design objective in this example. The panel is loaded on the right curved side with an uniform compressive load and a shear load equal to 30 % of the compressive load which is also applied on the longitudinal edges. The left curved edge is fully clamped and symmetry boundary conditions are applied on the longitudinal edges. Furthermore, the displacement in radial direction is suppressed on all edges.

3.1 Optimisation analysis

The fuselage panel is optimised for minimum mass. For the density of the composite and aluminium a value of 1600, respectively, 2800 kg/m3 has been applied. The buckling reserve factor should be larger than 1.0. A second constraint is set on the maximum stiffener (frames and stringers) displacement to be less than 5 % of the maximum skin displacement to prevent a global buckling mode of the panel.

The 9 design variables are the dimensions of the stringer, the stinger pitch, the skin thickness and the percentage 0 and 90 degree plies in the skin (Sp0degree and Sp90degree). The frame dimensions and frame pitch are set by other design criteria at the full barrel level and are therefore fixed. Since the stringer pitch is a design variable, the width of the panel will change during optimisation. To ensure a proper optimisation, the objective is in terms of the mass per square meter (mass ratio).

The optimisation converged in 10 maxi-cycles requiring 128 analyses. The optimised panel configuration is shown in figure 1. The minimum mass ratio reached is 6.5 kg/m2. The optimisation history is plotted in figure 2. The design variable values are given in table 6.

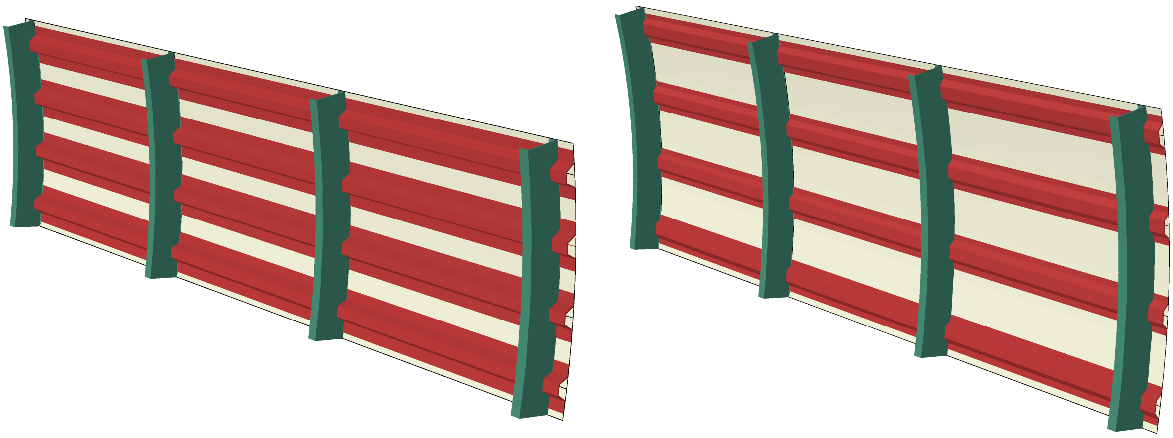

Figure 1 Initial (left) and optimal (right) fuselage panel design

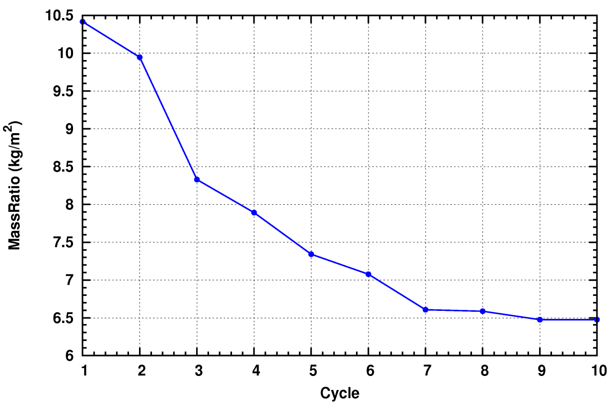

Figure 2 Mass ratio optimisation history for the fuselage panel

3.2 Sensitivity analysis of the initial and optimal design

The distribution functions assigned to all model parameters are provided in table 4. Conservative distribution functions were estimated using the following underlying ideas:

- An upper and lower bound have been estimated for each dimension and determines the three sigma (3σ) lower and upper bound reflecting a manufacturing tolerance. The estimated value can be found in the last column of the table. This type of scatter can best be represented by a normal distribution, i.e. symmetrical distribution with no preference for a lower or higher value.
- For a laminate angle and the stringer angle it has been assumed that the three sigma (3σ) lower and upper bound is 3 degrees. Again a Normal distribution seems to be the most appropriate one.
- For all thickness (skin, frame and stringer) a constant COV of 5 % has been applied, to take into account a smaller variation in thickness for thinner structures, e.g. for a laminate due to fewer number of layers.
- For all composite material properties an 8 % COV and a Normal distribution has been assumed. For the aluminium material properties a 4 % COV and a Normal distribution has been assumed. Both COV values are regarded upper bounds of the scatter seen in reality [6].
- For the load factor an extreme value Gumbel-max distribution has been assumed describing the scatter in the maximum load observed during each flight. A 15 % COV has been applied as scatter factor. Furthermore, the buckling load applied in the deterministic analysis is regarded as the limit load which is assumed to occur once during the lifetime of the aircraft. Assuming 100,000 flights the probability of occurrence is 1.e-5. i.e. the probability at which the load factor becomes 1.0 is 0.99999. This fully determines the load factor distribution.

The percentage 0 and 90 degree plies in the skin (Sp0degree and Sp90degree) are constants and determine the lay-up of the skin.

Table 4: Fuselage panel model parameter distributions

Model parameter	Distribution	Unit	COV	3σ
Angle 0	Normal(0, 1)	degree		3.
Angle 45	Normal(45, 1)	degree		3.
Angle 90	Normal(90, 1)	degree		3.
E1 Composite	Normal(157000, 12560)	MPa	0.08	
E2 Composite	Normal(8500, 680)	MPa	0.08	
E Aluminium	Normal(72000, 2880)	MPa	0.04	
Frame Height	Normal(83, 0.33)	mm		1.
Frame Lower Flange Width	Normal(22, 0.17)	mm		0.5
Frame Pitch	Normal(656, 3.3)	mm		10.
Frame Thickness	Normal(2, 0.1)	mm	0.05	
Frame Upper Flange Width	Normal(25, 0.17)	mm		0.5
G12 Composite	Normal(4200, 336)	MPa	0.08	
G13 Composite	Normal(4200, 336)	MPa	0.08	
G23 Composite	Normal(4200, 336)	MPa	0.08	
Load Factor	Gumbel(0.05134, 0.4094)		0.15	
Nu12 Composite	Normal(0.35, 0.028)		0.08	
Radius	Normal(2012, 3.3)	mm		10.
Skin Thickness	Normal(2, 0.1)	mm	0.05	
Stringer Angle	Normal(60, 1)	degree		3.
Stringer Height	Normal(25, 0.33)	mm		1.
Stringer Lower Flange Width	Normal(27, 0.17)	mm		0.5
Stringer Pitch	Normal(150, 3.3)	mm		10.
Stringer Thickness	Normal(2, 0.1)	mm	0.05	
Stringer Upper Flange Width	Normal(25, 0.17)	mm		0.5

A sensitivity analysis has been performed with this input. The buckling reserve factor should be larger than 1.0, which is the only failure function. The buckling reserve factor showed most sensitive for scatter in the load factor, skin thickness, composite modulus of elasticity in the fibre direction, stringer pitch, stringer thickness and modulus of elasticity of the aluminium, which are therefore the model parameters that are modelled as random variables in the remainder analyses.

3.3 Reliability analysis of the initial and optimal design

The obtained probability of failure for the initial design is about 8e-7 and predicted well with all methods applied, see table 5. MCS and LHS are not performed regarding the excessive amount of deterministic analyses required (about 14 million) to arrive at a solution of comparable accuracy. DS required over 13,000 simulations. ADIS is equally efficient as FORM and SORM, but much more robust and accurate in general. FORM and SORM show similar results indicating an approximately linear limit-state. The probability of failure of the optimal design has decreased to 4e-5. Again ADIS, FORM and SORM have comparable efficiency and are by far the most efficient methods. DS already requires about 11,000 simulations.

Table 5: Reliability analysis results of initial and optimal fuselage panel design

Probabilistic method	Pf	Number of analyses	Pf	Number of analyses
MCS/LHS	-	*14,000,000*	-	*1,200,000*
DS	7.9e-7	13,463	4.2e-5	10,938
ADIS	8.3e-7	86	4.3e-5	88
FORM	6.7e-7	56	3.5e-5	58
SORM	7.5e-7	95	4.1e-5	99

3.4 Reliability based optimisation

The probability of failure of the initial design is 8e-7 and for the deterministic optimum 4e-5. Suppose a POF of 1.e-6 (having a reliability index β of 4.75, see table 2) would be required, than the initial design would suffice, but the deterministic optimum would not be reliable enough. A reliability based optimisation is performed starting at the optimal design to meet the reliability constraint. The skin thickness, stringer thickness and stringer pitch are both design variable and random variable and are therefore modelled as random design variables. Both thicknesses have a fixed coefficient of variation and the pitch has a fixed standard deviation.

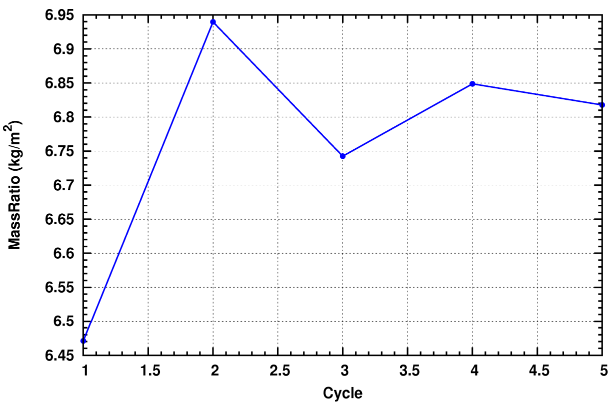

Figure 3 Mass evolution of the fuselage panel problem

The optimisation history is depicted in figure 3 and required 5 maxi-cycles requiring a total of 4269 deterministic analyses of which 4216 were required by the 67 reliability analyses. On average only 63 deterministic analyses were performed per reliability analysis, which again proofs the efficiency of ADIS. An optimum design lying close to the deterministic optimum, depicted in figure 4, is reached with a reliability index of β of 4.77 which is conform the required reliability. For the response surface of the Buckling reserve factor an R2 of 0.999 was obtained denoting a perfect local approximation of the most important part of the limit-state. The RBO values are given in table 6 together with the initial design and deterministic optimum.

The RBO design has a slightly higher mass ratio (6.82 kg/m2) than the optimal design (6.48 kg/m2), caused by a somewhat thicker skin thickness.

Table 6: Objective and design variable values for the fuselage panel

	Initial	**Optimal**	**RBO**	**Unit**
POF	8e-7	4.3e-5	1.E-6	
Mass Ratio	10.4	6.48	6.82	kg/m^2
Stringer Pitch	150	199.3	198.3	mm
Skin Thickness	2.0	2.108	2.317	mm
Sp0degree	0.25	0.163	0.170	%/100
Sp90degree	0.25	0.4	0.4	%/100
Stringer Lower Flange Width	27	15	15	mm
Stringer Upper Flange Width	25	30	29.6	mm
Stringer Height	25	15	15	mm
Stringer Thickness	2	1.63	1.63	mm
Stringer Angle	60	45	45	degree

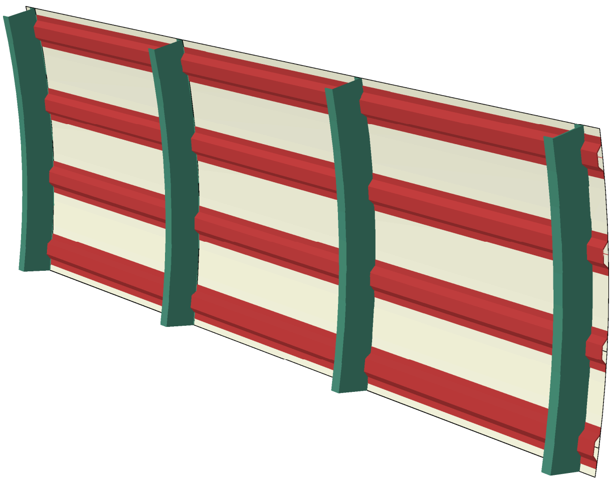

Figure 4 RBO optimal fuselage panel design

4 Conclusions

A new reliability-based optimisation scheme was presented based on a sampling probabilistic method and demonstrated on a composite fuselage panel.

Acknowledgement

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