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A symmetrical Boundary Element Formulation for Sound Transmission through Fuselage Walls - theory, implementation and test cases -

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ABSTRACT
In this paper a method to solve the sound transmission through a double wall structure is discussed. The finite element method is used to describe the structure and the boundary element method to describe the air and glasswool domains. The boundary element formulation used is a symmetrical one, instead of the more generally used non-symmetrical formulations. This boundary element formulation has been implemented by NLR in the mechanical analysis package B2000 and was coupled with the existing finite element code. The final model describing the coupled structural-acoustic problem consists of full, complex, symmetric and frequency dependent matrices. A solution strategy has been developed and implemented to solve this coupled structural-acoustic FEM/BEM problem. In the first part of this paper the boundary element model is evaluated by simple configurations. In part II of this paper the numerical results obtained in analyzing a double wall arrangement as was chosen as a common problem in the European cooperation programme BRAIN (Basic Research on Aircraft Interior Noise) are presented. The calculated results show a good agreement with the measurements.

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ABSTRACT

In this paper a method to solve the sound transmission through a double wall structure is discussed. The finite element method is used to describe the structure and the boundary element method to describe the air and glasswool domains. The boundary element formulation used is a symmetrical one, instead of the more generally used non-symmetrical formulations. This boundary element formulation has been implemented by NLR in the mechanical analysis package B2000 and was coupled with the existing finite element code. The final model describing the coupled structural-acoustic problem consists of full, complex, symmetric and frequency dependent matrices. A solution strategy has been developed and implemented to solve this coupled structural-acoustic FEM/BEM problem.

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1. INTRODUCTION

In aircraft industry one of the topics is the reduction of cabin noise. The noise in propeller aircrafts is mainly generated by the propulsion system. They produce a sound field in air that impinges on the outer surface of the fuselage. The vibrations caused by this sound field are transmitted through the fuselage wall and cause interior noise. The fuselage wall consists of a stiffened outer skin connected to frames and a trim panel which is mounted to the frames by means of connectors. The cavity in between the skin and trim is partly filled with glasswool and partly filled with air.

The cabin noise can be reduced by changing the double wall structure. For example, by choosing another type of glasswool or by adding a damping layer to the structure. To calculate the influence caused by these adjustments one must be able to calculate the noise level in the cabin due to an impinging acoustic field.

The aim of this study, which is carried out in the framework of the BRITE/AERO BRAIN (Basic Research on Aircraft Interior Noise) project is to get a better understanding of the fundamental mechanisms of noise transmission through double wall structures. It is aimed at identifying, understanding and quantifying these mechanisms and on this basis developing accurate models that describe the sound transmission process. One of the tasks was an experimental as well as a numerical analysis on a double wall arrangement that has been defined by the BRAIN partners to examine the accuracy of the different models. In figure 1 a schematic overview of the double wall set-up used in BRAIN is given.

In this paper the solution scheme to solve the sound transmission problem and numerical results obtained are discussed. To calculate the amount of acoustic power transmitted through the double wall structure, a combination of the finite (FEM) and boundary element method (BEM) is applied. The structure is modelled with existing shell and beam elements. For the modelling of the acoustic domains (air in- and outside the cavity and glasswool inside the cavity) a symmetrical boundary element formulation has been implemented and has been coupled with the existing finite shell elements.

In chapter 2 the symmetrical boundary element formulation will be discussed. With this formulation the air and glasswool domains are modelled. For both glasswool and air domains the same formulation can be applied, with the difference that the wavenumber is a complex quantity in the case of glasswool describing the energy dissipation of the material (see also ref. 2). The glasswool model used is a limp model, in which the stiffness of the fibre matrix (which is very low in case of glasswool) is neglected.

In chapter 3 the coupling of the BEM formulation with the existing FEM formulation will be discussed. The strategy used to solve the resulting coupled system of equations is discussed in section 3.2. After the solution of this system of equations a few post-processing steps have to be performed to obtain the quantities describing the sound transmission. This is covered in section 3.3.

In chapter 4 two testcases are treated to show the correctness of the code. The testcases are a double wall structure consisting of two completely clamped flat plates filled with air and glasswool, respectively.

2. SYMMETRICAL BEM FORMULATION

2.1 Introduction

In this section the main concept of the symmetrical boundary element formulation is discussed. For a more thorough description one is referred to (ref. 3).

In the boundary integral equation method two formulations can be distinguished to model the fluid, i.e. the direct method and the indirect method. In the direct method the problem is formulated in terms of variables with a direct physical meaning, one can use the pressure and normal derivative of the pressure (related to velocity). The indirect method uses variables with an indirect physical meaning like pressure jump and jump in the normal derivative of the pressure (velocity jump). The indirect method can be applied only when the fluid domains have the same properties (that is same Green’s function). However, the BRAIN problem consists of more than one fluid domain (glasswool, air) with different properties, resulting in different Green’s functions. So, only the direct method can be applied here.

The acoustic pressure in the air domains has to satisfy the Helmholtz equation

$$\Delta p + k^2 p = 0 \tag{2.1}$$

in which k is the wavenumber ($k=\omega/c$), where c is the speed of sound.

The main purpose of the glasswool in between the outer and inner fuselage of an aircraft is to provide thermal insulation. In addition, the material has also some acoustical damping properties. To get a correct model for the dynamic behaviour of a porous material one should take into account a full coupling between the elastic solid phase and the compressible fluid in the pores. In the framework of the BRAIN project, this has been done for the type of glasswool that is used in aircraft industry. Unfortunately, this mathematical model is not suitable for a boundary element implementation, as the dynamics of the material cannot be described by one single Green’s function. The dynamic behaviour can be reasonably approximated by a limp model, in which the stiffness of the solid fibres is neglected (ref. 2). As a consequence, the glasswool can be modelled as an equivalent fluid (so-called limp model) and is modelled by the Helmholtz equation having a complex wave number to account for the energy dissipation in the material. The wavenumber becomes a function of the porosity (h), viscous flow resistance (ϕ), air density (ρ) and bulk density (M).

$$k_{glasswool} = \frac{\omega}{c_p}\sqrt{(\kappa(\omega)\frac{h}{\rho})} \quad ; \quad \kappa(\omega) = 1 - \frac{\frac{\hat{i}\,\phi\,h}{\omega\,\rho}}{1 - \frac{\hat{i}\,h\,\phi}{\omega\,M}} \tag{2.2}$$

in which c_p is the isothermal speed of sound.

Due to the neglection of the stiffness of the fibre matrix, the resonance frequencies of this fibre matrix can not be described. Therefore the model becomes less accurate when the wavelength in the glasswool becomes smaller or in the neighbourhood of the thickness of the glasswool layer.

A distinction must be made between the different domains. There are bounded domains, the glasswool and air inside the cavity, and there are unbounded domains, the air beneath and above the cavity. For both domains a different boundary integral has to be solved, which will be discussed in the next two sections.

2.2 Boundary integral formulation for bounded domains

In order to obtain the pressure at a point on the boundary Γ of the fluid domain, the following boundary integral equation has to be solved:

$$\frac{p(r)}{2} = \int_{\Gamma}\left(G(r,r')\frac{\partial p(r')}{\partial n'} - \frac{\partial G(r,r')}{\partial n'}\,p(r')\right)d\Gamma(r') \tag{2.3}$$

where p is the pressure (fluctuation), n’ the normal on the boundary pointing outwards, r is the coordinate at the point on the boundary in which the pressure has to be calculated and r’ the coordinate of the integration point. G is the fundamental solution of the Helmholtz equation for a point source (Green’s function):

$$G(r,r') = \frac{e^{-\hat{i}\,k\,|r - r'|}}{4\,\pi\,|r - r'|} \tag{2.4}$$

in which k is the wavenumber, which is a complex quantity in the case of glasswool (2.2).
A similar integral equation can be obtained for the normal derivative of the pressure.

$$\frac{1}{2}\frac{\partial p(r)}{\partial n} = \int_{\Gamma}\left(\frac{\partial G(r,r')}{\partial n}\,\frac{\partial p(r')}{\partial n'} - \frac{\partial^2 G(r,r')}{\partial n'\partial n}\,p(r')\right)d\Gamma(r') \tag{2.5}$$

Both (2.3) and (2.5) are only valid in this form for smooth boundaries.

Taking the weak formulation of both integral equations, that is multiplying them by a weight function (w_1 and w_2 respectively) and integrating over the domain, results in:

$$\frac{1}{2}\int_\Gamma w_1(r)\,p(r)\,d\Gamma(r) = \int_\Gamma\int_\Gamma w_1(r)\,G(r,r')\,\frac{\partial p(r')}{\partial n'}\,d\Gamma(r')\,d\Gamma(r) - \int_\Gamma\int_\Gamma w_1(r)\,\frac{\partial G(r,r')}{\partial n'}\,p(r')\,d\Gamma(r')\,d\Gamma(r) \qquad (2.6)$$

and

$$\frac{1}{2}\int_\Gamma w_2(r)\,\frac{\partial p(r)}{\partial n}\,d\Gamma(r) = \int_\Gamma\int_\Gamma w_2(r)\,\frac{\partial G(r,r')}{\partial n}\,\frac{\partial p(r')}{\partial n'}\,d\Gamma(r')\,d\Gamma(r) - \int_\Gamma\int_\Gamma w_2(r)\,\frac{\partial^2 G(r,r')}{\partial n'\,\partial n}\,p(r')\,d\Gamma(r')\,d\Gamma(r) \qquad (2.7)$$

These equations are solved by the use of boundary elements. The boundary is hereby divided into elements and the pressure and its normal derivative are discretised according to:

$$p(r) = \sum_{i=1}^{n} N_i(\eta)\,P_i = N\,P \qquad (2.8a)$$

$$\frac{\partial p(r)}{\partial n} = \sum_{i=1}^{n} N_i(\eta)\,\frac{\partial P}{\partial n_i} = N\,\frac{\partial P}{\partial n} \qquad (2.8b)$$

where n is the number of nodes of one element. N is the vector containing the interpolation functions.

Note: variables are denoted in lower case, discretised variables in the i^{th} node are denoted in upper case with index i, vectors and matrices for all nodal degrees of freedom are denoted in upper case without index.

With the above discretisation and taking the weight functions w_{1i} and w_{2i} the same as the interpolation functions N_i (Galerkin method) yields the matrix-vector equations:

$$\frac{M}{2}\,P = V'\,\frac{\partial P}{\partial n} - K\,P \qquad (2.9)$$

$$\frac{M}{2}\,\frac{\partial P}{\partial n} = K^T\,\frac{\partial P}{\partial n} - D\,P \qquad (2.10)$$

where P is the vector containing the pressure degrees of freedom, ∂P/∂n the vector containing the normal pressure derivative degrees of freedom. V’, K and D are the so-called influence coefficient matrices. For example, the coefficient V'_{ij} describes the influence of a source of strength ∂P/∂n located in a node j on the pressure in a node i. In the same way the coefficient K_{ij} describes the influence of a dipole of strength P in a node j on the pressure in a node i. All three matrices (V’, K and D) are *full*, *complex* (in the sense of having an imaginary part) and *frequency dependent*. The matrix K is also *non-symmetric*, while V’ and D are *symmetric*.

The matrix M is a sort of mass matrix (from a numerical point of view, i.e. its coefficients

are found by multiplied interpolation functions). This matrix has a *banded structure*, is *symmetric* and *not frequency dependent*.
Rewriting both (2.9) and (2.10) results in:

$$M\,P = V'\,\frac{\partial P}{\partial n} + \left(\frac{M}{2} - K\right)P \tag{2.11}$$

and

$$P = -D^{-1}\left(\frac{M}{2} - K^{T}\right)\frac{\partial P}{\partial n} \tag{2.12}$$

Substituting (2.12) in (2.11) finally results in an expression describing the relation between the pressure and its normal derivative on the boundary of the fluid domain.

$$M\,P = \left[V' - \left(\frac{M}{2} - K\right)D^{-1}\left(\frac{M}{2} - K^{T}\right)\right]\frac{\partial P}{\partial n} \tag{2.13}$$

or

$$M\,P = T'(\omega)\,\frac{\partial P}{\partial n} \tag{2.14}$$

From the above it can be concluded that T' is a *full*, *complex*, *symmetric* and *frequency dependent* matrix. The ω indicating the frequency dependency will be left out most of the time for convenience. This is a symmetrical formulation, instead of the use of equation (2.11) or (2.12) directly, which would have resulted in non-symmetric matrices.

Noted here is that this is not the only symmetric formulation possible. From (2.9) and (2.10) also a symmetric formulation can be derived with P and ∂P/∂n interchanged. The reason for choosing the formulation of (2.14) instead of the other one will become clear in sections 3.1 and 3.2, where the coupling of these equations with a FEM model of the structure and the solution strategy of it are discussed.

The closed fluid domain in the BRAIN problem is bounded by a structure consisting of flexible walls and rigid side walls. The boundary conditions for this can be formulated as:

$$\frac{\partial p(r)}{\partial n} = \rho\,\omega^2\,u_s(r)\cdot n \qquad \text{on } \Gamma_f \tag{2.15a}$$

$$\frac{\partial p(r)}{\partial n} = 0 \qquad \text{on } \Gamma_r \tag{2.15b}$$

in which u_s stands for the displacement of the structure, ρ the density and n the vector normal to the fluid boundary. Γ_f is the part bounded by a flexible wall and Γ_r by a rigid wall. Taking the weak formulation of these boundary conditions and discretise this relation using the same interpolation functions as (2.8) yields:

$$M\,\frac{\partial P}{\partial n} = \rho\,\omega^2\,C\,U_s \tag{2.16}$$

in which C is the so-called couple matrix describing the relation between the displacements of the structure and pressure of the fluid. Substituting this relation in (2.14) yields the final matrix-vector equation:

$$P = \rho\,\omega^2\,T(\omega)\,C\,U_s \tag{2.17a}$$

with

$$T(\omega) = M^{-1}\left[V'(\omega) - \left(\frac{M}{2} - K(\omega)\right)D^{-1}(\omega)\left(\frac{M}{2} - K^T(\omega)\right)\right]M^{-1} \tag{2.17b}$$

The computation of the matrix T involve the discretization of a hyper-singular, a weakly singular and a regular integral operator. The hyper-singular integral operator is regularized by integration of parts. Once this operator has been regularized, only weakly singular and regular integrals remain (see ref. 4). It can be shown that the number of kernel evaluations needed for the computation of the coefficients of the matrix T are comparable with the costs of classical boundary element formulations (non-symmetrical) derived from (2.9) or (2.10).

2.3 Boundary integral formulation for unbounded domains

For an unbounded domain again (2.3) is used as the basis equation. Again Γ is the surface of the fluid part bounded by the structure (that is trim or skin panel, the fluid boundary condition in infinity (Sommerfeld) already has been taken into account in the formulation).
Equation (2.3) can be simplified, because the BRAIN problem consists of flat plates only. The second integral can then be eliminated, due to the fact that the normal derivative of the Green's function is zero for that case, i.e.

$$\frac{\partial G}{\partial n} = \nabla G \cdot n = G\,(1 - \hat{i}\,k)\,\frac{r \cdot n}{|r|^2} = 0 \tag{2.18}$$

with $G = e^{-ikr}/4\pi r$ and $r \cdot n = 0$ for flat surfaces.
Also the first integral only has to be solved for the trim panel, because the remainder part of the structures surface consists of a rigid wall, i.e.:

$$\frac{\partial p(r)}{\partial n} = 0$$

So,

$$\frac{p(r)}{2} = \int_{\Gamma_f} G(r,r')\,\frac{\partial p(r')}{\partial n'}\,d\Gamma(r') \tag{2.19}$$

Taking again the weak formulation of this equation and the same discretisation as (2.8), we obtain the matrix-vector equation:

$$M\,P = 2\,V'\,\frac{\partial P}{\partial n} \tag{2.20}$$

The boundary conditions again result in the same relation as (2.16). Substituting this relation in (2.20) finally results in the matrix-vector equation:

$$P = 2\,\rho\,\omega^2\,V(\omega)\,C\,U_s \tag{2.21a}$$

with

$$V(\omega) = M^{-1} V^{/}(\omega) M^{-1} \tag{2.21b}$$

Also V is a *full, complex, symmetric* and *frequency dependent* matrix.

3. SOLUTION STRATEGY

3.1 Coupled structural-acoustic problems

The coupled structural-acoustic problem is described by three matrix-vector equations. The first equation describes the dynamical behaviour of the structure (here a double wall structure), yielding:

$$(K_s(\omega) - \omega^2 M_s) U_s = C_b^T P_b + C_u^T P_u + F^{ext} \tag{3.1}$$

The left-hand-side consists of the stiffness (K) and mass (M) matrix of the structure, both symmetric, and the structural displacement vector U_s. The stiffness matrix is complex and frequency dependent taking into account the energy dissipation (damping) of the structure. For this, a special damping model was implemented, the so-called Augmented Hooks Law (AHL).

The right-hand-side expresses the loading on the structure and consists of three terms: the pressure loading of the fluid inside the cavity of the double wall, the pressure loading of the fluid outside the cavity and an external force distribution, respectively.
With indices u and b is meant the matrices and vectors of the unbounded and bounded domain, respectively.

The second equation describes the dynamical behaviour of the fluid inside the cavity, expressed by (2.17):

$$T^{-1} P_b = \rho\, \omega^2 C_b U_s \tag{3.2}$$

The third equation describes the dynamical behaviour of the fluid outside the cavity, expressed by (2.20):

$$V^{-1} P_u = \rho\, \omega^2 C_u U_s \tag{3.3}$$

in which the right-hand-sides express the pressure loading on the fluid due to accelerations of the structure.

The matrix-vector equation describing the complete coupled structural-acoustic problem then yields:

$$\begin{bmatrix} K_s(\omega) - \omega^2 M_s & C_b^T & C_u^T \\ C_b & -\dfrac{T^{-1}(\omega)}{\rho\, \omega^2} & 0 \\ C_u & 0 & -\dfrac{V^{-1}(\omega)}{\rho\, \omega^2} \end{bmatrix} \begin{Bmatrix} U_s \\ P_b \\ P_u \end{Bmatrix} = \begin{Bmatrix} F^{ext} \\ 0 \\ 0 \end{Bmatrix} \tag{3.4}$$

The solution of this system of equations is discussed in section 3.2.

In the above equations it has been assumed that the cavity can be described by only one matrix equation. This is true, also for the case where the cavity is partly filled with air and partly filled with glasswool. In that case we have two sub-domains described by similar equations (2.17):

$$P^{a} = T^{a} Q^{a} \quad ; \quad P^{g} = T^{g} Q^{g}$$

where Q is:

$$Q = \rho\, \omega^2 C_b U_s$$

and the superscripts "a" and "g" stand for air and glasswool, respectively.
Both parts have a mutual boundary (interface). The boundary conditions on that interface are:

$$P^{a} = P^{g} \tag{3.5}$$

representing force equilibrium at the interface, and:

$$U^{a} \cdot n = U^{g} \cdot n \, \frac{1 + \hat{i}\,\dfrac{M\,\omega}{\phi}}{h + \hat{i}\,\dfrac{M\,\omega}{\phi}}$$

representing the continuity of the mass flow over the interface (see expression 2.2 for an explanation of the symbols). The porosity h for the glasswool used in BRAIN equals 0.9955, therefore this equation can be simplified to:

$$U_n^{a} = U_n^{g}$$

Applying Euler's equation, this can be rewritten as:

$$\frac{\partial P^{a}}{\partial n} = -\frac{\rho^{a}}{\kappa^{g}(\omega)} \frac{\partial P^{g}}{\partial n} \tag{3.6}$$

with $\kappa(\omega)$ according to (2.2). The minus sign is caused by the opposite direction of the normal on the interface of both fluid (glasswool and air) domains.

On the interface there is a continuity of pressure, but a jump in the derivative of the pressure. Eliminating the degrees of freedom on the interface, by the use of (3.5) and (3.6), finally results in a new T matrix describing the whole cavity. This matrix is non-symmetric due to the pressure jump on the interface, however, after coupling it with the structure it becomes symmetric again.

3.2 Solution scheme

In this section the solution scheme used to solve equation (3.4) is discussed.
The first step is the elimination of the pressure degrees of freedom of (3.4), resulting in:

$$[K_s(\omega) - \omega^2 (M_s + \Delta M_b(\omega) + \Delta M_u(\omega))]\, U_s = F^{ext} \tag{3.7a}$$

with

$$\Delta M(\omega)_b = \rho\, C_b^{T}\, T(\omega)\, C_b \tag{3.7b}$$

$$\Delta M(\omega)_u = \rho\, C_u^T\, V(\omega)\, C_u \tag{3.7c}$$

where ΔM_b and ΔM_u can be seen as the "mass" matrices of the bounded and unbounded fluid domain, respectively, which are added to the mass matrix of the structure. Both matrices are frequency dependent and could also have been added, multiplied by ω^2, to the stiffness matrix. Part of the matrices will therefore give rise to an added stiffness effect and the other part will give rise to an added mass effect (their imaginary parts will give rise to an added damping effect), depending among others on the boundary conditions of the problem.

Eliminating the degrees of freedom out of the coupled system of equations (3.4) results in an equation containing the matrix T and V. As discussed in section 2 another symmetric formulation could have been applied, which however would have resulted in an inverse of the matrix T, which would be non-attractive from a computational point of view.

Both ΔM_b and ΔM_u are again full matrices. Therefore, the addition of both matrices will destroy the banded structure of the finite element matrices K and M and give rise to an enormous increase in solution time. To overcome this problem the equations are transformed by the use of a transformation matrix Φ, consisting of the damped (complex) modeshapes of the structure, a so-called modal base. The displacement vector then becomes:

$$U_s = \Phi\, \tilde{U}_s \tag{3.8}$$

$\tilde{U}_s$ contains the displacements of the structure in the so-called modal coordinates.
The transformed system of equations, pre- and post-multiplied by the transformation matrix, yields:

$$[\tilde{K}_s(\omega) - \omega^2 (\tilde{M}_s + \Delta\tilde{M}_b(\omega) + \Delta\tilde{M}_u(\omega))]\, \tilde{U}_s = \Phi^T F^{ext} \tag{3.9a}$$

with

$$\Delta\tilde{M}_b = \rho_b\, \Phi^T C_b^T\, T\, C_b\, \Phi \tag{3.9b}$$

$$\Delta\tilde{M}_u = \rho_u\, \Phi^T C_u^T\, V\, C_u\, \Phi \tag{3.9c}$$

and $\tilde{K}$ and $\tilde{M}$ the modal stiffness and mass matrix.
Besides the elimination of the destroyed band problem, this also leads to a reduction in the number of degrees of freedom, resulting in a faster solution of the system.

As stated above the modal base consists of the damped modeshapes of the uncoupled structural problem. This means that the eigenvalue problem of the damped structure has to be solved:

$$(K_s(\omega) - \omega_i^2 M_s)\, \phi_i = 0 \tag{3.10}$$

The problem arising now is that the stiffness matrix is frequency dependent, which makes the eigenvalue problem non-linear. However, we may assume that the shape of the modes do not depend very much on the frequency. In other words the shape will be more or less the same for the whole frequency domain under view. This in contrary to the eigenvalues, which will depend more strongly on the choice of ω.

The modeshapes now may be obtained once by selecting a ω-value for the calculation of the stiffness matrix K, e.g. the centre frequency of the frequency domain under view.
Another important fact is the need for a complex eigenvalue solver, as K is a complex matrix.

Therefore, the eigenvectors will be complex as well.

The final step is the solution of equation (3.9). This has to be done by means of a response analysis, with which the modal participation factors ($\tilde{U}_s$) are determined. Applying equation (3.8) yields the response of the structure in terms of the initial degrees of freedom. In order to obtain the response on the boundary and in the fluid domain a number of postprocessing steps have to be performed, which are discussed in the next section.

3.3 Post-processing

In section 3.2 is discussed how to obtain the response (displacements) of the structure for the coupled structural-acoustic problem. This section discusses how to obtain the response (pressure and its derivative) of the fluid domain, once the response of the structure has been calculated.

The response for the degrees of freedom of the boundary element mesh can be found by applying equations (3.2) and (3.3), for the bounded and unbounded domains, respectively. The normal derivative of the pressure on the boundary simply can be found from (2.15).

Once the pressure and its normal derivative have been obtained, the response can be calculated at an arbitrary point within the fluid domain. This is done with a similar equation as (2.3), but now for a point within the volume instead of one on the boundary, where (2.3) is valid.

$$p(r) = \int_{\Gamma} \left(G(r,r') \frac{\partial p(r')}{\partial n'} - p(r') \frac{\partial G(r,r')}{\partial n'} \right) d\Gamma(r') \tag{3.11}$$

So, in order to obtain the pressure for an arbitrary point within the volume, an integration over the boundary Γ of the fluid domain has to be performed. Therefore, the values of the pressure and normal derivative, as calculated by (3.2), (3.3) and (2.15), should be known. Discretising the integral equation results in:

$$P_{\text{point in volume}} = a^T \frac{\partial P}{\partial n} - b^T P \tag{3.12}$$

where P and its normal derivative are the vectors containing the values on the boundary.
Also, a similar integral equation as (3.11) can be derived for the derivative of the pressure at a point within the fluid domain.

The next step in the post-processing phase is the calculation of the sound intensity for points above the trim panel (so-called fieldpoints, see figure 1). The time averaged sound intensity is defined as:

$$I(x) = \frac{1}{T} \int_0^T p\, v\, dt \tag{3.13}$$

in which p and v stands for pressure and velocity at a point in the volume, respectively. This is a vector quantity. For a sinusoidal sound field the sound intensity component in n-direction can be calculated by (see ref. 5):

$$I_n = \frac{1}{2} \mathrm{Re}\,[\,P\,V_n^*\,] \qquad (3.14)$$

in which P and V are the complex pressure and velocity amplitudes. A * expresses the complex conjugate.
For a sinusoidal sound field, the velocity can be written as:

$$V_n = \frac{\hat{i}}{\omega\,\rho} \frac{\partial P}{\partial n} \qquad (3.15)$$

From this equation we see that the velocity requires the calculation of the derivative of the pressure. The derivative of the pressure can be approximated by a first order finite difference scheme:

$$\frac{\partial P}{\partial n} = \frac{P_2 - P_1}{\Delta} \qquad (3.16)$$

avoiding the time consuming explicit calculation of the derivative, which is an integral equation comparable to (3.11). So, the pressure in two points in n-direction are used to calculate the derivative. The approximation is valid as long as the distance Δ between the two pressure points is small compared to the wavelength.

Substituting (3.15) and (3.16) for the velocity amplitude and the mean of the pressure values at the two points, for the pressure amplitude, in (3.14), yields:

$$I_n = \frac{-1}{4\,\rho\,\omega\,\Delta} \mathrm{Re}\,[\,(P_1 + P_2)\,\hat{i}\,(P_2^* - P_1^*)] \qquad (3.17)$$

which easily can be simplified to:

$$I_n = \frac{1}{2\,\rho\,\omega\,\Delta}\,\mathrm{Im}[P_1\,P_2^*] = \frac{1}{2\,\rho\,\omega\,\Delta}\,(P_{2r}\,P_{1i} - P_{1r}\,P_{2i}) \qquad \left[\frac{W}{m^2}\right] \qquad (3.18)$$

in which the indices r and i stands for the real and imaginary parts, respectively.
Once the intensity has been calculated other quantities, such as transmission loss can be obtained as well.

4. Numerical results - Test problems

The symmetrical boundary element formulation has been implemented in the modular finite element program B2000 (ref. 6) and has been coupled with the existing finite element code as described in chapter 3. The boundary element implemented is a three nodes (linear) triangular element.

In this chapter two numerical examples (eigenvalue analysis) will be discussed to show the correctness of the implemented code. The structure is modelled with finite elements and the fluid with boundary elements. The results are compared with calculations in which the fluid has been modelled with finite acoustic elements, instead of boundary elements.

In both examples the structure consists of two identical parallel plates, representing the double wall, with a length of 1.46 m and a width of 0.76 m (see fig. 2), which are clamped at all the edges. The distance between the plates is 0.1 m. Each of the plates are modelled by

10*10 linear quadrilateral finite shell elements. The basic properties of the plates used are (aluminium):

Modulus of Elasticity - *7.0E+10* N/m^2
Poisson's ratio - *0.3*
Density - *2800.* *kg/m3*
Thickness - *0.0025* *m*

4.1 Double wall filled with air

In this first testcase the cavity between the plates is filled with air. The air surrounding the plates is left out, to be able to compare the results with calculations done using finite acoustic elements. The air properties used are:

Density - *1.2* kg/m^3
Speed of sound - *340* *m/s*

The air in the cavity has been modelled by 200 triangular elements on each plate (in such a way that the nodes coincide with the nodes of the structural finite element mesh) and 320 triangular elements on the four vertical faces closing the cavity between the plates.

The first five eigenfrequencies for the structure without the fluid (in vacuo) are given in the second column of table 1. The coupled eigenfrequencies are given in the third column. The fourth column shows the results for the calculation in which finite acoustic elements were used to model the fluid instead of boundary elements. The eigenfrequency is defined here as: the root of the complex eigenvalue divided by 2π.

A good agreement of the results is observed between both methods. The small differences are due to differences in the numerical methods, e.g integration rules, type of elements.

Table 1 Eigenfrequencies two plate configuration with air in between

MODE	VACUO	BEM	FEM
1	13.22	11.95	12.27
2	13.22	13.16	13.16
3	22.02	20.83	21.02
4	22.02	21.93	21.93
5	38.21	36.98	37.14

4.2 Double wall filled with glasswool

In this second testcase the cavity between the plates is filled with glasswool, instead of air, having the following properties:

Air density - *1.2* kg/m^3
Speed of sound - *290* *m/s*

Bulk density	- *2267.*	*kg/m*^{3}
Porosity	- *0.9955*	
Viscous flow resistance	- *2.3E4*	*N/(s m*^{2}*)*

The limp model used for the glasswool, has a complex wavenumber, as defined in equation (2.2), expressing the energy dissipation in the material. The κ(ω) term in this expression is frequency dependent, therefore the eigenvalue problem now becomes non-linear.
The same number of elements have been used as in section 4.1.

The five coupled eigenfrequencies obtained with the use of boundary elements are given in the third column of table 2. The uncoupled frequencies, which are the same as in table 1, are given in the first column. In the fourth column of table 2 the results are shown for the calculation in which finite limp elements were used to model the glasswool instead of boundary elements. The eigenfrequencies now have become complex too, due to the energy dissipation.

A good agreement of the results is observed between the two numerical methods. The small differences in the real parts are again due to differences in the two numerical methods, but also due to the non-linearity of the eigenvalue problem.

To solve the eigenvalue problem a start value of ω has to be specified to calculate the κ(ω) term. When for example a value of 30 Hz would have been chosen for the calculation of κ(ω), the first eigenfrequency would have been 7.84+i*0.190, demonstrating the dependency of the problem on the value of κ(ω). Especially the imaginary parts are strongly dependent on this value. Every eigenfrequency, for BEM as well as FEM, has been obtained by choosing a real value close to the real part of the eigenfrequency. However, the unknown complex eigenvalue should have been substituted instead, in order to obtain the correct eigenfrequency.

The frequency dependency is amplified by the boundary element method because of the exponential functions (by the Green’s function), which depend on the frequency, making it strongly frequency dependent.

Table 2 Eigenfrequencies two plate configuration with glasswool in between

MODE	VACUO	BEM	FEM
1	13.22	7.46+i*0.161	7.84+i*0.081
2	13.22	12.73+i*0.019	12.62+i*0.018
3	22.02	14.97+i*0.188	15.05+i*0.153
4	22.02	21.21+i*0.054	21.93+i*0.049
5	38.21	29.61+i*0.657	29.29+i*0.466

5. CONCLUSIONS

In this paper a method to solve the sound transmission through a double wall structure has been discussed. The finite element method has been used to describe the structure and the boundary element method to describe the air and glasswool domains.

The boundary element formulation used is a symmetrical one, instead of the more generally used non-symmetrical formulations. This boundary element formulation was implemented by NLR, in the finite element code B2000 (ref. 6), and was coupled with the existing finite element code.

The final model describing the coupled structural-acoustic problem consists of full, complex, symmetric and frequency dependent matrices. A solution strategy has been developed and implemented to solve this coupled structural-acoustic FEM/BEM problem. The method has been evaluated by simple configurations.

The main conclusions are:

- The symmetrical boundary element formulation in combination with a finite element formulation is a good method to solve the problem.
- The frequency dependency of numerical models is amplified by the boundary element method, due to exponential function in the 3D-Green's function.
- The implementation of the boundary element formulation is much more complex and time demanding than a finite element formulation, due to the need of solving (nearly) singular integrals.
- Special solution schemes and a lot of cpu-time are needed to solve this type of problems, due to the complex and frequency dependent matrices.
- The modal approach strongly reduces the number of degrees of freedom and preserves the efficiency of the banded structure of the finite elements matrices.

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