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Summary

The paper combines IMM and JPDA for tracking of multiple possibly maneuvering targets in case of clutter and possibly missed measurements while avoiding sensitivity to track coalescence. The effectiveness of the filter is illustrated through Monte Carlo simulations.

List of acronyms

IMM	Interacting Multiple Model
IMMJPDA	Interacting Multiple Model Joint Probabilistic Data Association
IMMJPDA*	Track-coalescence-avoiding Interacting Multiple Model Joint Probabilistic Data Association
IMMPDA	Interacting Multiple Model
JPDA	Joint Probabilistic Data Association
JPDA*	Track-coalescence-avoiding Joint Probabilistic Data Association
MHT	Multiple Hypothesis Tracking
NLR	Nationaal Lucht- en Ruimtevaartlaboratorium
NN	Nearest Neighbour
PDA	Probabilistic Data Association

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1 Introduction

We consider the problem of tracking multiple maneuvering targets in clutter with a proper combination of two well known approaches in target tracking: IMM and JPDA. Since each of these two solve complementary tracking problems one might expect that it should be useful to combine these two approaches. In literature the problem of combining IMM (Blom & Bar-Shalom, 1988) and JPDA (Bar-Shalom and Fortmann, 1988) has been studied by Bar-Shalom et al. (1992), DeFeo et al. (1997) and Chen and Tugnait (2001). Bar-Shalom et al. (1992) developed an IMMJPDA-Coupled filter for situations where the measurements of two targets are unresolved during periods of close encounter. In Blom & Bloem (2000) it has been shown that these IMMJPDA-Coupled filter equations are rather heuristic. Chen and Tugnait (2001) developed an IMMJPDA-Uncoupled fixed-lag smoothing algorithm with IMMJPDA uncoupled tracking as a special case. They also showed that the IMMJPDA of De Feo et al. (1997) does not account for ”interactions” between the target modes. All in all, in spite of the significant headway which has been made regarding the combination of IMM and JPDA, there is a lack of insight in the proper choices to be made when combining IMM and JPDA for multiple maneuvering target tracking.

In order to improve this situation, the paper studies the problem of combining IMM and JPDA following an approach that is based on recent new insight gained regarding the derivation of a track coalescence avoiding JPDA version (Blom & Bloem, 2000). The basis for this development is to embed the multi target tracking problem with possibly false and missing measurements into one of filtering for a linear descriptor system with random coefficients. In this paper this embedding approach is extended towards the development of various IMMJPDA filters, and it is shown how these compare with known IMMJPDA filters.

The paper is organized as follows. Section 2 develops the stochastic model for the tracking problem considered. Section 3 presents exact filter equations. Section 4 develops IMMJPDA filter equations. Section 5 develops the track coalescence avoiding IMMJPDA filter equations. Section 6 shows the effectiveness of the approach through Monte Carlo simulation results.

2 Stochastic modeling

This section describes the target model and the measurement model.

2.1 Target model

Consider M targets and assume that the state of the i-th target is modeled as a jump linear system:

$$x_{t+1}^i = a^i(\theta_{t+1}^i)x_t^i + b^i(\theta_{t+1}^i)w_t^i, \qquad i = 1, ..., M, \tag{1}$$

where x_t^i is the n-vectorial state of the i-th target, θ_t^i is the mode of the i-th target and assumes values from $\{1, .., N\}$, $a^i(\theta_t^i)$ and $b^i(\theta_t^i)$ are $(n \times n)$-matrices and w_t^i is a sequence of i.i.d. standard Gaussian variables of dimension n with w_t^i , w_t^j independent for all $i \neq j$ and w_t^i ,x_0^i, x_0^j independent for all $i \neq j$. Let $x_t \triangleq$ Col$\{x_t^1, ..., x_t^M\}$, $\theta_t \triangleq$ Col$\{\theta_t^1, ..., \theta_t^M\}$, $A(\theta_t) \triangleq$ Diag$\{a^1(\theta_t^1), ..., a^M(\theta_t^M)\}$, $B(\theta_t) \triangleq$ Diag$\{b^1(\theta_t^1), ..., b^M(\theta_t^M)\}$, and $w_t \triangleq$ Col$\{w_t^1, ..., w_t^M\}$. Then we can model the state of our M targets as follows:

$$x_{t+1} = A(\theta_{t+1})x_t + B(\theta_{t+1})w_t \tag{2}$$

2.2 Measurement model

A set of measurements consists of measurements originating from targets and measurements originating from clutter. Firstly the measurements originating from targets are treated. Subsequently the clutter measurements are randomly inserted between the target measurements.

A Measurements originating from targets

We assume that a potential measurement associated with state x_t^i (which we will denote by z_t^i) is modeled as a jump linear system:

$$z_t^i = h^i(\theta_t^i)x_t^i + g^i(\theta_t^i)v_t^i \qquad , i = 1, ..., M \tag{3}$$

where z_t^i is an m-vector, $h^i(\theta_t^i)$ is an $(m \times n)$-matrix and $g^i(\theta_t^i)$ is an $(m \times m)$-matrix, and v_t^i is a sequence of i.i.d. standard Gaussian variables of dimension m with v_t^i and v_t^j independent for all $i \neq j$. Moreover v_t^i is independent of x_0^j and w_t^j for all i,j. Next with $z_t \triangleq$ Col$\{z_t^1, ..., z_t^M\}$, $H(\theta_t) \triangleq$ Diag$\{h^1(\theta_t^1), ..., h^M(\theta_t^M)\}$, $G(\theta_t) \triangleq$ Diag$\{g^1(\theta_t^1), ..., g^M(\theta_t^M)\}$, and $v_t \triangleq$ Col$\{v_t^1, ..., v_t^M\}$, we obtain:

$$z_t = H(\theta_t)x_t + G(\theta_t)v_t \tag{4}$$

We next introduce a model that takes into account that not all targets have to be detected at moment t, which implies that not all potential measurements z_t^i have to be available as true measurements at moment t. To this end, let P_{d}^i be the detection probability of target i and let $\phi_{i,t} \in\{0,1\}$ be the detection indicator for target i, which assumes the value one with probability $P_{\mathrm{d}}^i > 0$, independently of $\phi_{j,t}$, $j \neq i$. This approach yields the following detection indicator vector ϕ_t of size M:

$$\phi_t \triangleq \mathrm{Col}\{\phi_{1,t}, ..., \phi_{M,t}\}.$$

Thus, the number of detected targets is $D_t \triangleq \sum_{i=1}^{M} \phi_{i,t}$. Furthermore, we assume that $\{\phi_t\}$ is a sequence of i.i.d. vectors.

In order to link the detection indicator vector with the measurement model, we introduce the following operator Φ: for an arbitrary (0,1)-valued M'-vector ϕ' we define $D(\phi') \triangleq \sum_{i=1}^{M'} \phi'_i$ and the operator Φ producing $\Phi(\phi')$ as a $(0,1)$-valued matrix of size $D(\phi') \times M'$ of which the ith row equals the ith non-zero row of Diag$\{\phi'\}$. Next we define, for $D_t > 0$, a vector that contains all measurements originating from targets at moment t in a fixed order.

$$\tilde{z}_t \triangleq \underline{\Phi}(\phi_t)z_t, \text{ where } \underline{\Phi}(\phi_t) \triangleq \Phi(\phi_t) \otimes I_m,$$

with I_m a unit-matrix of size m, and $\otimes$ the tensor product.

In reality, however, we do not know the order of the targets. Hence, we introduce the stochastic $D_t \times D_t$ permutation matrix χ_t, which is conditionally independent of $\{\phi_t\}$. We also assume that $\{\chi_t\}$ is a sequence of independent matrices. Hence, for $D_t > 0$,

$$\tilde{\tilde{z}}_t \triangleq \underline{\chi}_t\tilde{z}_t, \text{ where } \underline{\chi}_t \triangleq \chi_t \otimes I_m,$$

is a vector that contains all measurements originating from targets at moment t in a random order.

B Measurements originating from clutter

Let the random variable F_t be the number of false measurements at moment t. We assume that F_t has Poisson distribution:

$$p_{F_t}(F) = \frac{(\lambda V)^F}{F!}\exp(-\lambda V), \qquad F = 0, 1, 2, \ldots$$
$$= 0, \qquad \text{else}$$

where λ is the spatial density of false measurements (i.e. the average number per unit volume) and V is the volume of the validation region. Thus, λV is the expected number of false measurements in the validation gate. We assume that the false measurements are uniformly distributed in the validation region, which means that a column-vector v_t^* of F_t i.i.d. false measurements has the following density:

$$p_{v_t^*|F_t}(v^*|F) = V^{-F}$$

where V is the volume of the validation region. Furthermore we assume that the process $\{v_t^*\}$ is a sequence of independent vectors, which are independent of $\{x_t\}, \{w_t\}, \{v_t\}$ and $\{\phi_t\}$.

C Random insertion of clutter measurements

Let the random variable L_t be the total number of measurements at moment t. Thus,

$$L_t = D_t + F_t$$

With $\tilde{\mathrm{y}}_t \triangleq \mathrm{Col}\{\tilde{\tilde{z}}_t, v_t^*\}$, it follows with the above defined variables that

$$\tilde{\mathrm{y}}_t = \begin{bmatrix} \underline{\chi}_t \underline{\Phi}(\phi_t) z_t \\ \ldots\ldots\ldots\ldots \\ v_t^* \end{bmatrix}, \text{ if } L_t > D_t > 0 \tag{5}$$

whereas the upper and lower subvector parts disappear for $D_t = 0$ and $L_t = D_t$ respectively. With this equation, the measurements originating from clutter still have to be randomly inserted between the measurements originating from the detected targets. To do so, we first define target indicator and clutter indicator processes, denoted by $\{\psi_t\}$ and $\{\psi_t^*\}$, respectively. Let the random variable $\psi_{i,t} \in \{0,1\}$ be a target indicator at moment t for measurement i, which assumes the value one if measurement i belongs to a detected target and zero if measurement i comes from clutter. This approach yields the following target indicator vector ψ_t of size L_t:

$$\psi_t \triangleq \mathrm{Col}\{\psi_{1,t}, ..., \psi_{L_t,t}\}.$$

Let the random variable $\psi_{i,t}^* \in \{0,1\}$ be a clutter indicator at moment t for measurement i, which assumes the value one if measurement i comes from clutter and zero if measurement i belongs to an aircraft (thus $\psi_{i,t}^* = 1 - \psi_{i,t}$). This approach yields the following clutter indicator vector ψ_t^* of size L_t:

$$\psi_t^* \triangleq \mathrm{Col}\{\psi_{1,t}^*, ..., \psi_{L_t,t}^*\}.$$

In order to link the target and clutter indicator vectors with the measurement model, we make use of the operator Φ introduced before. With this the measurement vector with clutter inserted reads as follows:

$$\mathrm{y}_t = \left[\underline{\Phi}(\psi_t)^T \vdots \underline{\Phi}(\psi_t^*)^T\right] \tilde{\mathrm{y}}_t \;\text{ if } L_t > D_t > 0 \tag{6}$$

This, together with equation (2), (4) and (5), forms a complete characterization of our tracking problem in terms of stochastic difference equations.

3 Exact filter equations

Next we introduce an auxiliary indicator process $\tilde{\chi}_t$ as follows:

$$\tilde{\chi}_t \stackrel{\triangle}{=} \chi_t^T \Phi(\psi_t) \text{ if } D_t > 0.$$

Following the approach of Blom & Bloem (2000) equations (4), (5) and (6) can be transformed to:

$$\underline{\tilde{\chi}}_t \mathrm{y}_t = \underline{\Phi}(\phi_t) H(\theta_t) x_t + \underline{\Phi}(\phi_t) G(\theta_t) v_t \text{ if } D_t > 0 \tag{7}$$

Notice that (7) is a linear Gaussian descriptor system (Dai, 1989) with stochastic i.i.d. coefficients $\underline{\Phi}(\phi_t)$ and $\underline{\tilde{\chi}}_t$. From (7), it follows that for $D_t > 0$ all relevant associations and permutations can be covered by $(\phi_t, \tilde{\chi}_t)$-hypotheses. We extend this to $D_t = 0$ by adding the combination $\tilde{\chi}_t = \{\}^{L_t}$ and $\phi_t = \{0\}^M$. Hence, through defining the weights

$$\beta_t(\phi, \tilde{\chi}, \theta) \stackrel{\triangle}{=} \text{Prob}\{\phi_t = \phi, \tilde{\chi}_t = \tilde{\chi}, \theta_t = \theta \mid Y_t\},$$

where Y_t denotes the σ-algebra generated by measurements y_t up to and including moment t, then the law of total probability yields:

$$p_{\theta_t|Y_t}(\theta) = \sum_{\tilde{\chi},\phi} \beta_t(\phi, \tilde{\chi}, \theta) \tag{8}$$

$$p_{x_t,\theta_t|Y_t}(x, \theta) = \sum_{\tilde{\chi},\phi} \beta_t(\phi, \tilde{\chi}, \theta) p_{x_t|\theta_t,\phi_t,\tilde{\chi}_t,Y_t}(x \mid \theta, \phi, \tilde{\chi}) \tag{9}$$

Since

$$p_{x_t|\theta_t,Y_t}(x \mid \theta) = p_{x_t,\theta_t|Y_t}(x, \theta)/p_{\theta|Y_t}(\theta) \tag{10}$$

our problem is to characterize the right-hand terms in (9).
This has been accomplished in the following Theorem.

Theorem *Let* $p_{\theta_t|Y_{t-1}}(\theta) = \prod_{i=1}^M p_{\theta_t^i|Y_{t-1}}(\theta^i)$ *and let* $p_{x_t|\theta_t,Y_{t-1}}(x|\theta)$ *be Gaussian with mean* $\bar{x}_t(\theta) = Col\{\bar{x}_t^1(\theta^1), ..., \bar{x}_t^M(\theta^M)\}$ *and covariance* $\bar{P}_t(\theta) = Diag\{\bar{P}_t^1(\theta^1), ..., \bar{P}_t^M(\theta^M)\}$*, then*

$\beta_t(\phi,\tilde{\chi},\theta)$ satisfies for $\phi \neq \{0\}^M$:

$$\beta_t(\phi,\tilde{\chi},\theta) = \lambda^{(L_t - D(\phi))} \cdot \prod_{i=1}^{M} \left[f_t^i(\phi,\tilde{\chi},\theta^i)(1-P_d^i)^{(1-\phi_i)}(P_d^i)^{\phi_i} \cdot p_{\theta_t^i|Y_{t-1}}(\theta^i) \right] / c_t \tag{11}$$

with:

$$\begin{aligned} f_t^i(\phi,\tilde{\chi},\theta^i) = {} & [(2\pi)^m \mathit{Det}\{Q_t^i(\theta^i)\}]^{-\frac{1}{2}\phi_i} \cdot \\ & \cdot \exp\{-\frac{1}{2}\sum_{k=1}^{L_t}\left([\Phi(\phi)]_{*i}^T \tilde{\chi}_{*k} \mu_t^{ik}(\theta^i)^T [Q_t^i(\theta^i)]^{-1} \mu_t^{ik}(\theta^i)\right)\} \end{aligned} \tag{12.a}$$

where:

$$\mu_t^{ik}(\theta^i) \triangleq y_t^k - h^i(\theta^i)\bar{x}_t^i(\theta^i) \tag{12.b}$$

$$Q_t^i(\theta^i) \triangleq h^i(\theta^i)\bar{P}_t^i(\theta^i)h^i(\theta^i)^T + g^i(\theta^i)g^i(\theta^i)^T \tag{12.c}$$

whereas $[\Phi(\phi)]_{*i}$ and $\tilde{\chi}_{*k}$ are the i-th and k-th columns of $\Phi(\phi)$ and $\tilde{\chi}$, respectively. Moreover, $p_{x_t^i|\theta_t^i,Y_t}(x^i|\theta^i)$, $i \in \{1,...,M\}$, is a Gaussian mixture, while its overall mean $\hat{x}_t^i(\theta^i)$ and its overall covariance $\hat{P}_t^i(\theta^i)$ satisfy:

$$p_{\theta_t^i|Y_t}(\theta^i) = \sum_{\substack{\phi,\tilde{\chi},\eta \\ \eta^i=\theta^i}} \beta_t(\phi,\tilde{\chi},\eta) \tag{13.a}$$

$$\hat{x}_t^i(\theta^i) = \bar{x}_t^i(\theta^i) + W_t^i(\theta^i)\left(\sum_{k=1}^{L_t}\beta_t^{ik}(\theta^i)\mu_t^{ik}(\theta^i)\right) \tag{13.b}$$

$$\begin{aligned} \hat{P}_t^i(\theta^i) = {} & \bar{P}_t^i(\theta^i) - W_t^i(\theta^i)h^i(\theta^i)\bar{P}_t^i(\theta^i)\left(\sum_{k=1}^{L_t}\beta_t^{ik}(\theta^i)\right) + \\ & + W_t^i(\theta^i)\left(\sum_{k=1}^{L_t}\beta_t^{ik}(\theta^i)\mu_t^{ik}(\theta^i)\mu_t^{ik}(\theta^i)^T\right)W_t^i(\theta^i)^T + \\ & - W_t^i(\theta^i)\left(\sum_{k=1}^{L_t}\beta_t^{ik}(\theta^i)\mu_t^{ik}(\theta^i)\right)\left(\sum_{k'=1}^{L_t}\beta_t^{ik'}(\theta^i)\mu_t^{ik'}(\theta^i)\right)^T W_t^i(\theta^i)^T \end{aligned} \tag{13.c}$$

with:

$$W_t^i(\theta^i) = \bar{P}_t^i(\theta^i) h^i(\theta^i)^T [Q_t^i(\theta^i)]^{-1} \tag{13.d}$$

$$\beta_t^{ik}(\theta^i) = \sum_{\substack{\phi,\tilde{\chi},\eta \\ \phi\neq 0 \\ \eta^i=\theta^i}} \Phi(\phi)_{*i}^T \tilde{\chi}_{*k} \beta_t(\phi,\tilde{\chi},\eta)]/p_{\theta_t^i|Y_t}(\theta^i) \tag{13.e}$$

Proof: See Blom & Bloem (2002) ■

4 IMMJPDA filter

In this section the IMMJPDA filter algorithm is specified. To do so use is made of the IMM filter algorithm and of the Theorem. One cycle of the IMMJPDA filter algorithm consists of the following six steps.

IMMJPDA Step 1: For each target this comes down to the mixing/interaction step of the IMM algorithm (Blom & Bar-Shalom, 1988) for all $i \in \{1, \ldots, M\}$: Starting with the weights

$$\hat{\gamma}^i_{t-1}(\theta^i) \stackrel{\triangle}{=} p_{\theta^i_{t-1}|Y_{t-1}}(\theta^i), \quad \theta^i \in \{1, ..., N\}$$

the means

$$\hat{x}^i_{t-1}(\theta^i) \stackrel{\triangle}{=} E\{x^i_{t-1}|\theta^i_{t-1} = \theta^i, Y_{t-1}\}, \quad \theta^i \in \{1, ..., N\}$$

and the associated covariances

$$\hat{P}^i_{t-1}(\theta^i) \stackrel{\triangle}{=} E\{[x^i_{t-1} - \hat{x}^i_{t-1}(\theta^i)][x^i_{t-1} - \hat{x}^i_{t-1}(\theta^i)]^T \mid \theta^i_{t-1} = \theta^i, Y_{t-1}\}, \quad \theta^i \in \{1, ..., N\}$$

one evaluates the mixed initial condition for the filter matched to $\theta^i_t = \theta^i$ as follows:

$$\bar{\gamma}^i_t(\theta^i) = \sum_{\eta^i=1}^{N} \Pi_{\eta^i,\theta^i} \cdot \hat{\gamma}^i_{t-1}(\eta^i)$$

$$\hat{x}^i_{t-1|\theta^i_t}(\theta^i) = \sum_{\eta^i=1}^{N} \Pi_{\eta^i,\theta^i} \cdot \hat{\gamma}^i_{t-1}(\eta^i) \cdot \hat{x}^i_{t-1}(\eta^i)/\bar{\gamma}^i_t(\theta^i)$$

$$\hat{P}^i_{t-1|\theta^i_t}(\theta^i) = \sum_{\eta^i=1}^{N} \Pi_{\eta^i,\theta^i} \cdot \hat{\gamma}^i_{t-1}(\eta^i) \cdot$$
$$\cdot\Big(\hat{P}^i_{t-1}(\eta^i) + [\hat{x}^i_{t-1}(\eta^i) - \hat{x}^i_{t-1|\theta^i_t}(\theta^i)][\hat{x}^i_{t-1}(\eta^i) - \hat{x}^i_{t-1|\theta^i_t}(\theta^i)]^T\Big)/\bar{\gamma}^i_t(\theta^i)$$

with

$$\Pi_{\eta^i,\theta^i} \stackrel{\triangle}{=} Pr\{\theta^i_t = \theta^i \mid \theta^i_{t-1} = \eta^i\}$$
$$\bar{\gamma}^i_t(\theta^i) \stackrel{\triangle}{=} p_{\theta^i_t|Y_{t-1}}(\theta^i)$$
$$\hat{x}^i_{t-1|\theta^i_t}(\theta^i) \stackrel{\triangle}{=} E\{x^i_{t-1} \mid \theta^i_t = \theta^i, Y_{t-1}\}$$
$$\hat{P}^i_{t-1|\theta^i_t}(\theta^i) \stackrel{\triangle}{=} E\{[x^i_{t-1} - \hat{x}^i_{t-1}(\theta^i)][x^i_{t-1} - \hat{x}^i_{t-1}(\theta^i)]^T|\theta^i_t = \theta^i, Y_{t-1}\}$$

IMMJPDA Step 2: Prediction for all $i \in \{1, \ldots, M\}$, $\theta^i \in \{1, \ldots, N\}$:

$$\bar{x}^i_t(\theta^i) = a^i(\theta^i)\hat{x}^i_{t-1|\theta^i_t}(\theta^i) \tag{14.a}$$
$$\bar{P}^i_t(\theta^i) = a^i(\theta^i)\hat{P}^i_{t-1|\theta^i_t}(\theta^i)a^i(\theta^i)^T + b^i(\theta^i)b^i(\theta^i)^T \tag{14.b}$$

IMMJPDA Step 3: Gating, which is based on Bar-Shalom & Li (1995).
Evaluate for each i and $\theta^i$the crosscovariance as follows:

$$Q^i_t(\theta^i) = h^i(\theta^i)\bar{P}^i_t(\theta^i)h^i(\theta^i)^T + g^i(\theta^i)g^i(\theta^i)^T$$

Subsequently identify for each target the mode for which Det $Q^i_t(\theta^i)$ is largest:

$$\theta^{*i}_t = \underset{\theta^i}{\text{Argmax}} \ \{\text{Det } Q^i_t(\theta^i)\}$$

and use this to define for each target i a gate $G^i_t \in I\!R^m$ as follows:

$$G^i_t \overset{\triangle}{=} \{z^i \in I\!R^m; [z^i - h^i(\theta^{*i}_t)\bar{x}^i_t(\theta^{*i}_t)]^T Q^i_t(\theta^{*i}_t)^{-1}[z^i - h^i(\theta^{*i}_t)\bar{x}^i_t(\theta^{*i}_t)] \leq \gamma\}$$

with γ the gate size. If the j-th measurement y^j_t falls outside gate G^i_t; i.e. $\mathrm{y}^j_t \notin G^i_t$, then the j-th component of the i-th row of $[\Phi(\phi)^T\tilde{\chi}_t]$ is assumed to equal zero. This reduces the set of possible detection/permutation hypotheses to be evaluated at moment t for various ϕ to $\tilde{\mathcal{X}}_t(\phi)$.

IMMJPDA Step 4: Evaluation of the detection / association / mode hypotheses is based on the Theorem. For all $\phi \in \{0, 1\}^M$, $\tilde{\chi} \in \{0, 1\}^{D(\phi)\times D(\phi)}$, $\theta \in \{1, ..., N\}^M$:

$$\begin{aligned} \beta_t(\phi, \tilde{\chi}, \theta) &= \lambda^{(L_t - D(\phi))} \textstyle\prod_{i=1}^M [f^i_t(\phi, \tilde{\chi}, \theta^i)\cdot \\ &\qquad \cdot(1 - P^i_{\mathrm{d}})^{(1-\phi_i)}(P^i_{\mathrm{d}})^{\phi_i} \cdot \bar{\gamma}^i_t(\theta^i)]/c_t \qquad \text{for } \tilde{\chi} \in \tilde{\mathcal{X}}_t(\phi), \\ &= 0 \qquad\qquad\qquad\qquad\qquad \text{else} \end{aligned} \tag{15.a}$$

with $f^i_t(\{0\}^M, \{\}^{L_t}, \theta^i) = 1$ and for $\phi \neq \{0\}^M$:

$$\begin{aligned} f^i_t(\phi, \tilde{\chi}, \theta^i) &\cong [(2\pi)^m \text{Det}\{Q^i_t(\theta^i)\}]^{-\frac{1}{2}\phi_i} \\ &\quad \cdot \exp\{-\frac{1}{2}\sum_{k=1}^{L_t}[\Phi(\phi)^T_{*i}\tilde{\chi}_{*k}\mu^{ik}_t(\theta^i)^T[Q^i_t(\theta^i)]^{-1}\mu^{ik}_t(\theta^i)]\} \end{aligned} \tag{15.b}$$

where

$$\mu_t^{ik}(\theta^i) = \mathrm{y}_t^k - h^i(\theta^i)\bar{x}_t^i(\theta^i) \tag{15.c}$$

IMMJPDA Step 5: Measurement update equations are based on the Theorem. For all $i \in \{1, ..., M\}$, $\theta^i \in \{1, ..., N\}$:

$$\hat{\gamma}_t^i(\theta^i) \cong \sum_{\substack{\phi,\tilde{\chi},\eta \\ \eta^i=\theta^i}} \beta_t(\phi, \tilde{\chi}, \eta) \tag{16}$$

and using (13b, c, d, e) as approximate equations to evaluate $\hat{x}_t^i(\theta^i)$ and $\hat{P}_t^i(\theta^i)$.

IMMJPDA Step 6: Output equations:

$$\hat{x}_t^i = \sum_{\theta^i=1}^{N} \hat{\gamma}_t^i(\theta^i) \cdot \hat{x}_t^i(\theta^i)$$
$$\hat{P}_t^i = \sum_{\theta^i=1}^{N} \hat{\gamma}_t^i(\theta^i)(\hat{P}_t^i(\theta^i) + [\hat{x}_t^i(\theta^i) - \hat{x}_t^i][\hat{x}_t^i(\theta^i) - \hat{x}_t^i]^T)$$

Remark: It can be verified that the above IMMJPDA filter algorithm is similar to the IMMJPDA filter algorithm of Chen & Tugnait (2001). The main new element is that the above specification of IMMJPDA Steps 4 and 5 explicitly show the relation to the processes $\{\tilde{\chi}_t\}$ and $\{\phi_t\}$. In the sequel this relation is exploited for the development of a track coalescence avoiding IMMJPDA filter, for short IMMJPDA* filter.

5 IMMJPDA* filter

A shortcoming of JPDA is its sensitivity to track coalescence. With their JPDA* approach, Blom & Bloem (2000) have shown that hypothesis pruning can provide an effective track-coalescence avoidance. The JPDA* filter equations can be obtained from the JPDA algorithm by pruning per (ϕ_t, ψ_t)-hypothesis all less likely χ_t-hypotheses prior to measurement updating. In order to apply this approach to IMMJPDA the JPDA* hypothesis pruning strategy is now extended: evaluate all $(\phi_t, \psi_t, \theta_t)$ hypotheses and prune per $(\phi_t, \psi_t, \theta_t)$-hypothesis all less-likely χ_t-hypotheses. To do so, define for every ϕ, ψ and θ, satisfying $D(\psi) = D(\phi) \leq \text{Min}\{M, L_t\}$, a mapping $\hat{\chi}_t(\phi, \psi, \theta)$:

$$\hat{\chi}_t(\phi, \psi, \theta) \stackrel{\triangle}{=} \underset{\chi}{\text{Argmax}} \;\; \beta_t(\phi, \chi^T \Phi(\psi), \theta)$$

where the maximization is over all permutation matrices χ of size $D(\phi) \times D(\phi)$.
The pruning strategy of evaluating all (ϕ, ψ, θ)-hypotheses and only one χ-hypothesis per (ϕ, ψ, θ)-hypothesis implies that for $D(\phi) > 0$ we adopt the following pruned hypothesis weights $\hat{\beta}_t(\phi, \psi, \theta)$:

$$\begin{aligned}\hat{\beta}_t(\phi, \psi, \theta) &= \beta_t(\phi, \hat{\chi}(\phi, \psi, \theta)^T \Phi(\psi), \theta)/\hat{c}_t \\ &\qquad\qquad \text{if } D(\phi) = D(\psi) \leq \text{Min}\{M, L_t\} \\ &= 0 \qquad\qquad \text{else}\end{aligned}$$

with $\hat{c}_t$ a normalization constant for $\hat{\beta}_t$; i.e. such that

$$\sum_{\substack{\phi, \psi, \theta \\ D(\psi) = D(\phi)}} \hat{\beta}_t(\phi, \psi, \theta) = 1$$

By inserting these particular weights within IMMJPDA, we get IMMJPDA*.
One cycle of the IMMJPDA* filter algorithm consists of the following 7 steps:

IMMJPDA Step 1:* Mixing for all $i \in \{1, ..., M\}$, $\theta^i \in \{1, ..., N\}$: Equivalent to IMMJPDA step 1, section 5.

IMMJPDA Step 2:* Prediction for all $i \in \{1, ..., M\}$, $\theta^i \in \{1, ..., N\}$: Equivalent to IMMJPDA step 2, section 5.

IMMJPDA Step 3:* Gating: Equivalent to IMMJPDA step 3, section 5.

IMMJPDA Step 4:* Evaluation of the detection/evaluation hypotheses: Equivalent to IMMJPDA step 4, section 5.

IMMJPDA Step 5:* Track-coalescence hypothesis pruning: First evaluate for every (ϕ, ψ, θ) such that $D(\psi) = D(\phi) \leq \text{Min}\{M, L_t\}$:

$$\hat{\chi}_t(\phi,\psi,\theta) = \underset{\chi}{\text{Argmax}} \ \beta_t(\phi, \chi^T\Phi(\psi), \theta)$$

Next evaluate all $\hat{\chi}_t(\phi,\psi,\theta)$ hypothesis weights:

$$\begin{aligned}\hat{\beta}_t(\phi,\psi,\theta) &= \beta_t(\phi, \hat{\chi}_t^T(\phi,\psi,\theta)\Phi(\psi),\theta)/\hat{c}_t \quad && \text{if } 0 < D(\psi) = D(\phi) \leq \text{Min}\{M, L_t\} \\ &= \beta_t(\{0\}^M, \{\}^{L_t}, \theta)/\hat{c}_t && \text{if } D(\psi) = D(\phi) = 0 \\ &= 0 \qquad \text{else} \end{aligned}$$

where $\hat{c}_t$ is a normalizing constant for $\hat{\beta}_t$.

IMMJPDA Step 6:* Measurement update equations for all $i \in \{1, ..., M\}$, $\theta^i \in \{1, ..., N\}$:

$$\hat{\gamma}_t^i(\theta^i) \cong \sum_{\substack{\phi,\psi,\eta \\ \eta^i=\theta^i}} \hat{\beta}_t(\phi,\psi,\eta) \tag{17.a}$$

$$\hat{x}_t^i(\theta^i) \cong \bar{x}_t^i(\theta^i) + W_t^i(\theta^i)\left(\sum_{k=1}^{L_t} \hat{\beta}_t^{ik}(\theta^i)\mu_t^{ik}(\theta^i)\right) \tag{17.b}$$

$$\begin{aligned}\hat{P}_t^i(\theta^i) \cong\ & \bar{P}_t^i(\theta^i) - W_t^i(\theta^i)h^i(\theta^i)\bar{P}_t^i(\theta^i)\left(\sum_{k=1}^{L_t}\hat{\beta}_t^{ik}(\theta^i)\right) + \\ & +W_t^i(\theta^i)\left(\sum_{k=1}^{L_t}\hat{\beta}_t^{ik}(\theta^i)\mu_t^{ik}(\theta^i)\mu_t^{ik}(\theta^i)^T\right)W_t^i(\theta^i)^T + \\ & -W_t^i(\theta^i)\left(\sum_{k=1}^{L_t}\hat{\beta}_t^{ik}(\theta^i)\mu_t^{ik}(\theta^i)\right)\left(\sum_{k'=1}^{L_t}\hat{\beta}_t^{ik'}(\theta^i)\mu_t^{ik'}(\theta^i)\right)^T W_t^i(\theta^i)^T\end{aligned} \tag{17.c}$$

with:

$$W_t^i(\theta^i) \stackrel{\triangle}{=} \bar{P}_t^i(\theta^i)h^i(\theta^i)^T[Q_t^i(\theta^i)]^{-1} \tag{17.d}$$

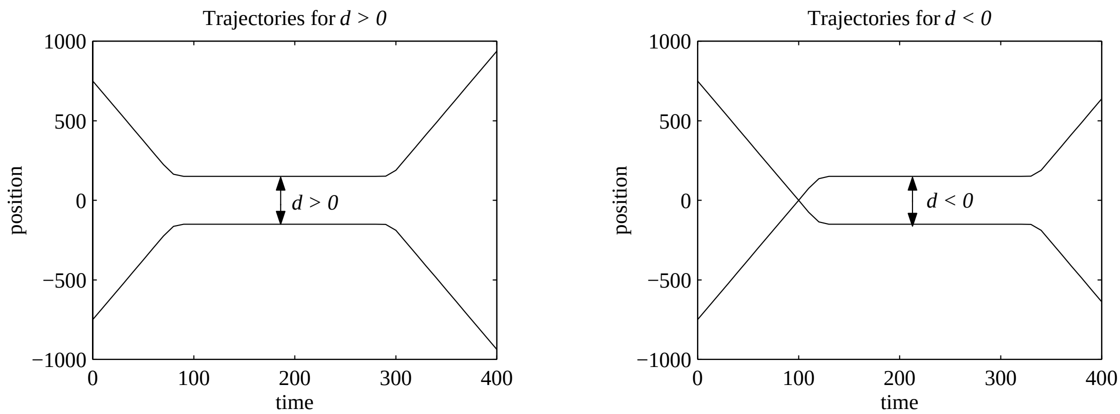
$$\hat{\beta}_t^{ik}(\theta^i) = \sum_{\substack{\phi,\psi,\eta \\ \phi,\psi\neq 0 \\ \eta^i=\theta^i}} [\Phi(\phi)_{*i}^T[\hat{\chi}_t(\phi,\psi,\eta)^T\Phi(\psi)]_{*k}\hat{\beta}_t(\phi,\psi,\eta)]/\hat{\gamma}_t^i(\theta^i) \tag{17.e}$$

where $[.]_{*k}$ is the k-th column of $[.]$.

IMMJPDA Step 7:* Output Equations: Equivalent to IMMJPDA step 6, section 5.

6 Monte Carlo simulations

In this section some Monte Carlo simulation results are given for the IMMPDA, IMMJPDA and IMMJPDA* filter algorithms. The simulations primarily aim at gaining insight into the behaviour and performance of the filters when objects move in and out close approach situations, while giving the filters enough time to converge after a manoeuvre has taken place. In the example scenarios there are two targets, each modeled with two possible modes. The first mode represents a constant velocity model and the second mode represents a constant acceleration model. Both objects start moving towards eachother, each with constant initial velocity V_{initial} (i.e. the initial relative velocity $V_{\text{rel, initial}} = 2V$). At a certain moment in time both objects start decelerating with -0.5 m/s^2 until they both have zero velocity. The moment at which the deceleration starts is such that when the objects both have zero velocity, the distance between the two objects equals d. After spending a significant number of scans with zero velocity, both objects start accelerating with 0.5 m/s^2 away from each other without crossing until their velocity equals the opposite of their initial velocity. From that moment on the velocity of both objects remains constant again (thus the final relative velocity $V_{\text{rel, final}} = V_{\text{rel, initial}}$). Note that $d < 0$ implies that the objects have crossed each other before they have reached zero velocity. Each simulation the filters start with perfect estimates and run for 40 scans. Examples of the trajectories for $d > 0$ and $d < 0$ are depicted in figures 1a and 1b respectively. For each target, the underlying model of the potential



1a. Trajectories examples for $d > 0$

1b. Trajectories examples for $d < 0$

Fig. 1 Trajectories examples for $d > 0$ and for $d < 0$

target measurements is given by (1) and (3)

$$x_{t+1}^i = a^i(\theta_{t+1}^i)x_t^i + b^i(\theta_{t+1}^i)w_t^i \tag{1}$$

$$z_t^i = h^i(\theta_t^i)x_t^i + g^i(\theta_t^i)v_t^i \tag{3}$$

Furthermore for $i = 1, 2$ and $\theta_t^i \in \{1,2\}$:

$$a^i(1) = \begin{bmatrix} 1 & T_s & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \qquad a^i(2) = \begin{bmatrix} 1 & T_s & \frac{1}{2}T_s^2 \\ 0 & 1 & T_s \\ 0 & 0 & 1 \end{bmatrix}$$

$$b^i(1) = \sigma_a^i \cdot \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \qquad b^i(2) = \sigma_a^i \cdot \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$h^i = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}, \qquad g^i = \sigma_m^i$$

$$\Pi = \begin{bmatrix} 1 - T_s/\tau_1 & T_s/\tau_1 \\ T_s/\tau_2 & 1 - T_s/\tau_2 \end{bmatrix}$$

where σ_a^i represents the standard deviation of acceleration noise and σ_m^i represents the standard deviation of the measurement error. For simplicity we consider the situation of similar targets only; i.e. $\sigma_a^i = \sigma_a$, $\sigma_m^i = \sigma_m$, $P_{\mathrm{d}}^i = P_{\mathrm{d}}$. With this, the scenario parameters are P_{d}, λ, d, V_{initial}, T_s, σ_m, σ_a, τ_1, τ_2, and the gate size γ. We used fixed parameters $\sigma_m = 30$, $\sigma_a = 0.5$, $\tau_1 = 500$, $\tau_2 = 50$, and $\gamma = 25$. Table 1 gives the other scenario parameter values that are being used for the Monte Carlo simulations.

Table 1 Scenario parameter values.

IMMPDA's $\lambda = 0.00001$ for scenarios 1 and 3

Scenario	P_{d}	λ	d	V_{initial}	T_s
1	1	0	Variable	7.5	10
2	1	0.001	Variable	7.5	10
3	0.9	0	Variable	7.5	10
4	0.9	0.001	Variable	7.5	10

During our simulations we counted track i "O.K.", if

$$\mid h^i \hat{x}_T^i - h^i x_T^i \mid \leq 9\sigma_m$$

and we counted track $i \neq j$ "Swapped", if

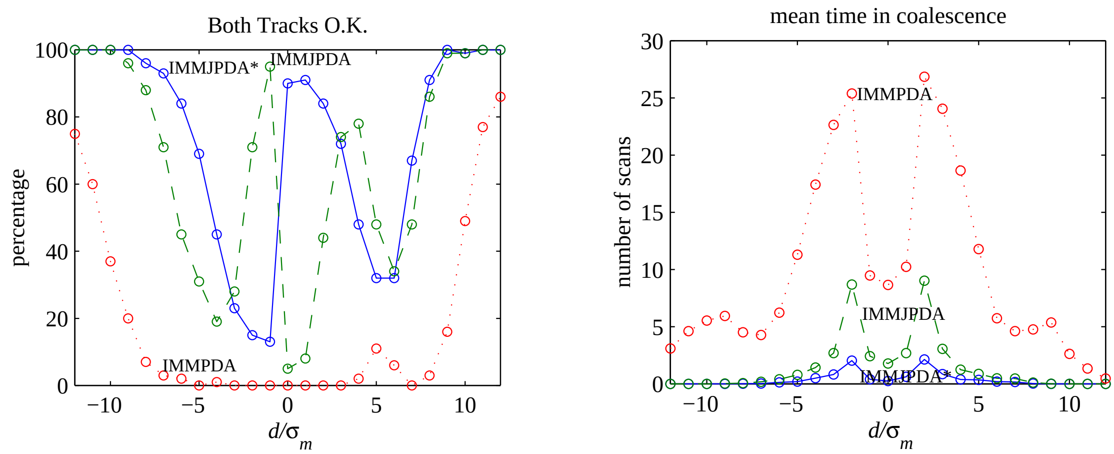
$$\mid h^i \hat{x}_T^i - h^j x_T^j \mid \leq 9\sigma_m$$

Furthermore, two tracks $i \neq j$ are counted "Coalescing" at scan t, if

$$\mid h^i \hat{x}_t^i - h^j \hat{x}_t^j \mid \leq \sigma_m \wedge \mid h^i x_t^i - h^j x_t^j \mid > \sigma_m$$

For each of the scenarios Monte Carlo simulations containing 100 runs have been performed for each of the tracking filters. To make the comparisons more meaningful, for all tracking mechanisms the same random number streams were used. The results of the Monte Carlo simulations for the three scenarios are depicted as function of the distance relative to σ_m in two types of figures, showing respectively:

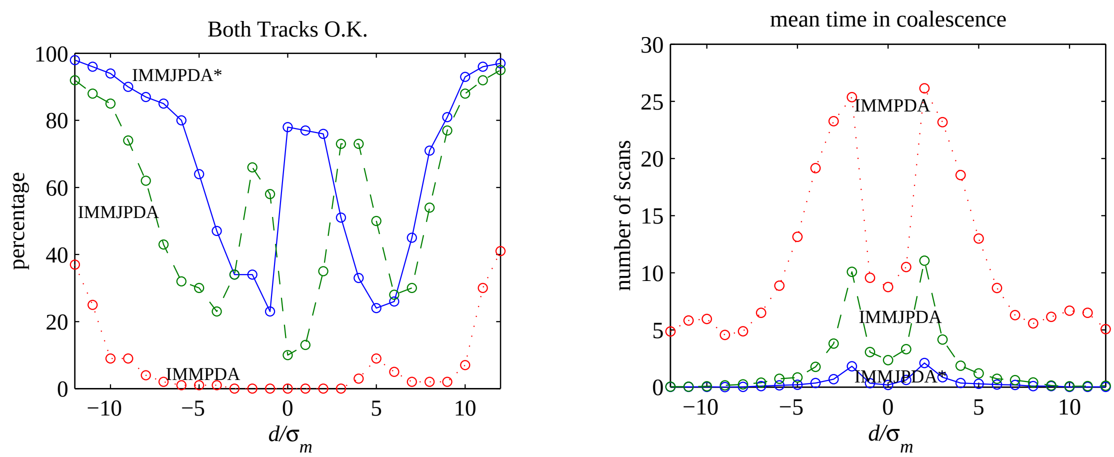
- The percentage of Both tracks "O.K. " (figures 2a, 3a, 4a, 5a)
- The average number of "coalescing" scans (figure 2b, 3b, 4b, 5b).



2a. "Both tracks O.K." percentage

2b. Average number of "coalescing" scans

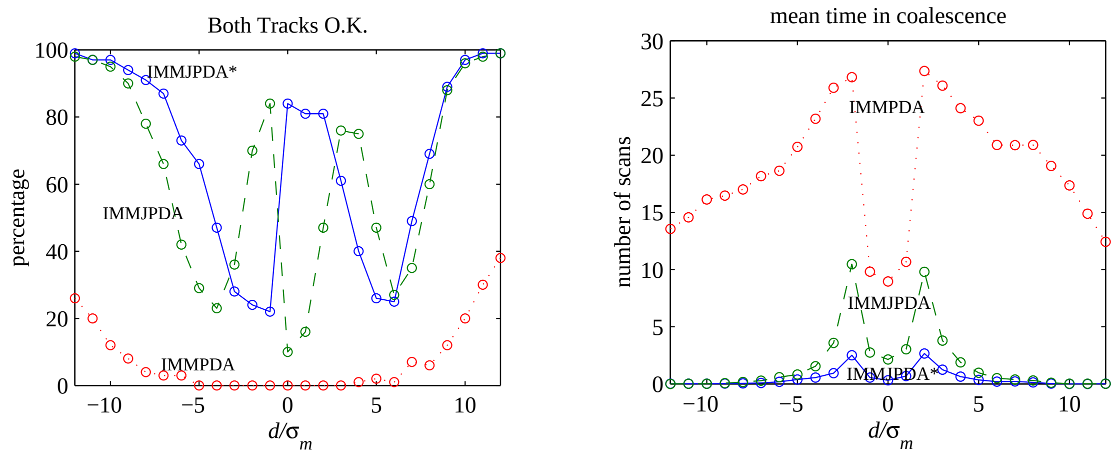
Fig. 2 Simulation results for scenario 1



3a. "Both tracks O.K." percentage

3b. Average number of "coalescing" scans

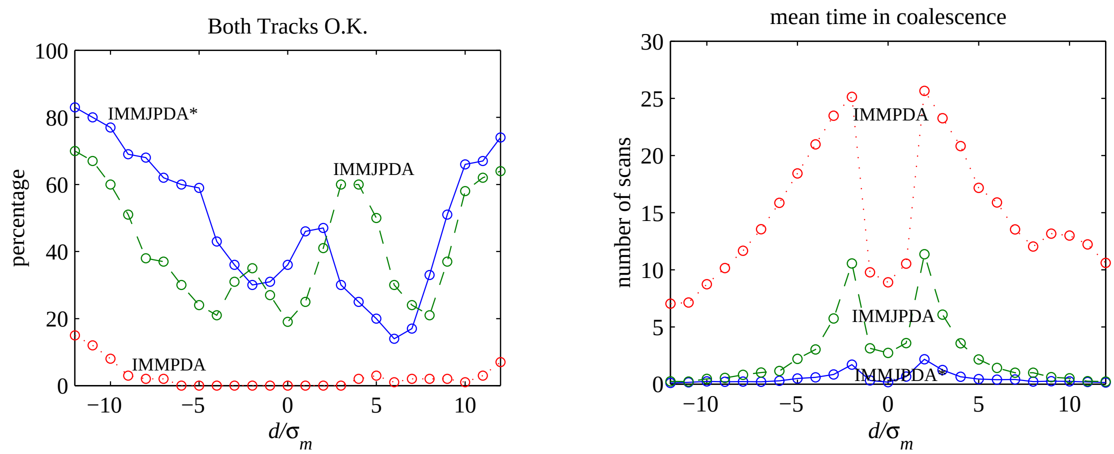
Fig. 3 Simulation results for scenario 2



4a. "Both tracks O.K." percentage

4b. Average number of "coalescing" scans

Fig. 4   Simulation results for scenario 3



5a. "Both tracks O.K." percentage

5b. Average number of "coalescing" scans

Fig. 5   Simulation results for scenario 4

For the examples considered, the simulation results show that both IMMJPDA* and IMMJPDA perform much better than IMMPDA. Moreover the results show that IMMJPDA* avoids track coalescence.

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