Nationaal Lucht- en Ruimtevaartlaboratorium

National Aerospace Laboratory NLR

Executive summary

The Generalized Extreme Value Statistical Method to Determine the GNSS Integrity Performance

Problem area
The actual development of satellite navigation systems is crucial and includes after the introduction of WAAS and EGNOS (augmentations of GPS), the planned modernization of GPS, and the newly designed European Galileo system. For safety of life applications of satellite navigation systems, in e.g. aviation, reliability in the sense of accuracy, availability and continuity and integrity is essential. In order to test the performance with regard to these characteristics a test methodology is required. This methodology should meet the requirement to be able to perform the analysis based on a limited amount of data collected within an acceptable observation time. This report describes a test method based on the application of the Generalized Extreme Value probability distribution for analyzing the integrity of receiver output data; the most demanding performance parameter.

Description of work
A practical method to analyze the receiver integrity output data has been developed. As a test case the method has been applied to data gathered during a test campaign for EGNOS. With this method an accurate estimate of the integrity can be made. The applied theory is described in this report and is based on the Generalized Extreme Value probability distribution.

Results and conclusions
From the test cases with EGNOS data, as presented in this report, it can be concluded that the estimation of the integrity based on test campaign data is possible indeed. The results of the test case show that the integrity did satisfy the requirements. These results also show that the probability is not Gaussian distributed at the tail. This must be taken into account in order to assess the performance data correctly.

Applicability
The new method can be used to analyze measurement data gathered during test campaigns for SBAS and for the future Galileo satellite navigation receivers. The developed software is used to estimate the integrity performance of a satellite navigation system being an important contribution to overall system reliability estimation.

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The Generalized Extreme Value Statistical Method to Determine the GNSS Integrity Performance

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Summary

For safety of life applications of satellite navigation systems, in e.g. aviation, reliability in the sense of accuracy, availability, continuity and integrity is essential. In order to test the performance with regard to these characteristics a test methodology is required. This methodology should meet the requirement to be able to perform the analysis based on a limited amount of data collected within an acceptable observation time. This paper describes a test method based on the Generalized Extreme Value probability function for analyzing the integrity of receiver output data; the most demanding performance parameter. The method takes into account the fact that generally the probability is not Gaussian distributed at the tail which turns out to be essential in order to estimate the correct integrity performance result.

Contents

Abbreviations

EGNOS	European Geostationary Navigation Overlay Service
GBAS	Ground Based Augmentation System
GEV	Generalized Extreme Value
GNSS	Global Navigation Satellite System
GPS	Global Positioning System
HAL	Horizontal Alert Limit
HMI	Hazardous Misleading Information
HPE	Horizontal Position Error
HPL	Horizontal Protection Level
ICAO	International Civil Aviation Organisation
MI	Misleading Information
NOP	Noord Oost Polder (North East Polder)
RAIM	Receiver Autonomous Integrity Monitoring
SBAS	Space Based Augmentation System
TTA	Time To Alert
VAL	Vertical Alert Limit
VPE	Vertical Position Error
VPL	Vertical Protection Level
XAL	HAL and/or VAL
XPE	HPE and/or VPE
XPL	HPL and/or VPL

The Generalized Extreme Value Statistical Method to Determine the GNSS Integrity Performance

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***Abstract*—For safety of life applications of satellite navigation systems, in e.g. aviation, reliability in the sense of accuracy, availability, continuity and integrity is essential. In order to test the performance with regard to these characteristics a test methodology is required. This methodology should meet the requirement to be able to perform the analysis based on a limited amount of data collected within an acceptable observation time. This paper describes a test method based on the Generalized Extreme Value probability function for analyzing the integrity of receiver output data; the most demanding performance parameter. The method takes into account the fact that generally the probability is not Gaussian distributed at the tail. Knowledge of the distribution at the tail is essential for estimating the correct integrity performance result.**

Keywords-component; SBAS; integrity; statistics;

I. INTRODUCTION

Service reliability of satellite navigation systems in terms of accuracy, integrity, availability and continuity is essential for safety of life applications such as civil aviation. Therefore its performance shall be tested versus the requirements for which a test methodology is required. This methodology shall be SMART (Specific, Measurable, Attainable, Relevant, Time-bound) and as such based on measurement data collected within an acceptable period of time. This paper describes a practical method of testing the integrity of SBAS (Space Based Augmentation System) based on test campaign data. For SBAS the protection level having a relation with the position error is the basis for integrity [1]. The applied method is based on the Generalized Extreme Value probability distribution function [2], [3], [4]. To demonstrate this test methodology Matlab software has been developed within the framework of NLR's internally funded Research and Development Programme on Satellite Navigation.

The GNSS (Global Navigation Satellite System) integrity performance requirement for the ICAO (International Civil Aviation Organisation) navigation service levels APV-I, APV-II and CAT-I is 2 x 10^{-7} per approach. Taking a duration of 150 seconds per approach [5] it turns out that an integrity failure may occur once per 75 x 10^{7} seconds or once per 23.8 years. Testing a GNSS system by collecting data over such a long period is far from practical and still insufficient from a statistical perspective. So one needs to invent a way to do integrity tests based on a limited amount of data to be collected within an acceptable observation time. The test method presented in this paper is based on the determination of the function of the probability of extreme values or block maxima per day. Once this probability distribution is known the integrity risk can be computed. The method is a black-box approach of the problem; therefore no system-internal knowledge of its performance at component level will be required.

The integrity tests for SBAS are based on the determined protection levels as a function of the position errors (the so-called Stanford diagram, e.g. [6], [7] and fig. 1). As a test case the method has been applied to data gathered with a Novatel Millennium receiver during a test campaign of three months at coordinates (lat. 52° 40' 31.51308", lon. 5° 55' 37.54884") at NLR Flevoland collecting GPS (Global Positioning System)/EGNOS (European Geostationary Navigation Overlay Service) data. The surrounding area of this measurement site was free of obstructions. With the method, presented in the paper, the integrity of the EGNOS receiver is determined. Local effects, such as multi-path, have not been studied into detail during this study. Furthermore it must be noted here that a possible effect of the Time To Alert has not been taken into account which means that the estimated integrity is conservative indeed. Although the effect of the Time To Alert on the integrity could very well be incorporated in the actual method, it was not implemented yet. Beside the demonstrated application to EGNOS receiver data, the method can also be applied to other SBAS and GBAS receivers or other GNSS systems such as a Galileo receiver.

II. THEORY

A. Introduction

An HMI (Hazardous Misleading Information) event occurs once the protection level is smaller than the alert limit while the position error is larger than the alert limit. A MI (Misleading Information) event occurs once the position error is larger than the protection level. The MI and HMI probabilities depend on two basic parameters: the actual position error XPE (being either Vertical Position Error or

Horizontal Position Error) and the determined protection level XPL (being either Vertical Protection Level or Horizontal Protection Level). Usually the test results are presented in a so-called Stanford diagram visualizing the occurrence of normal operation, unavailability, MI and HMI events. Fig. 1 shows a Stanford diagram filled with data gathered with a Novatel Millennium receiver obtained during the period 1 August 2007 through 31 October 2007. In case the protection level is smaller than the alert limit (20 m in fig. 1) while the position error is larger than the alert limit the position information is considered to be a Hazardous Misleading Information result. Since the probability of HMI is very low, it is not practical to state that the HMI probability is the ratio of the number of HMI results divided by the total number of measured samples. Very often, no HMI condition occurs during the tests. The resulting number of HMI conditions will then be zero and in that case the computed HMI probability on this basis will be zero as well, being obviously incorrect. Therefore, one needs to invent a way for obtaining a realistic estimate of the HMI probability even if no HMI events have occurred. The method developed for this purpose starts from the determination of two probability functions: the measured (it was decided not to make use of a fitted distribution for reasons discussed in the paper, see subchapter II.D.2) probability distribution of the protection level and an extreme value (block maxima) probability function related to the position error, together forming a two dimensional probability function.

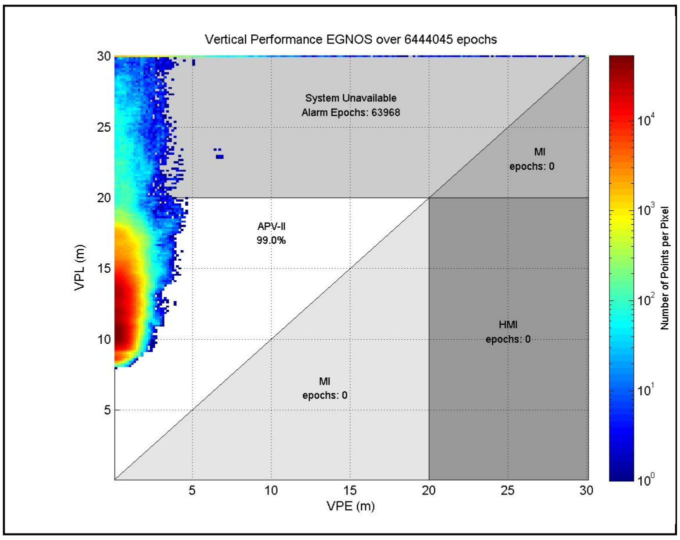

Figure 1. Stanford diagram for the aeronautical service level APV2 filled with a Novatel Millennium receiver data set, period 1 August 2007 through 31 October 2007. The vertical alert limit for APV2 is 20 m

B. *Problem definition*

The statistical problem can be described with the help of the Stanford diagram (fig. 1). The probability distribution of the data is not known beforehand and therefore must be fitted on the measured data. It is assumed here that a reliable probability distribution can be fitted through the measured data. It is also assumed here that especially the tail of the probability distribution can be fitted within a sufficient degree of accuracy. This is a requirement here because the probability of Misleading Information and of Hazardous Misleading Information depends heavily on the probability distribution in the tail region. This fitted probability distribution in the tail must be extrapolated towards the region of interest which is possible with sufficient confidence only once this fit is of a sufficient degree of accuracy.

Data points having a position error larger than the protection level are misleading, that are data occurring at the right of the diagonal line (fig. 1). The probability that Misleading Information occurs can be defined as follows:

$$P_{MI} = \int_{0}^{\infty} \int_{XPE}^{\infty} P_{data}(XPE, XPL) * dXPE * dXPL \qquad (1)$$

where P_{data} is the probability density of the data samples. It should be noted here that it is not necessary to know P_{data} over the entire area ($0 < XPE < \infty$, $0 < XPL < \infty$), it is sufficient to know this probability over the area ($XPL < XPE < \infty$, $0 < XPL < \infty$). This fact makes the Generalized Extreme Value (GEV) function an attractive candidate to compute P_{MI} as will be discussed in this chapter later on.

Data points being misleading and having a position error larger than the alert limit are considered to be hazardous, that are data occurring in the rectangular area at the right of the alert limit (fig. 1). The probability that Hazardous Misleading Information occurs can be defined as follows:

$$P_{HMI} = \int_{0}^{AlertLimit} \int_{AlertLimit}^{\infty} P_{data}(XPE, XPL) * dXPE * dXPL \qquad (2)$$

It should be noted here that it is sufficient to know P_{data} over the area (*Alert limit* $< XPE < \infty$, $0 < XPL <$ *Alert limit*). This fact makes also in this case the GEV function an attractive candidate for computing P_{HMI}.

The problem to be solved is to compute P_{MI} and P_{HMI} on the basis of a series of test data. In addition it is of much importance to compute the 95% confidence levels of these two performance parameters: P_{MI95} as well as P_{HMI95}.

C. *The Generalized Extreme Value function*

The extreme value theory is a powerful and robust framework to study the tail behavior of a distribution. The extreme value theory has found large applicability as has been reported e.g. in [2] and [3].

The Generalized Extreme Value (GEV) probability distribution has the following cumulative distribution function:

$$H(x;K,\sigma,\mu) = \exp\left[-\left(1+K\frac{x-\mu}{\sigma}\right)^{-1/K}\right],$$

$$for\ 1+K\frac{x-\mu}{\sigma} > 0 \qquad (3)$$

The probability density is expressed by:

$$f(x;K,\sigma,\mu) = \frac{1}{\sigma}\left(1+K\frac{x-\mu}{\sigma}\right)^{-1/K-1} *$$

$$\exp\left[-\left(1+K\frac{x-\mu}{\sigma}\right)^{-1/K}\right],\ for\ 1+K\frac{x-\mu}{\sigma} > 0 \qquad (4)$$

where K is called the extreme value index or shape parameter, μ is the location parameter and σ is the scale parameter. For K=0 the distribution is of the type I also called Gumbel, for K>0 it is of type II also called Fréchet and for K<0 it is of type III also called Weibull. The distribution has a finite left boundary for K>0, a finite right boundary for K<0 and extends to infinity in both directions for K=0. Note that the navigation errors (*XPE*) are not bounded, anyway not towards positive values; as a consequence K can not be negative.

The parameter x is the set of so called block maxima, for example the maximum water level per year, or in this case a parameter related to the maximum position error per day (the choice of one day is discussed in subchapter II.E). It is assumed that the block maxima are uncorrelated and each block is of an identical probability distribution. One way to estimate the parameters K, μ and σ is applying the 'maximum likelihood method' on the measured set of block maxima. During this study the Gauss-Newton iteration (e.g. [8]) is used to solve the maximum likelihood problem in a computationally efficient, reliable and accurate way.

In the vertical case the parameter x is defined as $(VPE/VPL)_{blockmaxima}$. The correctness of this choice will be proven in this paper. In the horizontal case the same probability function as for the vertical case is used in this study except that the probability is multiplied by x (x is the block maxima of the dimensionless radius x = $(HPE/HPL)_{blockmaxima}$).

D. *Probability distribution*

The problem to be solved is described by (1) and (2). These equations can only be evaluated once the two dimensional probability distribution $P_{data}(XPE,XPL)$ is known over the area of interest. To assess the determination of this two dimensional probability function, we will first study the probability distribution $P_{data}(XPE)$, then that of $P_{data}(XPL)$ and finally that of $P_{data}(XPE,XPL)$.

1) *Probability distribution $P_{data}(XPE)$*

Fig. 2 shows a histogram of the Vertical Position Error (VPE). This histogram shows that there is a significant bias of 40 cm having a probability of 96%. The horizontal bias is not further addressed here, but is 83 cm. It is not known if this bias is due to a systematic error in EGNOS or that the location of the antenna is wrongly surveyed or both. Another point which is of importance is the fact that the standard deviation of the Vertical Position Error is expected to increase linearly with the Vertical Protection Level since the protection level is in essence an estimate of the actual standard deviation of the position error. This point will be addressed in this chapter later on.

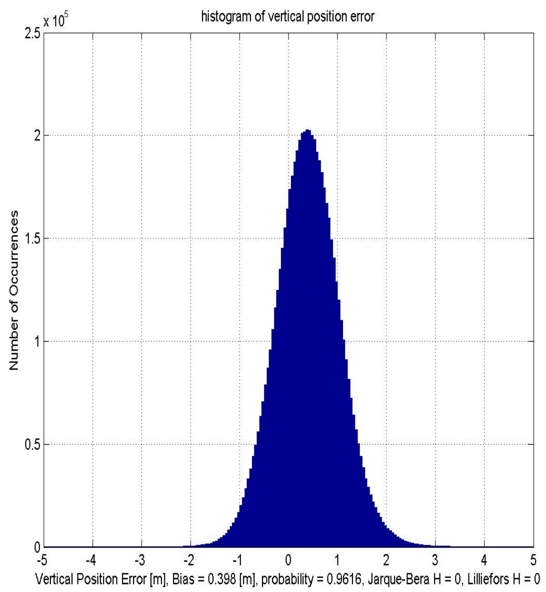

Figure 2. Histogram of Vertical Position Error of the Novatel Millennium data set

2) *Probability distribution $P_{data}(XPL)$*

Fig. 3 shows a histogram of the Vertical Protection Level (VPL). The figure shows that the VPL distribution is not of a known probability function. It is therefore decided to do the analysis with the measured VPL distribution as shown in fig. 3 throughout this study.

3) *Probability distribution $P_{data}(XPE,XPL)$*

Fig. 4 shows the increase in the Vertical Position Error standard deviation σ_{VPE} as function of VPL. The figure shows that there is a linear relationship between σ_{VPE} and VPL. Fig. 4 also shows that the fitted linear relationship of $\sigma_{VPE}(VPL)$ is not 0 at VPL = 0. The probability that this intercept is not of a zero value is 99.35%. It is now possible to introduce the following new parameter $VPE/(VPL+VPL_{intercept})$ which in practice can be approximated by the safety index VPE/VPL (the validity of this assumption is discussed in detail in [9]). A two dimensional probability distribution can now be constructed. The two required parameters are (1) VPL and (2) VPE/VPL. Fig. 5 shows the probability distribution of the safety index VPE/VPL. From this figure it is clear that the tail of the distribution is non-Gaussian. In addition it is not clear what type of probability distribution is valid for the tail. To compute the MI or HMI an extrapolation of this probability distribution is needed. Therefore an accurate probability distribution especially at the tail is a requirement. The Generalized Extreme Value function overcomes this problem as will be discussed in this chapter later on.

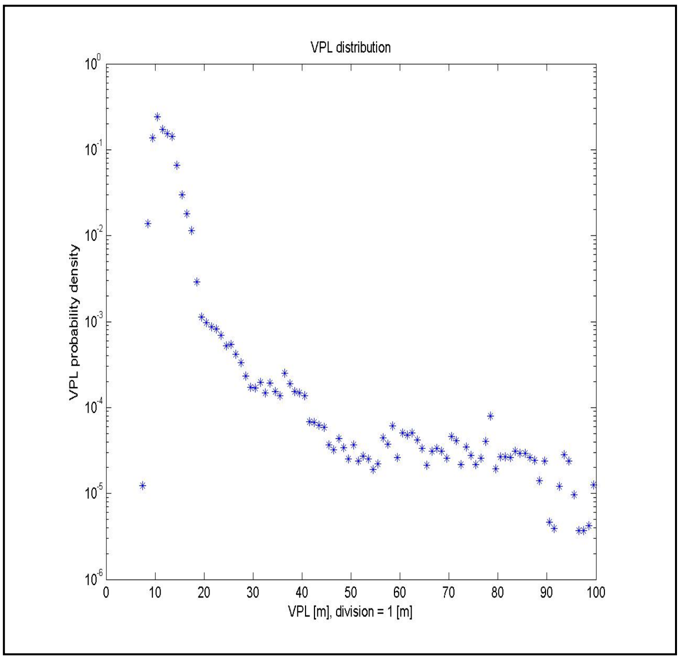

Figure 3. Probability distribution of the Vertical Protection Level of the Novatel Millennium data set, data of a value more than 100 m are left out

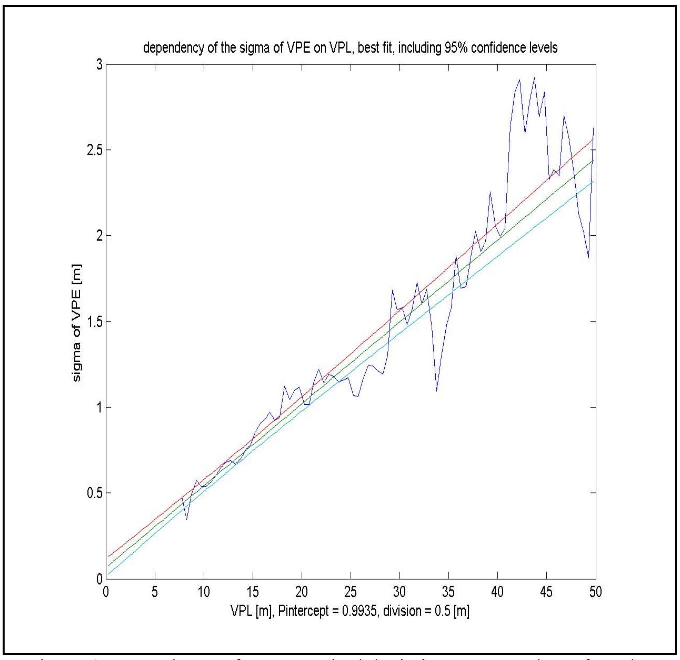

Figure 4. Dependency of VPE standard deviation on VPL, data of a value larger than 50 m are excluded in the fit (from the Novatel Millennium data set, the position bias is removed)

Fig. 6 shows the probability distribution of the safety index HPE/HPL. The shown probability function is of the Rayleigh type. From this figure it is clear that the tail of the distribution is not Rayleigh distributed. In addition it is not clear what type of probability distribution is valid for the tail. To compute the MI or HMI an extrapolation of this probability distribution is needed.

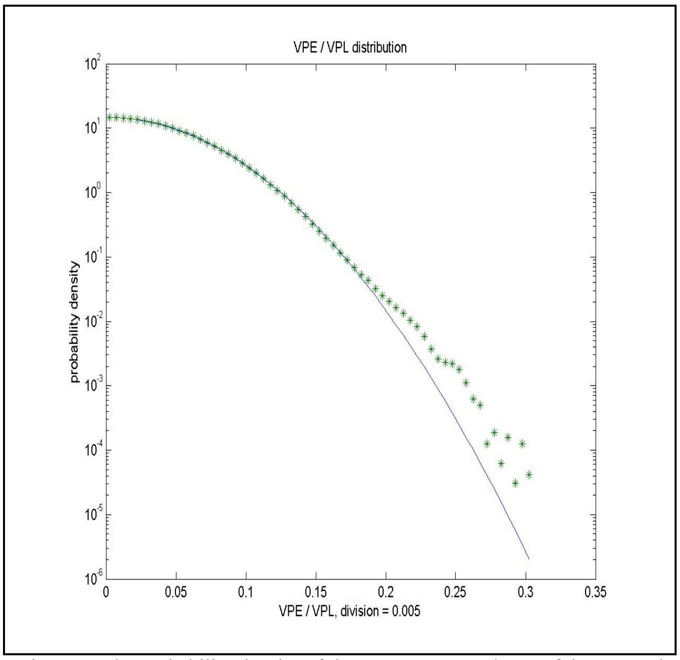

Figure 5. The probability density of the parameter VPE/VPL of the Novatel Millennium data set, the solid line is the Gaussian probability distribution

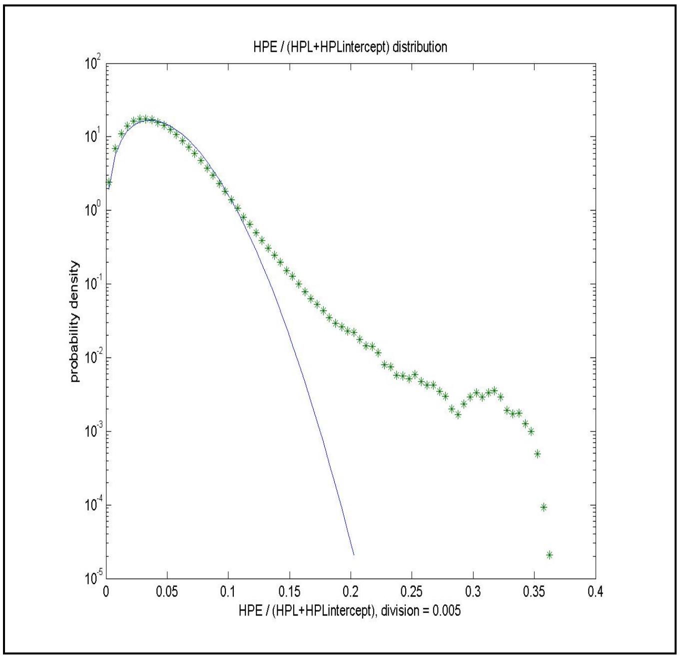

Figure 6. The probability density of the parameter HPE/HPL of the Novatel Millennium data set, the solid line is the Rayleigh probability distribution

E. *Application of the Generalized Extreme Value function*

Fig. 7 shows the Stanford diagram of the daily block maxima of the parameter *x* = VPE/VPL of the Novatel Millennium data set. A division of the data set into blocks of one day (24 hours) is favorable because of the following two reasons:

- the block maxima must be mutually uncorrelated, therefore the minimum block length must be of at least 6 up to 12 hours;
- each day there is a larger probability that the maximum error occurs during the daylight period when due to the sun the influence of the fluctuations in the ionosphere is worse than over night; so there possibly exists a daily periodicity.

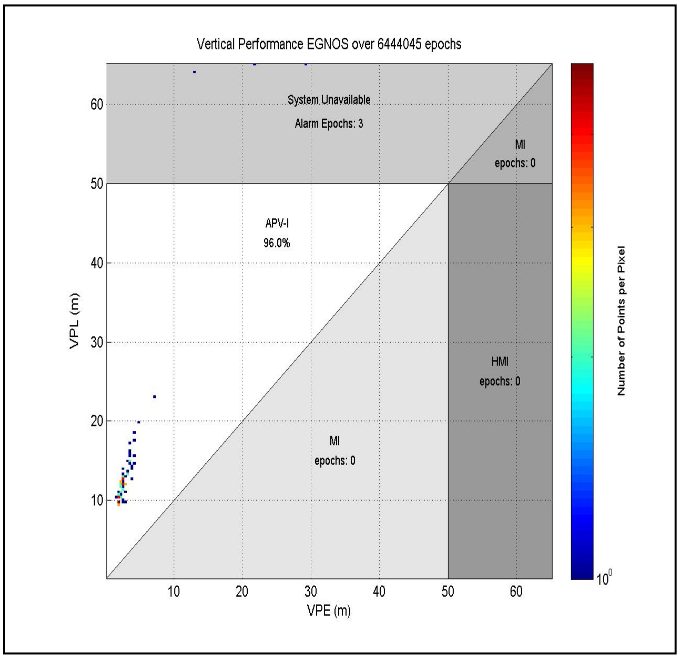

Figure 7. Stanford diagram filled with block maxima of $(VPE/VPL)_{blockmaxima}$ per day of the Novatel Millennium receiver data set

Fig. 7 also shows an approximate linear relation ship between VPE and VPL for the block maxima of the safety index $(VPE/VPL)_{blockmaxima}$ per day.

1) *The GEV fit through $(XPE/XPL)_{blockmaxima}$*

A fit is made through the cumulative distribution of the available block maxima of VPE/VPL after removal of the bias in VPE. Concerning the vertical case fig. 9 shows the resulting GEV probability density as function of VPE/VPL through the available block maxima together with 95% confidence intervals. To compute the confidence intervals it is required to determine the confidence intervals of the estimated parameters K, σ and μ. These confidence intervals result from the Gauss-Newton iteration method as described in e.g. [8]. The estimated GEV parameters are: $K = 0.17$, $\sigma = 0.023$ and $\mu = 0.19$. The 95% confidence interval of K is 0.13 and 0.22. From this result it can be concluded with sufficient significance that K > 0 and as such we are dealing with a GEV function or more specific a Fréchet function.

2) *MI probability computation*

From previous subchapters it can be concluded that the MI probability computation can very well be based on the daily block maxima of the parameter $(XPE/XPL)_{blockmaxima}$ which stands for either $(VPE/VPL)_{blockmaxima}$ or $(HPE/HPL)_{blockmaxima}$. From (1) it follows:

$$P_{MI} = \int_{0}^{\infty}\int_{x=1}^{\infty} P_{data}(x * XPL, XPL) * dx * dXPL \qquad (5)$$

So, to compute the MI probability, we need to compute the cumulative probability for $x = 1$ and for XPL of $(XPE/XPL)_{blockmaxima}$. From (1), (3) and (5) it follows that the MI probability is:

$$P_{MI} = \int_{0}^{\infty} H_X(1; K, \sigma, \mu) * dXPL$$

for XPL of $(XPE/XPL)_{blockmaxima}$,
X = vertical, horizontal (6)

Since we will rely on the measured probability distribution along the XPL axis (see the fig. 3) (6) can be rewritten into:

$$P_{MI} = \int_{0}^{1} H_X(1; K, \sigma, \mu) * dP_{measured}(XPL)$$

for XPL of $(XPE/XPL)_{blockmaxima}$,
X = vertical, horizontal (7)

This equation can be digitized as follows:

$$P_{MI} = \frac{1}{N_{block\max}} \sum_{i_{block\max}=1}^{i_{block\max}=N_{block\max}} H_X(1; K, \sigma, \mu)_{i_{block\max}}$$

for XPL of $(XPE/XPL)_{blockmaxima}$,
X = vertical, horizontal (8)

Where $N_{blockmax}$ is the number of block maxima of $(XPE/XPL)_{blockmaxima}$. It must be noted here that the XPE should not be calculated relative to the known position of the receiver but relative to the mean value of the measurements of the position. This is a requirement since the statistical results are sensitive to a possible bias which can easily cause wrong results and even into too optimistic results. A simulation shows (fig. 8) the effect of a possible bias on the μ and the σ of a Gumbel distribution. The random generated samples are Gaussian distributed. So the ideal situation is assumed here that the position of the receiver matches with the averaged position exactly.

Fig. 9 shows the GEV probability density as function of VPE/VPL together with confidence intervals extrapolated towards $(VPE/VPL)_{blockmaxima} = 1$. The MI probability is the cumulated GEV probability for $(VPE/VPL)_{blockmaxima} = 1$ and turns out to be $1.1x10^{-5}$ per day. The MI probability including 95% confidence turns out to be $4.8x10^{-5}$ per day.

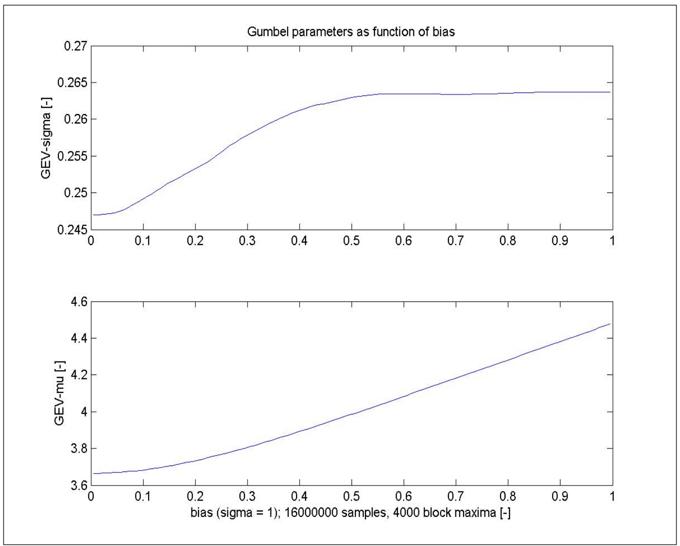

Figure 8. Simulation showing the effect of a possible bias

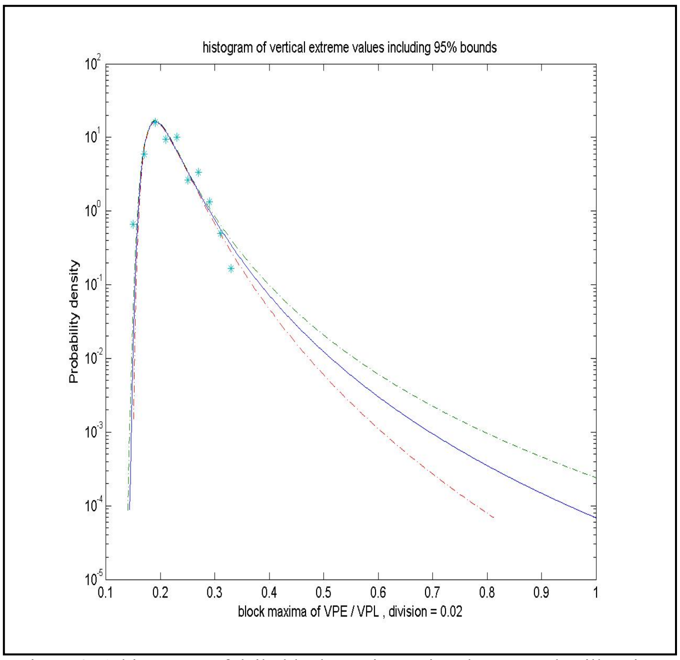

Figure 9. A histogram of daily block maxima using the Novatel Millennium data set with GEV fit including 95% confidence intervals extrapolated up to $(VPE/VPL)_{blockmaxima} = 1$

3) MI probability computation with bias

In practice a bias in the receiver position can very well be present and as such the question arises what the effect is of such a bias on the MI probability. The effect of the vertical bias can be computed starting from (8) resulting into:

$$P_{MI} = \frac{1}{2 * N_{block\max}} \sum_{i_{block\max}=1}^{i_{block\max}=N_{block\max}} \{H(1-\frac{bias}{VPL};K,\sigma,\mu) + H(1+\frac{bias}{VPL},K,\sigma,\mu)\}_{i_{block\max}}$$

for VPL of $(VPE/VPL)_{blockmaxima}$ (9)

The effect of the horizontal bias (only positive values) can be computed starting from (8) as well resulting into:

$$P_{MI} = \frac{\sum_{i_{block\max}=1}^{i_{block\max}=N_{block\max}} H_{horizontal}(1-\frac{bias}{HPL};K,\sigma,\mu)_{i_{block\max}}}{N_{block\max}}$$

for HPL of $(HPE/HPL)_{blockmaxima}$ (10)

4) HMI probability computation

The basic difference between the computation of HMI probability and the MI probability is that for the MI probability we only have to compute the cumulative probability for the parameter $x = (XPE/XPL)_{blockmaxima}$ at the constant value of $x = 1$. To compute the HMI probability however, we need to compute the cumulative probability for the varying x:

$$x = ALERTLIMIT/XPL$$

for $XPL < ALERTLIMIT$ and for XPL of $(XPE/XPL)_{blockmaxima}$ (11)

From (2) and (3) it follows that the HMI probability is:

$$P_{HMI} = \int_0^{ALERTLIMIT} H_X(\left(\frac{ALERTLIMIT}{XPL}\right);K,\sigma,\mu) * dXPL$$

for $XPL < ALERTLIMIT$ and for XPL of $(XPE/XPL)_{blockmaxima}$
X = vertical, horizontal (12)

Since we will rely on the measured probability distribution along the XPL axis (see fig. 3) (12) can be rewritten into:

$$P_{HMI} = \int_0^{P_{ALARMLIMIT}} H_X(\left(\frac{ALERTLIMIT}{XPL}\right);K,\sigma,\mu) * dP_{measured}(XPL),$$

for $XPL < ALERTLIMIT$ and for XPL of $(XPE/XPL)_{blockmaxima}$
X = vertical, horizontal (13)

This equation can be digitized as follows:

$$P_{HMI} = \frac{\sum_{i_{block\max}=1}^{i_{block\max}=N_{block\max}} H_X(\left(\frac{ALERTLIMIT}{XPL}\right);K,\sigma,\mu)_{i_{block\max}}}{N_{block\max}}$$

for $XPL < ALERTLIMIT$ and for XPL of $(XPE/XPL)_{blockmaxima}$
X = vertical, horizontal (14)

where $N_{block\max}$ is the number of block maxima of $(XPE/XPL)_{\text{blockmaxima}}$ for which *XPL* is less than the *ALERTLIMIT*.

5) Weighed cumulative probability fit

A way to improve the cumulative probability fit is by weighing the measurement samples by sorting the $(XPE/XPL)_{\text{blockmaxima}}$ in magnitude as follows:

$$w_i = i, \text{ while } (XPE/XPL)_i < (XPE/XPL)_{i+1} \quad \text{for } i = 1, 2, 3, .., N_{block\max} \tag{15}$$

where w_i is the weighing factor. This means that samples of the relative large $x_i = (XPE/XPL)_i$ values are more heavily weighted than that of the low x_i values. This weighing function $w_i = i$ can easily be included into the Gauss-Newton iteration procedure.

A weighed fit is made through the cumulative distribution of the available block maxima of VPE/VPL. Fig. 10 shows the resulting GEV probability density as function of VPE/VPL through the available block maxima together with confidence intervals for the vertical case. Comparing fig. 10 with fig. 9 it is clear that at x=1 the probability density becomes approximately a factor 10 smaller after weighing the measurement samples. It can therefore be concluded that weighing according to (15) will have a significant effect on the computed MI and HMI probabilities. It is advised to perform both analyses, with and without weighing and choose for the most conservative (most save) estimated HMI probability result.

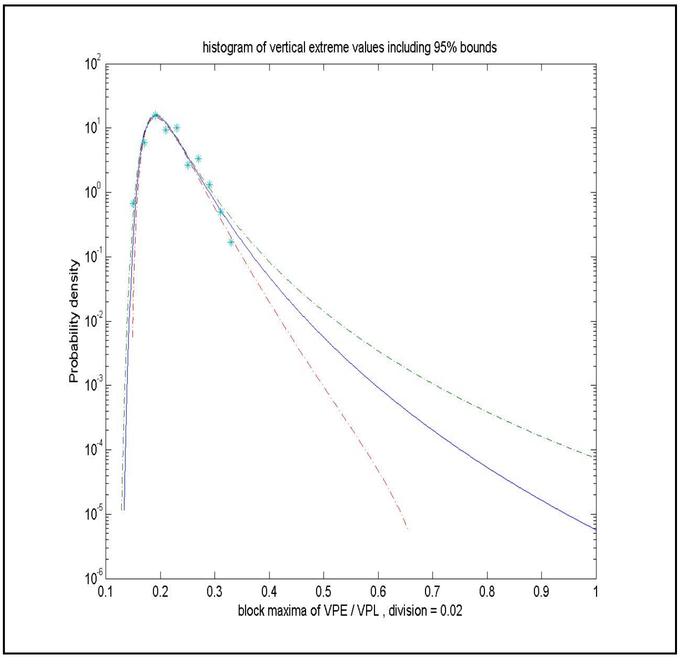

Figure 10. A histogram of daily block maxima using the Novatel Millennium data set with weighed GEV fit

The same weighing fit function has been applied to the cumulative distribution of the available block maxima of HPE/HPL. Fig. 11 shows the resulting GEV probability density as function of HPE/HPL through the available block maxima together with confidence intervals for the horizontal case.

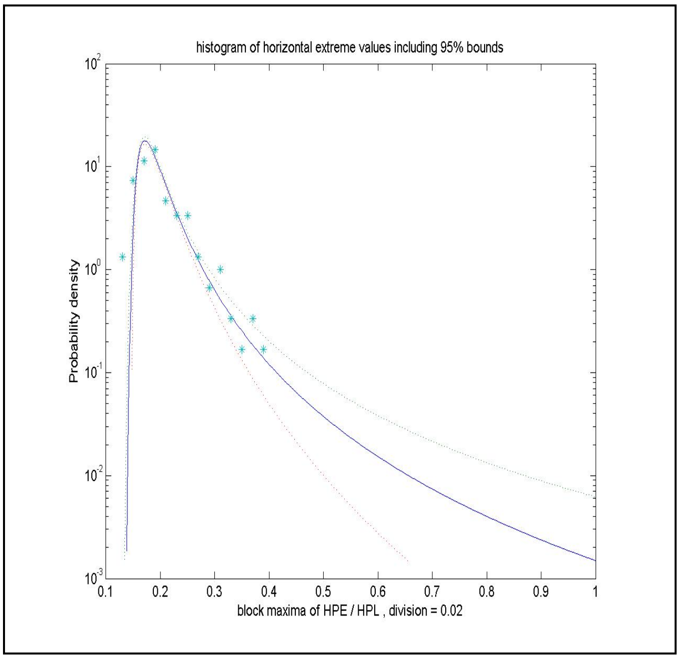

Figure 11. A histogram of daily block maxima using the Novatel Millennium data set with weighed GEV fit

III. Discussion of Test Results from the Novatel Millennium Receiver

As a test case EGNOS data, recorded at a measurement site at NLR Flevoland using a Novatel Millennium receiver, was analyzed. The data collection started at 0.00 hours UTC on 1 August 2007 and ended at 24.00 hours UTC on 31 October 2007.

A. MI probability

The MI probability will be computed by evaluating (8). The results are listed in table 1. To study the effect of a possible bias the MI probability has been computed as function of a bias by evaluating equation 14. Fig. 12 shows the MI probability according to the GEV distribution fit as function of a possible bias.

TABLE I. Estimated MI probabilities (vertical case)

$MI_{probability}$ per day	95% confidence per day	Remarks
$1.1x10^{-5}$	$4.8x10^{-5}$	non-weighed
$6.3x10^{-7}$	$1.2x10^{-5}$	weighed

B. HMI probability

The HMI probability will be computed by evaluating (14). The results are listed in table 2. The integrity performance as required by ICAO is $2x10^{-7}x24x3600/150 = 1.1x10^{-4}$ per day. From these figures it can be concluded that the ICAO requirements are satisfied for HMI in the vertical case.

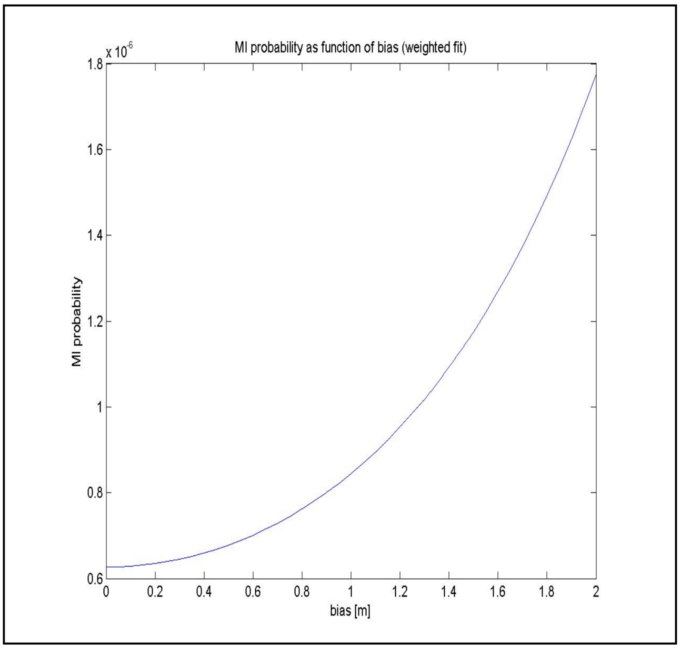

Figure 12. MI probability as function of the vertical bias (weighed fit)

TABLE II. ESTIMATED HMI PROBABILITIES (VERTICAL CASE)

Aeronautical service level	$\text{HMI}_{\text{probability}}$ per day	95% confidence	Remarks
APV-I	$1.8\text{x}10^{-7}$	$4.7\text{x}10^{-6}$	weighed
APV-II	$1.8\text{x}10^{-7}$	$5.0\text{x}10^{-6}$	weighed
CAT-I	$4.5\text{x}10^{-7}$	$1.1\text{x}10^{-5}$	weighed

C. Visualization of the probability distribution in the Stanford diagram

The Stanford diagram usually shows a 2-dimensional (VPE, VPL) histogram of the measured data set. This diagram can easily be converted into a 2-dimensional probability distribution which can on its turn be extended with the known GEV distribution towards the integrity region. Along the VPE axis this GEV distribution is described by equation 4 with x = VPE/VPL and along the VPL axis the measured distribution (fig. 3) is to be used. The resulting Stanford diagram is shown in fig. 13 visualizing the probability distribution in the (un)availability, MI and HMI regions for the APV-II aeronautical service.

IV. CONCLUSIONS

A test method has been developed for the analysis of receiver output data gathered during test campaigns of limited duration for GNSS receivers and is applicable also to the future Galileo receivers in order to assess the integrity performance of these satellite navigation systems. The method is based on the Generalized Extreme Value probability distribution function allowing a reliable extrapolation of probability functions towards the MI and HMI regions in the Stanford diagrams.

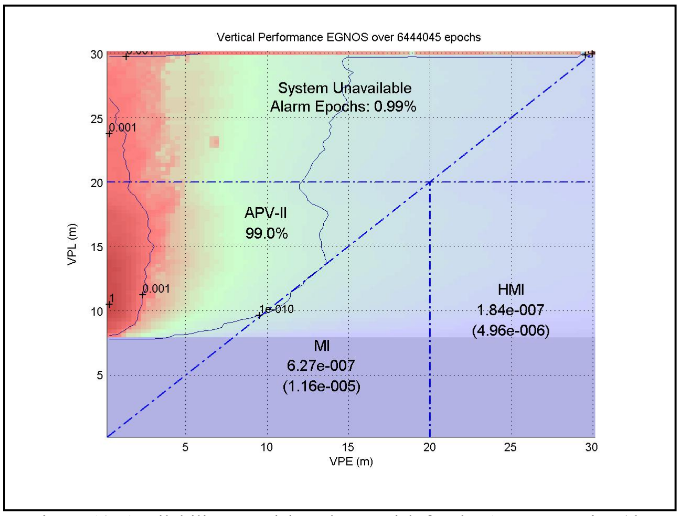

Figure 13. Availability, MI risk and HMI risk for the APV-II service (the values within brackets refer to 95% confidence one sided upper limits)

From the test case presented in this paper, it can be concluded that the estimation of the integrity, based on a few months of test campaign data, is realistic and reliable indeed. The results show that the obtained EGNOS receiver integrity satisfies the ICAO requirements for the APV-I, APV-II and CAT-I aeronautical service levels for the EGNOS configuration during the test.

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