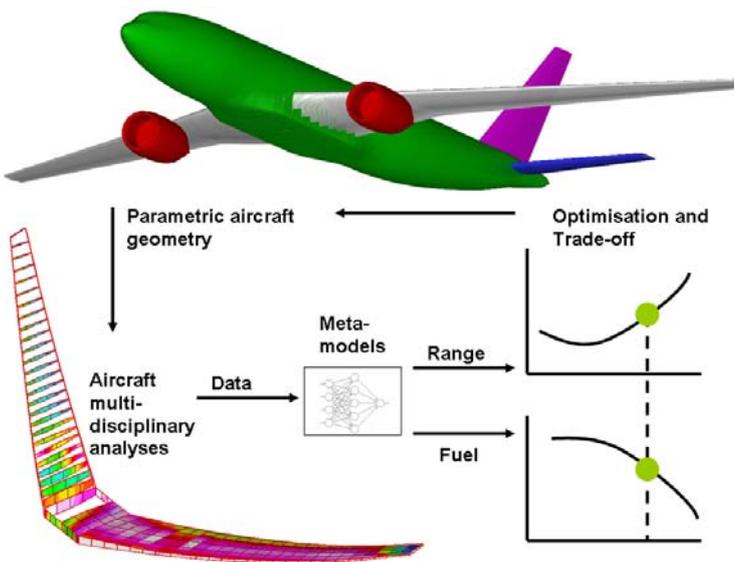




## Executive summary

# Multi-objective Optimisation of Aircraft Range and Fuel Consumption



The multidisciplinary wing design and multi-objective optimization process

### Problem area

Aircraft fuel consumption is a critical aspect and competitive attribute of new civil aircraft. Hence aircraft design progressively incorporates fuel consumption as a key objective. Proper assessment of fuel consumption in the early stages of aircraft design requires an integrated multidisciplinary engineering process, which includes closely coupled analyses of all key phenomena that determine the aircraft performance.

### Description of work

A system for integrated multi-disciplinary analysis and multi-objective optimization of transonic aircraft wings, with focus on the aerodynamics and structural mechanics disciplines, is applied in a multi-objective design optimization study of a 250 pax mid-range civil aircraft. Besides fuel consumption, also the range is considered as a design objective in this optimization study. To allow for extensive multi-disciplinary analysis evaluations in the optimization iterations, simplified and efficient models are applied for

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multi-objective optimization

fast and sufficiently accurate approximation of the multi-disciplinary analysis results.

### **Results and conclusions**

The results obtained by the multi-objective optimization approach provide a clear overview of the most interesting aircraft design points. The maximum achievable range found is about 7550 nm, and the maximum achievable fuel efficiency is about 38 km/l/pax, which are clear improvements compared to the 5484 nm and 32 km/l/pax of the initial reference aircraft configuration.

The computational time gained by the use of the simplified models comes at the cost of their possible limitation in accuracy. Predictions with the third order polynomial based model appeared to strongly overestimate fuel efficiency values

for the optimal design points. Because of the availability, in this study, of different simplified models for fuel efficiency, this effect could be easily discovered and mitigated by making use of the other models.

### **Applicability**

The resulting sets of optimal wing designs provide the design points that have the most profitable combination of range and fuel consumption. It is shown that these results can be achieved with quite limited computational effort, but also that they do depend on the accuracy of the predictions by the simplified models, and that adequate control of this accuracy is crucial for achieving reliable results. The latter aspect is subject of ongoing research.



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## Multi-objective Optimisation of Aircraft Range and Fuel Consumption

W.J. Vankan, E. Kessler and M. Laban

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## Summary

A system for integrated multi-disciplinary analysis and multi-objective optimisation of transonic aircraft wings is presented. The multi-disciplinary analysis focuses on the aerodynamics and structural mechanics disciplines, deploying computational fluid dynamics-based aerodynamics and finite element method-based structural mechanics tools for accurate, but computationally expensive simulations. These simulations are complemented with lower fidelity tools covering the other contributing disciplines, such as weight estimation and engine sizing. The multi-disciplinary analysis system is applied in a multi-objective optimisation study of aircraft range and fuel consumption. To allow for extensive multi-disciplinary analysis evaluations in the optimisation iterations, meta-models are used for fast and sufficiently accurate approximation of the multi-disciplinary analysis results. The resulting Pareto optimal sets of wing designs provide the design points that have the most profitable combination of range and fuel consumption. It is shown that these results can be achieved with quite limited computational effort, but also that they do depend on the accuracy of the predictions by the meta-models, and that adequate control of this accuracy is crucial for achieving reliable results.

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## Abbreviations

ann	artificial neural networks
CFD	Computational Fluid Dynamics
FEM	Finite Element Method
kriging-cC	kriging-constant-Cubic fit (kcc)
kriging-cE	kriging-constant-Exponential fit (kce)
kriging-cG	kriging-constant-Gauss fit (kcg)
kriging-lC	kriging- linear-Cubic fit (klc)
kriging-lE	kriging- linear-Exponential fit (kle)
kriging-lG	kriging-linear-Gauss fit (klg)
LoD	Lift over Drag (L/D)
MDA	Multidisciplinary Design Analysis
MTOW	Maximum Take-Off Weight
MAPE	Mean Absolute Percentage Error
NSGA	Non Dominated Sorting Genetic algorithm
poly0	0 <sup>th</sup> order polynomial fit
poly1	1 <sup>st</sup> order polynomial fit
poly2	2 <sup>nd</sup> order polynomial fit
poly3	3 <sup>rd</sup> order polynomial fit
poly4	4 <sup>th</sup> order polynomial fit
rbf	radial basis functions
RMSE	Root Mean Squared Error



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## 1 Introduction

Due to high oil prices and environmental concerns, fuel consumption is becoming a critical aspect, as well as a competitive attribute, of new civil aircraft. Hence aircraft design progressively incorporates fuel consumption as a key objective, already in the early design stages. Aircraft design requires an integrated multidisciplinary engineering process, which includes closely coupled analyses of all key phenomena that determine the aircraft performance. Besides the traditional performance aspects like range and speed, also fuel consumption is taken into account more prominently as design objective in this integrated multidisciplinary design process.

This paper describes an investigation of aircraft range and fuel efficiency, which is performed in part in the context of the European project VIVACE [1]. Fuel efficiency here represents the distance flown per unit fuel per unit payload. The investigation concerns the optimisation of transonic aircraft wings in the preliminary design phase. The design analyses in this investigation make use of a multidisciplinary design analysis (MDA) system that evaluates the aircraft characteristics as a function of a set of design parameters [2]. The evaluations comprise, among others, wing structural sizing and optimisation using finite element method (FEM) analyses, and cruise lift over drag performance using computational fluid dynamics (CFD) analyses. The design parameters include geometric wing planform parameters such as span, chord, sweep, as well as “aircraft operational parameters” such as maximum take-off weight (MTOW) and cruise altitude. With the design parameters inputs, the wing MDA system predicts the corresponding aircraft characteristics in terms of, among others, weight breakdown information, maximum range and fuel consumption. In order to effectively handle these different (and possibly conflicting) design objectives, multi-objective Pareto front [3] optimisation algorithms are used in the presented aircraft wing design investigation. The multi-objective Pareto front results directly provide the design information on which further trade-off considerations of the different objectives for the wing design can be based.

To limit the number of computationally expensive evaluations with the MDA system, the multi-objective optimisation iterations have been de-coupled from the MDA evaluations by making use of an advanced meta-modelling (or response surface) approach [4]. The meta-modelling approach allows for computationally efficient exploration of the aircraft characteristics in a pre-defined design domain. Different meta-modelling methods, such as polynomial regression, kriging models and neural networks, are used and their predictive accuracy is carefully checked and compared in order to achieve the best representation. Obviously, the results of the optimisation depend on the accuracy of the meta-models used, and therefore also require careful assessment and validation, as is shown in the present aircraft design optimisation study.

## 2 Multi-disciplinary design analyses

### 2.1 Introduction

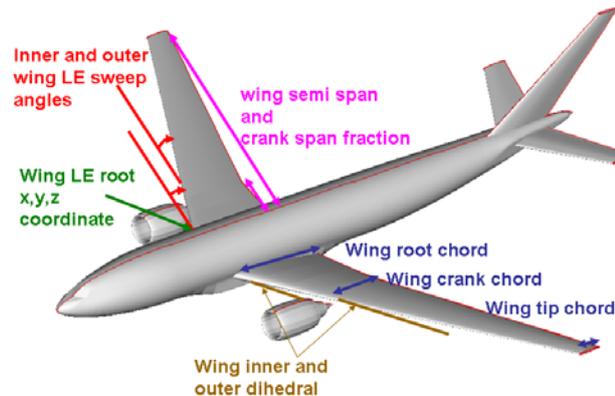
A generic multi-disciplinary analysis system and optimisation framework for design of aircraft wings has been developed at NLR during the last decade. In this system, the key disciplines for aeronautic design, aerodynamics and structural mechanics, take a central place. In addition, several other disciplines, like weight estimation and engine sizing, are included by means of models of various levels of fidelity. The main components of this MDA framework are the following:

- 1) A *Geometry module* providing the parametric external (aerodynamic) and internal (structural) shape of the configuration.
- 2) A *Weight and Balance module* keeping a record of all items contributing to the mass and centre of gravity of the configuration.
- 3) An *Engine Sizing module* sizing the propulsion system to meet the aircraft thrust requirements.
- 4) A *Structural Optimisation module* sizing the structural element thicknesses to arrive at a minimum weight primary wing structure.
- 5) An *Aerodynamic Performance module* predicting the aircraft lift over drag (L/D) performance.
- 6) A *Mission Analysis module* collecting the results from all contributing analysis disciplines and providing the aircraft mission range.

The exchange of the appropriate information among the different modules is achieved by reading/writing data from/to a central product database that contains the most relevant information of the aircraft being analysed [2]. The different modules are implemented as stand-alone executable programmes. The modules and the data exchange and interdependencies among the various disciplines will be described in more detail in the next sections.

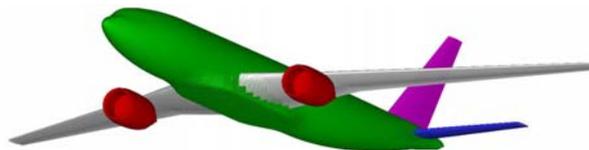
### 2.2 Geometry Generation

The *Geometry Generation module* is used to generate the global geometry of the considered aircraft, in particular the external (for aerodynamic analyses) and internal (for structural analyses) geometries of the wings. This present investigation focuses on wing planform modifications. The wing planform is modelled as a parametric double trapezium (Figure 1).



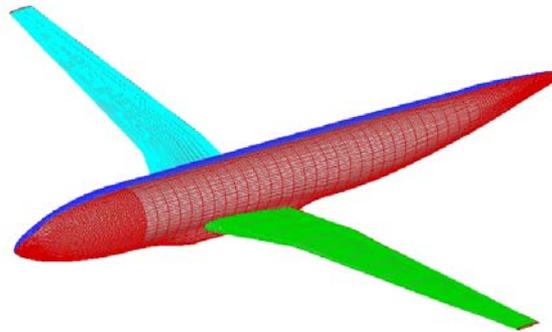
*Figure 1 Global geometry of the aircraft configuration considered in the present study; fuselage and tail are fixed; wing geometry is parametrically defined by the 12 wing design parameters indicated; besides the external geometry, for the wings also the internal geometry is generated*

The Geometry Generation module is fed by the 12 wing planform design parameters (Table 1). The geometries of the wings and their position and orientation on the fuselage are derived from these design parameters. The wing airfoil shape is defined for each wing section and is based on coordinates read from a database, which are not varied in this study. During the geometry generation the surfaces of the individual aircraft components (wings, nacelles, fuselage, stabilizer, fin) are computed and connected together (Figure 2).



*Figure 2 Aircraft components geometries connected together*

The external shape of the aircraft is computed to facilitate the CFD based aerodynamic computations. Currently, this retains the fuselage and wing components only. For this purpose, the intersection of the wing and fuselage is computed and the two are properly connected together, and the wing-tip is closed. The resulting aircraft wing/body geometry is delivered to the aerodynamics module as a number of surfaces (Figure 3).



*Figure 3 Aircraft external geometry defined as a set of surfaces that are provided to the CFD analyses. (The wing geometry shown here was generated with a low value of the outer wing sweep angle design parameter.).*

The wing structural topology is also generated, which is used in the FEM analysis for structural sizing. The considered structural elements comprise spars, ribs, covers and stringers. The wing structural layout comprises multiple ribs, oriented in flight direction at 50 cm equi-distant rib spacing intervals, and two spars. The wing covers are supported by the spars and ribs. The spar and rib layout take the engine attachment points and the leading-/trailing-edge movables into account. Figure 4 shows an example of the aircraft internal wing structural geometry.

The structural topology is “rubberized” and follows changes in wing planform as defined by the global wing design parameters. E.g., the number of ribs included depends on the wing span. The wing panels are stiffened using hat-type stringers supporting the upper-wing covers and Z-type stringers supporting the lower-wing covers. A number of physical stringers are lumped together in the analysis and are represented using a single bar type of element (green lines in Figure 4). The structural elements are represented by a set of structured surfaces and are delivered to the Structural Optimisation module for element thickness sizing.

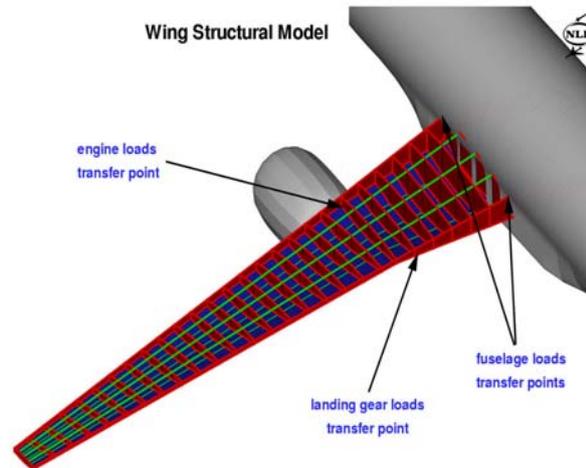


Figure 4 Aircraft internal wing structural geometry. Indications are given of where the external loads on the wing from the engines, landing gear and fuselage are located

Fuel is stored in the fuselage and centre-wing, inner-wing and outer-wing tanks. The geometry generation module computes the total fuel volume capacity of the configuration.

The engine nacelles are scaled according to the aircraft thrust requirements, obtained from the Engine Sizing module, and are properly positioned relative to the configuration.

### 2.3 Weight and balance

The Weight and Balance module is responsible for keeping a record of all items contributing to the mass and centre of gravity location of the aircraft. The contributing items are classified as follows.

- 1) *Structural Items*, like spars, ribs, stringers, skin panels etc. Wing weight is mainly determined by the wing planform, which depends on the wing design parameters. The actual wing structural weight also depends on the structural sizing, which is computed by the Structural Optimisation module described below. The structural weight of the fuselage and tail planes is assumed to remain fixed, irrespective of any wing planform changes.
- 2) *Non-structural items*, like systems, cabin furnishing, operator items, etc., i.e. items not belonging to the primary aircraft structure. For the wing, a fixed weight for leading- and trailing edge devices of 3000 kg is assumed. The weight of the wing tank sealing is modelled as 0.4% of the total wing tank fuel capacity. Fuselage and empennage are fixed during the design process, having total weights of 40000 kg and 3000 kg, respectively. The weight of the landing gear is modelled as 4.5% of the aircraft MTOW.
- 3) *Propulsion System*. The mass of the propulsion system varies as a function of the design parameters. This is due to the variable aircraft thrust requirement. The thrust requirement is linked to the wing aerodynamic characteristics and aircraft MTOW. Hence, propulsion

system weight is modelled as a ratio of the required take-off thrust (0.03 [kg/N]), and is calculated by the Engine Sizing module described below.

- 4) *Payload*. A payload of 35000 kg, representative for 250 passengers, is represented by a fixed mass.
- 5) *Fuel* stored in the wing tanks. Total weight available for fuel is computed as the difference between MTOW, and the aircraft's empty operating weight plus the payload. However, also the capacity of the fuel tanks, as obtained from the geometry module, is taken into account. In case this fuel capacity is lower than the above mentioned weight available for fuel, then the available fuel weight is set equal to the fuel capacity and the aircraft take-off weight is reduced accordingly.

The Weight and Balance module also assembles the individual mass components into load cases. For each load case, a full set of information comprising mass, centre of gravity, flight condition etc. is generated and written to the central database. It should be noted that the total mass is different in the different load cases due to consumption of fuel. From this information the driving scenarios are derived for the subsequent disciplinary analyses for the engine sizing, wing structural mass minimization and aerodynamic cruise performance.

#### **2.4 Engine sizing**

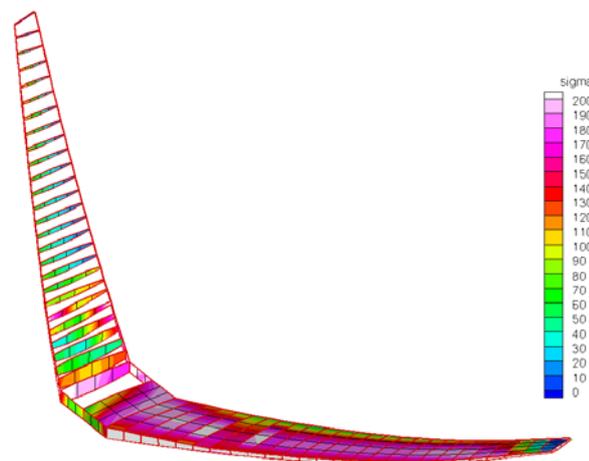
A standard "rubberized engine" model is used to calculate the size of the engine. As a worst case, the required thrust during the take-off condition is evaluated, assuming a one-engine-out failure condition and a standard limited runway length. The engines are sized accordingly.

#### **2.5 Structural Optimisation**

The Structural Optimisation module is responsible for sizing the thicknesses of the wing primary structural components: spars, ribs, skins. For this purpose, FEM analyses are well-suited and computationally efficient, provided that the number of elements is limited. The driving scenario for sizing of the structural components is currently limited to a single representative load case, i.e.: a +2.5g pull-up manoeuvre at MTOW, low-altitude/low-speed. The aircraft loading is configured such that the wing structure experiences maximum bending moments, i.e. maximum payload and maximum fuel in wing tanks. The Geometry Generation module delivers both the layout of the internal structural elements as well as the configuration external-shape for loads computations. The aerodynamic loading is based on a quasi three dimensional flow solution for the considered load case, where the flow solver is run for the prescribed manoeuvre lift coefficient. The aerodynamic surface pressures are translated into elementary force vectors on the aerodynamic wing surface grid. These force vectors are then mapped, using spline interpolation techniques, to the structural grid points on the

aerodynamics/structures interface. In addition, the non-structural mass items (i.e. landing gear, engines, LE-/TE-movables, servo systems, etc.) and fuel masses are identified and connected as discrete mass items to the nearest structural grid points. These discrete mass items contribute to the inertial loading of the structure during the considered load case. The result is a load card representing point mass items and external surface pressure loads.

The wing structural layout, as provided by the geometry module, is read into a special purpose algorithm. This algorithm meshes the covers, spars and ribs using quadrilateral elements (NASTRAN [5]: CQUAD4) and meshes the stringers using bar elements (NASTRAN: CBAR), combines groups of structural elements into design areas, connects non-structural mass items to the mesh, reads in the external (aerodynamic) loads and returns a bulk data deck file for the structural analysis. The structural analysis makes use of the FEM solver implemented in MSC-NASTRAN SOL101. The von Mises stresses in the (isotropic aluminium) element corner points are used to drive a local-level optimisation loop, which sizes all the elements' thicknesses. The structural optimisation objective is minimal weight of the structural components under the constraints that the maximum von Mises stress is below 200 MPa and element thicknesses are greater than 2 mm. The element thicknesses of the covers, spars and ribs are grouped into approximately 120 design areas, for each of which one thickness value is prescribed. The optimisation is performed using NASTRAN's native gradient based SOL200 optimiser, and requires about 5 to 15 iterations. The outcome of the NASTRAN-based optimisation process is the thicknesses for each of the design areas of the primary aircraft structure (Figure 5). Other details of this module are also given in [6].



*Figure 5 Illustration of the von Mises stress [MPa] distribution in the upper-wing covers and in the spars and ribs*

## 2.6 Aerodynamic Performance

Information concerning the aircraft aerodynamic behaviour is required for several load cases considered in the multi-disciplinary analysis. The Engine Sizing module requires inputs on take-off thrust requirements, which depends on aerodynamic drag of the aircraft. The Mission Analysis module would require aerodynamic lift over drag (L/D) performance information at several points of the mission profile. CFD based methods can accurately provide this information, but it would become computationally very expensive to compute multiple entries in the Mach-CL plane using CFD technology. As a compromise, CFD technology, based on an efficient solver of the full-potential equations in interaction with a boundary-layer solver for wing-body configurations, is used for the cruise condition only. Computationally inexpensive methods are used to complement this information for the other flight phases.

## 2.7 Mission Analysis

The Mission Analysis module combines mass, aerodynamic and engine data to evaluate the range performance of the aircraft. Mission range constitutes an important global-level design objective or constraint. Mission range is computed according to the Breguet range equation [7].

## 3 Aircraft Design Optimisation

The MDA system described above is used in an aircraft wing design optimisation study. Aircraft designs are pursued that have optimal performance for both range and fuel efficiency. As a starting point, a reference aircraft design is defined for which sensible values for the design parameters are based on estimates and expertise for the considered design targets. For the design optimisation study, there are many possible inputs, i.e. design parameters, to the wing MDA system. For example, the wing geometry can be defined by the following parameters:

*Table 1 Wing geometry parameters; values given here for the reference aircraft*

#	Wing Planform	
18.0000	-- Wing LE-Root x-coordinate	[m]
00.0000	-- Wing LE-Root y-coordinate	[m]
-1.2500	-- Wing LE-Root z-coordinate	[m]
12.0000	-- Wing Root Chord	[m]
33.0000	-- Wing-Inner LE-Sweep	[deg]
03.0000	-- Wing-Inner LE-Dihedral	[deg]
00.3000	-- Wing Crank Span Fraction	
07.0000	-- Wing Crank Chord	[m]

33.0000	-- Wing-Outer LE-Sweep	[deg]
04.0000	-- Wing-Outer LE-Dihedral	[deg]
30.0000	-- Wing Semispan	[m]
02.5000	-- Wing Tip Chord	[m]
# Wing Sections		
03.0000	-- Number of Wing Definition Sections	
00.0000	-- Wing Section 01 Span Fraction	
00.1400	-- Wing Section 01 t/c	
04.5000	-- Wing Section 01 Twist	[deg]
00.3000	-- Wing Section 02 Span Fraction	
00.0900	-- Wing Section 02 t/c	
01.0000	-- Wing Section 02 Twist	[deg]
01.0000	-- Wing Section 03 Span Fraction	
00.0900	-- Wing Section 03 t/c	
-1.5000	-- Wing Section 03 Twist	

In addition, also several aircraft level operational and weight breakdown parameters can be specified, for example:

*Table 2 Aircraft operational and weight breakdown parameters; values given here for the reference aircraft*

# Flight Conditions	
0.8000	-- Start-of-Cruise Mach Number
330.0000	-- Start-of-Cruise Flight Level
1.0000	-- Start-of-Cruise Loadfactor
0.5000	-- Manoeuvre Mach Number
15.0000	-- Manoeuvre Flight Level
2.5000	-- Manoeuvre Loadfactor
# Aircraft Weight Breakdown	
35000.0000	-- Payload Weight [kg]
30.0000	-- Payload CG X-Coordinate [m]
230000.0000	-- Maximum Take-Off Weight [kg]
180000.0000	-- Maximum Landing Weight [kg]

Furthermore, many settings for fuselage, fin, stabilizers, control surfaces, tanks, engines etc., are included, but are not varied in the present wing design study.

From the many results from the MDA simulations, different variables can be selected as relevant objective or constraint functions in aircraft design optimisation studies. For example the Breguet Range, Time at Landing on Scheduled Destination or Cruise Engine Specific Fuel Consumption are directly available from the MDA simulations. In the present wing design optimisation study we look for optimal overall range and fuel efficiency, and hence we take into account the Breguet range and the total fuel consumption for the calculation of the design objectives. Some more detail on these aspects of the wing MDA simulation system is given in [2][6].

To limit the scope of the present wing design optimisation study, four of the most relevant design parameters of the ones mentioned above have been selected as independent design variables: wing semi-span, outer wing leading-edge sweep angle, wing chords, and aircraft MTOW. The three wing chords (at root, crank and tip) are reduced to a single parameter, the wing chord scale factor, which linearly scales all three chords equally. All other design parameters of the MDA system are equal to their values for the reference aircraft and remain unchanged in this study. The geometric design variables determine the aircraft's wing planform geometry, and through that affect the aircraft's aerodynamic and structural mechanic behaviour. The MTOW determines the aircraft global sizing and fuel capacity. MTOW sets the value for the total aircraft weight, which is built up from a number of fixed weights (fuselage, fin, stabilizer weights, etc.;  $W_{fixed}$ ), several dependent weights (wing structural weight, engine weight, landing gear weights, etc.  $W_{dep}$ ), the payload ( $W_{payload}$ , fixed to 35000 kg, corresponding to 250 passengers), and the take-off fuel weight ( $W_{JTO}$ ). The dependent weights depend, via the manoeuvre wing structural loading, the required engine take-off thrust, the maximum landing weight, etc., on the design parameters span ( $s_p$ ), sweep ( $s_w$ ), chord ( $c_h$ ) and MTOW ( $W_{MTO}$ ). Hence there exists a non-linear relation between the take-off fuel weight and the design parameters span-sweep-chord-MTOW according to:

$$(1) W_{JTO} = W_{MTO} - W_{fixed} - W_{payload} - W_{dep},$$

$$(2) W_{dep} = f(s_p, s_w, c_h, W_{MTO}).$$

The amount of take-off fuel is one of the key determinants in Breguet's range equation that is applied for the range computation in the mission analysis of the wing MDO simulation system. This range computation assumes an actual fuel consumption of 96% of the take-off fuel weight.

In fact, in this study we apply a small correction to the normal Breguet range ( $R_B$ ), yielding the corrected Breguet range ( $R_{Bcorr}$ ), which takes into account the "virtual Lost Ranges" ( $R_{lost}$ ) related to extra fuel consumption during take-off, climb, etc.

$$(3) R_B = \frac{v_{cruise}}{c_{fs}} \times L_{OD-cruise} \times \ln \left( \frac{W_{MTO}}{W_{MTO} - W_{fc}} \right),$$

$$(4) R_{Bcorr} = R_B - R_{lost},$$

where  $c_{fs}$  is the engine's specific fuel consumption,  $v_{cruise}$  is the cruise speed, and  $L_{OD-cruise}$  is lift-over-drag in cruise. To be more specific, the computed Breguet range is based on the actual distance travelled, assuming the actual amount of fuel consumed ( $W_{fc}$ ) as follows:

$$(5) W_{fc} = W_{fTO} - W_{fhold} - W_{fdiversion} - W_{freserve},$$

where the reserve fuel ( $W_{freserve} = 0.04 * (W_{fTO} - W_{fhold} - W_{fdiversion})$ ) represents the amount of fuel that should in any case remain in the tanks after landing, and Hold fuel ( $W_{fhold} = 5000$  kg) and Diversion fuel ( $W_{fdiversion} = 5000$  kg) represent the amounts of fuel needed for the emergence cases of Hold and Diversion operations.

Hence the aircraft fuel efficiency  $\eta_f$  can be evaluated as a combination of range and actual fuel consumption, and is calculated by:

$$(6) \eta_f = \frac{R_{Bcorr}}{\left( \frac{W_{fc}}{n_{pax}} \right)},$$

and is expressed in [km/(l/person)]. These values can be easily compared to other fuel efficiency numbers as for example published for cars ( $\eta_f \sim 14$  for single person driving a middle class car).

The resulting multi-objective optimisation problem for aircraft range and fuel efficiency can be formulated as follows:

$$(7) \max_{(s_p, s_w, c_h, W_{MTO})} (R_{Bcorr}, \eta_f).$$

## 4 Meta-modelling

In order to evaluate the aircraft design objectives (i.e. range and fuel efficiency) in the considered design domain, a series of simulations is executed in a limited number of design points using the wing MDA system. These design points are generated in several subsequent sets of samples (fractional factorial design-of-experiments [8]) of the four dimensional design space (i.e., parameter space of the design parameters: wing semi-span, outer wing sweep angle, wing chord and aircraft MTOW). The semi-span is varied between 29 m and 32 m. The outer wing sweep angle is varied between 21 deg and 39 deg. The wing chords at 3 stations (wing root, crank and tip) are equally changed by one single chord scale factor, which is varied between 1.000 and 1.075. MTOW is varied between 150000 and 280000 kg. In total 99 design points are evaluated, yielding the values of range and fuel efficiency in these design points. As a

quick preliminary design assessment, these range and fuel efficiency values are ordered according to a basic Pareto ranking procedure [3] in order to obtain a first indication of the interesting design regions. The results of range, fuel efficiency and Pareto rank (indicated by color coding) for these 99 design points are given in Figure 6 below.

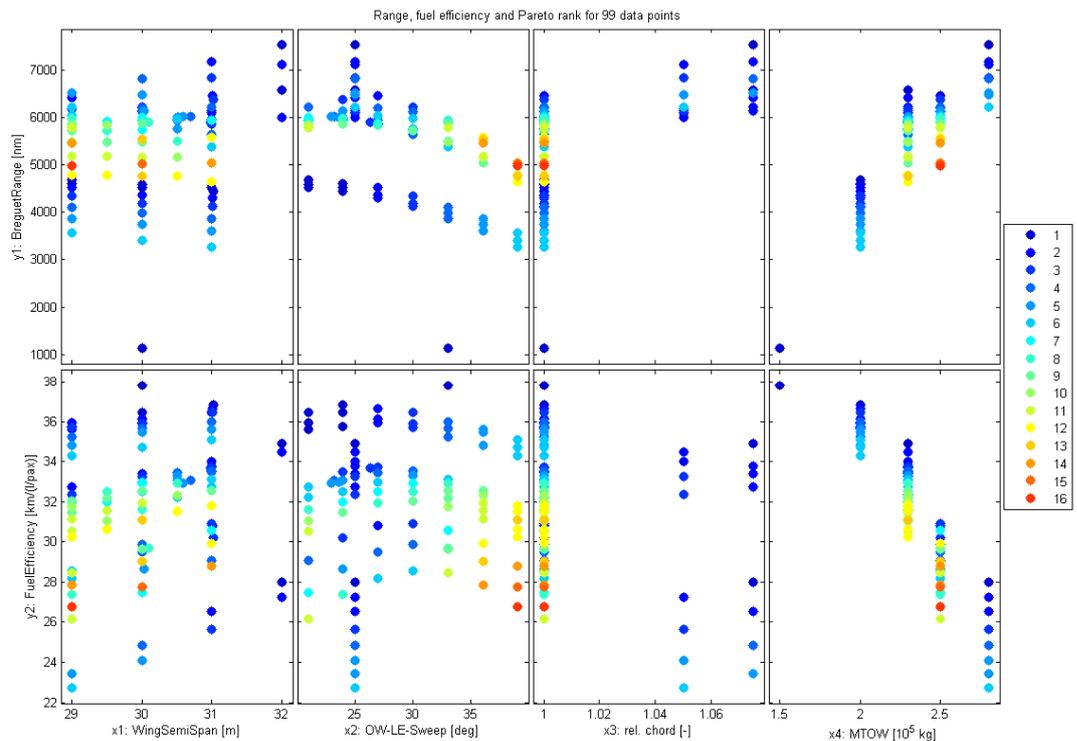


Figure 6 Results of range, fuel efficiency and Pareto rank (indicated by colour coding) for the 99 design points

The resulting data set with the values of the design parameters and of the range and fuel efficiency objectives in these 99 design points is then used to create the meta-models. The meta-models shall approximate as good as possible the objectives in each point of the parameter space. Polynomial functions of different orders (polyn), several kriging models (kriging-xy), neural networks (ann) and radial basis functions (rbf) fits are applied [4], compared, and the best fit functions are determined. These best fit functions are found through various cross-validation assessments on the data set, such that these functions' predictions of the design objectives (range, fuel efficiency) have the smallest residuals. The tests that are performed for this purpose evaluate the root mean squared (RMS) values of the residuals (or in other words, root mean squared errors, RMSE) in some of the most interesting data points, i.e. those data points having the best (lowest) Pareto rank values for the considered objectives (i.e., the dark blue points in (Figure 6)). The first cross-validation assessment uses the 9 rank-one data points as validation

points, i.e., the fits with each method are made on the remaining 90 points, and the RMS of the residuals (predicted value – actual value in data set) in the 9 validation points are calculated. This assessment indicates that the kriging-linear-Exponential (kle) [9] fit function provides the best fit for range. Fuel-efficiency is best represented by the third order polynomial (poly3) fit function. (99/9-column in Table 3 and Table 4 below). These assessments represent the accuracy of the fits in only a local region around the rank-one data points. In order to obtain a more global accuracy assessment we include some more validation points by adding the 11 Pareto rank-two data points to the cross-validation set (99/20-column in Table 3 and Table 4 below). Because this cross-validation set is rather large (20 out of 99 points), the validation fits are made on relatively small data sets (79 points), and thus will differ significantly from the “full” fits made on the complete data set (99 points). Therefore we also evaluate the RMS-residual from a leave-1-out experiment of this validation set (99/1/20-column in Table 3 and Table 4 below). Finally, as a more global accuracy assessment, we also performed a full leave-1-out experiment on the data set (99/1/99-column in Table 3 and Table 4 below). As an additional indication of the relative accuracy of the fits, we also include the Mean Absolute Percentage Error (MAPE) of the global leave-1-out residuals (99/1/99/%-column in Table 3 and Table 4 below).

For the different cross validation assessments we find reasonably consistent accuracies for most fit functions (in Table 3 and Table 4 below). The best RMSE or MAPE value found in each assessment is marked by the green shaded cells.

Based on the results of each of the assessments performed, it is concluded that the best fit for range is found by the kriging-linear-Exponential (kle) fit function. The radial basis function (rbf) also provides good results for the leave-1-out experiments (columns 99/1/20 and 99/1/99), but very poor fit quality according to the 99/20 experiments, and is therefore not selected as best fit.

*Table 3 For the range data: Accuracies of the different fit functions (identified in left column) for the different cross-validation tests (identified in first row by data set size and number of validation points). Values given are the root-mean-squares of the residuals (or prediction errors) in the validation points.*

fit function	RMSE				MAPE
	99/9	99/20	99/1/20	99/1/99	99/1/99/%
poly0	1824.8	1450.2	1464.0	993.2	18.5785
poly1	789.0	720.6	541.0	401.6	6.7994
poly2	739.3	509.2	460.8	234.1	3.7504
poly3	708.2	484.7	489.4	223.2	2.5825
poly4	757.5	720.8	521.5	237.7	2.6824

fit function	RMSE				MAPE
	99/9	99/20	99/1/20	99/1/99	99/1/99/%
kriging-cG	1386.0	1155.3	886.3	400.3	4.2159
kriging-cE	1297.2	730.4	913.8	414.1	4.2473
kriging-cC	1025.6	722.3	814.8	367.0	3.8202
kriging-IG	608.7	519.3	301.7	138.6	1.7258
kriging-IE	567.6	418.8	465.5	210.1	2.2546
kriging-IC	600.9	440.5	411.0	186.8	2.2124
ann	1175.3	1053.7	957.3	859.6	12.8121
rbf	784.1	5130.0	205.0	99.7	1.1252

For fuel efficiency, it can be concluded that the best fit is achieved by the third order polynomial (poly3) fit function.

*Table 4 For the fuel-efficiency data: Accuracies of the different fit functions (identified in left column) for the different cross-validation tests (identified in first row by data set size and number of validation points). Values given are the root-mean-squares of the residuals (or prediction errors) in the validation points.*

fit function	RMSE				MAPE
	99/9	99/20	99/1/20	99/1/99	99/1/99/%
poly0	4.648	4.182	3.909	3.259	8.464
poly1	1.984	1.499	1.368	0.995	2.368
poly2	0.722	0.544	0.258	0.264	0.637
poly3	0.269	0.401	0.223	0.143	0.313
poly4	0.766	2.908	0.488	0.256	0.480
kriging-cG	1.746	1.422	0.830	0.421	0.658
kriging-cE	2.435	1.289	0.947	0.431	0.389
kriging-cC	2.103	2.251	1.198	0.576	0.780
kriging-IG	1.590	1.358	0.939	0.443	0.484
kriging-IE	1.692	1.378	1.187	0.539	0.414
kriging-IC	1.778	1.404	1.305	0.607	0.634
ann	1.886	1.393	0.672	1.179	3.760
rbf	6.990	66.977	4.740	2.140	1.121

## 5 Aircraft optimisation results

A Pareto front optimisation of the aircraft's range and fuel efficiency is performed using a multi-objective genetic algorithm (based on epsilon-NSGA-II as described in [10]), where the best fits for range (kle) and for fuel efficiency (poly3) are used as objective functions. In this optimisation a population size of 99 individuals is used, where the 99 design points from the data set are used as the initial generation. The bounds of the search domain for the optimisation are set to the minimum and maximum values of the design parameters of the 99 design points. About 100 generations are evaluated by the genetic algorithm. The total number of meta-model objective function evaluations in this optimisation is then about 10.000, and takes about 20 seconds computational time on a standard PC (P-4, 2.8 GHz). If we compare this computation time with the computational time that would be needed to perform the 10.000 evaluations with the MDA system, which would be about 5000 hours, the significant gain in computation time is obvious. Note, however, that the MDA evaluations of the 99 design points did require some 50 hours of computational time. The resulting Pareto front solution (red diamonds in Figure 7) provides a set of clearly improved designs, as compared to the initial set of designs in the data set (black dots).

The fuel efficiency values in the Pareto points, as predicted by the poly3 meta-model, appear to be quite high (up to about 75) compared to the data set (fuel efficiency between 23 and 38). Although the poly3 fit clearly resulted as the best fit from the accuracy assessments (Table 4), it is also well known that polynomial models may become less reliable, in particular in case of higher polynomial orders and in case of extrapolation outside the "cloud of data points" [11]. Therefore the values for fuel efficiency in the Pareto points as predicted by the poly3 meta-model are now checked by the predictions of these values with a variety of other fits. For this purpose we use one of the best kriging fits (kle; see Table 4), and the ann and rbf fits. The predictions for fuel efficiency in the Pareto points by these fits are rather consistent (blue, green and magenta circles in Figure 7), and significantly lower than the poly3 predictions (red diamonds in Figure 7). We therefore conclude that the poly3 fit for fuel efficiency is not as accurate as expected, in particular near the bounds of the design space (i.e., semi span around 32 m, sweep around 21 deg, chord ratio around 1.075 and MTOW around 150000). In order to achieve a more reliable set of Pareto points, we rerun the optimisation several times now using for fuel efficiency subsequently the three fits kle, ann and rbf. The Pareto front results are shown in Figure 8.

The Pareto fronts found with these fits are close together, and much closer to the 99 points of the data set, and therefore probably more accurate approximations of the MDA results for fuel

efficiency. This illustrates that the result of the optimisation does depend on the quality of the fits used in the objective functions, and therefore requires careful treatment, and if possible verification, of these results.

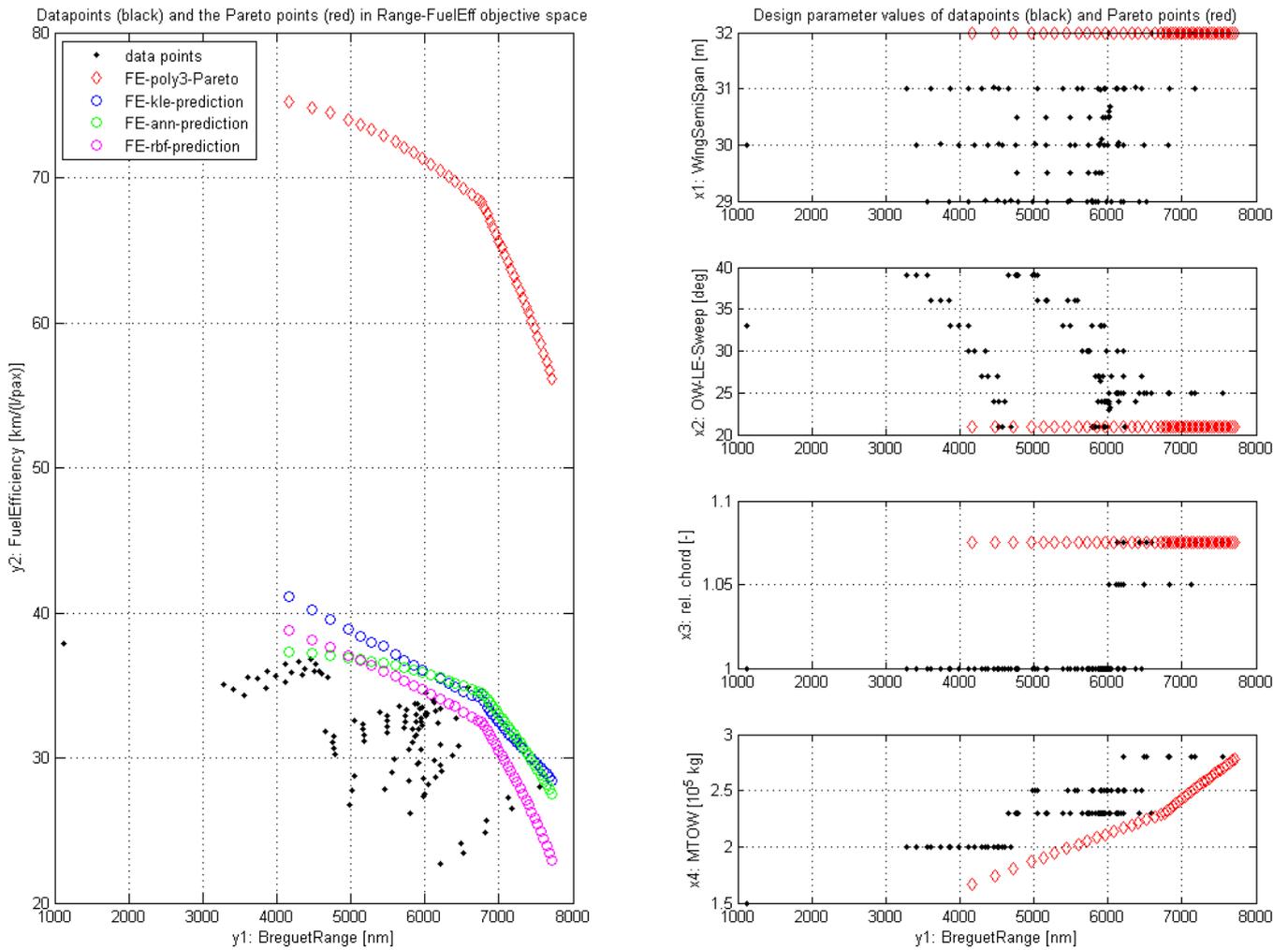


Figure 7 The 99 design points (black), the kle-poly3 Pareto front results (red) and the kle, ann, rbf predictions (blue, green, magenta) of these results in the objective space (left) and in parameter space (right)

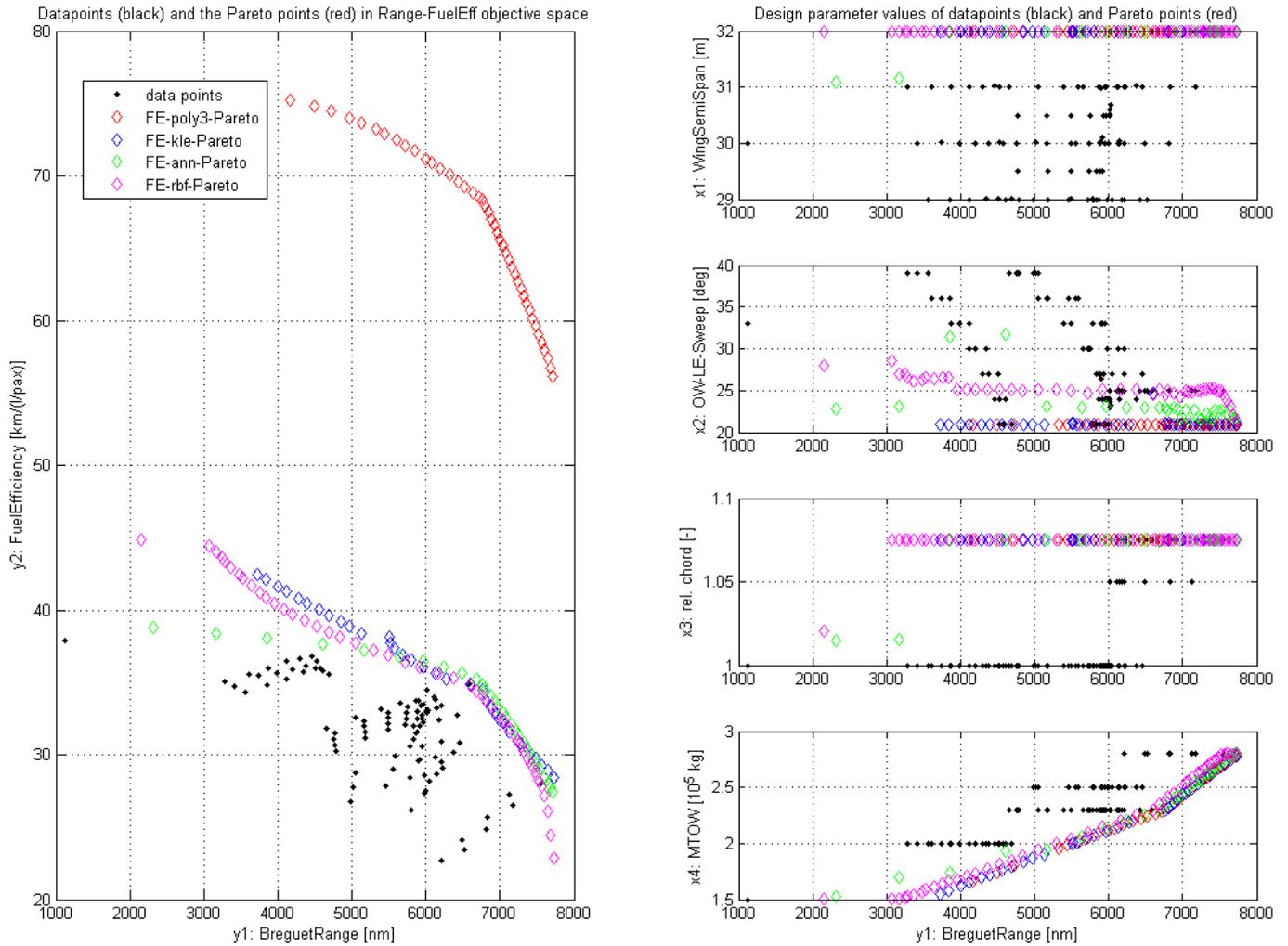


Figure 8 The 99 design points (black) and the kle-poly3 (red), kle-kle (blue), kle-ann (green) and kle-rbf (magenta) Pareto fronts in the objective space (left) and in the 4 parameter sub-spaces (right)

It should be noted that the location of the Pareto points in the parameter space (Figure 8, right panels) is not very different from the previous Pareto set found with the poly3 prediction of fuel efficiency (, right panels).

On the basis of the Pareto optimum design points that are found we select two suitable candidate optimum design points, which are evaluated with the accurate MDA system: one design point is expected to provide primarily a high range, and the other design point is expected to provide primarily high fuel efficiency (Table 5). The range and fuel efficiency values for these two design points as predicted by the fits indeed are as expected, as shown in Table 5 (only results of the kle fits are included here).

Table 5 MDO analysis result and meta-model prediction for the two candidate optimal design points

parameters				meta model(kle)		MDO analysis	
span	sweep	chord	MTOW	range	FuEff.	range	FuEff.
32.5	26.0	1.08	285000	7790.5	28.1	7593.5	28.0
32.0	21.0	1.075	200000	5510.8	38.1	4730.6	36.7

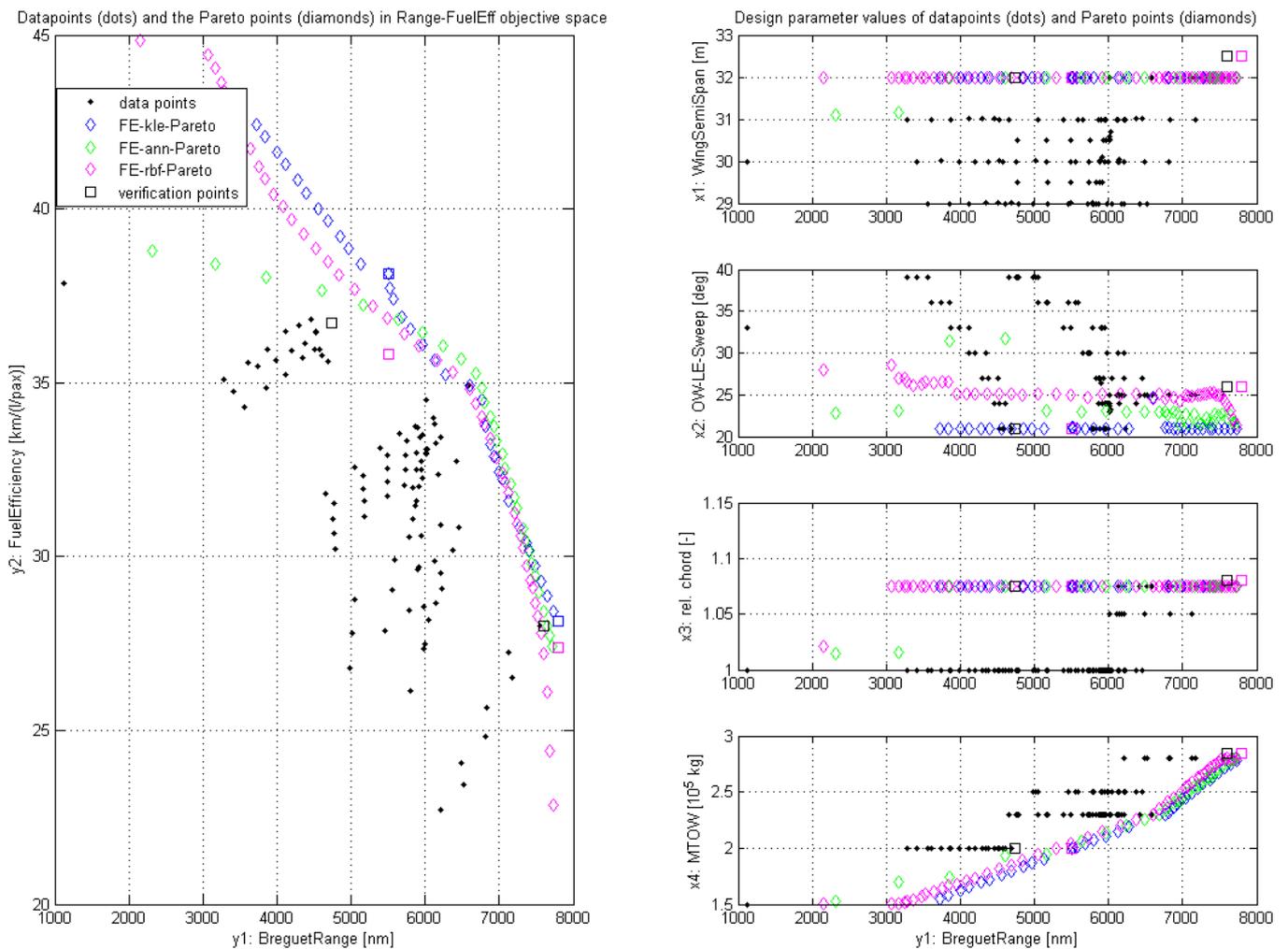


Figure 9 The 99 design points (black) and the Pareto front results with the kle, ann and rbf fits (blue, green, magenta diamonds) in the range vs. fuel efficiency objective space (left) and in the 4 parameter sub-spaces (right). Also the results for the two optimum design points are included (squares).

Also the MDA system yields the expected high values for range and fuel efficiency, respectively, in the two candidate optimum design points (Table 5). However, the difference between these values and the meta-models predictions are rather large, in particular for the

second point (differences of 780.2 nm and 1.4 km/(l/pax), respectively, for range and fuel efficiency predicted by the kle meta-models; Table 5 and Figure 9). Nevertheless, the Pareto points that are found with the meta-models do indicate the interesting design regions. After all, the two selected candidate optimum design points appear both to provide improved designs compared to the original data set (99 points), as these points are both additional Pareto optimal (rank 1) points (Figure 9).

From the graphs showing the design parameter values for the Pareto optimal design points (Figure 9, right panels) it is clear that an increasing range requires an increasing MTOW. Obviously, the increasing amount of fuel that is available with higher MTOW values allows to achieve higher ranges. For most Pareto optimal design points, the wing span and chord are found to be as high as possible. This can be expected according to aerodynamic efficiency considerations. The sweep angle appears to be between 21 and 25 degrees for the Pareto optimal design points which is probably related to the fixed cruise Mach value of 0.8 that is used in this design study.

## 6 Conclusions

An advanced integrated multidisciplinary design analysis system has been deployed in a multi-objective design optimisation study of aircraft range and fuel efficiency. Wing semi-span, sweep angle, chord and MTOW are used as the independent variables in this optimisation.

In order to efficiently search through the design space, appropriate meta-models of the considered design objectives are created. The creation of these meta-models requires a proper set of data that represents the design objectives in the considered design space. This data set is generated by a series of 99 evaluations of the aircraft range and fuel efficiency with the MDA system, which take in total about 50 hours of computation time on a standard PC. With this data set, several meta-models are created for both range and fuel efficiency, and by extensive cross-validation assessments the most accurate meta-models are identified. The computation time for the complete process of meta-model creation and cross-validation assessments is in the order of 1 hour on a standard PC. Once the meta-models are created, their evaluation takes only a small fraction of a second computation time. Compared to the approximately half hour computation time for a single evaluation with the MDA system, the time gained with the use of the meta-models is obvious, in particular in the case of the multi-objective optimisation design study, which requires many thousands of evaluations of the objective functions.

The computational time gained by the use of the meta-models comes at the cost of their possible limitation in accuracy. As is shown for the third order polynomial fit for fuel efficiency, the predictions of this meta-model appeared to strongly overestimate fuel efficiency values for the

Pareto optimal design points. Although this effect known for (high order) polynomial fits of sparsely sampled data sets, such predictions should be carefully considered and validated if possible. Because of the availability of the variety of meta-models for fuel efficiency, this effect could be easily discovered and mitigated by making use of these other meta-models for fuel efficiency in the optimisation. From the resulting Pareto optimal design points, two candidate optimal design points were selected, and the meta-model predictions in these points showed a reasonable correspondence with the results of the MDA. For the kriging-linear-exponential meta-models, relative prediction errors of about 10% and 5% were found for range and fuel efficiency, respectively. Moreover, the two selected candidate optimum design points appeared to be both additional Pareto optimal (rank 1) points when added to the original data set of 99 design points.

The key benefit of the multi-objective design optimisation approach applied in this study is that the multi-objective Pareto front results directly provide the design information on which further design trade off considerations can be based.

The resulting Pareto front provides a clear overview of the most interesting aircraft design points. The maximum achievable range found is about 7550 nm, and the maximum achievable fuel efficiency is about 38 km/l/pax. Obviously, for the Pareto optimal aircraft designs with increasing range values the fuel-efficiency drops, in particular for ranges higher than about 6500 nm. This clearly illustrates the trade-off decision that shall be made by the designer: decide either for an aircraft design allowing for a high range, or allowing for a high fuel efficiency. The advantage of the Pareto front for this trade-off decision is, obviously, that each of these designs is non-dominated, i.e. not worse than any other design point. Therefore, if for example it is decided that the desired range is 7000 nm, then directly the design for this range with the best fuel efficiency can be selected.

This process illustrates the possibilities offered by the multi-objective Pareto approach to efficiently investigate the considered design space, and selected optimum design points according to certain performance requirements. It leads the designer directly to the design areas of most interest for the considered design objectives.

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