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ABSTRACT
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Identification of the geometry of accelerometers in an arrangement

J.P.B. Vreeburg

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IDENTIFICATION OF THE GEOMETRY OF ACCELEROMETERS IN AN ARRANGEMENT

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Consider a linear accelerometer on a moving frame in a gravity field. Its geometry is described by the location of the seismic point on the frame, **R**, and the direction of its sensitive axis **s** . Ideal measurements give the ambient field component along **s** . The field strength, a vector quantity, at the seismic point is represented by **a** + **A.R** , where the components of vector **a** (3) and of matrix **A** (9) vary with time. Quantities **a** and **A** can be related to vector and tensor properties of the frame dynamics and the gravity field. The field on a free-floating rigid body under no forces can be described analytically, as a function of inertial properties and angular momentum. Measurements of this field can be used to resolve unknowns in the parameters that occur in its description. Equations are presented that relate the field to the geometrical properties of the accelerometer. Suggestions for applications, including uses on ISS, are given. Desirable is a portable instrument with high-quality accelerometers (gradiometer), and a free-float volume on ISS.

1. Introduction

The Wet Satellite Model test article that was launched with the MASER 5 sounding rocket carried a motion sensing instrument named 'ballistometer' [1]. It is an arrangement of 9 linear accelerometers the output of which can be processed to yield the linear and angular accelerations, and the angular rate of the arrangement.
Similar instruments are used on earth for the diagnosis of motion, e.g. in biodynamics.
Other arrangements of accelerometers constitute a gradiometer, an instrument for the determination of a gravity field [2]. Although the accelerometers (sensitivity in pg) in a gradiometer are very different from those in a motion sensing instrument, the basic theory is the same. The field that is measured can be represented by a vector **v** + [N]**R** , where **R** , a location, is different for the sensors in the arrangement. The anti-symmetric part of 3 x 3 matrix [N] is the skew-symmetric matrix formed from the angular acceleration of the arrangement; the symmetric part consists of terms with components of the gravity field gradient and the angular rate of the arrangement. The geometry of a single linear accelerometer is given by the three components of its location **R**, plus two that specify its sensitive direction. The output from an ideal accelerometer is then a function of 12 + 5 = 17 quantities. If the field is known, a sequence of different measurements can be processed to recover the geometric parameters. Evidently, field knowledge is relative; the data of interest in gravity field measurements are often orders of magnitude smaller than those in motion studies (and then neglected).

Either type of data may need to be determined on the International Space Station (ISS). With high performance accelerometers[3] available, one may envisage use of such to help determine the field that is to be measured by sensors of lower quality for calibration purposes, including the identification of their geometry. Generation of a motion field can be accomplished by any method; the difficulty is to achieve accuracy. The tidal base field on the ISS structure is expected to be of μg order [4] and higher magnitudes occur at higher frequencies. Rejection of the high frequency disturbances is accomplished by an isolation mount, essentially a free-floating rigid frame maintained at a central position in its enclosure. If the enclosure were large, the frame motion would not be restrained so much and, for extended periods, it could be under gravity forces only. Analytical solutions exist for the field at any location on the frame as a function of inertial properties and angular momentum. Now measurement data can be used to determine the geometry of the sensors on the body, (some of) its inertial properties[5] , or the angular momentum, depending on apriori information.
Conversely, if the geometry of an arrangement of accelerometers were known, as in a gradiometer, its data can be processed to recover the field parameters. The paper presents the equations that result from an arbitrary arrangement, not just the highly regular configurations that are so expensive to realise. It will be shown that the core of the problem becomes the determination of a vector from the values of three quadratic forms with this vector. Questions of sensitivity and accuracy, although crucial for viable instrumentation, have not been addressed.
On ISS there will be several systems of accurate accelerometers. The proper combination of data from these systems may already yield useful information on ISS field, motion and inertial properties.

2. Nomenclature

bold character, e.g. $\boldsymbol{\varepsilon} = [\ \varepsilon_1\ \varepsilon_2\ \varepsilon_3\ ]^T$ = column vector, also specifies a diagonal matrix, e.g. diag(**1**) = [1] = identity matrix

$$[A] = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{12} & A_{22} & A_{23} \\ A_{13} & A_{23} & A_{33} \end{bmatrix} = \text{symmetric matrix}$$

R = R**r** location vector
r, **s**, **t** = unit vectors
ρ = reference location
σ = acceleration field vector
Ω = rotation rate vector
ϕ = body force field potential

$\Delta\psi = \psi_0 - \psi$ = true ψ – known ψ

$$\Box\psi = \begin{bmatrix} \psi_{xx} & \psi_{xy} & \psi_{xz} \\ \psi_{xy} & \psi_{yy} & \psi_{yz} \\ \psi_{xz} & \psi_{yz} & \psi_{zz} \end{bmatrix} \text{, symmetric}$$

$$\{\boldsymbol{\Omega}\} = \begin{bmatrix} 0 & -\Omega_3 & \Omega_2 \\ \Omega_3 & 0 & -\Omega_1 \\ -\Omega_2 & \Omega_1 & 0 \end{bmatrix} \text{, skew-symmetric}$$

$$d\mathbf{u}/dt = \mathbf{u}' + \{\boldsymbol{\Omega}\}\mathbf{u} = \mathbf{u}' + \boldsymbol{\Omega} \times \mathbf{u}$$

$\mathbf{u}'$ = rate of change of **u** in a system rotating with **Ω**

3. Acceleration field on a moving base

In space gravity is represented by a potential field ϕ that can be expanded about a reference location **ρ** via the Taylor expansion formula, viz.

$$\phi(\boldsymbol{\rho}+\mathbf{R}) = \phi(\boldsymbol{\rho}) + (\mathbf{R}.\nabla)\phi_{/\boldsymbol{\rho}} + \tfrac{1}{2}(\mathbf{R}.\nabla)^2\phi_{/\boldsymbol{\rho}} + \ ---\ \quad (1)$$

If ϕ has considerable amplitude only for spatial wavelengths > **R** the higher terms in the expansion can be neglected. In the sequel this assumption is made in order to reduce the complexity of the formulas. The subscript $/\boldsymbol{\rho}$ means that the value is to be taken at location **ρ**. The second-order term in the expansion can be written as a quadratic form with the tensor of second derivatives of ϕ at location **ρ**, i.e.

$$(\mathbf{R}.\nabla)^2\phi_{/\boldsymbol{\rho}} = \mathbf{R}.\ \Box\phi_{/\boldsymbol{\rho}}\ \mathbf{R}$$

Note that for harmonic fields the trace of □ϕ is zero, and for point mass gravity potential, two eigenvalues are equal.
The field strength at location **R** is obtained by taking the gradient (in **R**) of (1) :

$$\nabla\phi(\boldsymbol{\rho}+\mathbf{R}) = \nabla\phi_{/\boldsymbol{\rho}+\mathbf{R}} = \nabla\phi_{/\boldsymbol{\rho}} + \Box\phi_{/\boldsymbol{\rho}}\ \mathbf{R} \quad (2)$$

i.e. a linear vector function of **R** . It is observed that the $/\boldsymbol{\rho}$ terms number nine (eight when accounting for the zero trace) different scalar coefficients, time-dependent in general. The determination of their values is the objective of (space) gravimetry.

Take location **ρ** as the origin of a co-ordinate system that has rotation rate **Ω**. The acceleration at location **R** in this co-ordinate system is composed of the body force field and the field induced by the motion[6] :

$$\boldsymbol{\sigma}(\mathbf{R}) = d^2\boldsymbol{\rho}/dt^2 + \nabla\phi_{/\boldsymbol{\rho}} + (\ \Box\phi_{/\boldsymbol{\rho}} + \{\boldsymbol{\Omega}'\} + \{\boldsymbol{\Omega}\}^2\ )\mathbf{R} + $$
$$+\ 2\{\boldsymbol{\Omega}\}\ \mathbf{R}' + \mathbf{R}'' \quad (3)$$

The last two terms, the Coriolis and the relative accelerations, are zero at locations that move with the system and so, $\boldsymbol{\sigma}(\mathbf{0}) = d^2\boldsymbol{\rho}/dt^2 + \nabla\phi_{/\boldsymbol{\rho}}$. If the reference location is at the center of mass of a physical system that moves freely in the body force field : $\boldsymbol{\sigma}(\mathbf{0}) = \mathbf{0}$.

4. Accelerometer data

A linear acceleration sensor measures at its seismic point **R** the component of the field along the sensitive direction **s** , i.e. the ideal output is $p_o = \mathbf{s}\ .\ \boldsymbol{\sigma}(\mathbf{R})$. Real sensors modify the ideal output by scale factor (sf) , bias and noise terms, and have an actual output p_a :

$$p_a = sf.\ p_o + bias + noise$$

The determination and modeling of sf, bias and noise is a discipline all by itself and will not be addressed. In the sequel p_a is taken p_o . The type of accelerometer is relevant for the terms that contribute to p_o . For rebalanced types, the location of the proof mass (the 'seismic' point) is held stationary by a control circuit. Consequently, with the reference point fixed with respect to the accelerometer housing, the Coriolis and the relative acceleration terms can be neglected to noise. Other types use (vibration) control to create known values of **R′** and **R″** .

The quantities that make up p_o are only known approximately. In the sequel, the true value of each will be indicated by subscript 0 , e.g. $\mathbf{R}_0$ represents the true location of the seismic point while **R** gives the known location. So, the output of an ideal, rebalanced linear accelerometer could be given by an equation

$$p_o = \mathbf{s}_0\ .\ \boldsymbol{\sigma}_0(\mathbf{R}_0) = \mathbf{s}_0\ .\ \boldsymbol{\sigma}(\mathbf{0})_0 + \mathbf{s}_0\ .\ (\ [A]_0 + \{\boldsymbol{\Omega}'\}_0\ )\ \mathbf{R}_0$$

where $[A] = \Box\phi_{/\boldsymbol{\rho}} + \{\boldsymbol{\Omega}\}^2$, a symmetric matrix.

The predicted output of this accelerometer, based on the knowledge of fields and vectors **R** and **s** , is :

$$p = \mathbf{s}\ .\ \boldsymbol{\sigma}(\mathbf{0}) + \mathbf{s}\ .\ (\ [A] + \{\boldsymbol{\Omega}'\}\ )\ \mathbf{R} \quad (4)$$

5. Error equations

The error terms for an ideal accelerometer are obtained from $\Delta\,[\mathbf{s}\,.\,\boldsymbol{\sigma}(\mathbf{R})] = \mathbf{s}_0\,.\,\boldsymbol{\sigma}_0(\mathbf{R}_0) - \mathbf{s}\,.\,\boldsymbol{\sigma}(\mathbf{R}) =$

$$(\mathbf{s} + \Delta\mathbf{s}).[\boldsymbol{\sigma}(\mathbf{R} + \Delta\mathbf{R}) + \Delta\,\boldsymbol{\sigma}(\mathbf{R}_0)] - \mathbf{s}\,.\boldsymbol{\sigma}(\mathbf{R})$$

$$\Delta\boldsymbol{\sigma}(\mathbf{0}) = \Delta d^2\boldsymbol{\rho}/dt^2 + \Delta\nabla\phi_{/\rho 0} + \Box\phi_{/\rho}\ \Delta\boldsymbol{\rho}$$

$$\Delta[A] = \{\boldsymbol{\Omega}\}\,\{\Delta\boldsymbol{\Omega}\} + \{\Delta\boldsymbol{\Omega}\}\{\boldsymbol{\Omega}\}$$

$$\Delta\,\boldsymbol{\sigma}(\mathbf{R}_0) = \Delta\boldsymbol{\sigma}(\mathbf{0}) + (\,\Delta[A] + \{\Delta\boldsymbol{\Omega}'\})\,\mathbf{R}_0 + 2\{\Delta\boldsymbol{\Omega}\}\mathbf{R}_0' + 2\{\boldsymbol{\Omega}\}\,\Delta\mathbf{R}_0' + \Delta\,\mathbf{R}_0''$$

for rebalanced type sensors the last three terms in the expression for $\Delta\,\boldsymbol{\sigma}(\mathbf{R}_0)$ should be negligible.

One finds for accelerometers of the rebalanced type :

$$p_o = \mathbf{s}_0\,.\,\boldsymbol{\sigma}_0(\mathbf{R}_0) = \mathbf{s}\,.\,\boldsymbol{\sigma}(\mathbf{R}) + \Delta\,[\mathbf{s}\,.\,\boldsymbol{\sigma}(\mathbf{R})] =$$

$$p + \Delta\mathbf{s}.[\boldsymbol{\sigma}(\mathbf{0}) + ([A]+\{\boldsymbol{\Omega}'\})\mathbf{R}] + \mathbf{s}.[([A]+\{\boldsymbol{\Omega}'\})\Delta\mathbf{R} + \Delta\mathbf{f}]$$

$$= \mathbf{s}_0.\,\boldsymbol{\sigma}(\mathbf{0}) + \mathbf{s}_0.\,([A] + \{\boldsymbol{\Omega}'\})\,\mathbf{R}_0 + \mathbf{s}.\,\Delta\mathbf{f} \qquad (5)$$

$$\Delta\mathbf{f} = \Delta\boldsymbol{\sigma}(\mathbf{0}) + (\Delta[A] + \{\Delta\boldsymbol{\Omega}'\})\mathbf{R} + 2\{\boldsymbol{\Omega}\}\mathbf{R}' + \mathbf{R}'' \qquad (6)$$

The field errors are collected in $\Delta\mathbf{f}$, and include the Coriolis and relative accelerations. The equation is accurate down to errors not much smaller than the magnitude of $\Box\phi_{/\rho}\ \Delta\boldsymbol{\rho}$ (or higher order field terms must be considered).

6. Sensor geometry components

Suppose that the field is known exactly, or, which comes to the same, much more accurately than the geometry of the accelerometer, or: $\Delta\mathbf{f} = \mathbf{0}$ in (5). A candidate would be the field from the free (tumbling) motion of an invariable body with precisely known inertial properties and angular momentum. It is then possible to recover the geometrical data $\mathbf{R}_0$ and $\mathbf{s}_0$ by processing the sensor output p_o . The procedure is shown for a rebalanced accelerometer and it is assumed that a reasonable initial estimate of the geometry is available.

For a suitable choice of origin one can make **r** and **s** not nearly parallel, and define a vector: $\mathbf{t}\sin\varphi = \{\,\mathbf{s}\,\}\,\mathbf{r}$ The orthogonal triad **s** , **t** and **{s} t** is used to make orthogonal matrix [Q] , viz.

$$[Q] = [\,\mathbf{s}\quad \mathbf{t}\quad \{\mathbf{s}\}\,\mathbf{t}\,] \qquad (7)$$

Similarly, $[Q_0] = [\,\mathbf{s}_0\quad \mathbf{t}_0\quad \{\mathbf{s}_0\}\,\mathbf{t}_0\,] = [Q]\,[U]$, where orthogonal matrix $[U] = [1] - \{\boldsymbol{\varepsilon}\} + \{\boldsymbol{\varepsilon}\}^2/(1+\cos\alpha) \approx [1] - \{\boldsymbol{\varepsilon}\}$, for $\sin\alpha = \sqrt{\boldsymbol{\varepsilon}.\,\boldsymbol{\varepsilon}}$, and $\boldsymbol{\varepsilon}.\,\boldsymbol{\varepsilon} \ll 1$ from the assumption of approximate knowledge. Then,

$$\mathbf{s}_0\,.\,\boldsymbol{\sigma}(\mathbf{0}) = [1\ 0\ 0]\,[Q_0]^T\,\boldsymbol{\sigma}(\mathbf{0}) = [1\ 0\ 0]\,[U]^T\,[Q]^T\,\boldsymbol{\sigma}(\mathbf{0})$$

$$\approx \mathbf{s}\,.\,\boldsymbol{\sigma}(\mathbf{0}) - \varepsilon_3\,\mathbf{t}\,.\,\boldsymbol{\sigma}(\mathbf{0}) + \varepsilon_2\,\mathbf{s}\,.\,\{\mathbf{t}\}\,\boldsymbol{\sigma}(\mathbf{0}) =$$

$$= \sigma(0)_1 - \varepsilon_3\,\sigma(0)_2 + \varepsilon_2\,\sigma(0)_3 \qquad (8)$$

The subscripts 1, 2 and 3 on σ(0) refer to components along **s** , **t** , and **{s} t** respectively

Term $\mathbf{s}_0\,.\,(\,[A] + \{\boldsymbol{\Omega}'\}\,)\,\mathbf{R}_0$ is treated likewise, and the result from (5) becomes, to first order :

$$\Delta p = p_o - p = [\,\Delta R\quad \Delta\varphi\quad \varepsilon_1\quad \varepsilon_2\quad \varepsilon_3\,]\,.$$

$$\begin{bmatrix} -\sin\varphi\,(\Omega'_2 + A_{13}) + \cos\varphi\,A_{11} \\ -R\{\cos\varphi\,(\Omega'_2 + A_{13}) + \sin\varphi\,A_{11}\} \\ R\sin\varphi\,(\Omega'_3 - A_{12}) \\ \sigma(0)_3 + R\{2\cos\varphi\,A_{13} + \sin\varphi\,(A_{11} - A_{33})\} \\ -\sigma(0)_2 - R\{\,\sin\varphi\,(\Omega'_1 - A_{23}) + 2\cos\varphi\,A_{12}\} \end{bmatrix} \qquad (9)$$

If the column vector in (9) is denoted **V**, a number of measurements 1,2,….,N yields the system:

$$[\Delta p_1\ \ \Delta p_2\ \ \text{- - -}\ \ \Delta p_N\,] =$$

$$[\,\Delta R\quad \Delta\varphi\quad \varepsilon_1\quad \varepsilon_2\quad \varepsilon_3\,]\,.\,[\,\mathbf{V}_1\quad \mathbf{V}_2\quad \text{- - -}\quad \mathbf{V}_N\,]$$

to be solved for the five geometrical error terms[7] . The analysis can be expanded to include other error terms with fixed value, e.g. parameters for bias or noise.

7. Field determination

The output of a sufficient number of accelerometers in a known configuration allows to determine the field components. A proposed algorithm requires [A] to be written as $\pm\,\{\mathbf{a}\}^2 + \mathrm{diag}(\mathbf{d})$. Particulars of this decomposition are explained in the Appendix. Any orientation of the base vectors of [A] can be chosen, i.e. write (4) :

$$p = \mathbf{s}.[V]^T[V]\boldsymbol{\sigma}(\mathbf{0}) + \mathbf{s}.[V]^T[V]([A]+\{\boldsymbol{\Omega}'\})[V]^T[V]\mathbf{R}$$

where [V] is an orthogonal matrix. Selection of [V], to achieve a desired $[V][A][V]^T$, requires that at least an approximate value of [A] is known. E.g. [V] could be selected such that the components of **a** after transformation have a certain magnitude. If [A] is calculated in a sequence, each new value allows to reset [V]. Evidently, the basis for the accelerometer geometry vectors **s** and **R** rotates with [V]. For convenience of notation [V] will not be shown explicitly; the transformation is assumed performed.

With vector $\mathbf{1}_{sR}$ defined as [$s_1 R_1 \quad s_2 R_2 \quad s_3 R_3$]^T :

$$p \pm \mathbf{s} .\{\mathbf{a}\}^2 \mathbf{R} = [\, \mathbf{s}^T \quad \mathbf{R}.\{\mathbf{s}\} \quad \mathbf{1}_{sR}^T \,] \,.\, \begin{bmatrix} \boldsymbol{\sigma}(\mathbf{0}) \\ \boldsymbol{\Omega}' \\ \mathbf{d} \end{bmatrix} \qquad (10)$$

i.e. the right hand side is written as the inner product of a 9-component sensor geometry vector with a field vector of 9 components. It is noted that for $\mathbf{s} . \mathbf{R} = 0$ a base vector along $\mathbf{s}$ or $\mathbf{R}$ will null $\mathbf{1}_{sR}$ which makes possible (3-D) arrangements that cannot measure $\mathbf{d}$ and that reduce the vectors to 6 components. Opportunities for reduction exist also in the fact that the angular acceleration $\boldsymbol{\Omega}'$ can be integrated to the angular rate $\boldsymbol{\Omega}$, but these issues will not be addressed.

$2\, \mathbf{s} .\{\mathbf{a}\}^2 \mathbf{R} = \mathbf{a}. (\{\mathbf{s}\}\{\mathbf{R}\} + \{\mathbf{R}\}\{\mathbf{s}\})\, \mathbf{a}$, a quadratic form in $\mathbf{a}$ and so N of (10) can be assembled into :

$$\begin{bmatrix} p_1 \pm \tfrac{1}{2}\mathbf{a}. (\{\mathbf{s}\}\{\mathbf{R}\} + \{\mathbf{R}\}\{\mathbf{s}\})_1\, \mathbf{a} \\ p_2 \pm \tfrac{1}{2}\mathbf{a}. (\{\mathbf{s}\}\{\mathbf{R}\} + \{\mathbf{R}\}\{\mathbf{s}\})_2\, \mathbf{a} \\ | \\ | \\ p_N \pm \tfrac{1}{2}\mathbf{a}. (\{\mathbf{s}\}\{\mathbf{R}\} + \{\mathbf{R}\}\{\mathbf{s}\})_N\, \mathbf{a} \end{bmatrix}_{N \times 1} =$$

$$= \begin{bmatrix} \mathbf{s}_1^T & \mathbf{R}.\{\mathbf{s}\}_1 & \mathbf{1}_{sR\,1}^T \\ \mathbf{s}_2^T & \mathbf{R}.\{\mathbf{s}\}_2 & \mathbf{1}_{sR\,2}^T \\ | & | & | \\ | & | & | \\ \mathbf{s}_N^T & \mathbf{R}.\{\mathbf{s}\}_N & \mathbf{1}_{sR\,N}^T \end{bmatrix}_{N \times 9} \begin{bmatrix} \boldsymbol{\sigma}(\mathbf{0}) \\ \boldsymbol{\Omega}' \\ \mathbf{d} \end{bmatrix}_{9 \times 1} \qquad (11)$$

Via singular value decomposition of the N x 9 array of geometry vectors, the system can be reduced to N – 9 equations for $\mathbf{a}$ only, each like $\mathbf{a}.$ [M] $\mathbf{a}$ = P with [M] and P linear combinations of ($\{\mathbf{s}\}\{\mathbf{R}\} + \{\mathbf{R}\}\{\mathbf{s}\}$)$_i$ and p_i respectively. If $N{-}9 \geq 3$, a solution for $\mathbf{a}$ can be calculated but particular difficulties appear[7] . The singular value decomposition works for any geometry array and so it is not necessary to align the sensors precisely; it suffices that the geometry is known. Observe that unless all $\mathbf{R}.\{\mathbf{s}\}_i = \mathbf{0}$, [M] is not simply transformed with [V] because of the vectors $\mathbf{1}_{sR}$.

Various special cases can be defined and analyzed using (10). For example consider two accelerometers with parallel sensitive directions, i.e. $\mathbf{s}_1 = \mathbf{s}_2$. Then, if the origin of the system is chosen at the midpoint between the seismic points, one has $\mathbf{R}_1 = -\mathbf{R}_2$, and addition of output gives: $p_1 + p_2 = 2\, \mathbf{s}_1 . \boldsymbol{\sigma}(\mathbf{0})$. Two aligned tri-axial sensors furnish all components of $\boldsymbol{\sigma}(\mathbf{0})$ and via (anti-) parallel displacement of a single tri-axial accelerometer the same result can be achieved. Such data are generated also by putting $\mathbf{R} = \mathbf{0}$, so the only advantage appears to be that the measurement location is not occupied by the instrument. On ISS, a portable instrument with this performance could be used to map the field $\boldsymbol{\sigma}(\mathbf{0})$ [4].

8. The $\sigma(0)$ field

Denote by C_0 the origin $\boldsymbol{\rho} = \mathbf{0}$ of the co-ordinate system for location vector $\boldsymbol{\rho}$. If C_0 is the location where $\boldsymbol{\sigma}(\mathbf{0}) = \mathbf{0}$ then $d^2\boldsymbol{\rho}/dt^2{}_{/\boldsymbol{\rho}=\mathbf{0}} + \nabla\phi_{/\boldsymbol{\rho}=\mathbf{0}} = \mathbf{0}$. Suppose that such C_0 is a certain point on ISS and define co-ordinate axes that move with ISS. Considering that ISS is not rigid, a point $\boldsymbol{\rho}$ on the ISS structure will vary periodically with time, typically with an ISS eigenfrequency. If one defines $\boldsymbol{\delta}g = \nabla\phi_{/\boldsymbol{\rho}} - \nabla\phi_{/\boldsymbol{\rho}=\mathbf{0}}$, the gravity gradient term, then :

$$\boldsymbol{\sigma}(\mathbf{0})_{/\boldsymbol{\rho}} = \boldsymbol{\rho}'' + 2\{\boldsymbol{\varpi}\}\, \boldsymbol{\rho}' + (\{\boldsymbol{\varpi}'\} + \{\boldsymbol{\varpi}\}^2)\boldsymbol{\rho} + \boldsymbol{\delta}g \qquad (12)$$

where $\boldsymbol{\varpi}$ is the angular rate of the ISS coordinate frame

If $\boldsymbol{\sigma}(\mathbf{0})$ is measured as a time series, (12) can become a fully known differential equation for $\boldsymbol{\rho}$ if the $\boldsymbol{\delta}g$ term is taken to be constant and the rate $\boldsymbol{\varpi}$ is known. If three locations $\boldsymbol{\rho}$ can be solved, C_0 can be constructed.

9. Applications

An all-accelerometer motion sensing system as the ballistometer has lower performance than an equally expensive system that includes gyroscopes. However the specific features of a ballistometer are attractive for back-up applications. Accelerometers of sufficient quality are available in chip technology and so can be accommodated with little impact on any system. On a spacecraft, distributed accelerometer packages would need to be provided with power and data connections. During nominal spacecraft operations the spacecraft motion can be reconstituted and processed for the identification of the geometry of the back-up ballistometer. After this has been accomplished, the instrument output gives intelligible motion information, useful when the regular system fails, or as an independent safety provision. The latter feature is relevant for service vehicles about ISS in order to diagnose inadvertent hazardous momentum build-up.

On ISS, accurate accelerometers will be installed at various locations. Selected (Fourier) components of the field are considered known and may thus serve to identify the geometry of the arrangement of these accelerometers. Then the ISS center of mass can be determined and tracked. Investment in a portable instrument opens wider options. The ambient ISS field could be mapped but also constructed motion fields can be diagnosed. Such constructed field, e.g. on a rotary device, serves to calibrate accelerometers for ISS or spacecraft use. Fields on a free-floating rigid frame have desirable properties, and a portable instrument carried by the frame can identify disturbances from airdrag and ambient gravity field. The dynamic range of a sensor is limited, therefore the motion field magnitude will be, for given sensitivity. Nevertheless, a gravity laboratory on ISS, being a reference enclosure with a floating gradiometer, could become a viable facility.

Already mentioned as a desirable service is calibration of accelerometers. Another could be the measurement of inertial properties of satellites, i.e. mass, center of mass location, and inertia tensor. It may be convenient to conduct such measurements in free-float and a portable instrument would be indispensable. The non-scalar inertial properties of an astronaut, constrained in a suitable reference posture, provide a valuable means to monitor this person's physical state.

10. Conclusions

Arrangements of accelerometers have been used to measure acceleration fields of very different magnitudes. The compensation of the 1-g terrestrial component by the orbital velocity of ISS results in an enormous increase in the useful dynamic range of the accelerometers. As a consequence, new sensors , optimized for space use, can provide various field data of important practical value. Theory to support the use of arrangements that do not need precision engineering is being developed.

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Appendix

Credited to Kelvin[8] is the decomposition of certain symmetric matrices as **aa** + diag (**d**) = **{a}**² + **a.a** [1] + diag (**d**) . The eigenvalues λ are solved from :

$$a_1^2 /(\lambda - d_1) + a_2^2 /(\lambda - d_2) + a_3^2 /(\lambda - d_3) = 1$$

and determine the eigenvectors :

$$[a_1 /(\lambda - d_1) \quad a_2 /(\lambda - d_2) \quad a_3 /(\lambda - d_3)]^T$$

which is easily verified by substitution. Observe that the sign of the components of **a** do not affect λ .
If two of λ are equal, $d_1 = d_2 = d_3$, and **a** is an eigenvector, as follows immediately from the dyadic decomposition of a symmetric matrix. The eigenvectors normal to **a** are undetermined.

The orthogonal transformation of a diagonal matrix results in a symmetric matrix, hence if diag(**d**) ≠ [1] :

$$[Q]\, \mathrm{diag}(\mathbf{d})\, [Q]^T = \{\mathbf{b}([Q])\}^2 + \mathrm{diag}(\boldsymbol{\mu}([Q])) \qquad (\text{A1})$$

where $[Q]\,[Q]^T = [1]$, $\mathbf{b}([1]) = \mathbf{0}$, and $\boldsymbol{\mu}([1]) = \mathbf{d}$,

for the matrices considered by Kelvin.

Conversely, a symmetric matrix with nonzero off-diagonal terms A_{ij} of which two or zero have negative values, can be written as (A1). The condition on A_{ij} is to be met either by the matrix or by its negative value, and so matrix $[V][A][V]^T \neq [1]$, with a free choice of [V], can generally be decomposed to have nonzero $\pm\{\mathbf{b}\}^2$ terms at off-diagonal.