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STOCHASTIC FATIGUE ANALYSIS OF AN F-16 FORWARD ENGINE MOUNT SUPPORT FITTING

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Fatigue analysis, to determine the lifetime of a component or complete aircraft structure, normally is based on deterministic models, in which the parameters are constants. In order to compensate for neglecting the natural variability of the parameters and other uncertainties a safety factor is applied. Another way of dealing with the variability of the parameters is by means of a stochastic analysis, adding an extra dimension to the deterministic analysis, by introducing a range of values that can occur with their chance of occurrence.

In the paper, the different steps of which a stochastic analysis consists are described. Furthermore, the stochastic concept will be applied to a realistic component: the forward engine mount support fitting of an F-16. This engine mount is a critical component with a small inspection interval, which is difficult to inspect. The stochastic nature of the various parameters of the crack growth model are examined using real life data and their stochastic importance is determined by means of a sensitivity analysis. This reveals the model parameters for which the variability should be taken into account and which therefore should be modelled as stochastic variables. These results are used in a stochastic Damage Tolerance analysis, revealing the very conservative nature of such an analysis due to the unknown initial flaw size. Finally, a stochastic crack growth approach is presented, dealing with this unknown initial flaw size in a stochastic way.

INTRODUCTION

Airworthiness regulations require that an aircraft manufacturer proves that the aircraft can be operated safely. This implies that safety critical components are replaced or repaired before they are damaged so much that safe operation of the aircraft can no longer be guaranteed. Different philosophies can be followed to prove that a component is safe. Which philosophy will be followed depends for example on the possibility to inspect a component. Sometimes customers prescribe the philosophy to be used to prove aircraft safety. Two philosophies form the basis for the approaches chosen by most manufactures: safe life and damage tolerance. Two additional philosophies are mentioned often: fail safe and durability. The latter however is used for not-safety-critical components.

The safe life philosophy is based on the concept that significant damage will not develop during the service life of a component. If a crack is initiated, or is already present, it will not grow fast enough to produce a significant reduction in strength. The life for which this is true is calculated and then checked by a suitable test program. The safe life of a component is obtained by factoring the life found by an appropriate safety factor. When the service life equals the safe life the component is replaced whether damage is evident or not. The result of a safe life calculation is a single number specifying the life of the component in e.g. flights or hours. The major



drawback of this approach is that components are taken out of service when it is likely that they still have a substantial remaining life.

With the damage tolerance philosophy it is recognised that damage exists and develops during the service life of a component. It says that a structure is designed to have an adequate life free from significant fatigue damage, but continued operation is permitted beyond the life at which such damage may develop. Safety is incorporated into this approach by the requirement that the damage is detected by routine inspection before it results in a dangerous reduction of the static strength. Two requirements are necessary for this approach to be successful: 1. One should be able to define a minimum crack size which will not go undetected at a routine inspection and 2. One should be able to predict the growth of such a crack during the time until the next inspection. On top of this it is assumed that cracks grow from flaws which are present in the material or from manufacturing faults. The result of a damage tolerance analysis is a curve presenting the crack length as function of the number of cycles.

Both approaches can be used for components which cannot be inspected, although the safe life approach is most suited. Such components need to be replaced when the critical life is reached. The critical life can be calculated via the safe life approach or the damage tolerance approach. In the latter case the component needs to be replaced when the first inspection is reached.

Apart from these design philosophies there are two addition philosophies often used. The first was introduced in the sixties and is called fail safe. The second is of a more recent date and is called: durability. The term fail safe is related to life prediction methods in two way's. First, it is used for a design philosophy developed in the early sixties which was the predecessor of the damage tolerance philosophy. It is similar to the damage tolerance philosophy except for the notion of an initial damage. Secondly the term fail safe is used to address structural redundancies like: alternate load path and crack stoppers.

The philosophies described are based on safety requirements. In case of a damage tolerant design, inspection intervals are defined so that a safe operation is ensured. Close to the end of the life of a component safe operation is only ensured with sufficient inspection. Inspection intervals as determined with the damage tolerance philosophy can become so short that it is cheaper to replace than to inspect. The economic life of a component is addressed by durability.

The above analyses are based on a so-called deterministic analysis. This means that the variability of the parameters used in the model is not taken into account, which introduces an uncertainty in the results of the model. In order to compensate for neglecting the variability of the parameters a safety factor is introduced.

Another way of dealing with the variability of the parameters is by means of a stochastic analysis, adding an extra dimension to the deterministic analysis, by introducing a range of values that can occur with their chance of occurrence. The different steps of a stochastic analysis are:

- Choice of random variables and their distribution functions
- Choice of failure function (here, component life)
- Solution of the stochastic problem (here, stochastic crack growth problem)

There exist several numerical techniques to solve the stochastic problem, all with their own limitations. The most simple and well known method is the Monte Carlo method, which will be applied here also. Other more advanced methods, which have been implemented at NLR also



(Ref. 1), are FORM, SORM and Importance Sampling methods, which require less computational effort than the Monte Carlo method.

This paper presents the results of a damage tolerance analysis of the forward engine mount support fitting of an F-16, determined deterministically and by means of a stochastic analysis. This engine mount is a critical component with a small inspection interval, which is difficult to inspect. Figure 1 gives a drawing of this component and its location in the aircraft. For the stochastic analysis the stochastic nature of the various parameters of the model are examined and their importance is determined by a sensitivity analysis. These results are used in a stochastic damage tolerance analysis. The damage tolerance method results in very conservative component lives due to the assumed initial flaw size.

From fracture surface investigations it is known that for virgin undamaged materials a considerable time is spent to initiate a crack. After initiation the crack will grow till failure. The ratio between crack initiation life and crack growth life depends on the geometry and loading. Material property data used in safe life analysis is often obtained from tests on narrow specimens. Such specimens fail shortly after a crack initiates. This implies that for a safe life analysis the crack growth part of the life is ignored. The damage tolerance philosophy on the other hand starts with a crack of minimal detectable size therefore ignoring the crack initiation life. This notion encouraged some manufactures to combine the advantages of both approaches. They use the safe life approach (with the proper material property data) to determine the crack initiation life and from that the initial inspection. The damage tolerance approach is used to determine subsequent inspection intervals.

In the last section an approach will be presented dealing with both the initiation and crack growth life simultaneously in a stochastic way.

DETERMINISTIC DAMAGE TOLERANCE ANALYSIS

The purpose of the damage tolerance analysis is to ensure safety. The analysis predicts the service life from an assumed worst case initial flaw size to the critical crack size. The inspection interval is half of the predicted service life, i.e. safety factor of two. The assumed worst case initial flaw size is the crack size that can be detected with the inspection procedure with a probability of 90%. A typical value is 0.05", but smaller values can be assumed in case of more advanced inspection methods.

The crack growth model consists of a damage accumulation model, describing the growth of the crack caused by the stress field. This relation depends on the crack growth rate, which is determined by a crack growth law, representing a relationship between the stress intensity range and the crack growth rate. The stress intensity is given by a separate expression depending on the type of crack and geometry of the component. Furthermore, the crack growth can be retarded or accelerated by previous load cycles, which is modelled by a so-called retardation model. Finally, some criteria determining failure of the component are incorporated.

The crack growth law used here to calculate the crack growth rate is the Forman equation:

$$\frac{da}{dn} = \frac{C \Delta K^m}{(1-R) K_c - \Delta K} \quad (1)$$



where,

- C_m = material parameters
- K_c = fracture toughness (material parameter)
- R = stress ratio $\sigma_{min}/\sigma_{max}$
- ΔK = stress intensity factor range $K_{max} - K_{min}$

Plasticity induced retardation is modelled here by means of the generalised Willenborg retardation model.

The above equation depends on the mode one stress intensity factor, K_I , which is the most important parameter in the analysis. Its value is determined for a through crack in a single edge notched plate (Ref. 2), which is the assumed initial flaw for this component.

The crack growth analysis will end when failure has occurred. This will be the case if:

- the crack length is larger than the critical crack length a_{cr}
- the stress intensity factor is higher than the critical stress intensity factor K_c
- the net section stress is higher than the yield limit

The stress state is plane stress, due to the small thickness.

The load spectrum used is typical for a fighter aircraft and consisted of 438 missions subdivided in eleven mission types of which 334 were unique and covered 500 flight hours.

The model parameters used, material type Al 2124, are:

- C = 6.48E-8 inch*ksi*sqrt(inch)/(cycle*(ksi*sqrt(inch)))^m
- m = 3.711
- K_c = 44. ksi*sqrt(in.)
- ΔK_{th} = 1.5 ksi*sqrt(in.) (ΔK threshold)
- W = 2.65 inch (plate width)
- R_{so} = 2.3 (overload shut-off ratio, from retardation model)
- σ_{ty} = 63. ksi (tensile yield strength)

From the fracture toughness the critical crack length is calculated yielding:

$$a_{cr} = 0.344 \text{ inch}$$

The initial crack length (flaw size) assumed in the damage tolerance analysis was:

$$a_0 = 0.1 \text{ inch}$$

The deterministic damage tolerance life computed with the above model for the applied load spectrum now yields:

$$T = 1216 \text{ flight hours}$$

The inspection interval becomes T/2:

$$T_{insp} = 608 \text{ flight hours}$$

PROBABILITY OF FAILURE

The stochastic method used here is the **Monte Carlo (MC)** method. The MC method is extensively used, due to its simplicity and accuracy. The basic idea behind this method is to calculate randomly values of the failure function (called simulations), by the use of random



values for its variables, based on their known **probability density function (PDF)**. This will result in a number of values for the damage tolerance life of the component from which the probability of failure can be calculated by means of:

$$P_f = \frac{\text{number of failures}}{\text{total number of simulations}} \quad (2)$$

The probability of failure is defined here as the chance that the life of the component (T) is less than 608 flight hours. This means that the structure has failed before the next inspection:

For every MC simulation a complete damage tolerance analysis is performed with varying values for the random variables (model parameters).

$$P_f = P(T < 608)$$

RANDOM VARIABLES

The choice which model parameter to treat as a random variable and which not (deterministic variable) depends on the variability of the parameter and its influence on the variability in the damage tolerance life of the component. All the independent parameters of the crack growth model are:

<i>Crack growth law:</i>	C, m, K _c
<i>Damage accumulation model:</i>	a ₀
<i>Retardation model:</i>	R _{so} , ΔK _{th} , σ _{ty}
<i>Stress intensity factor:</i>	W, σ

All the other parameters are functions of the above ones, e.g. the critical crack length a_{cr} or stress intensity factor K_I. Treating one of the independent parameters of the dependent one as random variables causes the dependent variable to be random too.

First the selected distribution functions for the various random variables will be discussed. After that the sensitivity of the solution for the variation in these variables will be examined, to determine which model parameters should be treated stochastically and which not.

In order to obtain the distribution function for a random variable sufficient data must be available. An extensive amount of damage tolerance material data can be found in reference 3. In this reference da/dn values are given at specific ΔK and R levels for different orientations obtained in various experiments. Furthermore, tables are given with values for K_c and σ_{ty} obtained in various experiments.

Variables C and m

In theory, from the da/dn versus ΔK data distribution functions can be found for the material parameters C and m (Eq. 1). However, it is not known which part of the variability in da/dn is caused by the random behaviour of C respectively m. Therefore, the random behaviour of C and m has been examined assuming that all the variability in da/dn was caused by C respectively m, or in other words, the other was held constant (deterministic). In figure 2 the calculated values for C have been plotted on log-normal probability paper, forming approximately a straight line. This is a good indication that the data set can indeed be described by this distribution function. Therefore, the distribution function of C can be described by:



$$C(\mu_C, \sigma_C) \sim \text{LN}(0.648\text{E-}7, 0.240\text{E-}7)$$

The coefficient of variation becomes:

$$\text{Cov} = \sigma_C/\mu_C = 37 \%$$

Similarly, the distribution function of m can be described by means of a normal distribution function:

$$m(\mu_m, \sigma_m) \sim \text{N}(3.711, 0.226)$$

The coefficient of variation is:

$$\text{Cov} = \sigma_m/\mu_m = 6.1 \%$$

Both distribution functions describe the same variability. Using both distribution functions with the determined values will overestimate the variability in da/dn . Using only one will overestimate the variability in C respectively m . As can be seen by the coefficient of variation, the variability in C is higher than the variability in m , which is also known from literature. In literature (Ref. 4) a correlation has been found between C and m . For the material type under view, Al 2124, no correlation relation could be found. Therefore, C and m are considered uncorrelated. In order to describe the variability in da/dn the above distribution functions for C and m have been used, with the same mean but a lower coefficient of variation, i.e.: 25 % respectively 3 %:

$$C(\mu_C, \sigma_C) \sim \text{LN}(0.648\text{E-}7, 0.162\text{E-}7)$$

$$m(\mu_m, \sigma_m) \sim \text{N}(3.711, 0.111)$$

Variable K_c

Values for the plane stress fracture toughness can also be obtained from reference 3. The distribution function for the plane stress fracture toughness can be approximated by a normal distribution function:

$$K_c(\mu_{K_c}, \sigma_{K_c}) \sim \text{N}(44., 4.4)$$

The coefficient of variation is:

$$\text{Cov} = \sigma_{K_c}/\mu_{K_c} = 10 \%$$

Variable σ

The stress spectrum used is based on measured stress spectrums and consisted of 438 missions of which 334 were unique and covered 500 flight hours. The variability in mission types is accounted for by the large number of missions. However, the order in which the various missions are flown in reality can differ from the order in the stress spectrum. In order to account for this last variation, the 438 missions are randomly ordered resulting in a new stress spectrum for each MC simulation. It will be shown that this influence may be neglected concerning the variability of the damage tolerance life.

Because the Dutch use consists of using all fighter planes for all mission types, one can argue that this load sequence is a reasonable representation of the real load sequence over a period of 500 flight hours and that sufficient variability is included in it.

For shorter periods of time, e.g. 50 flight hours, this however does not hold anymore. The variability in load sequence between different aircraft will then increase for decreasing periods and has to be taken into account.

Here, it is assumed that the load sequence of all aircraft can be described by the reference spectrum, due to the long period.



Variable σ_{ty}

Values for the tensile yield stress can again be obtained from reference 3. The distribution function for the yield stress is best approximated by a three-parameter Weibull distribution:

$$\sigma_{ty}(a, b, x_0) \sim W3(6.383, 18, 45.52)$$

Variable R_{so}

Insufficient data has been found on the overload shut-off ratio. It will be shown that the influence of the variability of this parameter on the variability of the damage tolerance life may not be neglected. However, due to the lack of data no distribution function can be determined. Because many material parameters are represented by the normal distribution function, also for R_{so} the normal distribution function is selected. The mean will be the deterministic value of 2.3. The only unknown remaining is the variation. Selecting a coefficient of variation of 10 %, which is the order of variation for many random variables, the standard deviation becomes 0.23.

$$R_{so}(\mu_{R_{so}}, \sigma_{R_{so}}) \sim N(2.3, 0.23)$$

Variables ΔK_{th} and W

Insufficient data has been found for the threshold stress intensity range ΔK_{th} , below which no crack growth does occur. Variability in dimensions due to the manufacturing process are also unknown, but can be estimated from the allowed tolerances. However, it will be shown that the influence of the variability of these two variables on the variability in the damage tolerance life can be neglected. Therefore, these variables can be treated as deterministic.

Variable a_0

The initial crack size in a damage tolerance analysis is determined by the method of inspection. It represents the crack size that can be detected with the inspection procedure with a probability of 90%, this to ensure safety which is the purpose of **Damage Tolerance Analysis (DTA)**. This crack length is a fixed value for a specific inspection method and therefore deterministic. However, a better approach would be to take into account all crack sizes with their probability of **not** detection, expressed by one minus the **Probability Of Detection (POD)** curve. In this way the fact that also smaller cracks than the 90% one can be found with the inspection method, although with a lower probability, is included in the analysis. Both initial crack sizes, fixed value and POD distribution, will be used in a stochastic DTA.

From reference 5 a POD curve has been selected, which had a probability of detection of 90% for a 0.1 inch crack length, which is the initial crack length in the deterministic analysis. The selected POD distribution is a log-normal distribution:

$$POD(\mu_a, \sigma_a) \sim LN(0.0695, 0.0166)$$

The best approach of course would be to take into account the real distribution of crack sizes. However, since these are unknown another approach will be outlined in the last section, where this crack size distribution is determined from failure data and taken into account stochastically in the crack growth analysis.

SENSITIVITY ANALYSIS

The influence of the variability of the various parameters on the damage tolerance life is investigated by treating one parameter at the time as a random variable, while keeping the other parameters deterministic. Altering the variability of the random variable will give an indication whether the variability of this parameter on the variability of the DT life is important or not, and thus whether this should be taken into account or not.

The distribution functions determined in the former section are used for the various parameters. For the plate width W and threshold stress intensity range ΔK_{th} a normal distribution function with a coefficient of variation of 10% has been used to demonstrate that both parameters can be treated deterministically, since insufficient information was available to determine their real distribution functions.

The results of the sensitivity analysis are depicted in figure 3. In this figure the DT life in flight hours is put against the chance that the life is less than a specific value. For every model parameter the cumulative distribution curve has been plotted. For some random variables the curves are steep, representing the small variability in the DT life caused by the variability of the parameter. Therefore, the steeper the curve the more the situation can be represented as deterministic; if the parameter would be deterministic the curve would be a straight vertical line in the figure. The vertical dashed line represents the deterministic inspection interval of 608 flight hours.

From this figure it can be concluded that the fatigue analysis is not very sensitive to variations in the threshold stress intensity range ΔK_{th} , the plate width W , the tensile yield strength σ_{ty} and the mission sequence σ , which can therefore be treated as deterministic variables.

The fatigue analysis is sensitive to variations in the material parameter C , the plain stress fracture toughness K_c , the initial crack size a_0 (POD) and the overload shut-off ratio R_{so} . The fatigue analysis is even very sensitive for variations in the material parameter m . In reality, i.e. using the real initial crack size distribution, the variability in crack size would cause the largest part of the variability of the life of the component.

These parameters are treated as stochastic variables. The variability caused by all these parameters simultaneously is also depicted in figure 3.

STOCHASTIC DAMAGE TOLERANCE ANALYSIS

The stochastic damage tolerance analysis is the same as the deterministic analysis, with the exception that now the variability in the model parameters C , m , K_c , R_{so} and a_0 are taken into account by means of their determined distribution functions:

$$\begin{aligned} C(\mu_C, \sigma_C) &\sim \text{LN}(0.648\text{E-}7, 0.162\text{E-}7) \\ m(\mu_m, \sigma_m) &\sim \text{N}(3.711, 0.1113) \\ K_c(\mu_{K_c}, \sigma_{K_c}) &\sim \text{N}(44., 4.4) \\ R_{so}(\mu_{R_{so}}, \sigma_{R_{so}}) &\sim \text{N}(2.3, 0.23) \end{aligned}$$

As explained a_0 is taken deterministic (0.1 inch) as well as stochastic (1-POD), therefore two separate analyses have been performed.

Figures 4 and 5 show the crack growth curves for the 1000 simulations for the deterministic, respectively, stochastic initial crack length. The critical crack has become also a random variable, because of the stochastic nature of K_c , and will depend on the current value of K_c in the simulation. This can be seen in the figures at the difference in end crack length of the different crack growth curves.

Figure 6a shows the cumulative probability density function for the critical crack length. The data points plotted on normal probability paper, figure 6b, show apart from the tails a straight line. The critical crack length a_{cr} thus can be described by a normal distribution:

$$a_{cr}(\mu_{acr}, \sigma_{acr}) \sim \text{N}(0.3424, 0.05331)$$



The critical crack does not depend on the initial crack length, therefore the a_{cr} distribution will be the same for both DT analysis.

Figure 7a shows the cumulative probability density functions for the DT life for both the deterministic respectively stochastic initial crack size. This figure furthermore shows that the mean DT life is larger in case of the stochastic a_0 . The vertical dashed line in the figure again represents the deterministic determined inspection interval of 608 flight hours.

The small fluctuations in the curves of figure 7a are caused by the fact that all the missions are different with respect to mission length and harmfulness and there is a high chance that the analysis will end on a heavy cycle. The small fluctuations are thus not due to the limited number of simulations, which also can be concluded from the fact that the fluctuations do not disappear for higher probabilities, which are very well approximated by the number of simulations used.

The DT life result for the deterministic a_0 also has been plotted on log-normal probability format in figure 7b. The result is a straight line, apart from the tails. A similar result has been obtained for the stochastic case. Therefore, the DT life T can be described by the log-normal distribution:

$$T(\mu_T, \sigma_T) \sim \text{LN}(1440, 810) \quad (\text{Deterministic } a_0) \quad (3a)$$

$$T(\mu_T, \sigma_T) \sim \text{LN}(2679, 1666) \quad (\text{Stochastic } a_0) \quad (3b)$$

The probability of failure P_f is defined here as the chance that the DT life T of the aircraft is less than the specified inspection interval t_{insp} . This means that the crack has grown to the critical crack length before the next inspection.

$$P_f = P(T < t_{insp}) \quad (4)$$

This probability of failure for both analyses (deterministic a_0 and stochastic a_0) can be read from figure 7a or calculated by equation 3 for any specific inspection interval. For an inspection interval of 608 flight hours this results in a probability of failure of 8.3% for the deterministic a_0 and 1.0% for the stochastic a_0 , which is unrealistically high.

It should be emphasised that the reader should keep in mind that the presented results are based on the assumption that the initial crack size is assumed to be present in the component at the start of the interval, this to ensure safety. In general, the real initial crack size will be much smaller. Furthermore, the time for the crack to grow to the assumed initial size (initiation time) is large compared to the time for the crack to grow from the initial size to failure. Therefore, for the largest part of the total life (initiation + growth to failure) of the component no crack will be found. This approach thus is very conservative.

In the next section an approach will be presented in which the realistic initial crack size distribution is determined from failure data and can be used in the stochastic crack growth analyses.

IMPROVED STOCHASTIC LIFE APPROACH

The advantages of a stochastic approach depend on the design philosophy adopted. We will discuss the different design philosophies and what will be the benefit of taking a parameter stochastic rather than deterministic.

The safe life approach

In the safe life approach the life to failure is determined for a large series of coupons or components. This leads to a distribution function of life to failure. This function is used to

determine the life that more than 99.9% of the components will survive. This is then called the 'safe life'. The component must be replaced as soon as the safe life is reached.

It is clear that stochastics is used to determine the safe life from the experimental results. For the application of the safe life approach no stochastics is used.

The damage tolerance approach

In the damage tolerance approach an initial damage is assumed, the damage growth starting from the initial damage to a critical damage is calculated. The time it takes the crack to grow from initial to critical damage is used to specify inspection intervals. In this approach the stochastics can be brought in for different parameters as shown in this paper.

In the damage tolerance approach the size of the initial damage is determined by the inspection capabilities. As a typical 90/95 value 0.05" initial crack size is used often. Instead of a conservative value the real probability of detection (POD) of the inspection method can be used to reduce the conservatism, as shown in this paper also.

Equivalent Initial Flaw Size (EIFS)

The damage tolerance approach has an inherent strange assumption in it that leads to very conservative results. The initial damage size is based on the POD of the inspection method. In other words the inspection method determines the initial crack size and not the damage that actually may be expected in the structural component. This observation has led to the concept of 'equivalent initial flaw size', EIFS. The critical crack length and life of the component are used to determine the crack size at time zero by means of a backwards crack growth calculation. This approach leads to an equivalent initial flaw size, which is not the true flaw size as the name already indicates, due to the fact that the crack growth model used is not valid in the small crack size region anymore. Performing this backwards crack growth calculation for many crack length/life combinations results in an EIFS distribution.

The only way the EIFS concept can be used is by assuming that the EIFS distribution has a more general validity, not only for a particular component but for several or all components. This, however, is not proven yet and it is doubtful whether it is really true.

Improved stochastic approach

The idea of starting from a failure distribution function and calculating backwards the crack growth, can be used in another way (see figure 8). The backward calculation is not proceeded till the time zero as in the EIFS approach, but till a reference crack length (A_{ref}) determined by the inspection method used or a value which is regarded as sufficiently small without reducing the strength and safe operation of the component (i.e. small cracks are often present in structures). From the resulting distribution function at this reference crack size, which is a function of time, the initial inspection time (T_{init}) is derived based on a certain percentage of time (e.g. 1%), since cracks of these sizes are present in the structure. In this way the crack initiation phase is accounted for in a natural way.

This is illustrated in figure 8, by the distribution function labelled PDF-T. The distribution function of crack lengths at this time (figure 8, label PDF-Aref) is now used in an upward crack growth analysis including a repeat inspection scheme, depicted by the crosses in figure 8 for one crack growth curve. The inspections are simulated by the POD function valid for the component. Depending on the repeat inspection scheme and POD function, the crack will be found or not.

Once a crack has been found in the numerical simulation of the crack growth, the component is repaired or replaced and the new component will have again an initiation period (T_{init}) in which a new crack will initiate. From this time again a crack growth analysis can be performed.



The crack growth calculation stops when the component has failed or when the economic life time of the component has been reached, depicted by T_{economic} in the figure.

Performing many of such crack growth simulations finally results in a certain Probability Of Failure (POF). This POF can be obtained for different repeat inspection schemes till a required level of POF is obtained. In this way an optimal inspection scheme can be determined.

The backward and upward crack growth calculations are done deterministically, because all the variability is already included in the failure distribution. In this way the upward crack growth calculation leads to the same failure distribution as originally started with, which is a prerequisite.

In order to perform such an analysis, the failure distribution of the component has to be known. This failure distribution function can be determined from experimental data by means of a **Weibull analysis**. If only a very limited number of failure data is available, a conservative lower bound of the real failure distribution can be determined by means of a **Weibays** or **Weibest analysis**, based on non-failure data, which can be updated during the service lifetime of the component with field data.

In a Weibull analysis a Weibull distribution function is used to describe the failure mode. The two-parameter Weibull distribution function (cumulative density function) yields:

$$F(t) = 1 - e^{-\left(\frac{t}{\eta}\right)^\beta} \quad (5)$$

where:

t = time to failure in cycles

β = measure of the speed of the failure mechanism (shape parameter)

η = measure of the characteristic life (scale parameter)

The unknown values of the parameters β and η are determined from a set of available data points, which may consist of failed and unfailed sample times. Various methods exist to fit these two parameters, of which the method of maximum likelihood is used in general. This method is based on a so-called likelihood function, describing the probability of obtaining the observed data. The two parameters β and η are now found by maximizing the likelihood function. Once β has been obtained, η can be determined from:

$$\eta = \left(\frac{\sum_{i=1}^N x_i^\beta}{n} \right)^{\frac{1}{\beta}} \quad (6)$$

where:

n = Number of failed samples

N = Total number of samples

This last equation can also be used to estimate η in case no failure points exist. Then, for β a value is assumed, based on historical failure data or engineering judgment (β normally lies normally in the range of 2 to 6). For the number of failures (n) different values can be selected.



Set to one a so-called Weibays method is obtained, where it is assumed that the first failure is assumed to be imminent resulting in a 63% lower confidence bound on the true Weibull. A less conservative approach is selecting $n=0.693$ resulting in a 50% lower confidence bound on the true Weibull, a so-called Weibest method is obtained.

CONCLUSIONS

In this paper, the different steps of which a stochastic analysis consists are demonstrated on a forward engine mount support fitting of an F-16. This engine mount is a critical component with a small inspection interval, which is difficult to inspect. The stochastic nature of the various parameters of the crack growth model is examined using real life data and their stochastic importance is determined by means of a sensitivity analysis. This reveals the model parameters, the material parameters C and m , the fracture toughness K_{Ic} , the initial crack size a_0 and the overload shut-off ratio R_{so} , for which the variability should be taken into account and which therefore should be modelled as stochastic variables. These results have been used in a stochastic Damage Tolerance analysis, revealing the very conservative nature of such an analysis due to the unknown initial flaw size distribution.

An improved stochastic approach has been presented in which the realistic initial crack size distribution is determined from failure data and which includes the initiation as well as the crack growth life.

ACKNOWLEDGEMENTS

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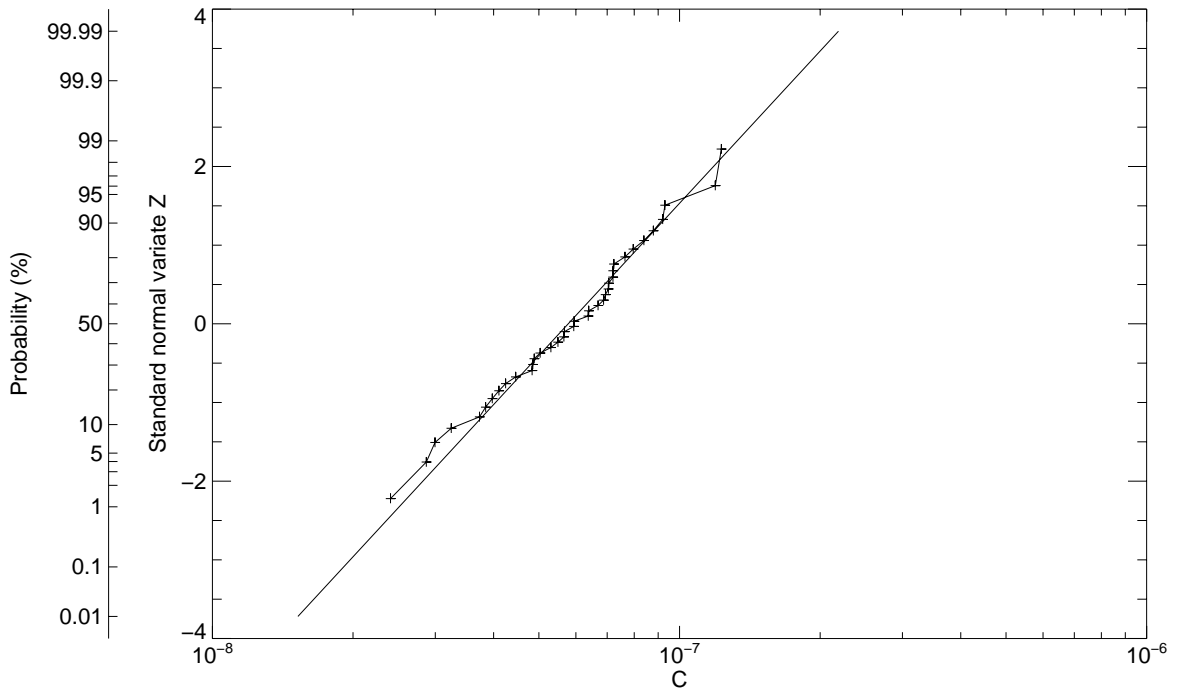
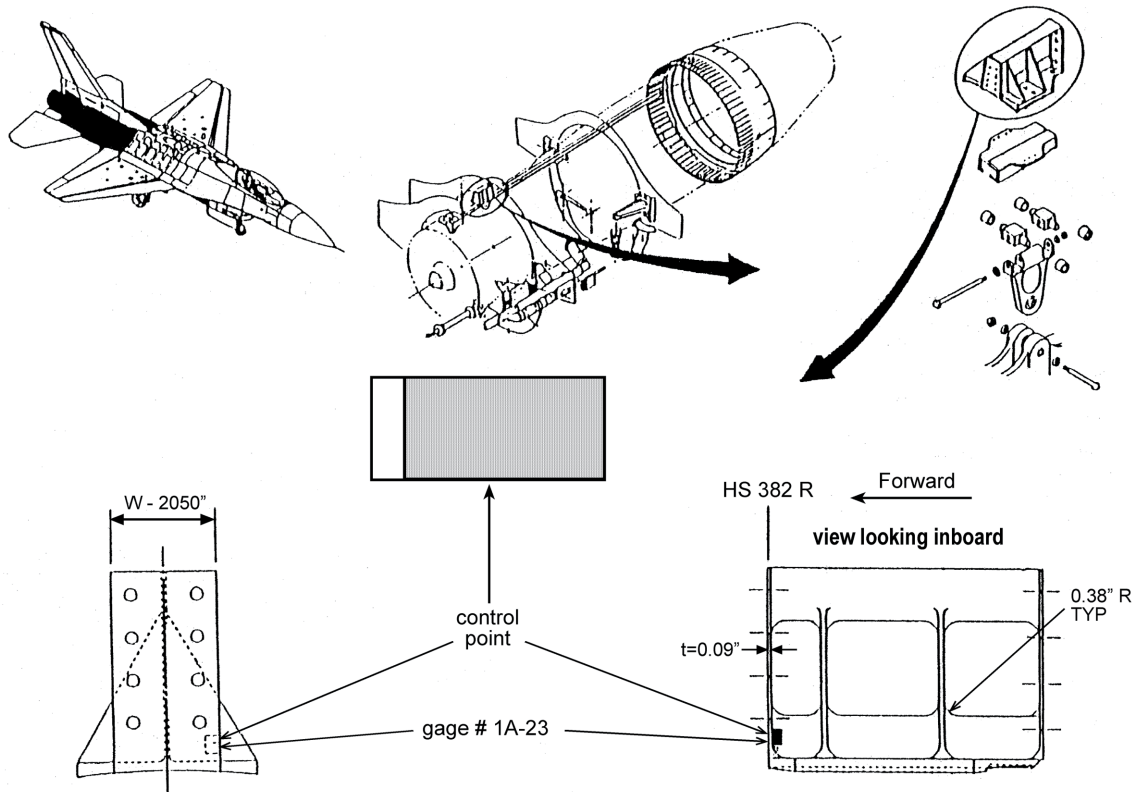


Figure 2. Material parameter C data points on log-normal probability paper

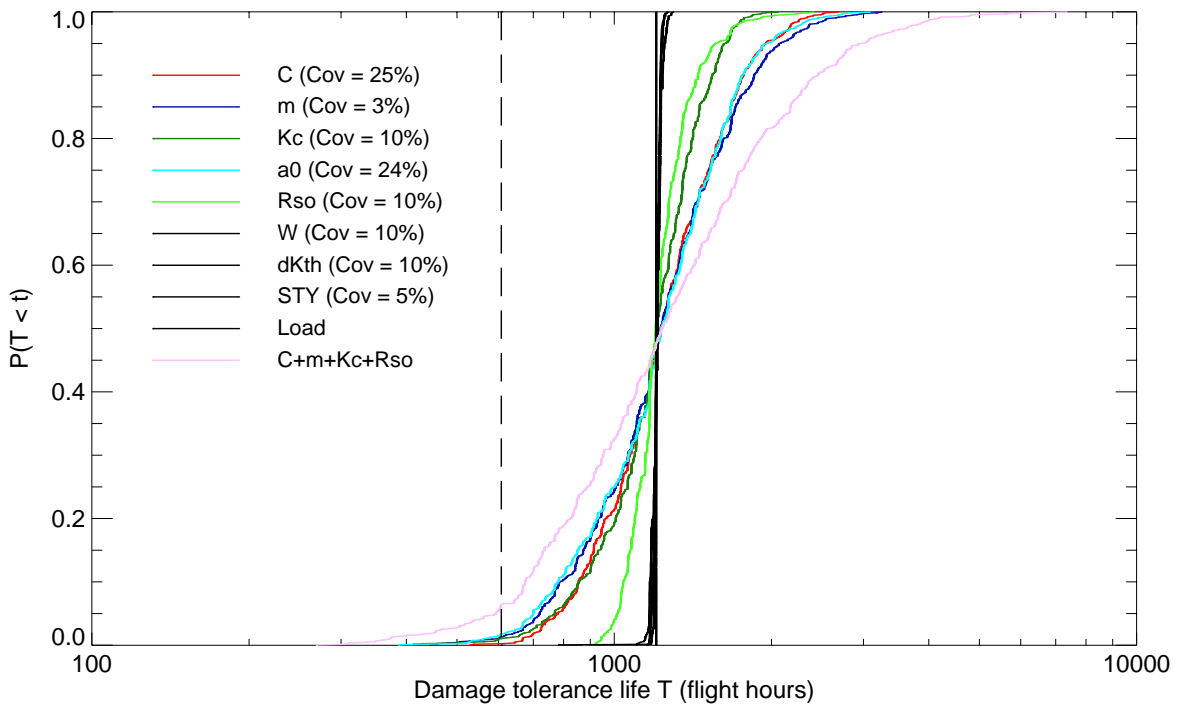


Figure 3. Cumulative distribution curves of the damage tolerance life T for various random variables

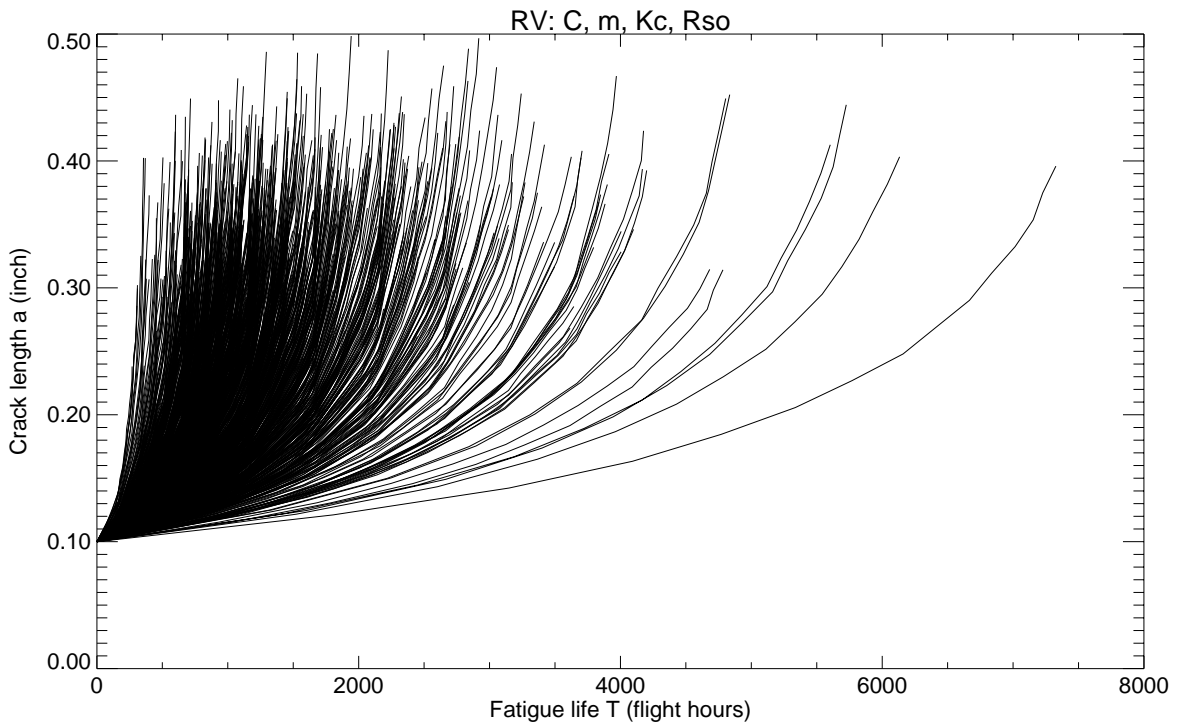


Figure 4. Damage tolerance crack growth curves for 1000 MC simulations; $a_0 = 0.1$ inch

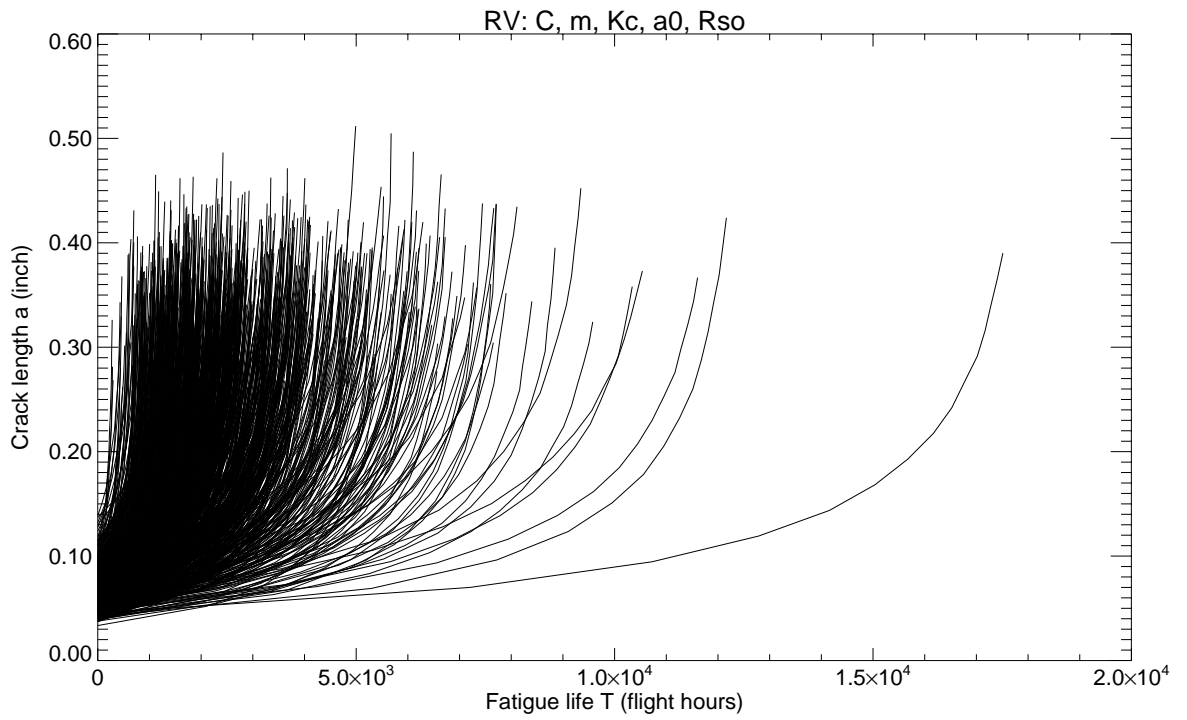


Figure 5. Damage tolerance crack growth curves for 1000 MC simulations; $a_0 = \text{POD}$

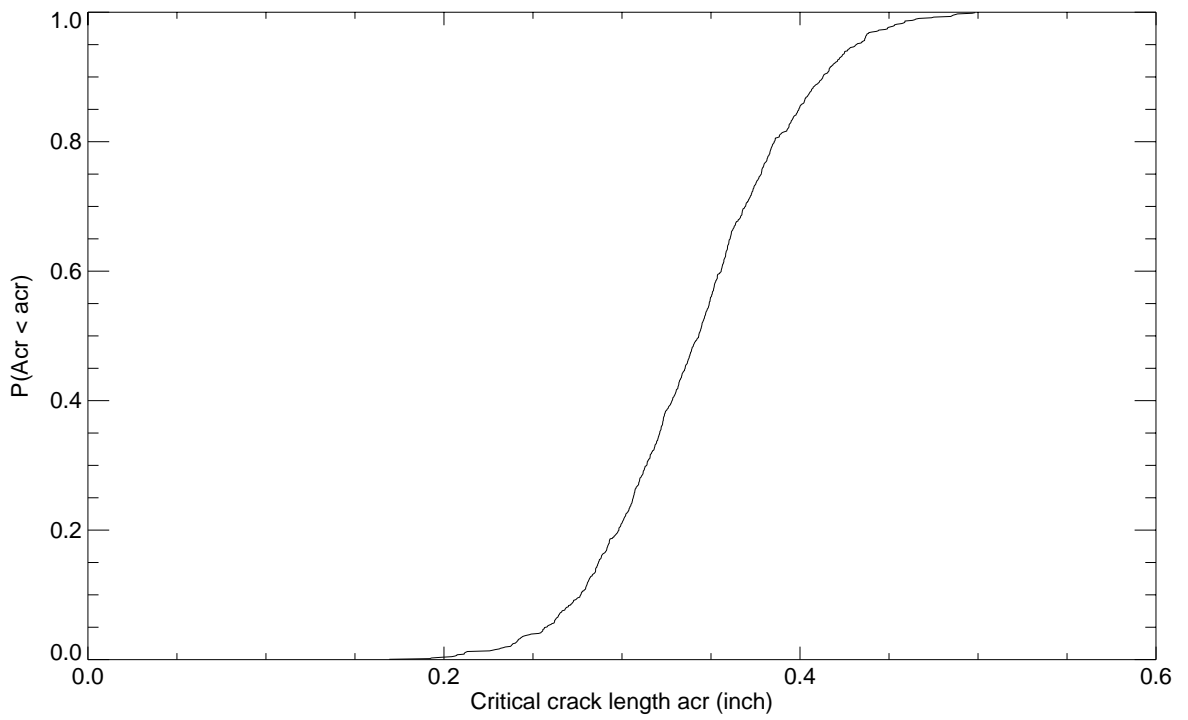


Figure 6a. Cumulative probability function for the critical crack length

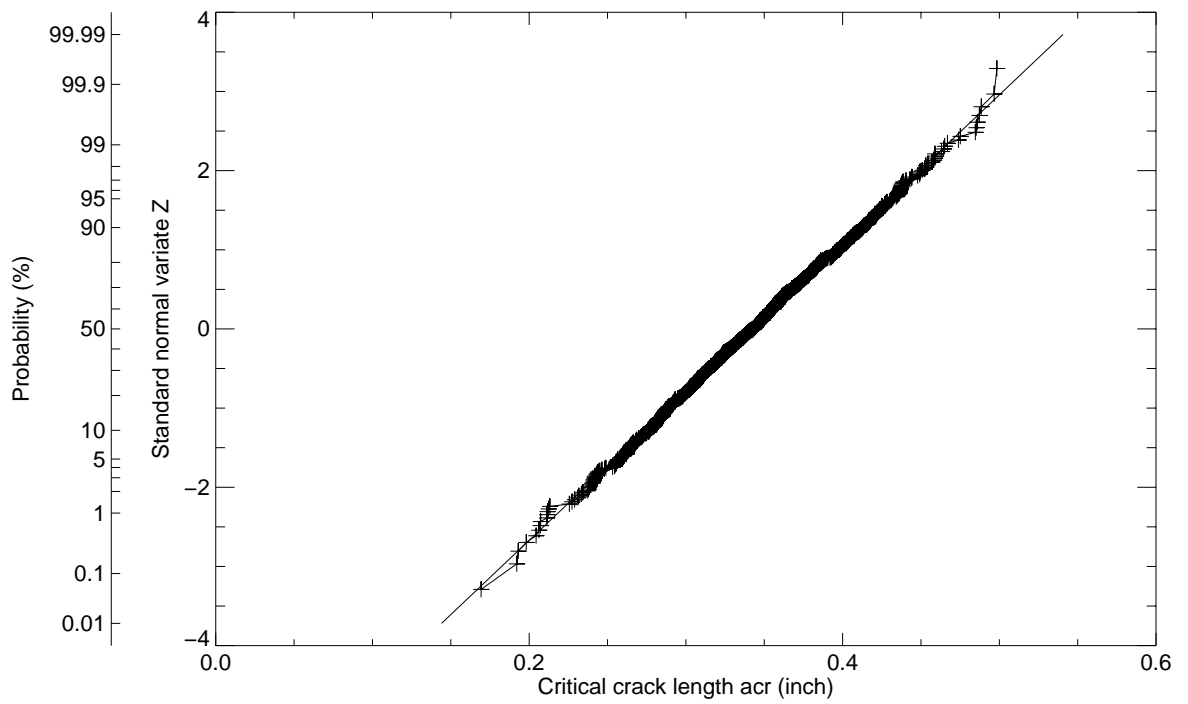


Figure 6b. Critical crack length data points on normal probability paper

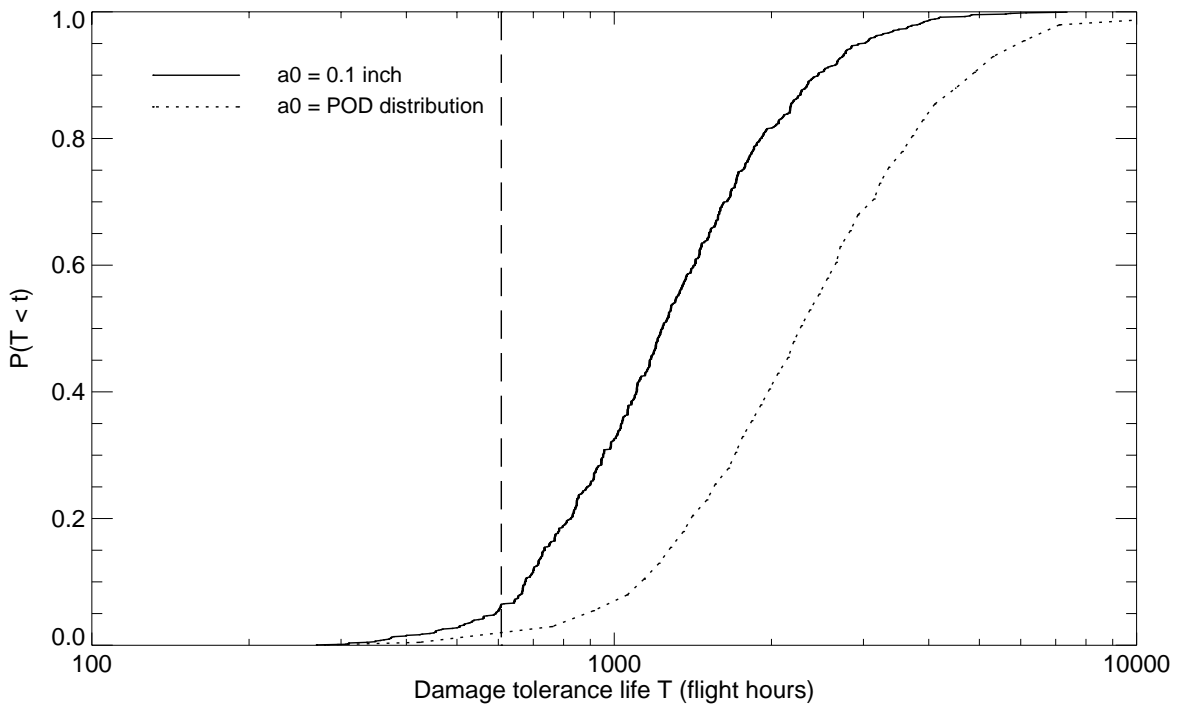


Figure 7a. Cumulative probability function for the damage tolerance life T

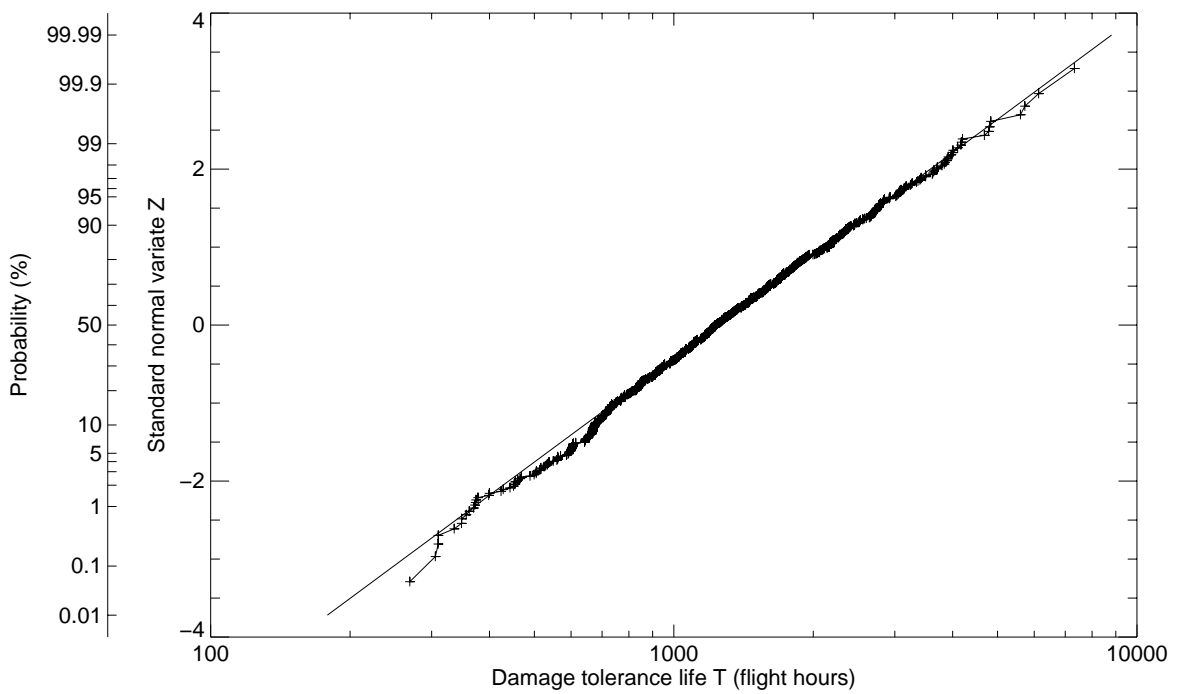


Figure 7b. Damage tolerance life data points on log-normal probability paper; $a_0 = 0.1$ inch

