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Physical aspects of separation in three-dimensional flows

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ABSTRACT
The global characteristics of separated flows around three-dimensional objects, including the associated topology of the wall streamlines, are discussed. The problem of the definition of separation in the three-dimensional case is considered. The emphasis is on the physical aspects, with the aim to demonstrate the large and essential differences between two- and three-dimensional separations

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Physical Aspects of Separation in Three-Dimensional Flows

Abstract

The global characteristics of separated flows around three-dimensional objects, including the associated topology of the wal1 streamlines, are discussed. The problem of the definition of separation in the three-dimensional case is considered. The emphasis is on the physical aspects, with the aim to demonstrate the large and essential differences between two- and three-dimensional separations.

Introduction

As the seminar is to honor Jan van Ingen at the occasion of his retirement as professor at the Technical University in Delft, it seems appropriate to start with some personal recollections. My more closer contact with Jan van Ingen started in 1973, when 1 requested him to become the supervisor of the PhD thesis I wanted to write about work done at NLR on three-dimensional turbulent boundary layers. After the doctoral degree ceremony we remained in good contact. This close contact has been very profitable, in my conviction, for both the aerodynamic department at NLR and his research group at the university. 1 am looking back with satisfaction to various research projects carried out here in Delft, which were initiated in concert. 1 thank him for the stimulating discussions about various snbjects of boundary layer research, his cooperative attitude and his fellowship during these years.

In the circumstances 1 would like to start going back to the past, viz. the three-dimensional turbulent boundary layer experiment, which was part of my PhD thesis (Van den Berg, 1976). In this experiment the flow on an infmite swept wing was simulated by a swept flat plate. Figure 1 shows a sketch of the slightly curved streamlines at the boundary layer edge and the much more strongly curved wal1 streamlines, or more precisely: the integral curves of the skin friction vectors. When viewing the flow normal to the leading edge, it is clear that the wal1 Streamline parallel to it, labelled ‘separation line’ in the sketch, constitutes a banier for the wal1 streamlines from upstream. Figure 2 is a photograph of the oil flow pattem, which visualizes the wal1 streamlines on the test plate surface (Elsenaar, van den Berg, Lindhout, 1975).

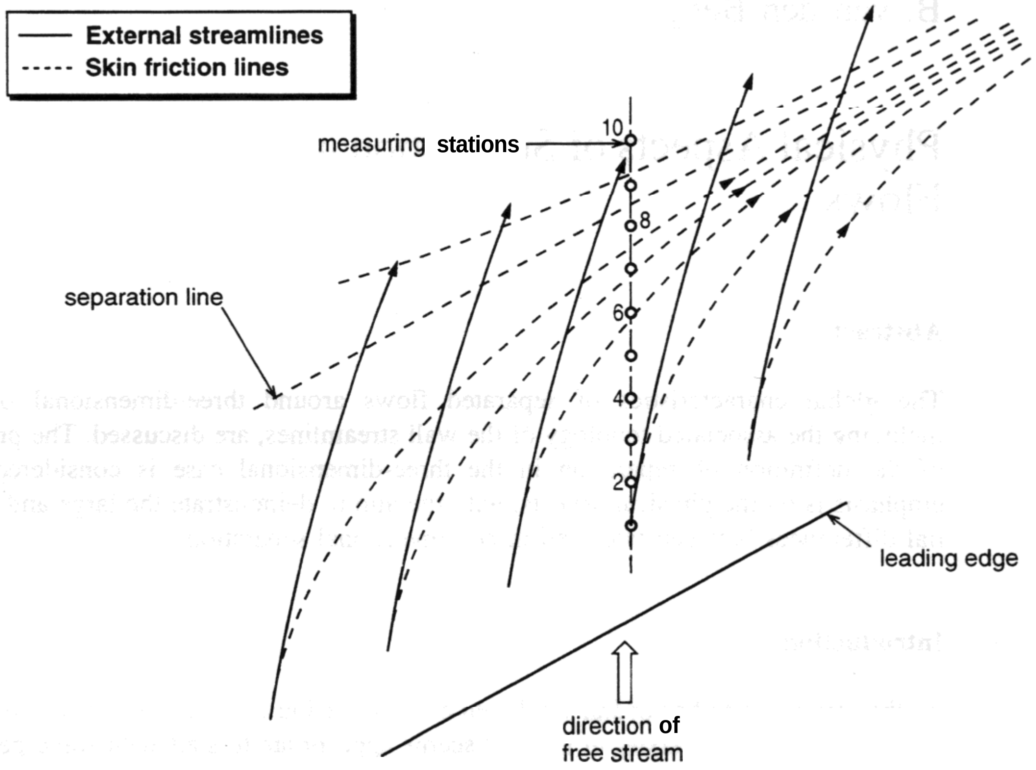

Fig. 1 Three-dimensional boundary layer flow on an infinite swept wing

The oil flow pattem does not suggest extremely low flow velocities downstream of the separation line, as in twodimensional separations where the expression “dead-air region” is common for that part of the flow. Tbis is confírmed by the plot of the skin friction magnitude measurement data in figure 3. It is seen that the skin friction magnitude reaches a minimum near sepamtion, but that it is far from zero there. As argued in my thesis (see also Van den Berg, 1975) the variation of the velocity component parallel to the skin friction (which is approximately equal to the velocity - magnitude close to the wall) is to fust order as predicted by the two-dimensional law of the wal1 at the corresponding skin friction. Consequently the skin friction data do not indicate low velocities in the wal1 region. The special feature of the flow is the streng variation of the direction of the velocity near the three-dimensional separation line, as appears f'rom the perpspective velocity vector plot in figure 4, based on the boundary layer measurement data at station 7, situated close to the sepamtion line (see figure 2). The velocity vector plots at the stations downstream of the separation line are not essentially different, apart from the fact that the limit velocity vector direction or skin friction direction exceeds then, of course, the direction of the separation line.

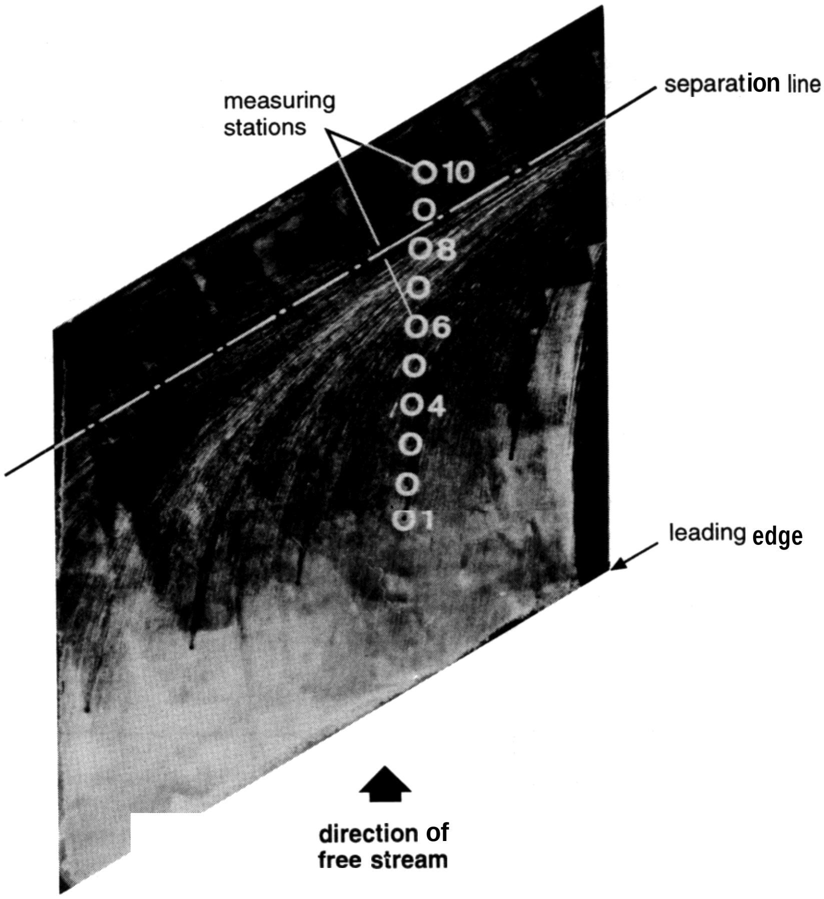

Fig. 2 *Photograph of the oil flow pattern on the swept wing surface*

The characteristics of two-dimensional separation are wel1 known: the skin friction is zero at separation, downstream reverse flow occurs and consequently the viscous shear layer thickness starts to increase strongly. When considering the flow in figure 2 normal to the swept leading edge, the skin friction component in that direction becomes zero at separation and reverse flow occurs indeed in this viewing direction. However, when viewing the flow parallel to the leading edge and forgetting that the flow is an infínite swept wing flow, it is difficult to see what is special about the wall Streamline labelled separation line. The wal1 streamlines converge to that wal1 streamline, but the same is true for many other wal1 streamlines. This is the subject of the paper: the special features of three-dimensional flow separation, or rather: what a very special case two-dimensional separation is.

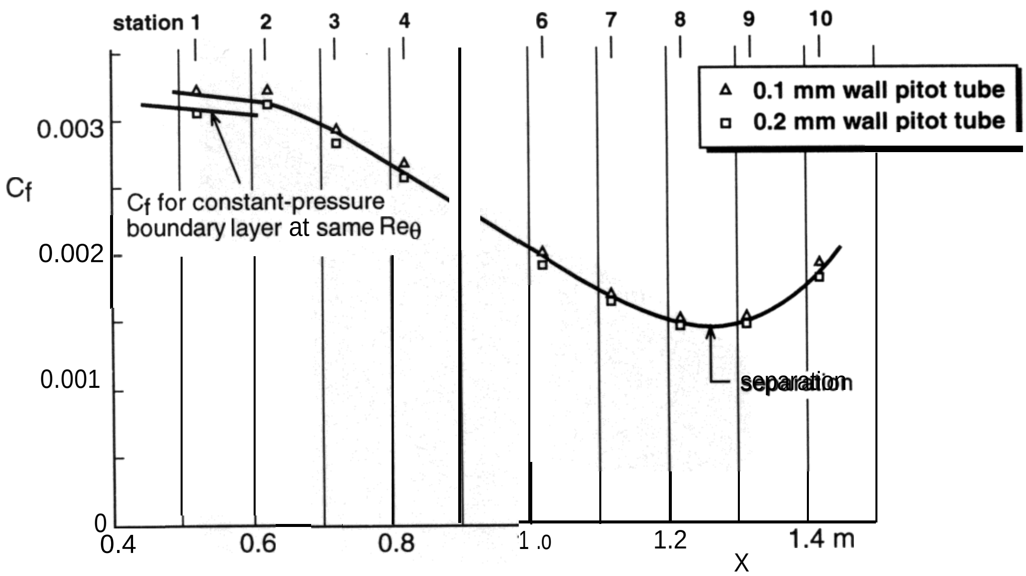

Fig. 3 *Variation of the magnitude of the skin friction with distance from the leading edge of the swept wing*

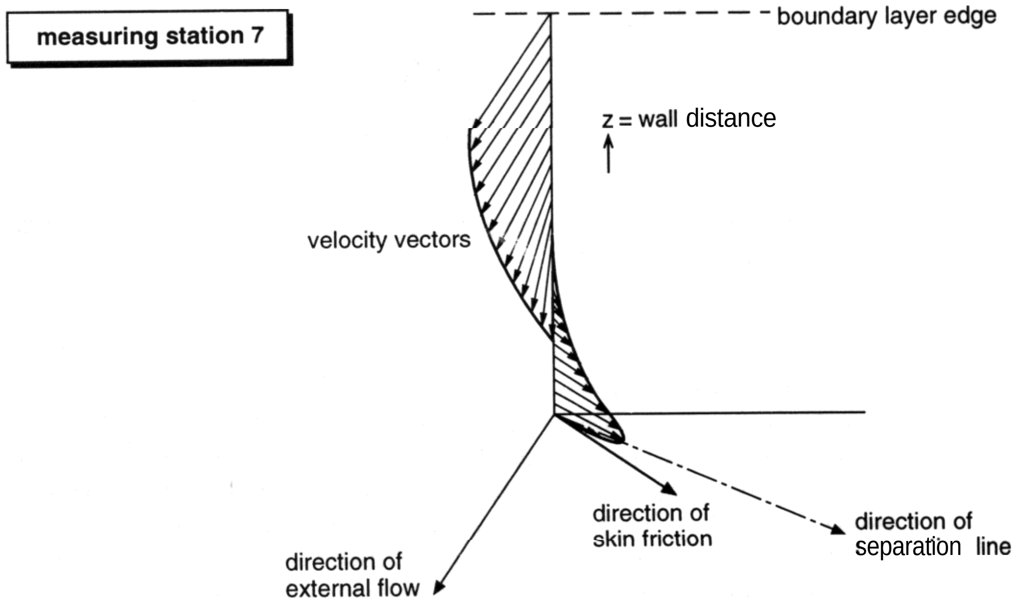

Fig. 4 Perspective velocity vector plot at a station close to the three-dimensional separation line

Topological features of separated flows

When considering flow separation on three-dimensional bodies, it can not be avoided to begin with a discussion about the features of the topology of the wall streamlines or skin friction envelopes. In the three-dimensional case the skin friction magnitude $\tau_w = 0$ only at singular points and not generally along a line on the surface. Close to such a point the skin friction components $\tau_{w,x}$ and $\tau_{w,y}$ in the orthogonal directions x and y may be written in case of regular behaviour: $\tau_{w,x} = a_{xx}\,x + a_{xy}\,y$ and $\tau_{w,y} = a_{yx}\,x + a_{yy}\,y$, retaining only the lowest-order terms in the series expansion. Higher-order singularities have been thoroughly investigated by Professor Bakker and his co-workers at Delft University (Bakker, 1988) and tbeir treatment, which includes a systematic investigation of possible flow topologies by applying the qualitative theory of differential equations, is far beyond the scope of this review.

a) Node of attachment
$a_{xx}=1$
$a_{yy}=2$
$a_{xy}=a_{yx}=0$

b) Spiral node or focus of separation
$a_{xx}=-1$
$a_{yy} = -1$
$a_{xy} = -1$
$a_{yx} = 1$

c) Saddle of separation
$a_{xx} = -2$
$a_{yy} = 1$
$a_{xy} = a_{yx} = 0$

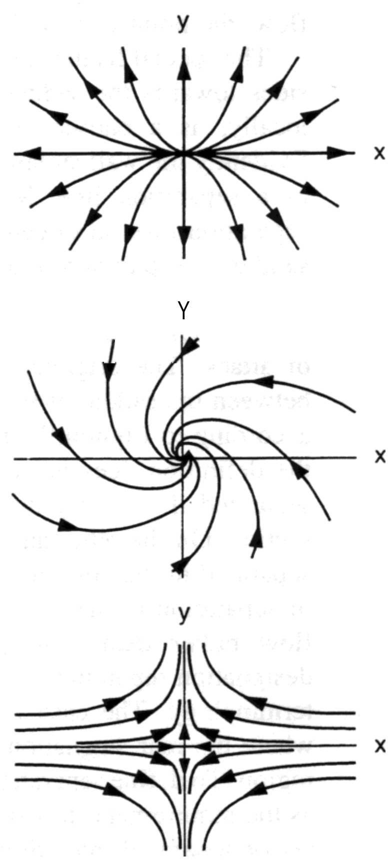

Fig. 5 Typical patterns of wall streamlines **neer** *singular points in three-dimensional flows*

First-order singular points of different type are obtained dependent on the sign and magnitude of the coefficients $a_{xx}, a_{xy}, a_{yx}, a_{yy}$ (e.g. Lighthill, 1963). Figure 5 shows some typical wall streamline pattems near a singular point for some values of the coefficients. The figure comprises examples of the three well-known basic types of singular points: a node, a focus and a saddle. Figure 5a shows anode of attachment, from which an infinite number of wall streamlines arise, as occurs at a stagnation point. By changing the sign of the arrows in the sketch a node of separation is obtained. Unless the local flow is axisymmetric, all wall streamlines, except two, touch each other at the node. Near a spiral node or focus, see figure 5b, the wall streamlines approach the singular point asymptotically in spirals. The presence of a focus of separation generally marks the development of a vortex in the flow above the surface. In the case of a saddle, see figure 5c, the wall streamlines do not pass through the singular point, except along two lines. Along one line the skin friction vector is directed towards the singular point and along the other it is away from it. Dependent on the relative magnitude of these skin friction vectors and the associated near-wall flow, the point is a saddle of separation or a saddle of attachment.

The special feature of a saddle point is that it partitions the flow coming from both sides towards the saddle. The two wall streamlines leaving the saddle act, at least locally, as a barrier between wall streamlines coming from opposite diitions. Evidently the wall streamlines leaving a saddle point have some of the characteristics of a separation line. More precisely Lighthill (1963) has proposed to defiie as separation lines in three-dimensional flows: the wall streamlines, which issue from a saddle of separation and disappear in a node of separation. Figure 6 gives a very simple example of a separation in three-dimensional flow (all examples will be kept simple). The figure shows the separation on a spheroid at (a moderately large) angle of attack. The original sketch is from Eichelbrenner (1957). The wall streamlines between the saddle of separation D and the node of separation C are separation lines according to Lighthill's definition, which seems perfectly acceptable here. However, the definition does not cover all possible flow cases, which one would like to call separated if not only the surface flow is considered, as will be argued in the next section. On the other hand, cases might be included which one would not like to call separated in the normal sense, see e.g. figure 7. The wall streamlines from the saddle of separation to the nodes of separation in this figure mark the beginning of a wake flow rather than a separation region. From the angle of surface topology the designation separation line may be correct, but it would be a quite uncommon physical terminology. The case is comparable to that in an attached two-dimensional flow where the rear stagnation point is not generally called the beginning of a separation region. To define separation, additional physical requirements are normally made, such as the requirement in two-dimensional flows that reverse flow occurs. Physical featu-res of separated three-dimensional flows will be discussed in the next section.

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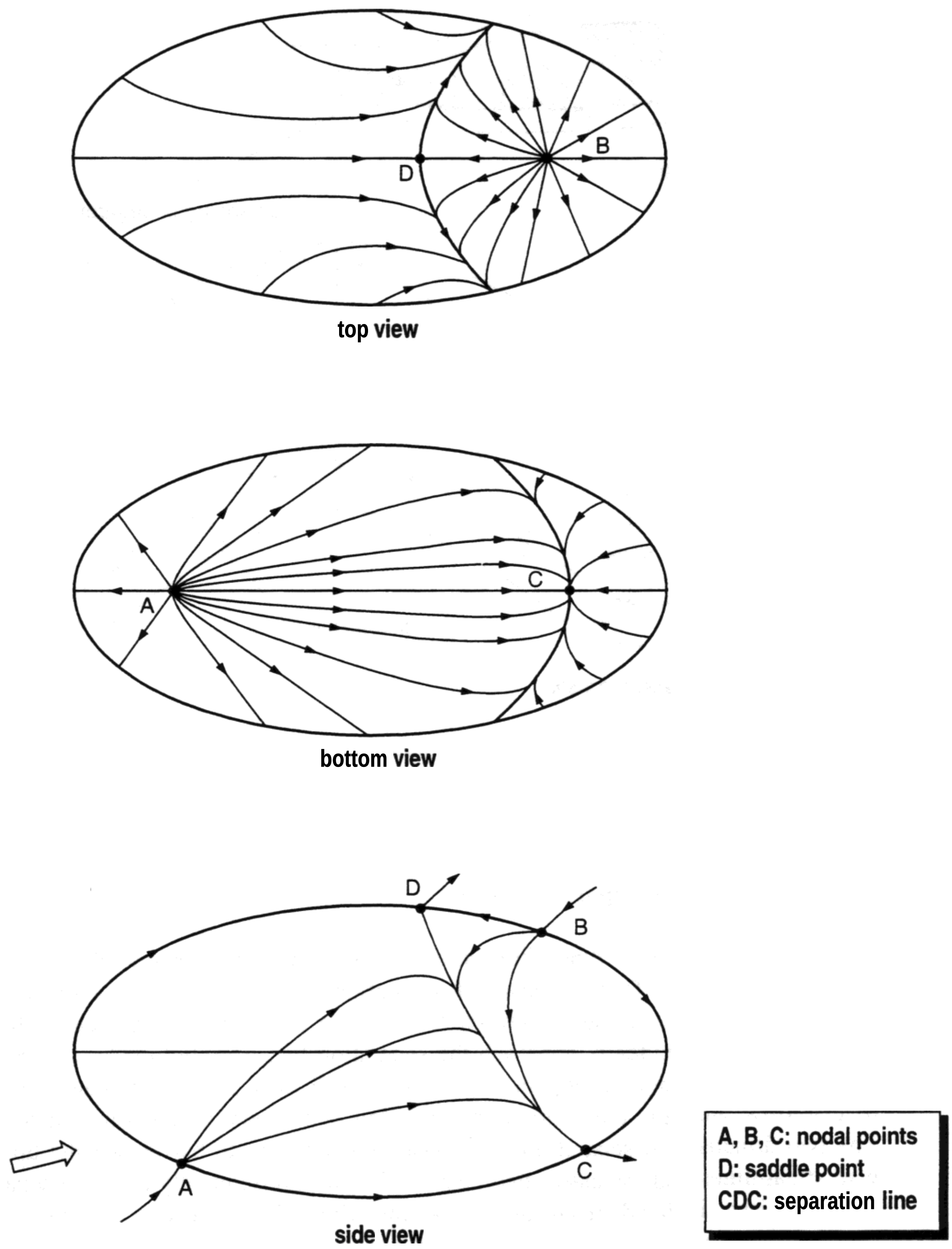


Fig. 6 Wall streamlines on an inclined spheroid

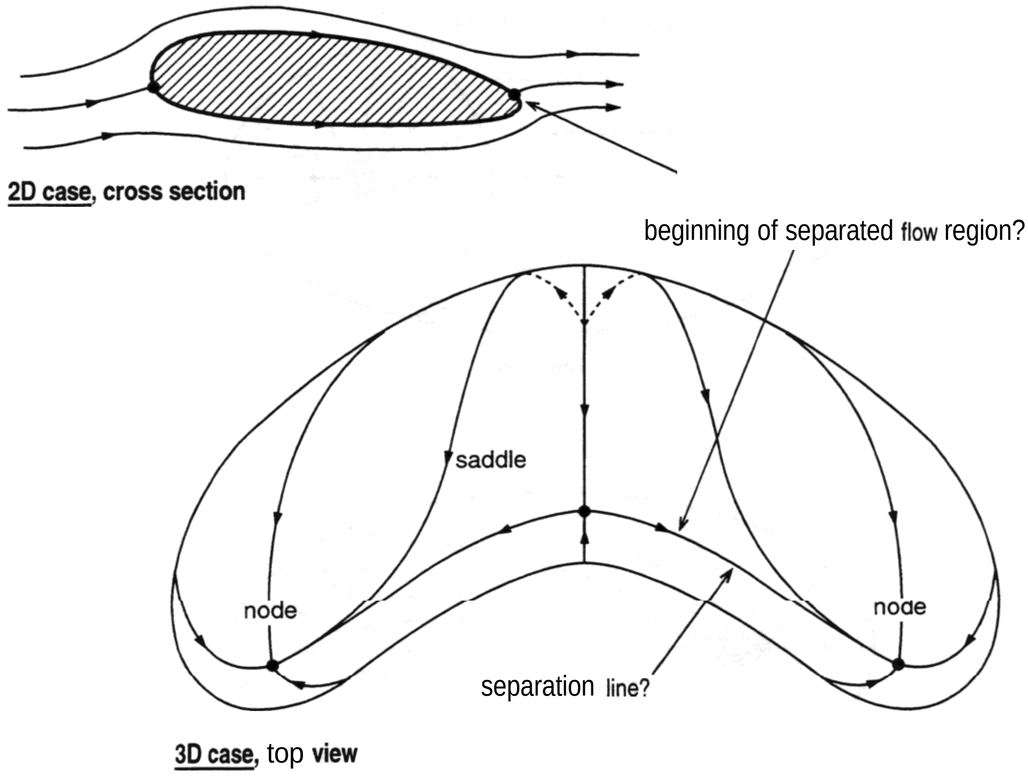


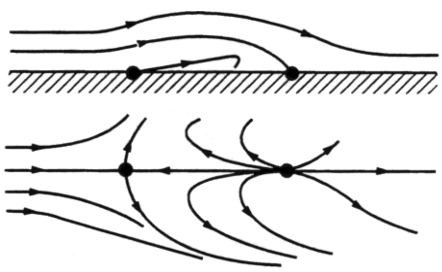
Fig. 7 *Incompleteness of definitions of separated flow by means of wall properties only*

Physical features of separated flows

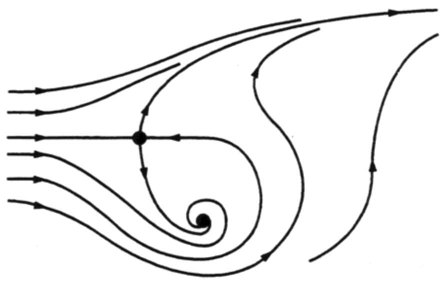
The physical features of three-dimensional separations differ considerably from those in two dimensions. Two-dimensional separation is associated with a reverse flow region downstream of the sepamtion line. In the three-dimensional case, the wall streamlines only gradually approach the separation line from both sides, as shown in figure 6, and it is not so evident that the near-wall flow at one side can be called a reverse flow. Actually the notion of reverse flow is not very clear in three dimensions. The two-dimensional separation line constitutes a complete banier for the upstream wall streamlines and the associated near-wall flow. Consequently a closed sepamtion region exists, isolated from the remaining flow. The question whether closed separation regions also exist in three-dimensional flows will be discussed extensively in the following.

Some simple wall Streamline pattems near one or two nodes and saddles are sketched in figure 8. The patterns are well known and have been named as indicated. For the vortex type and owl-face pattern (figures 8b and c), it is evident that the wall streamlines issuing from the saddle points and disappearing in the foci, do not constitute a banier. Actually all wall streamlines directed towards the saddle point from downstream originate upstream, which means that the whole surface is accessible from upstream. There is no closed separation region here. The bubble type (figure 8a) provides an example where the wall streamlines issuing from the saddle indeed constitutes a complete banier. However, this holds only for the wall streamlines and generally not for any other streamline away from the wall, since the external Streamline attaching to the downstream node normally does not originate at the upstream saddle, as indicated in figure 8a. A closed bubble only occurs when the Streamline issuing from the saddle happens to reattach at the downstream node, which is not a likely event. Also in this case, therefore, the separation region is not closed normally.

a) Bubble type
1 saddle
1 node



b) Vortex type
1 saddle
1 focus



c) Owl-face
2 saddies
2 foci

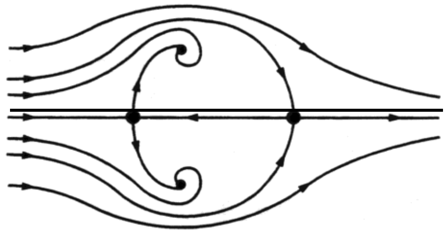
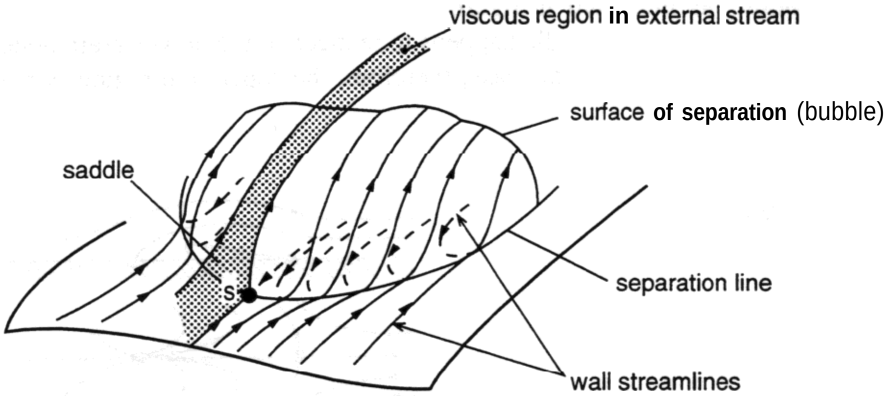
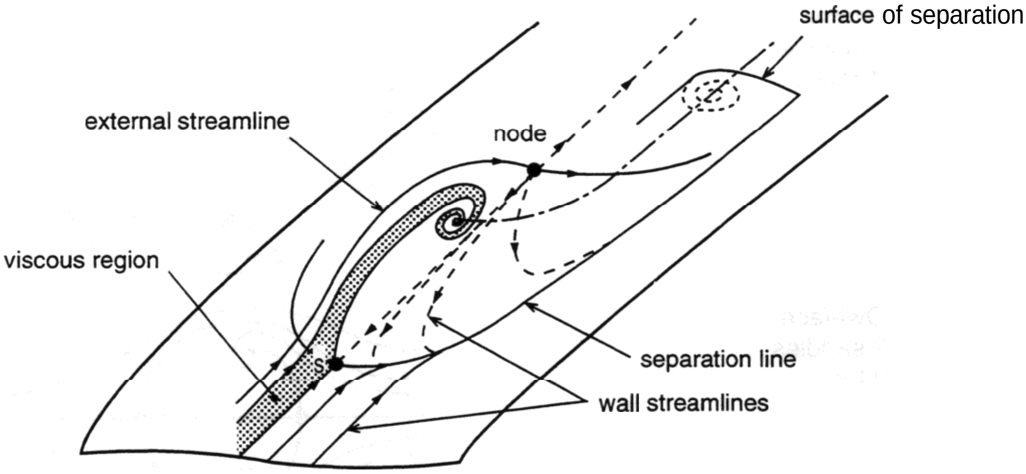


Fig. 8 Some *simple wall Streamline pattems near one or two nodes and saddles*

The open nature of nearly all three-dimensional separations appears not always to be fully appreciated, notwithstanding that it was stressed already in a number of earlier publications (e.g. Hunt et al., 1978; Homung et al., 1984). This may be partly due to the perspective sketch shown in the upper part of figure 9, which is often used to clarify the bubble type separation. Although probably differently meant in the original publication (Maskell, 1955). the sketch is easily misinterpreted, suggesting the presence of a closed three-dimensional separation region. That this is not necessarily so becomes clear when extending the sketch in downstream direction, as illustrated in the lower part of figure 9. There the surface of separation is rolling up into a vortical



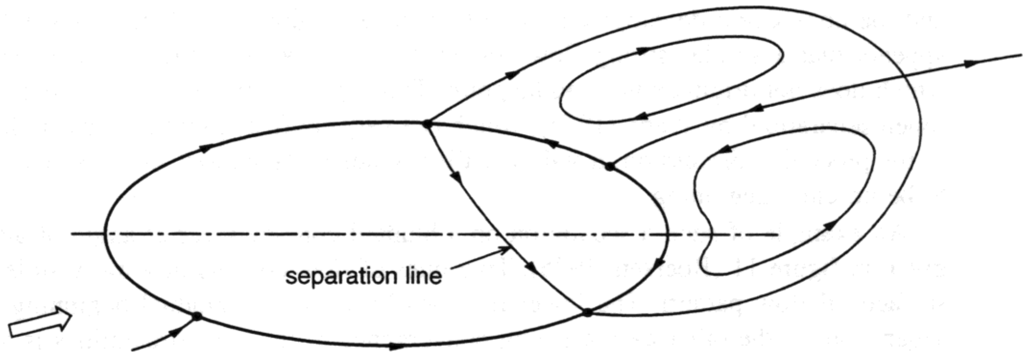
Customary sketch of bubble type separation
(after Maskell. 1955)



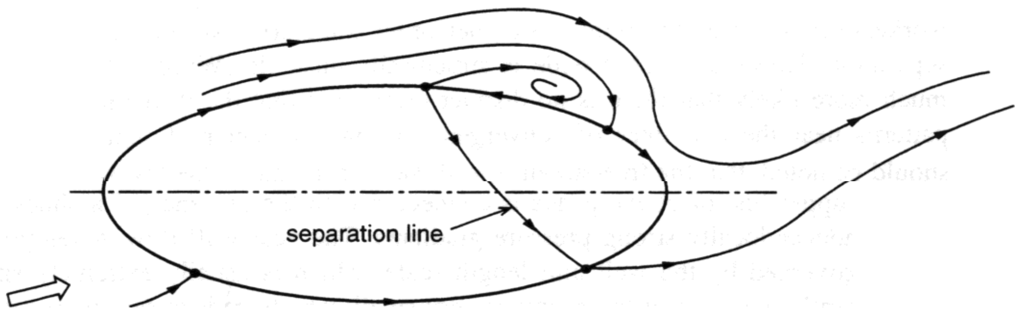
Same sketch as above with downstream extension, showing open nature of separation

Fig. 9 Perspective sketch of three-dimensional separation of bubble type structure, which is developing far downstream into two trailing vortices as behind a lifting body. The vortical flow contains the separated viscous layer. An inviscid streamline is stagnating at the node and creates a new boundary layer flow. This is the three-dimensional flow picture near a saddle and a downstream node, as it occurs normally in practice. Evidently no closed separation region is present here. The whole region downstream of the separation line is accessible from upstream. In this respect the name 'bubble-type' separation (figure 8a) is misleading.

For a concrete example the flow around a spheroid at angle of attack in figure 6 will be considered again. Now, however, attention will not be focused on the wall streamlines, but on the flow away from the surface. In figure 10a a sketch is given of a closed three-dimensional separation region, as one might picture behind a bluff body like a spheroid. In non-axisymmetric flows, however, the correct picture is more likely as given in the lower sketch, figure 10b, where an open 'bubble-type' separation similar to the one discussed in the preceding paragraph is shown. Far downstream the



a) Improbable pattern: closed separation region



b) More typical pattern: open separation

Fig. *10 Sketches of separated flow patterns behind an inclined spheroid* separated viscous shear layers will have developed into two (perhaps weak) trailing vortices. As bodies at angle of attack produce some lift, (weak) trailing vortices should indeed be expected. The flow pattem resembles in principle more that around a slender wing at angle of attack than a twodimensional bluff body separation. The flow pattem sketched in figure 10b (and figure 6) is one of the possibilities. A flow with two saddles instead of one occurs at smal1 angles of attack (Eichelbrenner, 1957). In practice often gradually developing separations along lines of convergence occur on slender spheroids (e.g. Meier et al., 1983), which will be discussed hereafter.

Two-dimensional separation is accompanied with a strong growth of the viscous layer thickness. Alternatively one can say that the shear layer separates from the surface at the zero skin friction line. In three dimensions the viscous shear layer thickness growth is generally due to strong flow convergence. If the local thickness increase becomes large, one can say again alternatively that a free shear layer is developing separate from the surface. A strong flow convergence generally occurs along the wall Streamline between a saddle of separation and a node of separation, i.e. the separation line according to Lighthill's definition. Figure 9 provides an example of such a strong flow convergence along the wall streamline leaving a saddle point and the consequent development of free shear layer along a surface of separation. It appears that a similar strong flow convergence can occur along a wall streamline, which does not originate in a saddle point. This type of separation has been called an 'open separation' by Wang (1976), which is a very confusing terminology in the light of the preceding discussion. It will be called 'gradual separation' hem, as there seems to be no clear beginning.

An example of such a separation on a blunted cone-cylinder at angle of attack is given in figure 11 (Boersen, 1975). Two lines of flow convergence are visible in the surface oil flow pattem. The lower line clearly shows the gradual beginning of the separation. In the other case a strong convergence of the wall streamlines is seen to exist and a very distinct line of convergence, includmg a fairly distinct beginning. The latter pattem suggests that the origin of the line of convergence might be after all a special, identifiable point. More spocifically it has been proposed by Wu and co-workers (1988). that the origin is a higher-order singularity, such as a saddle-node of, separation. However, such a node is structurally unstable (Winkel, 1996). It seems much more likely that there is no distinct origin and that the difference in the flow pattems near the two lines of convergence is only a matter of scale of events. It should be noted, that the free-stream speed was supersonic in the test shown in figure 11. The upper line of convergence is connected with the presence of a shock wave, which induces locally strong pressure gradients. The near-wall flow development is further govemed by the wall unit length scale, which is usually extremely small. - Consequently the scale of the events is very small, which leads to the suggestion that the convergence of wall streamlines is strong. The importance of the size of the viewing window is illustrated in figure 12 (taken from Homung et al., 1984).

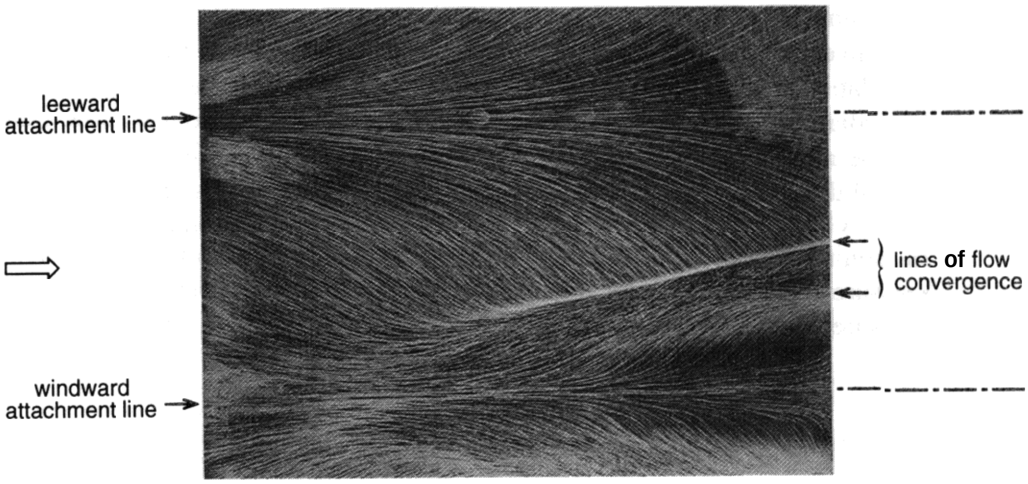


Fig. 71 Oil flow pattern on the unwrapped surface of an inclined cylindrical afterbody (Boersen, 1975)

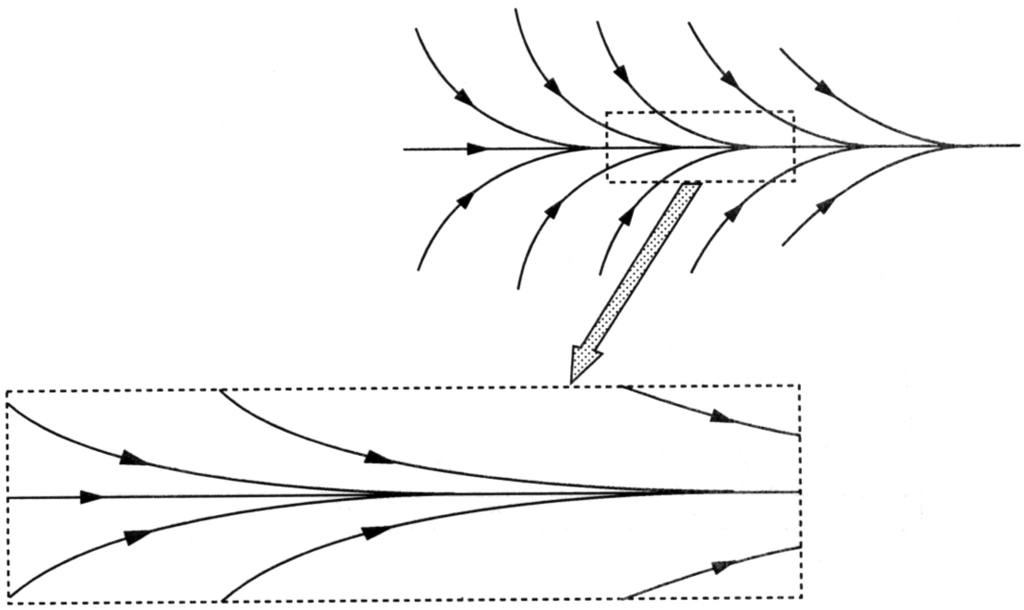


Fig. 12 Effect of the size of the viewing window on the apparent strength of convergence (Hornung & Perry, 1984)

It remains surprising that, as appears from many surface flow visualizations, lines of convergence often are very distinct lines in the suiface flow patterns. Also in calculations pronounced convergence of wall streamlines is frequently found. It is not immediately evident why this should be so. An attempt to provide some explanation will be made here with the help of figure 13. In two-dimensional flows the effect of a local disturbance, like a smal1 protuberance on the surface is known to be smoothed by the viscous flow interacting with the extemal flow. The protuberance causes a local thickening of the boundary layer and the external inviscid flow will induce a suction pressure at the top of the local thickening. The successive favourable and adverse local pressure gradients cause a decreased and increased growth of the boundary layer

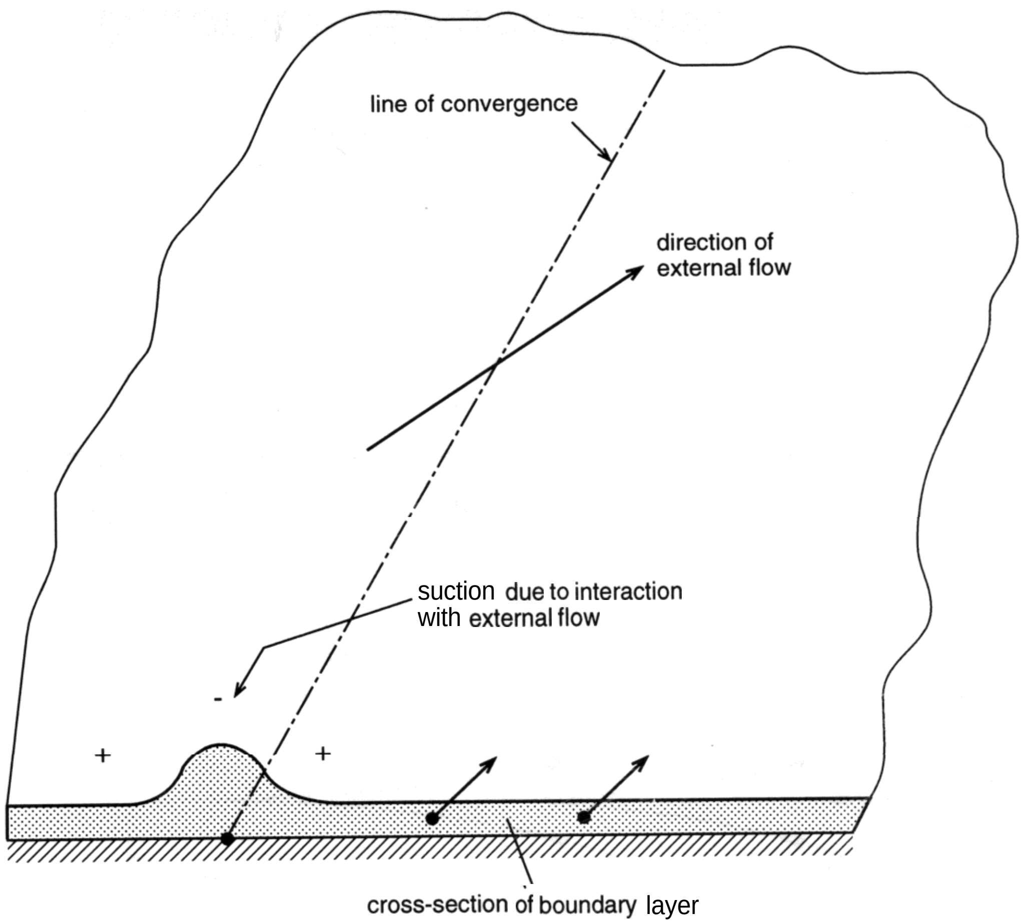


Fig. 13 Sketch *illustrating the effect of the external flow on the thickened boundary layer near a line of convergence*

displacement thickness respectively, reducing the original local thickening. In a three-dimensional flow a suction pressnre may stil1 be expected at the top of a local region with increased boundary layer displacement thickness, as along a line of convergence, at least when the external flow is not precisely parallel to that line (see fignre 13). However, the suction pressure induced by the external flow will increase the convergence of streamlines in the boundary layer, i.e. it will now augment the local boundary layer thickening already present. This means that once a line of convergence with increased bonndary layer thickness exists, the interaction witb the extemal flow will tend to magnify the amount of convergence and the associated boundary layer thickness increment. The fairly sudden development of lines of convergences may perhaps thus be explained. The pressure gradients towards the center of the convergence region will, of course, also affect the surface oil flow visualizations. The pressure forces will drive oil towards the center and might lead so to an oil flow pattern which snggests a more strong convergence than actually exists.

Definition of separation

As follows from the previous observations, a defmition of a separation line, which is in all respects satisfactoty, is not evident in three-dimensional flows. The only strict definition is that from Lighthill: a separation line is a wall Streamline between a saddle and a node of separation. Along such a line generally a strong flow convergence occnrs, but it appears that the same may occur along a wall Streamline not originating from a saddle. Further downstream there is no distinction between the two flows, so that it seems difftcult to call one separated and not the other, only on the basis of the surface topology: separation is a phenomenon connected to the whole flow.

It has been argued (Maskell, 1955) that separation lines are characterized by being envelopes of wall streamlines, and that stream surface bifurcation takes place at separation lines (Hornung et al., 1984). The notion may be inspired by the results of surface flow visualizations as in figure 11, which indicate that the wall streamlines approach the line of convergence very fast indeed. Being an envelope wonld imply that singnlar behaviour occurs along separation lines. Such singular behavionr may be a result of bonndary layer type calculations due the approximations made and the way the calculations are performed. Typically such calculations yield for a given pressnre distribution: $\tau_{w,x} \sim x^{1/2}$, where the surface coordinate x is the distance from the separation line. As $\tau_{w,y}$ is not generally zero at the separation line, the calculation results lead to wall streamlines tonching in a cusp-like marmer the separation line, which is a singular line then. Singular behaviour is not be expccted in physical reality, however, and will not occur generally in Navier-Stokes (see e.g. Bradshaw, 1979). In reality and in correct calculations $\tau_{w,x} \sim x$, so all wall streamlines converging to a 'separation line' either approach it asymptotically or touch it at the downstream node of separation. Note that tangent wall streamlines occur at any node (figure 5a), so that there is nothing special about that.

As the wall streamlines approaching the line between a saddle and a node, only touch it at tbe downstream node, it is in fact not clear why strong flow convergence should necessarily take place along such a wall Streamline. In practice strong flow convergence may be likely between a saddle of separation and a node of separation, as the flow is directed away from the smface at both singular points, stil1 it is not evident that it should occur.

If the definition is to comprise both 'separation lines' described above, than only a definition based on physical features seems possible. This leads to defining separation lines as lines along which the rate of growth of the boundary layer becomes large, say $d\delta/ds = O(1)$ or larger, or along which the shear layer departs from the surface (see e.g. Bradshaw, 1978, who includes in the discussion separation in unsteady flows, where similar definition problems occur). The definition is, of course, in principle an unprecise one. Fortunately in practice the definition is less vague than one might expect, as the lines of convergence in three-dimensional flows often have such a distinct character.

Concluding remarks

The concept 'separated flow', which has such a clear meaning in two-diiensional flows, is less well defined in three-dimensional flows. The concept has to be adapted essentially for the more genera1 three-dimensional case. Separation, in the sense that streamlines are leaving the smface, occurs only at singular points. A precise definition of separation lines is not possible, at least if the concept of separation is not exclusively connected to the smface topology, but also to features of the flow away from the smface. A more vague definition, based on a large growth of tbe boundary layer, as given in the preceding section, seems most practical.

The expression 'separation region' bas no significance in three-dimensional flows. The reason is that closed separation regions, isolated from the rest of the flow, do not exist as a rule in three dimensions. Consequently a specific region comprising the separated flow can not be defined. One can only identify viscous regions, which have a large thickness locally and can be viewed as separated viscous shear layers developing in the surrounding inviscid flow, often into vortical structures. Such a separated flow structure is well known on slender wings, bot it occurs also, perhaps in a less clear way, on bluff bodies. Of course, in nearly axisymmetric flow the conditions may in practice resemble those in a closed separation, bot strictly the above statement remains true in all flows not precisely axisymmetric or otherwise quasi-two-dimensional.

It will be evident from the foregoing that two-dimensional separations, with a distinct separation line and a closed separation region, are very special cases of separation. In view of this, one may ask what is the use of the many experimental investigations on two-dimensional separations, while these do not occur normally in reality and while their features are so exceptional. Such a feature is, for instance, the presence of "dead-air regions" with little or no flow occutring. This is in contrast with three-dimensional separations, where viscous shear layers away from the surface are mixing with inviscid external flow and where no low-velocity regions are present normally. The separated viscous shear layers generally develop into three-dimensional vortical flows, which are known to have very specific turbulence properties. Consequently empirical turbulence data obtained in two-dimensional separated flows have little relevance for three-dimensional flows.

Separation of an infinite swept wing flow, discussed in the introduction, is one of the few cases where some similarity with two-dimensional separation seems to exist, but the similarity is largely apparent rather than real. Instead of a 'dead-air region', a new boundary layer develops parallel to the separation line in the infinite swept wing case, which means that the separated flow regions are not similar at all. What holds for experiments is true also for calculations. It can be questioned how useful the development of good turbulence models for two-dimensional separated flows is, if these flows are quite different from three-dimensional separated flows, and if the latter are the flows which occur in practice.

To conclude, the main purpose of this paper has been to emphasize the essential differences between separations in two-dimensional and three-dimensional flows and to question whether, considering that the fact that real flows are three-dimensional, two-dimensional separated flows deserve the attention which they seem to have had up to now in the research.

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