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## Approximate modelling and multi objective optimisation in aeronautic design

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## **Summary**

An investigation of efficient approximation methods for computationally expensive objective functions in aeronautic multi-disciplinary design and multi objective optimisation is presented. Several approximation methods based on curve fitting using polynomials and artificial neural networks are considered. A comparison of these approximation methods in terms of the achieved quality and accuracy and the required computational cost is presented. The approximation models have been successfully applied in a preliminary design and multi objective optimisation study of a blended wing body aircraft.



## List of acronyms

2-OP	Second order polynomial approximation method
3-OP	Third order polynomial approximation method
3-1-ANN	Three inputs, one output ANN approximation method
3-5-ANN	Three inputs, five outputs ANN approximation method
ANN	Artificial neural network
BWB	Blended wing body
CFD	Computational fluid dynamics
EC	European Commission
EU	European Union
GA	Genetic algorithm
GM	Gradient based optimisation method
ICT	Information and communication technology
MDO	Multi-disciplinary design and optimisation
MOB	Project acronym for EU project: A Computational Design Engine Incorporating Multi-Disciplinary Design and Optimisation for Blended Wing Body Configuration
MOO	Multi objective optimisation
RMSE	Root mean squared error



### List of symbols

$C_d$	Drag coefficient
$C_{dl}$	Aerodynamic performance
$C_l$	Lift coefficient
$M_p$	Pitching moment
$M_{pA}$	Absolute pitching moment ("flight mechanics in-balance")
$M_r$	Roll moment
$M_t$	Total wing moment
$M_y$	Yaw moment
$\alpha$	Angle of attack
$s$	Number of design points in training set
$F_i$	Fitted approximation model
$a$	Approximation method indicator



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## 1 Introduction

Aeronautic design is an area that typically involves a number of different technical disciplines, ref. 1. Some of these disciplines, such as aerodynamics, structural mechanics and flight mechanics, strongly rely on simulation models and large computational analyses. Usually the aeronautic design process includes optimisation analysis of certain design objectives. Such a multi-disciplinary design and optimisation (MDO) process normally leads to multiple, non-linearly related objectives that arise from the different disciplines' analyses results. These objectives can only be properly dealt with when treated separately in a multi objective optimisation (MOO) approach. Such an MOO approach involves highly frequent evaluation of the design objectives, and therefore requires reliable and accurate, but also efficient representations of the objective functions. The objective functions in aeronautic MDO, however, often require computationally expensive design evaluations. Therefore approximation models are used for efficient representation of these objective functions.

This paper presents an investigation of efficient approximation methods for computationally expensive objective functions in aeronautic MDO. Several approximation methods based on different mathematical techniques, such as curve fitting using polynomial functions or artificial neural networks, are considered. A comparison of these approximation methods in terms of the achieved quality and accuracy, and the required computational cost will be presented.

The aeronautic MDO case to which the approximation methods are applied is the design of a blended wing body (BWB) aircraft. The BWB design data and some of the design analysis tools used in this study are taken from the EC supported project MOB, ref. 6, where a detailed design study of the BWB aircraft configuration is performed. In this paper some results of an MOO study of the BWB design, in which the different approximation models have been applied, will be shown.



## 2 Multi objective optimisation and approximate modelling

Today, a variety of optimisation methods, ranging from traditional gradient based optimisation methods (GM) to genetic- or evolutionary algorithms (GA), are widely available, refs.3, 4. Most GM are typically designed for single objective optimisation, while GA are more suitable for multi objective optimisation. There are however certain possibilities to apply constrained GM to multi objective optimisation problems. In general, the advantage of GA is that such algorithms have good global search capabilities, while GM easily get stuck in a local optimum. GM, on the other hand, are generally more efficient than GA in terms of the number of objective functions evaluations that is required for finding an optimum. Still the number of function evaluations in either method for MOO is too large for the computationally expensive design evaluations in aeronautic multi-disciplinary design. For example in the design case considered here, the preliminary design of a BWB aircraft taken from the MOB project, the objective functions are based on results of the CFD simulation of the aerodynamic behaviour of the BWB in cruise flight. These CFD simulations typically take one to several hours of computation on a standard workstation (MIPS R12000), which is too expensive in the case of MOO where the number of objective functions evaluations is in the order of 1000 or higher. Hence a computationally much cheaper approximation model for the objective functions is required.

The approximation models that are evaluated in this study are based on traditional polynomial functions and on artificial neural networks (ANN). The different models are applied to the BWB design case and the quality of the approximation and the efficiency in terms of required input data are compared for the different methods. The approximation models require representative datasets of the “true” objective functions values in the desired design space. These representative datasets are obtained by the design evaluations, i.e., the CFD simulations of the air flow about the BWB, for a number of variants of the BWB. Both the design space, which is spanned by the considered design parameters, and the objective functions space are multi-dimensional, and the approximation model provides a mapping between these two spaces. To fit the approximation model properly to the representative dataset, this dataset is divided into two separate datasets for “fitting” or “training”, and for validation, respectively.

In the MOB project a detailed MDO study, including high fidelity aerodynamics, structural mechanics and flight mechanics analyses and classical single objective response surface optimisation, is conducted on a new BWB aircraft configuration. Besides the MOB project, also a preliminary design study of the BWB, in which somewhat simplified analysis are considered, but where multi objective (instead of single objective) optimisation is applied. The present paper deals with this preliminary design study of the BWB. In this preliminary design study some key properties of



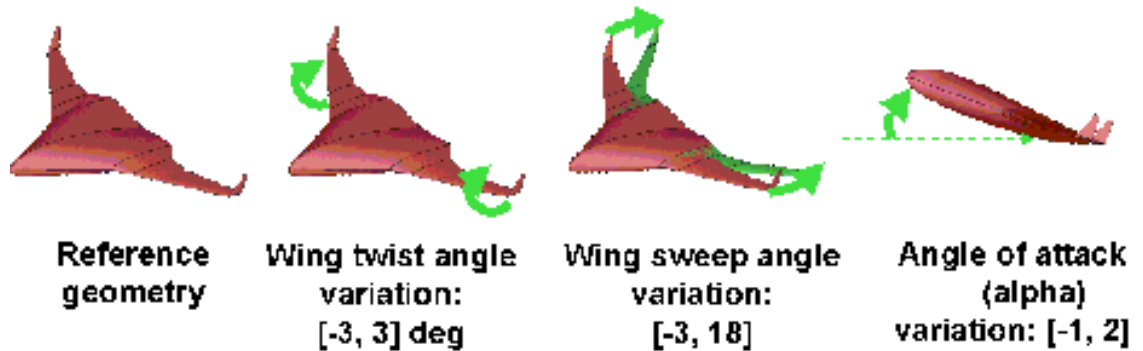


Fig. 1 Illustration of the BWB reference configuration, and the design parameters twist, sweep, and angle alpha used in the BWB preliminary design study.

the BWB in cruise flight are considered: aerodynamic performance ( $C_{dl}$ ) structural mechanical wing loading ( $M_t$ ) and untrimmed pitching moment ( $M_{pA}$ ). These properties are considered as the objective functions in the MOO, and can be derived directly from the design analysis results that come out of the CFD simulation of the BWB at cruise flight by the following equations:

$$C_{dl} = \frac{C_d}{C_l} ; M_t = \sqrt{M_r^2 + M_y^2} ; M_{pA} = |M_p| \quad (1)$$

The design analysis results that are used in these objective functions are the aerodynamic lift and drag coefficients  $C_l$  and  $C_d$ , and the roll, yaw and pitching moments  $M_r$ ,  $M_y$  and  $M_p$ , in the centre of mass of the BWB.

Three design parameters of the BWB have been selected as the design variables in the preliminary design study: wing twist, wing sweep and angle of attack in cruise flight. A parameter study has been conducted in which these design parameters are varied relative to a fixed, pre-defined reference configuration of the BWB, as illustrated in figure 1.

Discrete perturbation values of the three design variables have been used in this parameter study: seven values uniformly distributed in the range  $[-3, 3]$  degrees for twist perturbation, twelve values non-uniformly distributed in the range  $[-3, 18]$  degrees for sweep perturbation, and four values for perturbation of  $\alpha$  :  $[-1, 2]$ . Thus a total number of 336 design variants have been generated for which the design analysis results have been evaluated by CFD simulation. Twelve of these simulations gave unrealistic results and were rejected.



### 3 Approximate models for the BWB and MOO

Because of the high computational cost of the evaluation of the design analysis results ( $C_l$ ,  $C_d$ ,  $M_p$ ,  $M_r$  and  $M_y$ ), and the large number of evaluations required by the MOO process, these design analysis results are approximated by computationally efficient approximation models. The following approximation methods have been applied to the five individual design analysis results: second order and third order polynomial fits (identified as 2-OP and 3-OP, respectively), and a 2-layer perceptron ANN, ref. 2, with 3 inputs, 7 hidden nodes and 1 output (identified as 3-1-ANN). In addition, a fourth approximation method (identified as 3-5-ANN) is used, which is based on a single two-layer perceptron ANN with 3 inputs, 7 hidden nodes and 5 outputs for all the design analysis results simultaneously. The ANNs have all sigmoidal activation functions on the hidden layer and linear activation functions on the output layer. For the second and third order polynomial fits, the “fitting” data subsets must contain at least 10 and 20 data points, respectively, in order to avoid an under-determined fit. In the training of the ANNs, 80 % of the points are used for training and 20 % for validation.

Starting point for the approximation models is the existing data set with 324 data points, which is available from the parameter study with CFD simulations of the BWB in cruise flight. This data set consists of the values of the three design parameters (the inputs) and the five design analysis results (the outputs) in each design point. Both the input and the output data have been scaled to the  $[-1,1]$  range. For the input data this is always easily done when the boundaries of the design parameters space have been fixed. To scale the output data either the minima and maxima of the output data used in the approximation or prior knowledge has to be exploited. In order to do a valid comparison we created here ‘prior knowledge’ by taking the minima and maxima of the full data set (324 data points) to scale the output.

The test procedure is as follows: given a number  $s \in \{10, 20, 30, \dots, 100\}$ , ten sets of  $s$  design points are drawn at random. For each of those ten sets a fit ( $F_i$ ;  $i \in \{1, 2, \dots, 10\}$ ) to the data is computed with each of the four approximation methods ( $a$ ;  $a \in \{1, 2, 3, 4\}$ ). For each these 40 fits ( $F_{i_a}$ ) the root mean squared error ( $RMSE_{i_a}$ ) using all 324 data points of the existing dataset is computed. Then the average  $RMSE$  for the ten sets per approximation method ( $RMSE_a$ ) is computed. In figure 2 the  $RMSE_a$  for each of the five design analysis results for all data points are displayed for the different methods.

Clearly, for very ‘cheap’ data sets, i.e., containing only few (e.g.  $s = 10$  or  $20$ ) data points, and thus requiring only few ‘expensive’ design evaluations,  $RMSE_a$  is of order  $10^0$  or  $10^{-1}$ , i.e., none of the approximation methods is very adequate. The second order polynomial method (2-OP) gives

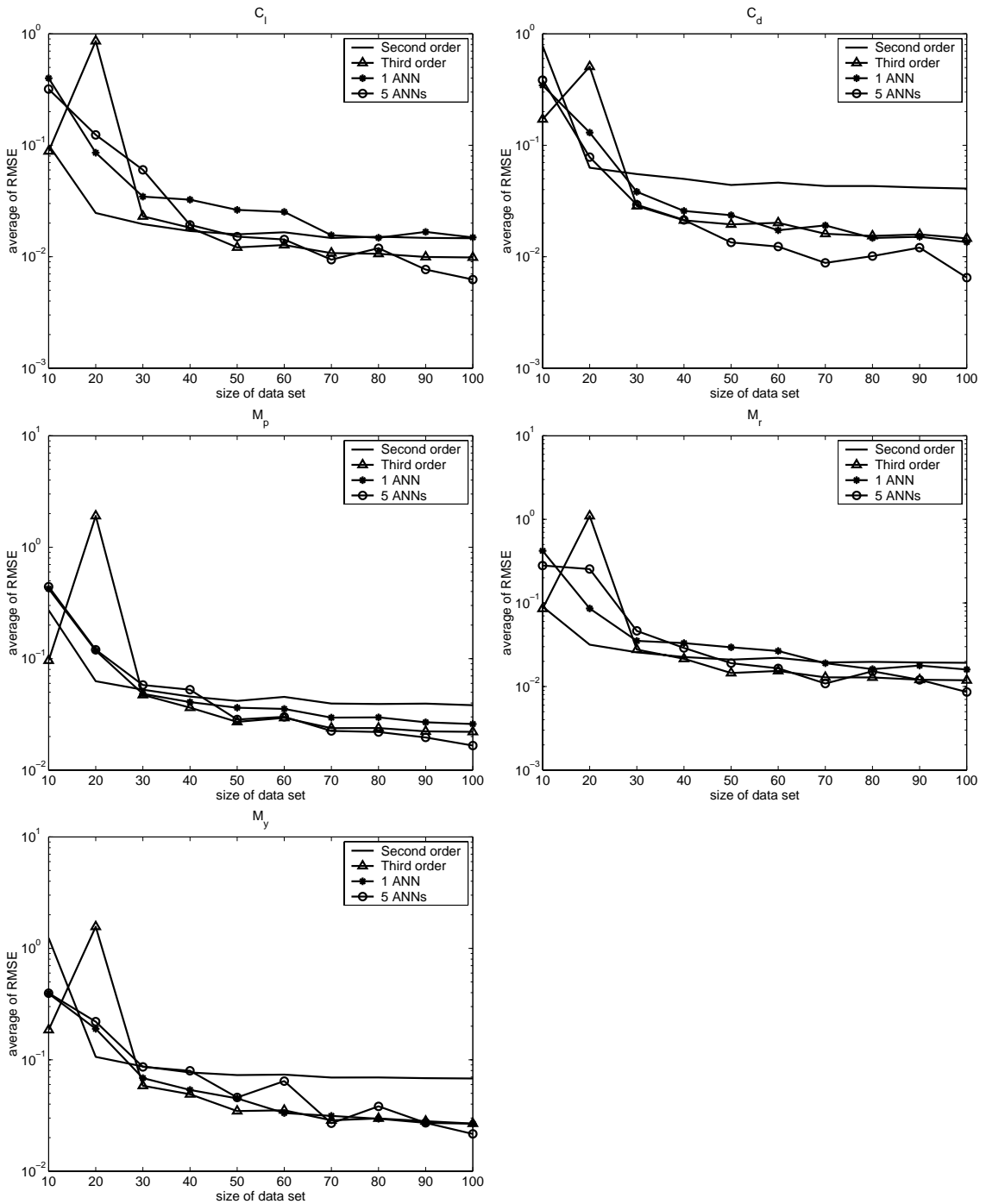


Fig. 2  $RMSE_a$  plots for the four different approximation methods for each of the five analysis results,  $C_l$ ,  $C_d$ ,  $M_p$ ,  $M_r$  and  $M_y$ , respectively.



reasonable results for moderate datasets (e.g.  $s = 20$  or  $30$ ) but is limited in maximum accuracy, even for large data sets. For more accurate approximation, the larger datasets (e.g.  $s > 40$ ) are needed and the 3-1-ANN approximation gives the best results, and seems to improve for growing datasets.



#### **4 MOO of BWB aircraft**

From the dataset available from the BWB parameter study, different approximation models can be generated and applied in the MOO analysis, in which GA and GM based MOO methods can be used, ref. 7. The MOO analysis, which leads to the so-called Pareto front, ref. 3, requires up to thousands of objective functions evaluations, and can be performed within few seconds on a standard PC when the approximation models are used.

In the present study two approximation models have been applied in the MOO analysis: one based on the third order polynomial fit (3-OP), and one on the three-inputs-five-outputs-ANN (3-5-ANN), and both trained with a set of 100 data points from the parameter study dataset. These approximation models are applied in an MOO analysis of the BWB objective functions using a GM based minimax optimisation method, ref. 7. The Pareto fronts found in the MOO analysis using the two approximation models are roughly the same (figure 3).

Furthermore, to qualify the error of both approximations in the points of the Pareto fronts obtained using the two approximation models, the inputs, i.e. the BWB design parameter values, for the points of one front, are filled in in the other approximation model, and vice versa. Moreover, another, more accurate ANN (3-1-ANN) model, which is based on one ANN per design result with three inputs, one output and between 5 and 12 hidden nodes, has also been used to verify the points in the Pareto fronts. The results of all three approximations of both Pareto front points are given in figure 4. The reasonable correspondence in the results of the different approximations indicates that these approximation models are a reasonable representation of the underlying functions of the true design results.

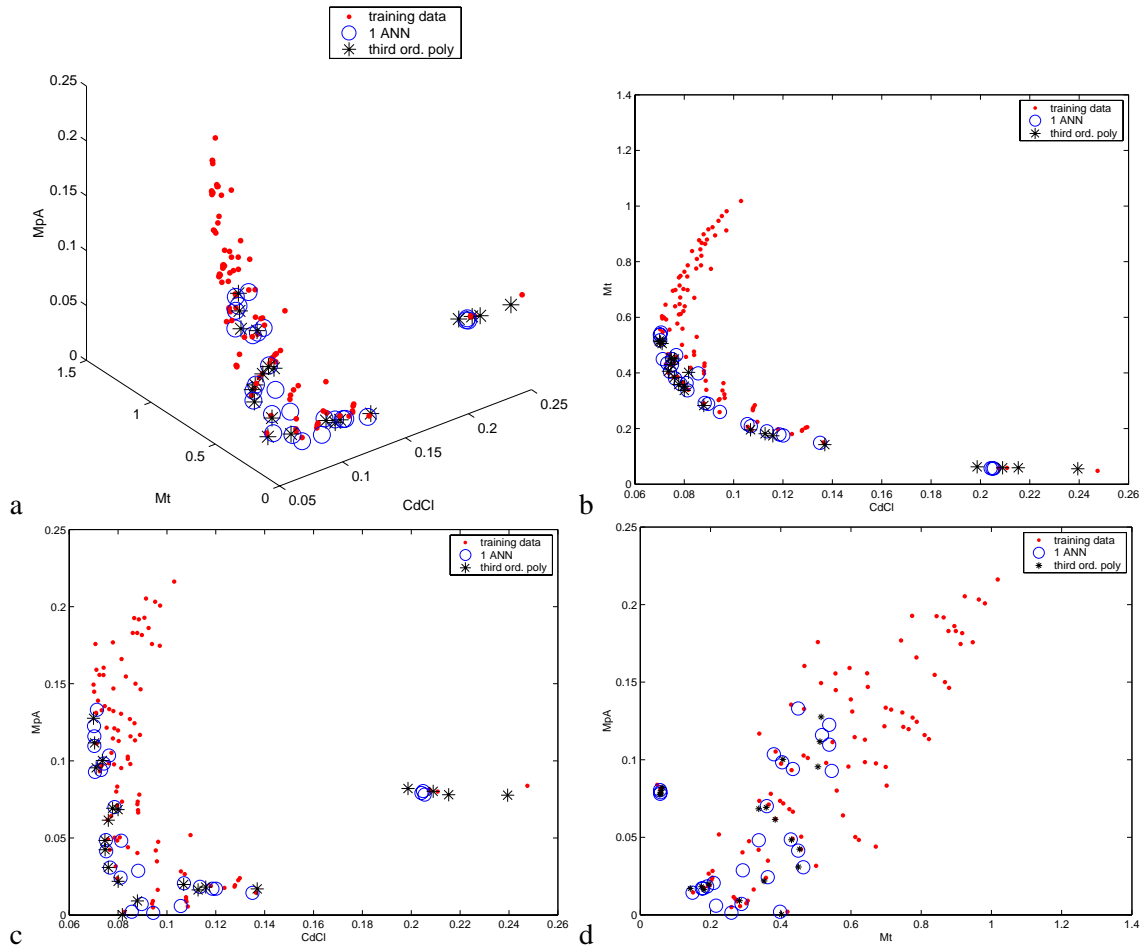


Fig. 3 The Pareto fronts found with the 3-OP and 3-5-ANN approximation models; panel a: plotted in the 3-D space spanned by the 3 objectives  $C_{dI}$ - $M_t$ - $M_{pA}$ ; panels b, c and d: projections of the Pareto fronts in the  $C_{dI}$ - $M_t$ ,  $C_{dI}$ - $M_{pA}$ , and  $M_t$ - $M_{pA}$  planes of the objective space, respectively.

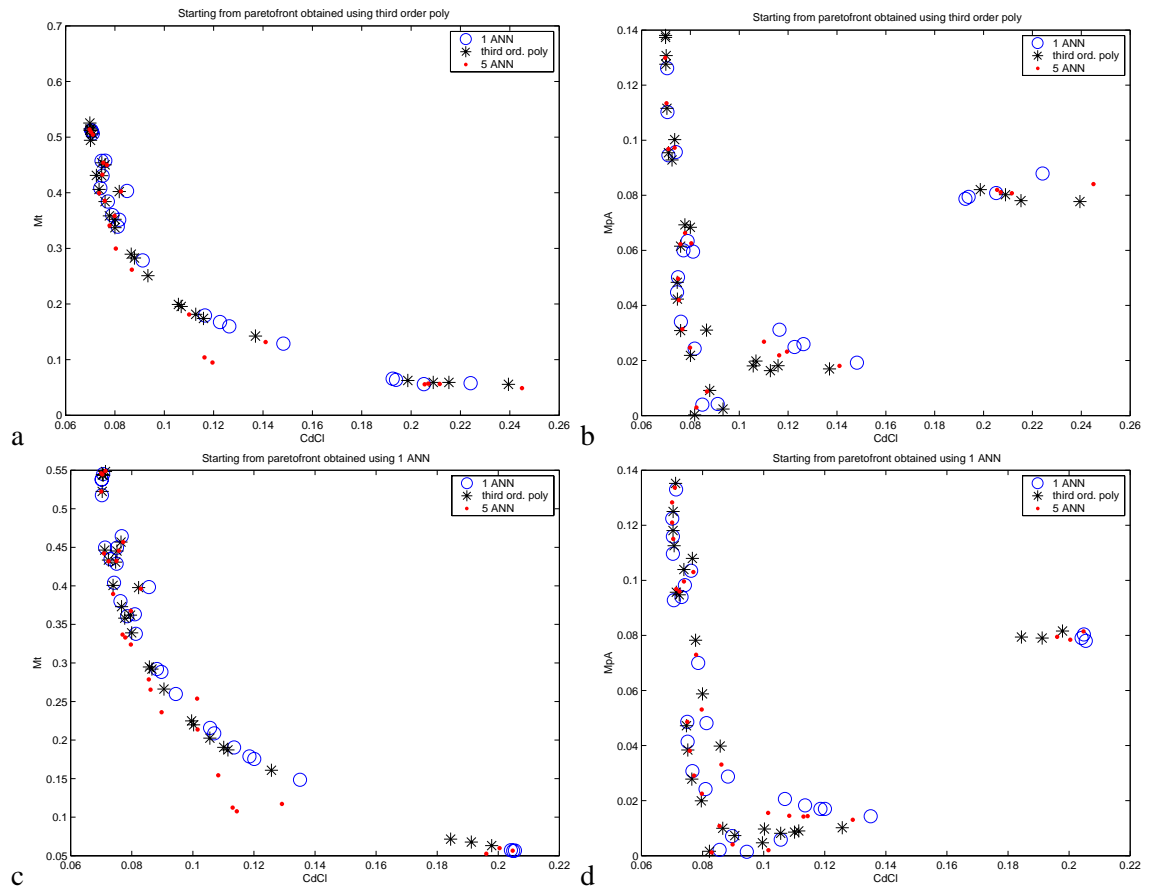


Fig. 4 Comparison of the points in the Pareto fronts found with the 3-OP, 3-5-ANN and 3-1-ANN approximation models; panel a and b: 3-OP Pareto front points also evaluated by 3-5-ANN and 3-1-ANN, plotted in the  $C_{dl}$ - $M_t$  and  $C_{dl}$ - $M_{pA}$  projections of the objective space; panel c and d: 3-5-ANN Pareto front points also evaluated by 3-OP and 3-1-ANN, plotted in the  $C_{dl}$ - $M_t$  and  $C_{dl}$ - $M_{pA}$  projections of the objective space.

## 5 Discussion and conclusions

The present paper shows a comparative study of different approximation models for design objectives in aeronautic MDO. The data sets used in this study are based on results of CFD simulations of a realistic preliminary design case of a BWB aircraft. Polynomial functions of different orders could be fitted to the considered data with reasonable accuracy. This indicates that the underlying functions of the true design results are reasonably smooth.

For higher accuracy of the approximation, ANN based approximation models proved more appropriate, in particular when one ANN for each of the design results was used (the 3-1-ANN model). In the case of less smooth datasets, ANN based approximation models are expected to be more effective, because of the absence of fixed functional behaviour.

The MOO computations, requiring thousands of objective functions evaluations, can be performed with reasonable accuracy within seconds on a standard PC, instead of many hours of computation time if the CFD analysis would be performed directly in the MOO computation.

The design points in the Pareto fronts found from the MOO analyses could be validated by evaluation of the design results by CFD analysis. Furthermore, improvement of the approximation models could then be achieved by incorporating these CFD results into the training dataset. Iterative continuation of this procedure will enhance the approximation in the interesting design areas ever further, while computational effort is kept within reasonable and controllable limits. This is subject of current and near future further investigations.

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