



NLR-TP-2006-683

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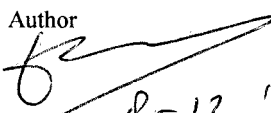
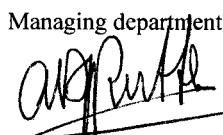
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This report contains a paper published in Automatica, Vol 42, January 2006.

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Customer: National Aerospace Laboratory NLR
Working Plan number: 2005 AT.1.A
Owner: National Aerospace Laboratory NLR
Division: Air Transport
Distribution: Unlimited
Classification title: Unclassified
September 2006

Approved by:

Author  8-12-'06	Reviewer Anonymous peer reviewers	Managing department  22/12/06
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Exact Bayesian filter and Joint IMM Coupled PDA tracking of maneuvering targets from possibly missing and false measurements [★]

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Abstract

This paper represents the problem of tracking multiple manoeuvring targets from possibly missing and false measurements as one of filtering for a jump-linear descriptor system with stochastic i.i.d. coefficients. This particular representation serves as an instrument in the characterization of the exact Bayesian filter. Subsequently, novel finite dimensional filter algorithms are developed through introducing approximations to the exact Bayesian solution. One filter approximation assumes conditionally Gaussian density of the joint target state given the joint target manoeuvre mode and the algorithm is referred to as Joint IMM Coupled PDA (JIMMCPDA). The specialty of this filter algorithm is that both the IMM step and the PDA step are performed jointly over the modes and states of all targets. Subsequently, the CPDA track coalescence avoiding hypothesis pruning approach of (Blom & Bloem, 2000) is extended to bring the joint target modes into account. The resulting filter algorithm is referred to as track-coalescence-avoiding Joint IMM Coupled PDA. The two novel algorithms are compared to IMMJPDA and IMMPDA through Monte Carlo simulations.

Key words: Bayesian estimation, Descriptor system, False measurements, Jump linear model, Missing measurements, Multitarget tracking, Stochastic hybrid system, Sudden maneuvers.

1 Introduction

We consider the problem of tracking multiple maneuvering targets from possibly missing and false measurements, with the aim to develop novel combinations of two well known approaches in target tracking: Interacting Multiple Model (IMM) algorithm and multi target versions of Probabilistic Data Association (PDA). Because each of these two solve complementary tracking problems it is of significant interest to combine both approaches. Since the publication of the IMM algorithm (Blom, 1984; Blom & Bar-Shalom, 1988), a large variety of IMM extensions and applications have appeared for many practical applications (e.g. see the overview in Mazor et al., 1998). One of the successful extensions is the combination of IMM with PDA (Houles & Bar-Shalom, 1989) as an effective approach to realize good tracking performance as long as targets stay sufficiently separated from each other. For closely spaced targets several tracking algorithms have been developed through combinations of IMM and Multiple Hypothesis Tracking (MHT) kind of approaches towards measurement to track assignment (e.g. Blackman and Popoli, 1999, pp.1090-1108; Kirubarajan et

al., 1996; Blackman et al., 1999; Leung et al., 1999; Koch, 2000). There is a remarkable lack of similarly successful approaches in tracking closely spaced targets with IMM and a multi target version of PDA.

Combinations of IMM and the well known JPDA (Bar-Shalom & Fortmann, 1988) have initially been studied along two heuristic directions. Bar-Shalom et al. (1992) heuristically developed an IMMJPDA-Coupled filter for situations where the measurements of two targets are unresolved during periods of close encounter. The filters of the individual targets are coupled through cross-target-covariance terms. The filtering results obtained have not been very encouraging to continue this development. De Feo et al. (1997) combined JPDA with an approximation of IMM. The first sound combination of IMM and JPDA has been developed by Chen & Tugnait (2001). Focus of this development was on comparing fixed-lag IMMJPDA smoothing versus IMMJPDA filtering but not on tracking closely spaced maneuvering targets.

The current paper aims to improve this situation by characterizing the exact Bayesian filter, and by developing novel target tracking combinations of IMM and PDA. To accomplish this, the descriptor system modeling approach of Blom & Bloem (2000) is adopted. First the problem of tracking multiple jump-linear targets is represented into one of filtering for a jump-linear descriptor system with stochastic i.i.d.

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coefficients. Subsequently this descriptor system representation is used as an instrument for the characterization of exact Bayesian equation for the conditional density of the joint maneuver modes and states of all targets. From this, finite dimensional filter algorithms are developed through introducing the following two successive approximations towards the exact Bayesian equation:

- Assume conditionally Gaussian density of the joint target states given the joint targets manoeuvre modes. The resulting filter algorithm is referred to as Joint IMM Coupled PDA (JIMMCPDA). The specialty of this filter algorithm is that both the IMM step and the PDA step is performed jointly over all target modes and states. An initial version of this has been presented in Blom & Bloem (2003).
- Extend and apply track coalescence avoiding hypothesis pruning approach of Blom & Bloem (2000). The resulting filter algorithm is referred to as JIMMCPDA* where the * stands for track-coalescence-avoiding. An initial version of this has been presented in Blom & Bloem (2004).

The paper is organized as follows. Section 2 defines the problem considered. In section 3, an exact Bayesian equation of the evolution of the conditional density for the joint states and modes of the multiple targets is developed. Section 4 and section 5 develop the JIMMCPDA and the JIMMCPDA* filter equations respectively. Section 6 shows the effectiveness of the novel filters through Monte Carlo simulation results. Section 7 draws conclusions.

2 Problem formulation

The problem of tracking multiple linear Markovian mode switching targets in false and missed detections is formulated in terms of filtering for a jump linear descriptor system with both Markovian switching and i.i.d. coefficients. The equations for the joint targets state x_t and the potential measurements z_t satisfy

$$x_t = A(\theta_t)x_{t-1} + B(\theta_t)w_t \quad (1)$$

$$z_t = H(\theta_t)x_t + G(\theta_t)v_t \quad (2)$$

The underlying model components of (1) are as follows:

$$x_t \triangleq \text{Col}\{x_t^1, \dots, x_t^M\},$$

$$\theta_t \triangleq \text{Col}\{\theta_t^1, \dots, \theta_t^M\},$$

$$A(\theta) \triangleq \text{Diag}\{a^1(\theta^1), \dots, a^M(\theta^M)\},$$

$$B(\theta) \triangleq \text{Diag}\{b^1(\theta^1), \dots, b^M(\theta^M)\},$$

$$w_t \triangleq \text{Col}\{w_t^1, \dots, w_t^M\},$$

where x_t^i is the n -vectorial state of the i -th target at moment t , θ_t^i is the mode of the i -th target at moment t and assumes values from $\mathbb{M} = \{1, \dots, N\}$ according to a transition probability matrix Π^i , $a^i(\theta^i)$ and $b^i(\theta^i)$ are $(n \times n)$ - and $(n \times n')$ -matrices, w_t^i is a sequence of i.i.d. standard Gaussian variables of dimension n' with w_t^i , w_t^j independent for all $i \neq j$ and w_t^i , x_0^i , x_0^j independent for all $i \neq j$. With this, x_t is

a vector of size Mn , $A(\theta)$ and $B(\theta)$ are of size $Mn \times Mn$ and $Mn \times Mn'$ respectively, and $\{\theta_t^i\}$ assumes values from \mathbb{M}^M according to transition probability matrix $\Pi = [\Pi_{\eta, \theta}]$. If targets switch mode independently of each other, then:

$$\Pi_{\eta, \theta} = \prod_{i=1}^M \Pi_{\eta^i, \theta^i}^i, \text{ for every } (\eta, \theta) \in \mathbb{M}^M$$

The coefficients in eq. (2) are as follows:

$$H(\theta) \triangleq \text{Diag}\{h^1(\theta^1), \dots, h^M(\theta^M)\},$$

$$G(\theta) \triangleq \text{Diag}\{g^1(\theta^1), \dots, g^M(\theta^M)\},$$

$h^i(\theta^i)$ is an $(m \times n)$ -matrix,

$g^i(\theta^i)$ is an $(m \times m')$ -matrix,

$$v_t \triangleq \text{Col}\{v_t^1, \dots, v_t^M\},$$

where v_t^i is a sequence of i.i.d. standard Gaussian variables of dimension m' with v_t^i and v_t^j independent for all $i \neq j$. Moreover v_t^i is independent of x_0^j and w_t^j for all i, j .

We next describe the relation between the potential measurement vector z_t and the measurement vector $y_t \triangleq \text{Col}\{y_{1,t}, \dots, y_{L_t,t}\}$, where $y_{i,t}$ denotes the i -th m -vectorial measurement at moment t and L_t denotes the number of measurements at moment t . Because y_t contains a random mixture of D_t target measurements and $L_t - D_t$ false measurements, the relation between z_t and y_t can be characterized by the following pair of equations for the target and false measurements respectively:

$$\begin{aligned} \underline{\Phi}(\psi_t)y_t &= \underline{\chi}_t \underline{\Phi}(\phi_t)z_t, \text{ if } D_t > 0, \\ &= \{\} \quad \quad \quad \text{, if } D_t = 0 \end{aligned} \quad (3.a)$$

$$\begin{aligned} \underline{\Phi}(\psi_t^*)y_t &= v_t^* \quad \quad \quad \text{, if } L_t > D_t, \\ &= \{\} \quad \quad \quad \text{, if } L_t = D_t \end{aligned} \quad (3.b)$$

where $\underline{\Phi}$, ψ_t , ϕ_t , ψ_t^* , v_t^* are explained below.

First we explain the target measurement eq. (3.a). This equation has stochastic i.i.d. coefficients $\underline{\Phi}(\psi_t)$ and $\underline{\chi}_t \underline{\Phi}(\phi_t)$.

$\psi_t \triangleq \text{Col}\{\psi_{1,t}, \dots, \psi_{L_t,t}\}$ is the target indicator vector, where $\psi_{i,t} \in \{0,1\}$ is a target indicator at moment t for measurement i , which assumes the value one if measurement i belongs to a detected target and zero if measurement i is false.

In order to let ψ_t select the correct measurements by simple matrix multiplication, a matrix operator Φ is defined, producing $\Phi(\psi')$ as a $(0,1)$ -valued matrix of size $D(\psi') \times M'$ of which the i th row equals the i th non-zero row of $\text{Diag}\{\psi'\}$, where $D(\psi') \triangleq \sum_{i=1}^{M'} \psi'_i$ for an arbitrary $(0,1)$ -valued M' -vector ψ' . To take into account the measurement vector size m , $\Phi(\psi_t)$ needs to be "inflated" to the proper size of $D_t m$ by means of the Kronecker product with I_m . To this end, $\underline{\Phi}(\psi') \triangleq \Phi(\psi') \otimes I_m$ with I_m a unit-matrix of size m , and \otimes the Kronecker product. Hence $\underline{\Phi}(\psi_t)y_t$ is a column vector that contains all correct measurements from y_t .

$\phi_t \triangleq \text{Col}\{\phi_{1,t}, \dots, \phi_{M,t}\}$ is the detection indicator vector, where $\phi_{i,t} \in \{0,1\}$ is the detection indicator for target i , which assumes the value one with probability $P_d^i > 0$, independently of $\phi_{j,t}$, $j \neq i$, where P_d^i denotes the detection probability of target i . $\{\phi_t\}$ is a sequence of i.i.d. vectors, and $D_t \triangleq \sum_{i=1}^M \phi_{i,t}$ denotes the number of detected targets. By using the matrix operator Φ , $\Phi(\phi_t)H(\theta_t)x_t$ is a column vector of potential detected measurements of targets in a fixed order.

The detected target measurements in the observation vector y_t are in random order. Hence the potential detected measurements of targets need to be randomly mixed. To perform this by a simple matrix multiplication, a sequence of independent stochastic permutation matrices $\{\chi_t\}$ of size $D_t \times D_t$ is defined and assumed to be independent of $\{\phi_t\}$. To take into account the measurement vector size m , χ_t needs to be "inflated" to the proper size of $D_t m$ by means of the Kronecker product with I_m . To this end, $\underline{\chi}_t \triangleq \chi_t \otimes I_m$ with I_m a unit-matrix of size m , and \otimes the Kronecker product. Hence $\underline{\chi}_t \Phi(\phi_t)H(\theta_t)x_t$ is a column vector of potential detected measurements of targets in random order.

Next we explain the false measurement eq. (3.b). v_t^* is a column vector of F_t i.i.d. false measurements within a given volume V . The prior density of these false measurements is assumed to be uniform on V . The number of false measurements at moment t , F_t , is assumed to be Poisson distributed

$$p_{F_t}(F) = \frac{(\lambda V)^F}{F!} \exp(-\lambda V), \quad F = 0, 1, 2, \dots$$

$$= 0, \quad \text{else}$$

where $\lambda > 0$ is the spatial density of false measurements (i.e. the average number per unit volume). Thus λV is the expected number of false measurements in volume V .

$\psi_t^* \triangleq \text{Col}\{\psi_{1,t}^*, \dots, \psi_{L_t,t}^*\}$ is a false indicator vector of size L_t ($= F_t + D_t$) with $\psi_{i,t}^* = 1 - \psi_{i,t}$. To select the false measurements by matrix multiplication, the matrix operator Φ is used again. Hence $\Phi(\psi_t^*)y_t$ is a column vector that contains all false measurements from y_t .

3 Exact equation for the conditional density

In this section a Bayesian characterization of the conditional density¹ $p_{x_t, \theta_t | Y_t}(x, \theta)$ is given where Y_t denotes the σ -algebra generated by measurements y_t up to and including moment t . This characterization is in the form of an extension of the single recursive equation of Elliot et al. (1996) for Markov jump linear filtering. For our derivation however we follow a Bayesian approach rather than the reference probability method (Elliott et al., 1995).

¹ More precisely it is a generalized probability density function (De Groot, 1970, p. 19)

In preparation to the Bayesian approach, eq. (3.a) is first transformed following Blom & Bloem (2000). Because χ_t has an inverse, (3.a) yields:

$$\begin{aligned} \underline{\chi}_t^T \Phi(\psi_t) y_t &= \Phi(\phi_t) z_t, \quad \text{if } D_t > 0 \\ &= \{\}, \quad \text{if } D_t = 0 \end{aligned} \quad (4)$$

By introducing an auxiliary indicator matrix process $\tilde{\chi}_t$ of size $D_t \times L_t$, as follows: $\tilde{\chi}_t \triangleq \underline{\chi}_t^T \Phi(\psi_t)$, (4) becomes:

$$\begin{aligned} \tilde{\chi}_t y_t &= \Phi(\phi_t) z_t, \quad \text{if } D_t > 0 \\ &= \{\}, \quad \text{if } D_t = 0 \end{aligned} \quad (5)$$

where the size of $\tilde{\chi}_t$ is $D_t m \times L_t m$ and the size of $\Phi(\phi_t)$ is $D_t m \times M m$. The right-hand side of (5) shows that all relevant combinations of detected potential target measurements can be covered by ϕ_t hypotheses. The left-hand side of (5) shows that all relevant selections of sets of target originating measurements out of the set of all measurements, can be covered by $\tilde{\chi}_t$ hypotheses. Thus from (5), it follows that all relevant measurement-to-target associations can be covered by $(\phi_t, \tilde{\chi}_t)$ -hypotheses. We extend this to $D_t = 0$ by adding the combination $\phi_t = \{0\}^M$ and $\tilde{\chi}_t = \{\}^{L_t}$.

Theorem 1. For any $\phi \in \{0,1\}^M$, such that $D(\phi) \triangleq \sum_{i=1}^M \phi_i \leq L_t$, the following recursive equation holds true for the conditional density $p_{x_t, \theta_t | Y_t}(x, \theta)$:

$$\begin{aligned} p_{x_t, \theta_t | Y_t}(x, \theta) &= \\ &= c_t \sum_{\phi \in \{0,1\}^M} \left[\lambda^{M-D(\phi)} \prod_{i=1}^M \left[(1 - P_d^i)^{(1-\phi_i)} (P_d^i)^{\phi_i} \right] \right. \\ &\quad \cdot \sum_{\tilde{\chi}} N_{mD(\phi)} \{ \tilde{\chi} y_t; \Phi(\phi) H(\theta) x, \Phi(\phi) G(\theta) G(\theta)^T \Phi(\phi)^T \} \\ &\quad \cdot \int_{\mathbb{R}^{Mn}} N_{Mn} \{ x; A(\theta) x', B(\theta) B(\theta)^T \} \\ &\quad \cdot \sum_{\eta \in \{1, \dots, N\}^M} [\Pi_{\eta\theta} p_{x_{t-1}, \theta_{t-1} | Y_{t-1}}(x', \eta)] dx' \end{aligned} \quad (6)$$

with normalization c_t , $N_K\{;\bar{x}, \bar{p}\}$ a K -dimensional Gaussian with mean \bar{x} and covariance \bar{P} , and 2nd sum running over all $\tilde{\chi} = \chi \Phi(\psi)$ with χ a $D(\phi) \times D(\phi)$ permutation matrix and $\psi \in \{0,1\}^{L_t}$ such that $D(\psi) = D(\phi)$.

Proof: The law of total probability yields:

$$p_{x_t, \theta_t | Y_t}(x, \theta) = \sum_{\phi, \tilde{\chi}} p_{x_t, \theta_t, \phi_t, \tilde{\chi}_t | Y_t}(x, \theta, \phi, \tilde{\chi}) \quad (7)$$

Together with eqs. (2) and (5), Bayes theorem (De Groot, 1970, p. 28) yields:

$$p_{x_t, \theta_t, \phi_t, \tilde{\chi}_t | Y_t}(x, \theta, \phi, \tilde{\chi}) =$$

$$= c'_t N \{ \underline{\tilde{X}}_{Y_t}; \underline{\Phi}(\phi) H(\theta) x, \underline{\Phi}(\phi) G(\theta) G(\theta)^T \underline{\Phi}(\phi)^T \} \cdot p_{x_t, \theta_t, \phi_t, \tilde{\chi}_t | Y_{t-1}}(x, \theta, \phi, \tilde{\chi}) \quad (8)$$

JPDA's derivation (Bar-Shalom & Fortmann, 1988) yields

$$p_{x_t, \theta_t, \phi_t, \tilde{\chi}_t | Y_{t-1}}(x, \theta, \phi, \tilde{\chi}) = c''_t \lambda^{M-D(\phi)} \cdot \prod_{i=1}^M \left[(1 - P_d^i)^{(1-\phi_i)} (P_d^i)^{\phi_i} \right] \cdot p_{x_t, \theta_t | Y_{t-1}}(x, \theta) \quad (9)$$

IMM's basic derivation (Blom, 1984, App. A) yields:

$$p_{x_t, \theta_t | Y_{t-1}}(x, \theta) = \int_{\mathbb{R}^{Mn}} N_{Mn} \{ x; A(\theta) x', B(\theta) B(\theta)^T \} \cdot \sum_{\eta \in \{1, \dots, N\}^M} [\Pi_{\eta\theta} p_{x_{t-1}, \theta_{t-1} | Y_{t-1}}(x', \eta)] dx' \quad (10)$$

Substituting (10) in (9), the result in (8), then in (7), and rearranging the summation over $\tilde{\chi}$ yields eq. (6). ■

Eq. (6) is a recursive equation for the exact Bayesian solution of tracking multiple targets from possibly false and missing measurements. From eq. (6) follows that if the initial density is a Gaussian mixture, then the exact conditional density solution of recursive equation (6) is a Gaussian mixture, the number of Gaussians increasing exponentially with time.

4 Joint IMM Coupled PDA filter

In this section we develop an algorithm which keeps the number of Gaussians limited to one per $\theta \in \{1, \dots, N\}^M$. For this we assume a conditional Gaussian $p_{x_t | \theta_t, Y_{t-1}}(x | \theta)$ in the following Theorem.

Theorem 2. For each $\theta \in \{1, \dots, N\}^M$, let $p_{\theta_t | Y_{t-1}}(\theta) > 0$ and let $p_{x_t | \theta_t, Y_{t-1}}(x | \theta)$ be Gaussian with mean $\bar{x}_t(\theta)$ and covariance $\bar{P}_t(\theta)$ and let $\beta_t(\phi, \tilde{\chi}, \theta)$ be defined by

$$\beta_t(\phi, \tilde{\chi}, \theta) \triangleq \text{Prob}\{\phi_t = \phi, \tilde{\chi}_t = \tilde{\chi}, \theta_t = \theta | Y_t\}.$$

Then

$$\beta_t(\phi, \tilde{\chi}, \theta) = F_t(\phi, \tilde{\chi}, \theta) \lambda^{(M-D(\phi))} \cdot \left[\prod_{i=1}^M (1 - P_d^i)^{(1-\phi_i)} (P_d^i)^{\phi_i} \right] \cdot p_{\theta_t | Y_{t-1}}(\theta) / c_t \quad (11)$$

with c_t such that it normalizes $\beta_t(\phi, \tilde{\chi}, \theta)$, and with:

$$F_t(\phi, \tilde{\chi}, \theta) = [(2\pi)^{mD(\phi)} \text{Det}\{Q_t(\phi, \theta)\}]^{-\frac{1}{2}} \cdot \exp\left\{-\frac{1}{2} \mu_t^T(\phi, \tilde{\chi}, \theta) Q_t(\phi, \theta)^{-1} \mu_t(\phi, \tilde{\chi}, \theta)\right\}, \text{ if } \phi \neq \{0\}^M \\ = 1, \quad \text{if } \phi = \{0\}^M \quad (12.a)$$

where

$$\mu_t(\phi, \tilde{\chi}, \theta) = \tilde{\chi}_{Y_t} - \underline{\Phi}(\phi) H(\theta) \bar{x}_t(\theta) \quad (12.b)$$

$$Q_t(\phi, \theta) \triangleq \underline{\Phi}(\phi) (H(\theta) \bar{P}_t(\theta) H(\theta)^T + G(\theta) G(\theta)^T) \underline{\Phi}(\phi)^T \quad (12.c)$$

Moreover, $p_{x_t | \theta_t, Y_t}(x | \theta)$ is a Gaussian mixture, with overall weight $p_{\theta_t | Y_t}(\theta)$, mean $\hat{x}_t(\theta)$ and overall covariance $\hat{P}_t(\theta)$ satisfying:

$$p_{\theta_t | Y_t}(\theta) = \sum_{\phi, \tilde{\chi}} \beta_t(\phi, \tilde{\chi}, \theta) \quad (13)$$

$$\hat{x}_t(\theta) = \bar{x}_t(\theta) + \sum_{\phi \neq 0} K_t(\phi, \theta) \left(\sum_{\tilde{\chi}} \beta_{t|\theta}(\phi, \tilde{\chi}) \mu_t(\phi, \tilde{\chi}, \theta) \right) \quad (14.a)$$

$$\hat{P}_t(\theta) = \bar{P}_t(\theta) + \sum_{\phi \neq 0} K_t(\phi, \theta) \underline{\Phi}(\phi) H(\theta) \bar{P}_t(\theta) \left(\sum_{\tilde{\chi}} \beta_{t|\theta}(\phi, \tilde{\chi}) \right) + \sum_{\phi \neq 0} K_t(\phi, \theta) \left(\sum_{\tilde{\chi}} \beta_{t|\theta}(\phi, \tilde{\chi}) \mu_t(\phi, \tilde{\chi}, \theta) \mu_t^T(\phi, \tilde{\chi}, \theta) \right) \cdot K_t^T(\phi, \theta) - \left(\sum_{\phi \neq 0} K_t(\phi, \theta) \left(\sum_{\tilde{\chi}} \beta_{t|\theta}(\phi, \tilde{\chi}) \mu_t(\phi, \tilde{\chi}, \theta) \right) \right) \cdot \left(\sum_{\phi' \neq 0} K_t(\phi', \theta) \left(\sum_{\tilde{\chi}'} \beta_{t|\theta}(\phi', \tilde{\chi}') \mu_t(\phi', \tilde{\chi}', \theta) \right) \right)^T \quad (14.b)$$

with:

$$K_t(\phi, \theta) = \bar{P}_t(\theta) H(\theta)^T \underline{\Phi}(\phi)^T Q_t(\phi, \theta)^{-1} \text{ if } \phi \neq 0, \\ = 0 \quad \text{else} \quad (15.a)$$

$$\beta_{t|\theta}(\phi, \tilde{\chi}) = \beta_t(\phi, \tilde{\chi}, \theta) / p_{\theta_t | Y_t}(\theta) \quad (15.b)$$

Proof: (Outline) If $p_{x_t | \theta_t, Y_{t-1}}(x | \theta)$ is Gaussian with mean $\bar{x}_t(\theta)$ and covariance $\bar{P}_t(\theta)$, then the density $p_{x_t | \phi_t, \tilde{\chi}_t, \theta_t, Y_t}(x | \phi, \tilde{\chi}, \theta)$ is Gaussian with mean $\hat{x}_t(\phi, \tilde{\chi}, \theta)$ and covariance $\hat{P}_t(\phi, \theta)$ satisfying for $\phi \neq 0$,

$$\hat{x}_t(\phi, \tilde{\chi}, \theta) = \bar{x}_t(\theta) + K_t(\phi, \theta) [\tilde{\chi}_{Y_t} - \underline{\Phi}(\phi) H(\theta) \bar{x}_t(\theta)] \\ \hat{P}_t(\phi, \theta) = \bar{P}_t(\theta) - K_t(\phi, \theta) \underline{\Phi}(\phi) H(\theta) \bar{P}_t(\theta)$$

and for $\phi = 0$:

$$\hat{x}_t(\phi, \tilde{\chi}, \theta) = \bar{x}_t(\theta) \\ \hat{P}_t(\phi, \theta) = \bar{P}_t(\theta)$$

Hence, $p_{x_t|\theta_t, Y_t}(\cdot | \theta)$ is a Gaussian mixture, and all equations follow from a lengthy but straightforward evaluation of this mixture. ■

Theorem 2 provides a conditional characterization for the joint targets modes and states. Here we use this to obtain a recursive algorithm by assuming that, for each $\theta \in \{1, \dots, N\}^M$, the conditional density $p_{x_t|\theta_t, Y_{t-1}}(x | \theta)$ is approximated by a single Gaussian density on \mathbb{R}^{Mn} . We refer to this recursive algorithm as the JIMMCPDA (Joint IMM Coupled PDA) filter, which consists of the following six subsequent steps.

JIMMCPDA Step 1: Interaction

For all $\theta \in \{1, \dots, N\}^M$, starting with the weights $p_{\theta_{t-1}|Y_{t-1}}(\theta)$, the means $\hat{x}_{t-1}(\theta)$ and the associated covariances $\hat{P}_{t-1}(\theta)$, one evaluates the mixed initial condition for the filter matched to $\theta_t = \theta$ as in IMM:

$$\begin{aligned} p_{\theta_t|Y_{t-1}}(\theta) &= \sum_{\eta \in \{1, \dots, N\}^M} \Pi_{\eta, \theta} \cdot p_{\theta_{t-1}|Y_{t-1}}(\eta) \\ \hat{x}_{t-1|\theta_t}(\theta) &= \sum_{\eta \in \{1, \dots, N\}^M} \mu_{t-1|t}(\eta|\theta) \hat{x}_{t-1}(\eta) \\ \hat{P}_{t-1|\theta_t}(\theta) &= \sum_{\eta \in \{1, \dots, N\}^M} \cdot \mu_{t-1|t}(\eta|\theta) \cdot \left(\hat{P}_{t-1}(\eta) + \right. \\ &\quad \left. + [\hat{x}_{t-1}(\eta) - \hat{x}_{t-1|\theta_t}(\theta)] \cdot [\hat{x}_{t-1}(\eta) - \hat{x}_{t-1|\theta_t}(\theta)]^T \right) \\ \mu_{t-1|t}(\eta|\theta) &= \Pi_{\eta, \theta} \cdot p_{\theta_{t-1}|Y_{t-1}}(\eta) / p_{\theta_t|Y_{t-1}}(\theta) \end{aligned}$$

JIMMCPDA Step 2: Prediction for all $\theta \in \{1, \dots, N\}^M$:

$$\begin{aligned} \bar{x}_t(\theta) &= A(\theta) \hat{x}_{t-1|\theta_t}(\theta) \\ \bar{P}_t(\theta) &= A(\theta) \hat{P}_{t-1|\theta_t}(\theta) A(\theta)^T + B(\theta) B(\theta)^T \\ \bar{Q}_t(\theta) &= H(\theta) \bar{P}_t(\theta) H(\theta)^T + G(\theta) G(\theta)^T \end{aligned}$$

JIMMCPDA Step 3: Gating, based on Bar-Shalom & Li (1995). Let $\bar{Q}_t^i(\theta)$ be the i -th $m \times m$ diagonal block matrix of $\bar{Q}_t(\theta)$, then identify for each target the mode for which $\text{Det } \bar{Q}_t^i(\theta)$ is largest:

$$\theta_t^{*i} = \underset{\theta}{\text{Argmax}} \{ \text{Det } \bar{Q}_t^i(\theta) \}$$

and define for each target i a gate $G_t^i \in \mathbb{R}^m$ as follows:

$$\begin{aligned} G_t^i \triangleq \{ z^i \in \mathbb{R}^m; [z^i - h^i(\theta_t^{*i}) \bar{x}_t^i(\theta_t^{*i})]^T \cdot \\ \cdot \bar{Q}_t^i(\theta_t^{*i})^{-1} [z^i - h^i(\theta_t^{*i}) \bar{x}_t^i(\theta_t^{*i})] \leq \nu \} \end{aligned}$$

with ν the gate size. If the j -th measurement y_t^j falls outside gate G_t^i , i.e. $y_t^j \notin G_t^i$, then the j -th component of the i -th row of $[\Phi(\phi)^T \tilde{\chi}]$ is assumed to equal zero at moment t . This reduces the set of possible detection/permutation hypotheses to be evaluated at moment t for various ϕ to $\tilde{\mathcal{X}}_t(\phi)$.

JIMMCPDA Step 4: Evaluation of the detection/permutation hypotheses by adapting the P_d^i in (11) for reduced detection probability due to the limited gate size ν :

$$\begin{aligned} \beta_t(\phi, \tilde{\chi}, \theta) &= F_t(\phi, \tilde{\chi}, \theta) \lambda^{(L_t - D(\phi))} \cdot p_{\theta_t|Y_{t-1}}(\theta) \cdot \\ &\quad \cdot \left[\prod_{i=1}^M \left(1 - P_d^i \cdot \text{Chi}_m^2(\nu) \right)^{(1 - \phi_i)} \cdot \right. \\ &\quad \left. \cdot \left(P_d^i \cdot \text{Chi}_m^2(\nu) \right)^{\phi_i} \right] / c_t \quad \text{for } \tilde{\chi} \in \tilde{\mathcal{X}}_t(\phi), \\ &= 0 \quad \text{else} \end{aligned}$$

where $F_t(\phi, \tilde{\chi}, \theta)$ is evaluated using eqs. (12.a,b,c), $\text{Chi}_m^2(\cdot)$ is the Chi-squared cumulative distribution function with m degrees of freedom, and c_t normalizes $\beta_t(\phi, \tilde{\chi}, \theta)$.

JIMMCPDA Step 5: Measurement update.

Evaluate equations (13) - (15).

JIMMCPDA Step 6: Output:

$$\begin{aligned} \hat{x}_t &= \sum_{\theta \in \{1, \dots, N\}^M} p_{\theta_t|Y_t}(\theta) \cdot \hat{x}_t(\theta) \\ \hat{P}_t &= \sum_{\theta \in \{1, \dots, N\}^M} p_{\theta_t|Y_t}(\theta) (\hat{P}_t(\theta) + [\hat{x}_t(\theta) - \hat{x}_t] \cdot [\hat{x}_t(\theta) - \hat{x}_t]^T) \end{aligned} \quad (16)$$

$$(17)$$

5 Track-coalescence-avoiding JIMMCPDA filter

Blom & Bloem (2000) have shown that CPDA is sensitive to track coalescence because CPDA merges over permutation hypotheses, and that a suitable hypothesis pruning may provide an effective countermeasure by pruning per (ϕ_t, ψ_t) -hypothesis all but the most likely χ_t -hypotheses prior to measurement updating. The physical explanation for why this is working has been given by Koch and Van Keuk (1997) for two targets: "If targets move closely spaced for a longer period of time, it seems to be reasonable to represent the pdf by a symmetric form invariant against a permutation of the objects." The CPDA hypothesis pruning strategy of Blom & Bloem (2000) is extended to JIMMCPDA as follows: evaluate all $(\phi_t, \psi_t, \theta_t)$ hypotheses and prune per $(\phi_t, \psi_t, \theta_t)$ -hypothesis all but the most-likely χ_t -hypotheses. For every ϕ, ψ and θ , satisfying $D(\psi) = D(\phi) \leq \text{Min}\{M, L_t\}$, the most likely χ hypothesis satisfies the mapping $\hat{\chi}_t(\phi, \psi, \theta)$:

$$\hat{\chi}_t(\phi, \psi, \theta) = \underset{\chi}{\text{Argmax}} \beta_t(\phi, \chi^T \Phi(\psi), \theta) \quad (18)$$

where the maximization is over all permutation matrices χ of size $D(\phi) \times D(\phi)$.

The pruning strategy of evaluating all (ϕ, ψ, θ) -hypotheses and only one χ -hypothesis per (ϕ, ψ, θ) -hypothesis implies

that we adopt the following pruned hypothesis weights:

$$\begin{aligned}\hat{\beta}_t(\phi, \psi, \theta) &= \beta_t(\phi, \hat{\chi}(\phi, \psi, \theta)^T \Phi(\psi), \theta) / \hat{c}_t, \\ &\quad \text{if } 0 < D(\phi) \leq \text{Min}\{M, L_t\} \\ &= \beta_t(\{0\}^M, \{0\}^{L_t}, \theta) / \hat{c}_t, \text{ if } D(\phi) = 0 \\ &= 0, \text{ else}\end{aligned}\quad (19)$$

with \hat{c}_t a normalization constant for $\hat{\beta}_t$; i.e. such that

$$\sum_{\substack{\phi, \psi, \theta \\ D(\psi)=D(\phi)}} \hat{\beta}_t(\phi, \psi, \theta) = 1 \quad (20)$$

By inserting this particular hypothesis pruning step after step 4 of JIMMCPDA and adapting the subsequent measurement update step, then we get JIMMCPDA*. The resulting cycle of the JIMMCPDA* filter algorithm consists of the following 7 steps:

JIMMCPDA Steps 1-4: Interaction*

Equivalent to JIMMCPDA Steps 1-4 in section 5.

JIMMCPDA Step 5: Hypothesis pruning.*

For every (ϕ, ψ, θ) determine $\hat{\beta}_t(\phi, \psi, \theta)$ by evaluating eqs. (18)-(20).

JIMMCPDA Step 6: Measurement update*

For all $i \in \{1, \dots, M\}$, $\theta^i \in \{1, \dots, N\}$:

$$\hat{p}_{\theta^i|Y_t}(\theta) \cong \sum_{\phi, \psi} \hat{\beta}_t(\phi, \psi, \theta) \quad (21)$$

$$\hat{x}_t(\theta) \cong \bar{x}_t(\theta) + \sum_{\phi \neq 0} K_t(\phi, \theta) \left(\sum_{\psi} \hat{\beta}_t(\phi, \psi, \theta) \hat{\mu}_t(\phi, \psi, \theta) \right) \quad (22)$$

$$\begin{aligned}\hat{P}_t(\theta) &\cong \bar{P}_t(\theta) + \\ &- \sum_{\phi \neq 0} K_t(\phi, \theta) \underline{\Phi}(\phi) H(\theta) \bar{P}_t(\theta) \cdot \left(\sum_{\psi} \hat{\beta}_t(\phi, \psi, \theta) \right) + \\ &+ \sum_{\phi \neq 0} K_t(\phi, \theta) \left(\sum_{\psi} \hat{\beta}_t(\phi, \psi, \theta) \hat{\mu}_t(\phi, \psi, \theta) \hat{\mu}_t(\phi, \psi, \theta)^T \right) \cdot \\ &\cdot K_t^T(\phi, \theta) - \left(\sum_{\phi \neq 0} K_t(\phi, \theta) \left(\sum_{\psi} \hat{\beta}_t(\phi, \psi, \theta) \hat{\mu}_t(\phi, \psi, \theta) \right) \right) \cdot \\ &\cdot \left(\sum_{\phi' \neq 0} K_t(\phi', \theta) \left(\sum_{\psi'} \hat{\beta}_t(\phi', \psi', \theta) \hat{\mu}_t(\phi', \psi', \theta) \right) \right)^T\end{aligned}\quad (23)$$

$$\hat{\beta}_{t|\theta}(\phi, \psi) = \hat{\beta}_t(\phi, \psi, \theta) / \hat{p}_{\theta^i|Y_t}(\theta) \quad (24)$$

$$\hat{\mu}_t(\phi, \psi, \theta) = \mu_t(\phi, \hat{\chi}(\phi, \psi, \theta)^T \Phi(\psi), \theta) \quad (25)$$

and $K_t(\phi, \theta)$ satisfies (15.a).

JIMMCPDA Step 7: Output equations*
Equivalent to JIMMCPDA Step 6.

6 Monte Carlo simulations

In this section we perform Monte Carlo simulations for two targets flying the 2D trajectory patterns as pictured in Figure 1 and in Figure 2. The filter algorithms evaluated are IMMJPDA, JIMMCPDA, JIMMCPDA* and an IMPDA which updates an individual track using PDA by assuming the measurements from the adjacent targets as false. The target trajectories in Figure 1 are from Chen & Tugnait (2001). We refer to this as **scenario R0**.²

In addition to this we simulate targets that start and stop maneuvering simultaneously as depicted in Figure 2. From 0 to 20s, targets 1 and 2 fly at a speed of 400 m/s in a straight line in south and north direction respectively. From 20 to 35s, both targets make a coordinated turn to the east. From 35 s to 55s, both targets fly in a straight line to the east. From 55s to 70s, targets 1 and 2 make a coordinated turn to the north and to the south respectively. From 70s to 90s, targets 1 and 2 fly in a straight line to the north and to the south respectively. Of the simultaneously manoeuvring targets we consider seven scenarios, which differ in the initial position of Target 1 only:

Scenario R1: Target 1 starts at (0,11820m) and target 2 starts at (0,-11820m).

Scenario R2/R2': Same as R1 but initial position of target 1 is shifted 200/100m to the south.

Scenario R3/R3': same as R1 but initial position of target 1 is shifted 200/100m to the north.

Scenario R4/R4': Same as R1 but initial position of target 1 is shifted 200/100m to the east.

Similar as in Chen & Tugnait (2001), the target motion models are patterned after Houles & Bar Shalom (1989). The motion models for the two targets are identical. In each mode the state of the target is position, velocity and acceleration in each of the two Cartesian coordinates (x, y) . Three modes for $\{\theta_t^i\}$ are adopted. The corresponding system matrices $a^i(\theta^i)$ and $b^i(\theta^i)$, for $\theta^i \in \{1, 2, 3\}$ and $i \in \{1, 2\}$, are defined as:

$$a^i(\theta^i) = \begin{bmatrix} a_1^i(\theta^i) & 0 \\ 0 & a_2^i(\theta^i) \end{bmatrix}, \quad b^i(\theta^i) = \begin{bmatrix} b_1^i(\theta^i) & 0 \\ 0 & b_2^i(\theta^i) \end{bmatrix}$$

² On a 3D version of scenario R0, Chen & Tugnait (2001) compared IMMJPDA filtering against IMMJPDA smoothing, but not against another filtering approach.

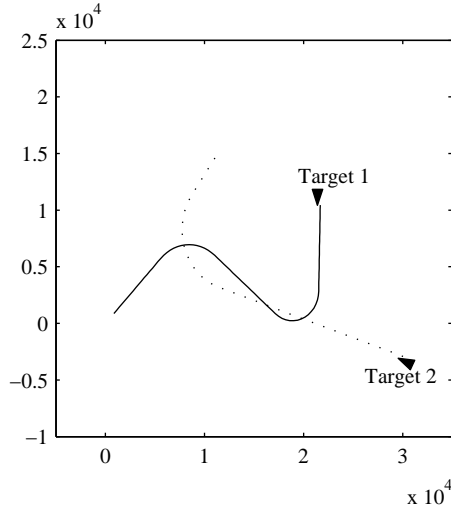


Fig. 1. Scenario R0: Trajectories from Chen & Tugnait (2001)

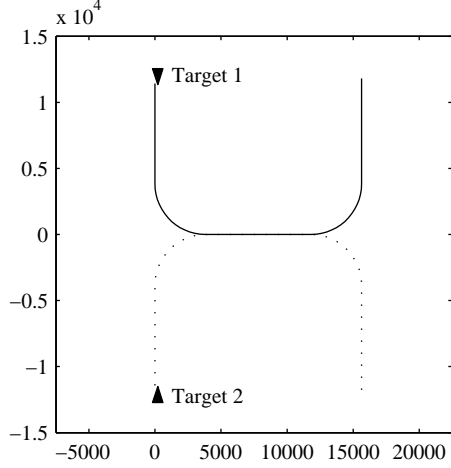


Fig. 2. Scenario R1: Simultaneously manoeuvring targets

$$a_j^i(1) = \begin{bmatrix} 1 & T_s & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad a_j^i(2) = a_j^i(3) = \begin{bmatrix} 1 & T_s & \frac{1}{2}T_s^2 \\ 0 & 1 & T_s \\ 0 & 0 & 1 \end{bmatrix}$$

$$b_j^i(k) = \sigma_a^i(k) \cdot \begin{bmatrix} \frac{1}{2}T_s^2 \\ T_s \\ 1\{k \neq 1\} \end{bmatrix}, \quad k = 1, 2, 3$$

where T_s is the sampling period.

- **Model 1:** nearly constant velocity model with zero mean perturbation in acceleration. The standard deviation of the process noise is $\sigma_a^i(1) = 5m/s^2$.
- **Model 2:** Wiener process acceleration (nearly constant acceleration motion). The standard deviation of the process noise is $\sigma_a^i(2) = 7.5m/s^2$.
- **Model 3:** Wiener process acceleration (model with large acceleration increments, for the onset and termination of maneuvers). The standard deviation of the process noise is $\sigma_a^i(2) = 40m/s^2$.

The initial model probabilities for the two targets are identical: $\hat{\gamma}_0^i(1) = 0.8$, $\hat{\gamma}_0^i(2) = 0.1$, $\hat{\gamma}_0^i(3) = 0.1$. The mode switching probability matrix for each of the two targets is also identical and is given by:

$$\Pi^i = \begin{bmatrix} 0.8 & 0.1 & 0.1 \\ 0.1 & 0.8 & 0.1 \\ 0.1 & 0.1 & 0.8 \end{bmatrix}$$

The sensor measurement coefficients for target i are

$$h^i(\theta^i) = \begin{bmatrix} h_1^i(\theta^i) & 0 \\ 0 & h_2^i(\theta^i) \end{bmatrix}, \quad g^i(\theta^i) = \begin{bmatrix} g_1^i(\theta^i) & 0 \\ 0 & g_2^i(\theta^i) \end{bmatrix}$$

$$h_j^i(\theta^i) = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}, \quad g_j^i(\theta^i) = \sigma_m, \quad j \in \{1, 2\}$$

The standard deviation σ_m of the measurement error is $\sigma_m = 20m$. The sensor is assumed to be located at the coordinate system origin. The sampling interval $T_s = 1s$ and it was assumed that the probability of detection $P_d = 0.997$. For generating false measurements in simulations the clutter was assumed to be Poisson distributed with expected number of $\lambda = 1 \times 10^{-6}/m^2$. The gates for setting up the validation regions for the measurements were based on the threshold $\nu = 25$.

For each of the scenarios Monte Carlo simulations containing 100 runs have been performed for each of the tracking filters. To make the comparisons more meaningful, for all tracking algorithms the same random number streams were used. During our simulations we counted track i "O.K.", if

$$|h^i \hat{x}_T^i - h^i x_T^i| \leq 9\sigma_m$$

with $|\cdot|$ the l_2 -norm. We counted track $i \neq j$ "Swapped", if

$$|h^i \hat{x}_T^i - h^j x_T^j| \leq 9\sigma_m$$

We counted track i and j as "Coalescing Tracks" if at three or more consecutive observation moments:

$$|h^i x_t^i - h^j x_t^j| > 9\sigma_m \quad \wedge \quad |h^i \hat{x}_t^i - h^j \hat{x}_t^j| \leq \sigma_m$$

The results of the Monte Carlo simulations are given in four types of Tables:

- % of both tracks "O.K." or "Swapped", in Table 1.
- % of both tracks "O.K.", in Table 2.
- % of "Coalescing" tracks, in Table 3.
- average CPU time per scan, in Table 4.

IMMJPDA performs much better than IMMPPDA except for scenario R1, i.e. the scenario where the two targets fly very close to each other for some time. Under scenario R1 the



performance of IMMJPDA suffers from serious track coalescence. Because JIMMCPDA is in theory more close to the exact Bayesian filter than IMMJPDA is, one would expect a significant improvement in performance of JIMMCPDA over IMMJPDA. The results however falsify this expectation for all scenarios except scenario R2. Most remarkable is the dramatic decrease in performance by JIMMCPDA for scenarios where the two targets come closer than 200m to each other, i.e. R1 (0m), R2' (100m), R3' (100m) and R4' (100m). These scenarios have in common that they cause JIMMCPDA to be caught in a situation where it has strongly coupled uncertainty about which target is gone in which direction. As a result of this JIMMCPDA increases its covariance and then diverges. The permutation hypothesis pruning of JIMMCPDA* however mitigates this problem effectively.

Table 1
% Both tracks "O.K." or "swapped".

	R0	R1	R2	R2'	R3	R3'	R4	R4'
IMMPDA	94	1	15	0	23	1	18	2
IMMJPDA	97	0	92	91	98	94	98	90
JIMMCPDA	97	0	97	15	93	16	97	15
JIMMCPDA*	97	97	96	96	98	97	98	98

Table 2
% Both tracks "O.K.".

	R0	R1	R2	R2'	R3	R3'	R4	R4'
IMMPDA	94	0	1	0	17	0	8	0
IMMJPDA	97	0	71	42	96	59	98	39
JIMMCPDA	97	0	83	4	91	5	97	10
JIMMCPDA*	97	46	80	39	95	32	98	78

Table 3
% Coalescing tracks.

	R0	R1	R2	R2'	R3	R3'	R4	R4'
IMMPDA	2	99	82	98	77	98	71	99
IMMJPDA	0	74	0	0	0	0	0	1
JIMMCPDA	0	99	0	51	5	43	1	59
JIMMCPDA*	0	0	0	0	0	0	0	0

Table 4
Average CPU time per scan (in seconds).

	R0	R1	R2	R2'	R3	R3'	R4	R4'
IMMPDA	0.18	0.17	0.17	0.17	0.16	0.16	0.16	0.17
IMMJPDA	0.20	0.74	0.20	0.21	0.19	0.19	0.18	0.21
JIMMCPDA	0.33	1.38	0.33	1.23	0.36	1.30	0.30	1.27
JIMMCPDA*	0.31	0.29	0.31	0.29	0.28	0.28	0.27	0.27

Track continuity comparison of JIMMCPDA* vs. JIMMCPDA shows a dramatic improvement for scenario R1, significant improvements for scenarios R2, R2', R3' and R4',

and equal performance for scenarios R0, R3 and R4. Moreover, JIMMCPDA* is the only algorithm of the four considered which performs constantly high and at a constant and reasonable CPU load.

7 Conclusion

This paper has developed novel Bayesian equations for tracking multiple maneuvering targets from possibly false and missing measurements. The approach taken is to first characterize this tracking problem into one of filtering for a jump-linear descriptor system with stochastic i.i.d. coefficients. This representation served as an instrument to derive the exact Bayesian filter equation and two novel tracking algorithms. The latter two are referred to as JIMMCPDA and JIMMCPDA*, where the * refers to a track-coalescence-avoiding version. These novel algorithms handle both the IMM and the PDA jointly over all target modes and all data associations. Through Monte Carlo simulations for a simple but demanding example, the effectiveness of the novel filters relative to IMMJPDA of Chen & Tugnait (2001) has been evaluated.

The theoretically "better" JIMMCPDA appears to be more sensitive to track coalescence than IMMJPDA. Consequently there are significant situations where JIMMCPDA performs less well than IMMJPDA. However, the hypothesis pruning method developed for JIMMCPDA* appears to work so effectively in avoiding track coalescence, that JIMMCPDA* realizes a significantly improved track continuation performance over IMMJPDA for demanding encounters.

There are several interesting extensions possible for the jump-linear descriptor framework and the novel exact and approximate filters. For example to develop a track coalescence avoiding IMMJPDA version (Blom & Bloem, 2002), to incorporate the target initiation model of Mušicki et al. (1994), or to incorporate unresolved measurement models of Chang & Bar-Shalom (1984) or Koch & Van Keuk (1997).

Acknowledgments

The authors would like to thank anonymous reviewers for their valuable comments and suggestions which significantly helped in improving the paper.

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