Low-frequency closed-form expressions for crosstalk between twisted wire pairs

NLR – Netherlands Aerospace Centre

Netherlands Aerospace Centre

NLR is a leading international research centre for aerospace. Bolstered by its multidisciplinary expertise and unrivalled research facilities, NLR provides innovative and integral solutions for the complex challenges in the aerospace sector.

NLR's activities span the full spectrum of Research Development Test & Evaluation (RDT & E). Given NLR's specialist knowledge and facilities, companies turn to NLR for validation, verification, qualification, simulation and evaluation. NLR thereby bridges the gap between research and practical applications, while working for both government and industry at home and abroad.

NLR stands for practical and innovative solutions, technical expertise and a long-term design vision. This allows NLR's cutting edge technology to find its way into successful aerospace programs of OEMs, including Airbus, Embraer and Pilatus. NLR contributes to (military) programs, such as ESA's IXV re-entry vehicle, the F-35, the Apache helicopter, and European programs, including SESAR and Clean Sky 2.

Founded in 1919, and employing some 650 people, NLR achieved a turnover of 71 million euros in 2016, of which three-quarters derived from contract research, and the remaining from government funds.

For more information visit: **www.nlr.nl**

Low-frequency closed-form expressions for crosstalk between twisted wire pairs
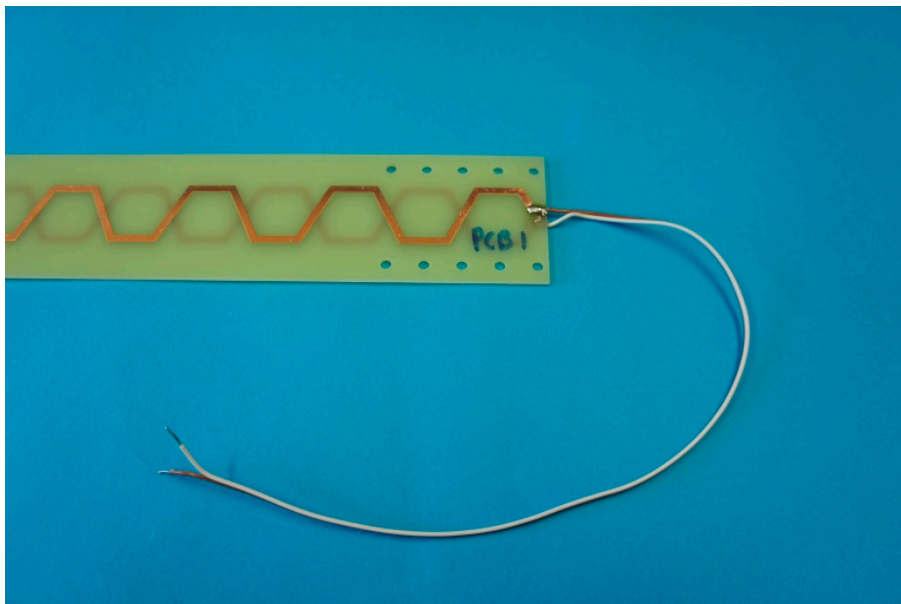
Problem area

The implication of the recent More Electric Aircraft technology development is that more and more functions on board airplanes are being performed by electric systems (instead of pneumatic, hydraulic or mechanical systems). As a consequence, the amount of cabling increases drastically, up to multiple hundreds of kilometers for Airbus A380 types of aircraft. Since the available space for cabling is actually decreasing, electromagnetic coupling or crosstalk will remain a very important EMC aspect when designing Electrical Wiring Interconnection Systems. When the impedances of the systems that are connected are lower than the characteristic impedances of the transmission lines, crosstalk will be dominated by inductive coupling. This type of coupling can be reduced by using twisted wire pairs, instead of straight wire pairs. Simplified expressions of crosstalk between twisted wire pairs can assist in the estimation of the effect of application of twisted wire pairs.

Description of work

In this paper, a low-frequency analysis is applied to a simplified model for the twisting of wire pairs that is applicable under a wide separation assumption. This analysis leads to closed-form expressions for crosstalk between two twisted wires above a ground plane. The expressions are validated by performing measurements of twisted wire pairs manufactured on PCB boards, for which the twists are very well controlled. Comparisons are made of crosstalk levels between wire pairs with equal twist rates, and crosstalk levels between wire pairs with unequal twist rates.

Results and conclusions

From the cascaded Multi-Conductor Transmission line equations, closed-form expressions are derived for crosstalk between twisted wire pairs in two specific cases, which both comprise two twisted wire pairs above a ground plane. In the first case, the twist rate of the two pairs is equal, while in the second case the twist rate of one of the two pairs is doubled.
Both the low-frequency analysis and the measurements show that for the case with equal twist rate, the crosstalk behaviour is similar to that of straight, untwisted wire pairs. On the contrary, in the case where one twist rate is doubled, crosstalk between the wire pairs is completely cancelled up to linear order in frequency. Due to the very low level of inductive crosstalk in such cases, a connector with pins of a few millimeters length may dominate crosstalk behaviour (this also depends on the level of capacitive crosstalk).

Applicability

The derived closed-form expressions are useful in understanding crosstalk behaviour when replacing untwisted wire pairs by twisted wire pairs. Moreover, the two cases that were investigated form a best and worst case scenario. Combinations of similar twisted pairs with different twist rates will yield crosstalk values in between these bounds obtained by the two cases that were analysed.

Low-frequency closed-form expressions for crosstalk between twisted wire pairs

AUTHOR(S):

J.H.G.J. Lansink Rotgerink   NLR
J. Verpoorte   NLR

NLR - Netherlands Aerospace Centre

This report is based on a presentation held at the 2016 ESA workshop on Aerospace EMC, Valencia, 23-25 May 2016, proceedings are found on: https://doi.org/10.1109/AeroEMC.2016.7504555.

The contents of this report may be cited on condition that full credit is given to NLR and the author(s).

CUSTOMER	Netherlands Aerospace Centre
CONTRACT NUMBER	-----
OWNER	NLR
DIVISION NLR	Aerospace Systems
DISTRIBUTION	Unlimited
CLASSIFICATION OF TITLE	UNCLASSIFIED

APPROVED BY :		
AUTHOR	**REVIEWER**	**MANAGING DEPARTMENT**
J.H.G.J. Lansink Rotgerink	J. Verpoorte	M. Keuning
DATE 0 9 0 1 1 8	DATE 0 9 0 1 1 8	DATE 1 5 0 1 1 8

Contents

This page is intentionally left blank.

LOW-FREQUENCY CLOSED-FORM EXPRESSIONS FOR CROSSTALK BETWEEN TWISTED WIRE PAIRS

J.H.G.J. Lansink Rotgerink[(1)], J. Verpoorte [(1)]

[(1)] *Netherlands Aerospace Centre, Voorsterweg 31, 8316 PR Marknesse, The Netherlands,*
{Jesper.Lansink.Rotgerink, Jaco.Verpoorte}@nlr.nl

ABSTRACT

Crosstalk between two twisted wire pairs of equal and unequal twist rate is analysed and compared. Low-frequency approximations to the Multi-conductor Transmission Line equations are used to derive closed-form expressions for near-end crosstalk. Such analysis on cable configurations with different twist scenarios gives insight into sensitivity on relevant twist parameters as well as dependencies of crosstalk on all other model parameters. Results show that crosstalk between twisted pairs of equal twist rate behaves similar to that between untwisted wire pairs. On the contrary, an ideal combination of twisted pairs, by for instance doubling the twist rate in one of the pairs, causes crosstalk to vanish up to linear order. The performed analysis and derived closed-form expressions agree to measured crosstalk results and can lead to good understanding of upper and lower boundaries for crosstalk in different cable configurations.

1. INTRODUCTION

Twisting of wire pairs has been considered to be an effective way to decrease the amount of unintentional electromagnetic (EM) coupling between cables. Therefore twisting is widely applied in for instance aviation, automotive industries and data networks. EM coupling referred to as crosstalk is of high importance for the electromagnetic compatibility (EMC) of an entire aircraft. Therefore it plays an important role in the design rules for installation of cable systems applied by cable manufacturers. Guidelines for the routing, harnessing and twisting of cables require accurate as well as practical relations between crosstalk and model parameters. The majority of cables inside an aircraft is either shielded, twisted or both.

Crosstalk between twisted wire pairs (TWPs) is a frequently studied problem. Analysis on Multi-conductor Transmission Lines (MTLs) can be performed by the methods described by Clayton R. Paul [1]. Paul also introduces ways to include shielding [2] and twisting [3] into these MTL equations. By solving the MTL equations with exact chain parameter matrices, the behaviour of different types of transmission lines (TLs) can be computed. Sensitivity with respect to for instance twist parameters can be investigated by applying statistics to these models [5]. As opposed to

the use these very accurate models, simpler low-frequency analysis leading to closed-form expressions can be used to gain knowledge on crosstalk in a more practical and easier way. Already in [6] and [7] practical crosstalk expressions with respect to model parameters were derived for unshielded wire pairs and shielded wires. The current paper describes the derivation of such closed-form expressions that directly relate the amount of EM coupling to all model parameters for near-end differential-mode crosstalk between TWPs.

Two configurations that might be observed as best and worst cases are considered. The first consists of two twisted wire pairs with equal twist rate that are perfectly aligned. In a second case the amount of twists in one pair is doubled. Since twisting is mainly used to reduce inductive coupling, the focus of this paper will be on that, as opposed to capacitive coupling. All results are validated by measurements of twisted pairs constructed on a PCB (see Fig. 1). Geometric parameters are very well controlled for these PCB twisted pairs, which is useful for comparison with models and to increase knowledge on crosstalk between twisted pairs. When similar low-frequency analysis is performed on the case of a single wire above a ground plane versus a twisted wire pair, treated by Paul [4], the obtained low-frequency expressions are equal.

The second section of this paper describes the general crosstalk models for our analyses, after which in section 3 the low-frequency crosstalk is analysed for the different wiring configurations. Section 4 shows results of measurements, simulations and analyses.
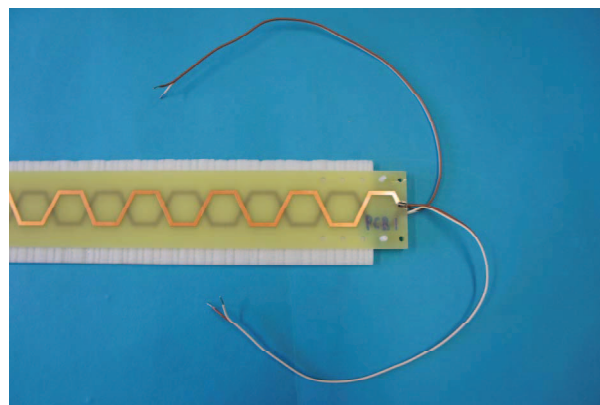
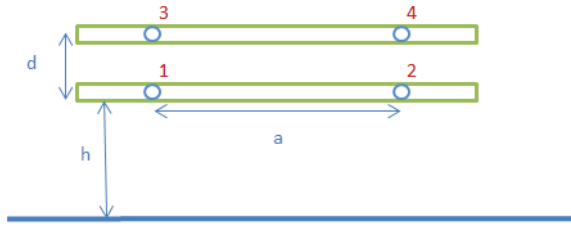*Figure 1. Twisted pairs printed on PCBs used for crosstalk measurements.*

Figure 2. Cross section for modelling of two twisted pair PCBs above a perfectly conducting ground plane.

2. CROSSTALK MODELLING

Near-end crosstalk is defined as the ratio of the voltage at the source side on the receptor or victim transmission line over that on the generator or culprit line. Hence for crosstalk prediction a solution from the multi-conductor transmission line equations for the voltage in each conductor is required.

Consider an MTL of length ℓ with n conductors parallel above an infinite, perfectly conducting ground plane. If $\mathbf{V}_0$ represents the vector containing the voltages in each conductor with respect to the ground plane, then near-end crosstalk is defined as:

$$\gamma_{NE} = \frac{\mathbf{U}_2^T \mathbf{V}_0}{\mathbf{U}_1^T \mathbf{V}_0}, \qquad (1)$$

in which the vectors $\mathbf{U}_1$ and $\mathbf{U}_2$ are used to select the voltages corresponding to respectively culprit and victim transmission line conductors from the vector of voltages. The cross section given in Fig. 2 is used to model the twisted pair PCBs, in which a voltage source is included between wires 1 and 2, i.e. the culprit pair. Then $\mathbf{U}_1 = \left(-1, 1, 0, 0\right)^T$ and $\mathbf{U}_2 = \left(0, 0, -1, 1\right)^T$.

Equation (1) implies that to compute crosstalk levels between transmission lines the voltage in each conductor is required. This is obtained by solving the MTL equations [1]. One solution is to compute chain parameter matrices that relate the voltages and currents at the end of the TL to those at the beginning of the TL:

$$\begin{bmatrix} \mathbf{V}_\ell \\ \mathbf{I}_\ell \end{bmatrix} = \hat{\mathbf{\Phi}}\left(\ell\right) \begin{bmatrix} \mathbf{V}_0 \\ \mathbf{I}_0 \end{bmatrix} = \begin{bmatrix} \hat{\mathbf{\Phi}}_{11}\left(\ell\right) & \hat{\mathbf{\Phi}}_{12}\left(\ell\right) \\ \hat{\mathbf{\Phi}}_{21}\left(\ell\right) & \hat{\mathbf{\Phi}}_{22}\left(\ell\right) \end{bmatrix} \begin{bmatrix} \mathbf{V}_0 \\ \mathbf{I}_0 \end{bmatrix}. \quad (2)$$

Solution of the MTL equations in terms of voltage of each conductor with respect to ground plane can then be obtained by solving the currents from [1]:

$$\left[\hat{\mathbf{\Phi}}_{12} - \hat{\mathbf{\Phi}}_{11}\mathbf{Z} - \mathbf{Z}\hat{\mathbf{\Phi}}_{22} + \mathbf{Z}\hat{\mathbf{\Phi}}_{21}\mathbf{Z} \right]\mathbf{I}_0 = \\ -\left[\hat{\mathbf{\Phi}}_{11} - \mathbf{Z}\hat{\mathbf{\Phi}}_{21} \right]\mathbf{V}_S, \qquad (3)$$

and incorporating the terminal conditions to calculate the voltages:

$$\mathbf{V}_0 = \mathbf{V}_S - \mathbf{Z}\mathbf{I}_0. \qquad (4)$$

Voltage sources, assumed only to be present in the near-end termination network, are represented by the vector $\mathbf{V}_S$. The matrix $\mathbf{Z}$ is the impedance matrix modelling all impedances in the termination network. It is assumed to be equal for the near- and far-end side. The argument for the chain parameter matrices has been dropped to simplify notation.

The chain parameter matrices carry the cross sectional information of the transmission lines. Chain parameters are constant along the length of cables when their cross section is uniform. In case of twisted wire pairs the latter doesn't hold and a cascade of uniform pieces of transmission line is applied [3].

The above implies that to estimate crosstalk in cables it is required to have cross sectional knowledge in the form of per-unit-length (PUL) parameters, as well as termination matrices modelling the networks at both ends of the transmission lines. To incorporate twisting also properties along the length of the TL are needed.

2.1. Per-Unit-Length parameters

All cross sectional information of a specific cable configuration is incorporated into per-unit-length parameters of a transmission line: capacitance, inductance, resistance and conductance. The last two are assumed to be zero in this paper, whereas the first two are represented by matrices of dimension n. The inductance matrix for a cable configuration with cylindrical conductors like that in Fig. 2 is given by:

$$\mathbf{L} = \begin{bmatrix} l_{11} & l_{12} & l_{13} & l_{14} \\ l_{12} & l_{22} & l_{23} & l_{24} \\ l_{13} & l_{23} & l_{33} & l_{34} \\ l_{14} & l_{24} & l_{34} & l_{44} \end{bmatrix}, \qquad (5)$$

in which:

$$l_{ii} = \frac{\mu_0}{2\pi}\ln\left(2h_i/r_i\right)$$

$$l_{ij} = \frac{\mu_0}{4\pi}\ln\left(1 + 4h_i h_j / s_{ij}^2\right).$$

Here h_i is the height of wire i above the ground plane, r_i its radius and s_{ij} the distance between wires i and j. Assuming homogeneous media, by which for simplicity the relative permittivity of the PCB dielectric material is neglected, the capacitance matrix can be found by inverting (5):

$$\mathbf{C} = \mu_0 \varepsilon_0 \mathbf{L}^{-1}. \qquad (6)$$

2.2. Termination network

For the configuration of two wire pairs, either twisted or untwisted, terminations of both transmission lines will be like illustrated in Fig. 3. By the use of Norton equivalent representation techniques the corresponding impedance matrix can be derived:

$$\mathbf{Z} = \begin{bmatrix} \mathbf{Z}_c^* & \varnothing \\ \varnothing & \mathbf{Z}_v^* \end{bmatrix}. \qquad (7)$$
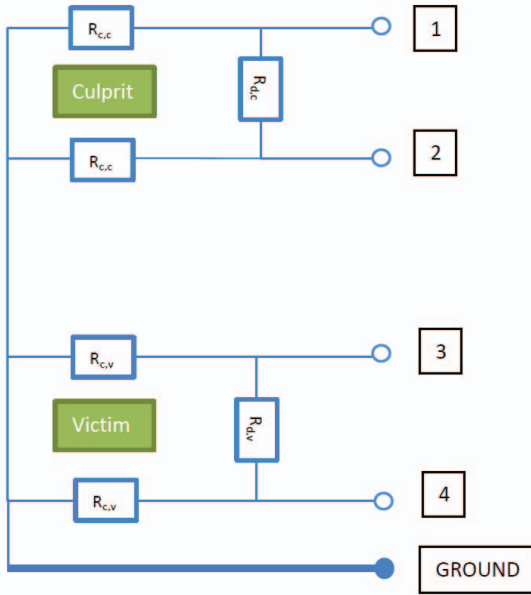
Figure 3. Termination network for two wire pairs above a perfectly conducting ground plane.

Here:

$$\mathbf{Z}_m^* = R_{d,m} \begin{bmatrix} c_m + \kappa_m & c_m - \kappa_m \\ c_m - \kappa_m & c_m + \kappa_m \end{bmatrix}.$$

in which $c_m = \left[R_{d,m} \left(R_{d,m}/R_{c,m} + 2 \right) / R_{c,m} \right]^{-1} + \kappa_m$ and $\kappa_m = \left[2 \left(R_{d,m}/R_{c,m} + 2 \right) \right]^{-1}$. Here $m \in \{v, c\}$ represents victim or culprit. The variable $R_{d,m}$ is the differential-mode and $R_{c,m}$ the common-mode impedance of either culprit or victim wire pair.

2.3. Twisted wire pairs

Besides the termination networks, the chain parameter matrix carrying all cross sectional information models the entire behaviour of the MTL. The difficulty with twisted pair cables is the nonuniformity along the length of the line, which implies varying PUL parameters. Transmission lines that are uniform along there length are a necessity to the presented way of solving MTL equations. To incorporate twisting in these MTL models twisted pairs are represented by a cascade of uniform pieces of transmission line. Specifically, the sections in between the twists of a TWP are represented by pieces of straight wire pair. The chain parameter matrix corresponding to the entire MTL is formed by cascading such matrices for each unifom section.

Consider for instance a situation of two twisted pairs above a perfectly conducting ground plane. Assume that in between the twists of a TWP the TL is uniform and the twist sections have zero length. Then in general the total chain parameter matrix equals [3]:

$$\Phi_T = \prod_{k=1}^{n_S} \mathbf{P}_k \Phi_S \left(\ell_k \right), \qquad (8)$$

in which n_S is the total amount of uniform sections in the MTL, $\mathbf{\Phi_S}$ the chain parameter matrix of one such uniform section and $\mathbf{P_k}$ permutation matrices. The latter are used to keep track of the positions of the conductors after twisting.

If these twisted pairs have equal twist rates and the twists form in total n_L perfectly aligned loops, then the corresponding chain parameter matrix can be given by:

$$\Phi_T = \begin{cases} \Phi_S \left(\mathbf{P}\Phi_S \right)^{n_L - 1} & n_L = \text{odd} \\ \left(\mathbf{P}\Phi_S \right)^{n_L} & n_L = \text{even} \end{cases}, \qquad (9)$$

in which, since at each interchange section both pairs obtain a twist:

$$\mathbf{P} = \begin{bmatrix} \tilde{\mathbf{P}} & \varnothing \\ \varnothing & \tilde{\mathbf{P}} \end{bmatrix}, \qquad \tilde{\mathbf{P}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}. \qquad (10)$$

If the second pair has twice the amount of loops in the first TWP, $n_L = 2m_L$, the chain parameter matrix of this cabling configuration is given by:

$$\Phi_T = \mathbf{P}_f \Phi_S \mathbf{P}_1 \Phi_S \left(\mathbf{P}_2 \Phi_S \mathbf{P}_1 \Phi_S \right)^{m_L - 1}, \qquad (11)$$

in which $\mathbf{P}_2$ is equal to $\mathbf{P}$ given in (10) and:

$$\mathbf{P}_1 = \begin{bmatrix} \tilde{\mathbf{P}}_1 & \varnothing \\ \varnothing & \tilde{\mathbf{P}}_1 \end{bmatrix}, \qquad \tilde{\mathbf{P}}_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}. \qquad (12)$$

Moreover $\mathbf{P}_f$ is the final permutation matrix and therefore depends on whether the two wire pairs have an even or odd amount of loops. By definition the second wire pair will always have an even amount of loops and therefore for m_L is even we have $\mathbf{P}_f = \mathbf{P}_2$ and for odd m_L we have $\mathbf{P}_f = \mathbf{P}_1$.

With termination networks and cross sectional information contained via the PUL parameters in the chain parameter matrices, the entire MTL is characterised. This is used in the next section to analyse crosstalk behaviour.

3. LOW-FREQUENCY CROSSTALK ANALYSIS

After obtaining the chain parameter matrix for an MTL, (3) can be solved by Taylor expansion to derive closed-form low-frequency expressions for near-end crosstalk. This is similar to analysis performed in [6] for crosstalk between straight wire pairs (SWPs). The chain parameter for the configuration of two SWPs above a ground plane can be approximated to first order for low frequencies as follows:

$$\Phi = \mathbf{1}_{2n_C} - j\omega\ell\mathbf{A}, \qquad \mathbf{A} = \begin{bmatrix} \varnothing & \mathbf{L} \\ \mathbf{C} & \varnothing \end{bmatrix}. \quad (13)$$

Here $\mathbf{1}_{2n_C}$ is the identity matrix of dimension $2n_c$. If this chain parameter matrix is substituted into (3) according to the structure in (2), the obtained low-frequency expressions for inductive and capacitive crosstalk read:

$$\gamma_{NE,ind} \approx \frac{1}{8} j\omega\ell\, \kappa_c^{-1} R_{d,c}^{-1} \left((l_{13}+l_{24}) - (l_{14}+l_{23}) \right)$$

$$\gamma_{NE,cap} \approx \frac{1}{2} j\omega\ell\, \kappa_v R_{d,v} \left((c_{14}+c_{23}) - (c_{13}+c_{24}) \right). \quad (14)$$

In the next sections similar expressions are derived for configurations involving twisted pairs. Therefore the approximation in (13) is also applied to the uniform sections of TWPs, represented by $\boldsymbol{\Phi}_S$.

3.1. Twisted pairs with equal twist rate

Consider the situation of two twisted pairs with an equal amount of loops n_L, that has been introduced in section 2.3. The corresponding chain parameter matrix is given in (9). If the approximation given in (13) is applied to $\boldsymbol{\Phi}_S$ then the corresponding low-frequency first order approximation for $\boldsymbol{\Phi}_T$ when n_L is even equals:

$$\begin{aligned} \boldsymbol{\Phi}_T &\approx \mathbf{P}^{n_L} - j\omega\ell_S \sum_{k=0}^{n_L-1} \mathbf{P}^{n_L-k}\mathbf{A}\mathbf{P}^k \\ &= \mathbf{1}_{2n_C} - j\omega\ell_S m\left(\mathbf{A}+\mathbf{PAP}\right), \end{aligned} \quad (15)$$

in which $m = n_L/2$. Here it is used that $\mathbf{P}^2$ equals the identity matrix. Similarly, for odd n_L it holds that:

$$\boldsymbol{\Phi}_T = \mathbf{1}_{2n_C} - j\omega\ell_S \left(m\mathbf{A} + (m-1)\mathbf{PAP} \right), \quad (16)$$

in which $m = (n_L+1)/2$.

The obtained total chain parameter matrix should be substituted into (3), after which by Taylor expansions an approximation for the conductor currents can be found. After incorporation of the termination conditions the following voltages are obtained for an even amount of loops:

$$\begin{aligned} \mathbf{V}_0 &\approx \frac{V_S}{2}\mathbf{U}_1 + \\ &\frac{V_S}{4} j\omega\ell \left[\left(\mathbf{L}+\tilde{\mathbf{P}}\mathbf{L}\tilde{\mathbf{P}}\right)\mathbf{Z}^{-1} - \mathbf{Z}\left(\mathbf{C}+\tilde{\mathbf{P}}\mathbf{C}\tilde{\mathbf{P}}\right) \right]\mathbf{U}_1 \end{aligned} \quad (17)$$

By use of (1) this yields the following expression for near-end crosstalk:

$$\gamma_{NE,ind} \approx \frac{1}{4} j\omega\ell_S m \kappa_c^{-1} R_{d,c}^{-1} \left(l_{13}+l_{24}-l_{14}-l_{23} \right)$$

$$\gamma_{NE,cap} \approx j\omega\ell_S m \kappa_v R_{d,v} \left(c_{14}+c_{23}-c_{13}-c_{24} \right). \quad (18)$$

A similar expression can be derived for an odd amount of loops. It turns out that both can be rewritten to:

$$\gamma_{NE,ind} \approx \frac{1}{8} j\omega\ell_S n_L \kappa_c^{-1} R_{d,c}^{-1} \left(l_{13}+l_{24}-l_{14}-l_{23} \right)$$

$$\gamma_{NE,cap} \approx \frac{1}{2} j\omega\ell_S n_L \kappa_v R_{d,v} \left(c_{14}+c_{23}-c_{13}-c_{24} \right). \quad (19)$$

These expressions for inductive and capacitive near-end crosstalk equal those for untwisted wire pairs in (14) when the interchange sections have zero length, by which $\ell = n_L \ell_S$.

3.2. Twisted pairs with unequal twist rate

As opposed to the previous cable configuration, consider the situation in which the amount of loops in the second pair is multiplied by two, by which the chain parameter matrix is equal to (11).
Define for ease of notation:

$$\mathbf{F}_1 = \left[\mathbf{C} + \tilde{\mathbf{P}}_2\mathbf{C}\tilde{\mathbf{P}}_1\tilde{\mathbf{P}}_2\tilde{\mathbf{P}}_1 + \tilde{\mathbf{P}}_2\tilde{\mathbf{P}}_1\mathbf{C}\tilde{\mathbf{P}}_2\tilde{\mathbf{P}}_1 + \tilde{\mathbf{P}}_2\tilde{\mathbf{P}}_1\tilde{\mathbf{P}}_2\mathbf{C}\tilde{\mathbf{P}}_1 \right]$$

$$\mathbf{G}_1 = \left[\mathbf{L} + \tilde{\mathbf{P}}_2\mathbf{L}\tilde{\mathbf{P}}_1\tilde{\mathbf{P}}_2\tilde{\mathbf{P}}_1 + \tilde{\mathbf{P}}_2\tilde{\mathbf{P}}_1\mathbf{L}\tilde{\mathbf{P}}_2\tilde{\mathbf{P}}_1 + \tilde{\mathbf{P}}_2\tilde{\mathbf{P}}_1\tilde{\mathbf{P}}_2\mathbf{L}\tilde{\mathbf{P}}_1 \right].$$

Again a low-frequency approximation can obtained for the presented chain parameter matrix, which yields:

$$\boldsymbol{\Phi}_{T,11} = \boldsymbol{\Phi}_{T,22} = \mathbf{1}_{n_C} \quad (20)$$

and for even m_L:

$$\boldsymbol{\Phi}_{T,12} = -j\omega\ell_S \frac{m_L}{2}\mathbf{G}$$

$$\boldsymbol{\Phi}_{T,21} = -j\omega\ell_S \frac{m_L}{2}\mathbf{F}, \quad (21)$$

whereas for odd m_L:

$$\boldsymbol{\Phi}_{T,12} = -j\omega\ell_S \left[\frac{m_L-1}{2}\mathbf{G} + \mathbf{L} + \tilde{\mathbf{P}}_1\mathbf{L}\tilde{\mathbf{P}}_1 \right]$$

$$\boldsymbol{\Phi}_{T,21} = -j\omega\ell_S \left[\frac{m_L-1}{2}\mathbf{F} + \mathbf{C} + \tilde{\mathbf{P}}_1\mathbf{C}\tilde{\mathbf{P}}_1 \right]. \quad (22)$$

If these expressions are substituted into (3) this leads to:

$$\mathbf{V}_0 \approx \frac{V_s}{2}\left[\mathbf{I}_n + \frac{m_L}{2} j\omega\ell\left(\mathbf{G}\mathbf{Z}^{-1}-\mathbf{Z}\mathbf{F}\right) \right]\mathbf{U}_1, \quad (23)$$

for an even amount of loops in the first wire pair m_L. When this number is odd:

$$\begin{aligned} \mathbf{V}_0 &\approx \frac{V_s}{2}\left[\mathbf{I}_n + \frac{m_L-1}{2} j\omega\ell_S\left(\mathbf{G}\mathbf{Z}^{-1}-\mathbf{Z}\mathbf{F}\right) \right]\mathbf{U}_1 + \\ &\frac{V_s}{4} j\omega\ell_S \left[\left(\mathbf{L}+\mathbf{P}_1\mathbf{L}\mathbf{P}_1\right)\mathbf{Z}^{-1} - \mathbf{Z}\left(\mathbf{C}+\mathbf{P}_1\mathbf{C}\mathbf{P}_1\right) \right]\mathbf{U}_1. \end{aligned} \quad (24)$$

For both cases it holds that when crosstalk is calculated from these estimated voltages as defined in (1), it is obtained that near-end crosstalk vanishes up to linear order:

$$\gamma_{NE} = 0 + O\left(\omega^2 \ell_S^2\right). \quad (25)$$

Thus for ideally aligned twisted pairs of which one has double the amount of loops, the crosstalk behaviour becomes at least quadratic with respect to frequency.

3.3. Single wire versus twisted wire pair

The analysis presented in the previous sections can also be applied to the simple twisted pair configuration presented by Paul [4]. This comprises a single wire and a twisted wire pair containing m_L loops, above a perfectly conducting ground plane. The analysis yields the following expression for near-end crosstalk when m_L is even:

$$\gamma_{NE} \approx j\omega\ell_S nR \frac{(c_{12} + c_{13})}{2},$$

in which $n = m_L/2$. When m_L is odd the result is:

$$\gamma_{NE} \approx \frac{j\omega\ell_S}{2} \left[R\left(nc_{12} + (n-1)c_{13}\right) + \frac{(l_{13} - l_{12})}{R} \right].$$

in which $n = (m_L + 1)/2$. These results equal the low-frequency expressions given by Paul.

4.   RESULTS

To validate the low-frequency expressions measurements were performed. Therefore both twisted pair configurations presented in this paper have been realised on PCBs (see Fig. 1). These PCBs resemble actual twisted pair behaviour without uncertainties in model parameters. Therefore simulation results should match measurements closely.

For simplicity the cross section of the PCBs was approximated by that given in Fig. 2. The radius of the wires r was taken equal to a quarter of the width of a PCB trace: r equalled 0.5 mm. The separation distance d was 2.5 mm, whereas the height above ground plane h equalled 8 cm. The distance between the traces a on a PCB was 16 mm and the length of the sample ℓ was 50 cm. For the situation involving TWPs with equal twist rate, the amount of loops was equal to 25.

The results of measurement, simulation and low-frequency analysis for this configuration is shown in Fig. 4. MTL simulations with exact chain parameter matrices are given in blue and red for respectively straight wire pairs and twisted wire pairs. It is observed that the crosstalk behaviour between the twisted wire pairs is completely similar to that between straight wire pairs, caused by the fact that the loops of the twisted pairs are perfectly aligned. The only difference in crosstalk levels appears from the slight decrease in effective length of the transmission line. In between twists the sections of TWPs are indeed perfectly uniform, but the interchange sections have a non-zero length. Therefore crosstalk for TWPs is decreased by the ratio of total length of uniform sections and the total transmission line length, in this case 14 mm over

20 mm, in comparison to SWPs. When low-frequency approximations of the chain parameter matrices are used the yellow line is obtained, of which the corresponding crosstalk expressions coincide with those of SWPs. The obtained closed-form expression (19), shown in purple, is also similar to that of SWPs (14). Most importantly, measurements given in green indicate that the modelling of the twisted pairs is correct.

Crosstalk can be greatly reduced by taking an ideal twisted pair combination in which for instance the amount of twists in the second pair is double the amount of the first pair. This is regardless of the alignment of the TWPs. Fig. 5 shows the results for the situation where the second TWP contains 50 loops. The result of MTL simulations with exact chain parameter matrices is given in red. Indeed crosstalk levels are much lower than that of straight wire pairs, given in blue. Moreover, as predicted by (25), the linear behaviour of crosstalk with respect to frequency disappeared. Measurement results still show both higher and linear crosstalk for this ideal combination of twisted pairs. This indicates that between the TWPs crosstalk reduction is so high, that the tiny traces underneath the solder at both ends of the TL (used to connect to measurement equipment, see Fig. 1) become dominant. These traces together form 10 mm of straight wire pair and after inclusion in the simulation, crosstalk results indeed coincide with measurements (yellow respectively green line). The closed-form expressions of SWPs (purple) of 10 mm length (14) therefore become applicable to this cable configuration.
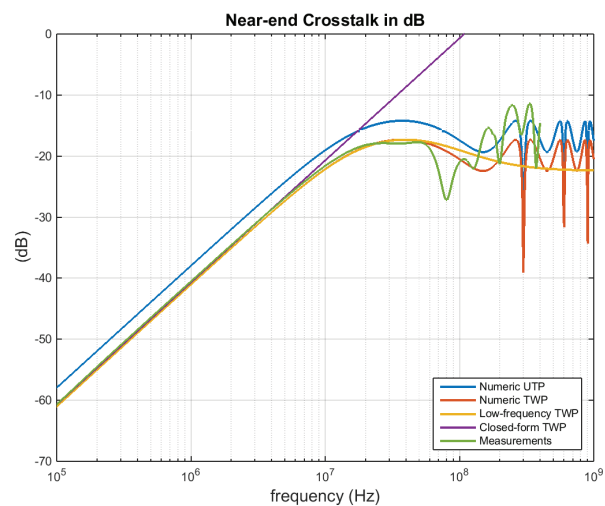

Figure 4. Simulation results for near-end crosstalk between two PCB twisted pairs with equal twist rate. Numerical solution of the MTL equations with exact chain parameter matrices for straight wire pairs (SWP, blue line) and twisted wire pairs (TWP, red line) are given. The yellow line illustrates the solution when using low-frequency approximation in (13). The closed-form expression for this cable configuration yields the purple line. Measurements are also included in green.
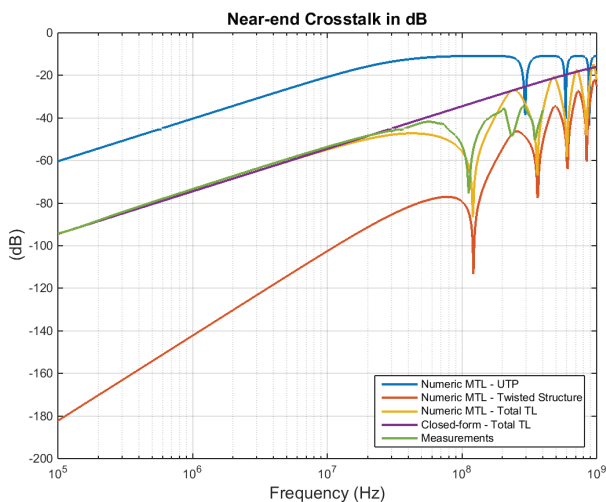
Figure 5. Simulation results for near-end crosstalk between two PCB twisted pairs, in which the second has a doubled twist rate. Numerical MTL simulations for straight wire pairs (SWP, blue line) and twisted wire pairs (TWP, red line) are given. The yellow line incorporates the 10 mm connection SWPs at both ends. These dominating pieces lead to a simple closed-form expression for this cable configuration (purple line). Measurements are also included in green.

5. CONCLUSIONS

Low-frequency approximation methods have been used to analyse crosstalk between two twisted pairs with equal and unequal twist rate. These linearisations yield closed-form expressions for different cable configurations that directly relate crosstalk levels to all model parameters. Such analysis is less accurate than direct MTL simulation and covers only low-frequency behaviour. On the other hand application to different twist scenarios gives insight into dependencies and sensitivities on all model parameters. Therefore derivation of such expressions is especially relevant for engineers in for instance aerospace industry, for creating design rules for cable installation.

Validation of theoretical results has been performed by the measurements of PCBs that resemble twisted pairs. Comparison between measurement and simulations reveals a good match. Moreover results show that crosstalk between twisted pairs of equal twist rate behaves similar to that between straight (untwisted) wire pairs. On the contrary, an ideal combination of twisted pairs by for instance doubling the twist rate in one of the pairs causes crosstalk to vanish up to linear order and crosstalk in wire pairs for connection to measurement equipment becomes dominant. This illustrates the applicability of the derived closed-form expressions. Appearance of the two presented extremes seems quite unlikely in practical cable bundles with TWPs., though the performed analysis and derived

closed-form expressions can lead to good understanding of upper and lower boundaries for crosstalk in different cable configurations. Combinations of twisted pairs with different twist rates will always imply crosstalk levels in between these bounds.

6. ACKNOWLEDGEMENT

7. REFERENCES

1. C.R. Paul, "Analysis of multiconductor transmission lines," New York, John Wiley & Sons, 1994.

2. C.R. Paul, "Transmission-line modeling of shielded wires for crosstalk prediction", *IEEE Transactions on Electromagnetic Compatibility*, vol. 23, no. 4, pp. 345-351, November, 1981.

3. C.R. Paul, "Prediction of crosstalk involving twisted pairs of wires-part I: a transmission-line model for twisted-wire pairs", *IEEE Transactions on Electromagnetic Compatibility*, vol. 21, no. 2, pp. 92-105, May, 1979.

4. C.R. Paul, "Prediction of crosstalk involving twisted pairs of wires-part II: a simplified low-frequency prediction model", *IEEE Transactions on Electromagnetic Compatibility*, vol. 21, no. 2, pp. 105-114, May, 1979.

5. G. Spadacini, S.A. Pignari, "Numerical assessment of radiated susceptibility of twisted-wire pairs with random nonuniform twisting", *IEEE Transactions on Electromagnetic Compatibility,* vol. 55, no. 5, 956-964, October, 2013.

6. J.H.G.J. Lansink Rotgerink, H. Schippers, "Crosstalk modelling of unshielded wire pairs", *International Symposium on Electromagnetic Compatibility (EMC Europe)*, Gothenburg, Sweden, September 1-4, 2014, pp. 641-646.

7. J.H.G.J. Lansink Rotgerink, H. Schippers, "Generic prediction of crosstalk between shielded wires", *IEEE International Symposium on Electromagnetic Compatibility (EMC)*, Dresden, Germany, August 16-22, 2015, pp. 616-621.