



## Executive summary

# A Qualitative Introduction to the Vortex-Ring-State, Autorotation, and Optimal Autorotation

### Problem area

The main objective of this paper is to provide the reader with some qualitative insight into the areas of Vortex-Ring-State (VRS), autorotation, and optimal autorotation.

### Description of work

First this paper summarizes the results of a brief VRS literature survey, where the emphasis has been placed on a qualitative description of the following items: conditions leading to VRS flight, the VRS region, avoiding the VRS, the early symptoms, recovery from VRS, experimental investigations, and VRS modeling. The focus of the paper is subsequently moved towards the autorotation phenomenon, where a review of the following items is given: the maneuver, the height-velocity zones, and factors affecting autorotation. Finally the paper concludes by providing a literature

survey relative to single-engine helicopter optimal autorotation, and its associated problem formulation as a nonlinear, constrained, optimal control problem.

### Results and conclusions

Presenting a complete survey of a field as diverse as helicopter VRS, autorotation, and optimal trajectories in autorotation is a daunting task. Hence the review is from a common qualitative approach, with emphasis on concepts rather than on details.

### Applicability

This survey was essentially tailored for researchers interested in designing control systems, for helicopter flight in the VRS and autorotation, such as automatic VRS avoidance, automatic recovery from VRS flight, and automatic autorotation.

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S. Taamallah

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**A Qualitative Introduction to the Vortex-Ring-State, Autorotation, and Optimal Autorotation**

**Nationaal Lucht- en Ruimtevaartlaboratorium**, National Aerospace Laboratory NLR

Anthony Fokkerweg 2, 1059 CM Amsterdam,  
P.O. Box 90502, 1006 BM Amsterdam, The Netherlands

Telephone +31 20 511 31 13, Fax +31 20 511 32 10, Web site: [www.nlr.nl](http://www.nlr.nl)



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S. Taamallah


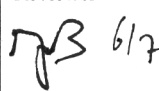
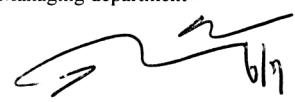
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# A Qualitative Introduction To The Vortex-Ring-State, Autorotation, And Optimal Autorotation

Skander Taamallah \*<sup>†</sup>

\* Avionics Systems Department, National Aerospace Laboratory (NLR)  
1059 CM Amsterdam, The Netherlands ([staamall@nlr.nl](mailto:staamall@nlr.nl)).

<sup>†</sup> Delft Center for Systems and Control (DCSC)  
Faculty of Mechanical, Maritime and Materials Engineering  
Delft University of Technology, 2628 CD Delft, The Netherlands.

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optimal control; optimal autorotation

**Abstract:** The main objective of this paper is to provide the reader with some qualitative insight into the areas of Vortex-Ring-State (VRS), autorotation, and optimal autorotation. First this paper summarizes the results of a brief VRS literature survey, where the emphasis has been placed on a qualitative description of the following items: conditions leading to VRS flight, the VRS region, avoiding the VRS, the early symptoms, recovery from VRS, experimental investigations, and VRS modeling. The focus of the paper is subsequently moved towards the autorotation phenomenon, where a review of the following items is given: the maneuver, the height-velocity zones, and factors affecting autorotation. Finally the paper concludes by providing a literature survey relative to single-engine helicopter optimal autorotation, and its associated problem formulation as a nonlinear, constrained, optimal control problem.

## 1 Introduction

This paper summarizes the results of a brief literature survey, of relevant work in the open literature, covering the areas of the VRS, autorotation, and optimal autorotation. Due to time and space constraints, only published accounts relative to standard helicopter configurations will be covered, omitting thus other types such as tilt-rotor, side-by-side, tandem, and co-axial. Presenting a complete survey of a field as diverse as helicopter VRS, autorotation, and optimal trajectories in autorotation is a daunting task. Hence the review is from a common qualitative approach, with emphasis on concepts rather than on details.

The paper is organized as follows: in Section 2, a review of the four rotor operating conditions in vertical flight is given. In Section 3, the VRS is presented, including a review of aspects affecting the VRS, the VRS region, and VRS modeling. In Section 4, a review of published accounts in the field of

autorotation, aspects affecting the maneuver, and the associated height-velocity diagram are provided. In Section 5, a literature survey relative to the optimal autorotation problem, and its solution through constrained optimal control, is presented. Finally, conclusions and future directions are presented in Section 6.

As a final introductory note, many interesting and important contributions or foundational works related to the VRS and autorotation have not been surveyed in this paper. In this, and many other respects, we sincerely ask for the kind understanding of readers and authors alike.

## 2 Vertical flight

Before addressing the areas of VRS and autorotation, we start by giving a quick review of the four rotor operating conditions in vertical flight, see Fig. 1 for a schematic representation.

Fig. 2 shows the momentum theory<sup>1</sup> solutions for a main rotor in vertical climb or descent. The lines  $V_c = 0$ ,  $V_c + v_i = 0$ , and  $V_c + 2v_i = 0$  divide the  $(V_c, v_i)$  plane<sup>2</sup> into four regions. The area of the plane right of line  $V_c = 0$  defines the normal working state rotor. The area of the plane between lines  $V_c = 0$  and  $V_c + v_i = 0$  defines the VRS region. The area of the plane between lines  $V_c + v_i = 0$  and  $V_c + 2v_i = 0$  defines the turbulent wake state. Finally the area left of line  $V_c + 2v_i = 0$  defines the windmill brake state [99]. We provide next a succinct review of those four regions, a much more detailed discussion can be found in [89].

<sup>1</sup>Momentum theory refers to the conservation of mass, momentum, and energy in the case of an inviscid, incompressible, steady, irrotational, and 1-D flow [89]

<sup>2</sup>With  $V_c$  being the climb velocity,  $v_i$  the main rotor induced velocity, and  $v_h$  the main rotor induced velocity in hover

- The normal working state region  $0 \leq V_c/v_h$ . It includes climb and hover. Here the velocity throughout the main rotor flow field is always downwards, hence a wake model with a definite slipstream<sup>3</sup> is valid for this rotor state, resulting in good estimates of rotor performance, in climb and hover, by momentum theory [89].
- The VRS region  $V_{tr}/v_h \leq V_c/v_h < 0$ , where  $V_{tr}$ <sup>4</sup> refers to the transition velocity between the VRS and the turbulent wake state regions. Over the years, several transition velocities or transition velocity ranges have been reported, for example in [89, 115]. There is indeed no clear-cut value for  $V_{tr}$  as can be seen from measurements scatter reported in Fig. 3, where the figure shows the universal empirical induced velocity curve. This curve can be constructed on the basis of estimates of the profile power coefficient. Hence the induced velocity always shows some scatter, due to errors in the profile power calculation, and other aspects such as tip loss and blade twist [89]. Further in the VRS region, a definite slipstream does not exist anymore, since the flow in the far wake inside and outside the slipstream are in opposite direction. At first for low descent rates  $-0.5 \leq V_c/v_h < 0$ , momentum theory is still valid [89]. As the descent rate increases  $V_{tr}/v_h \leq V_c/v_h < -0.5$ , the flow becomes turbulent and has large recirculation, resulting in rotor vibrations and degraded control [89]. In this region momentum theory is not valid anymore.
- The turbulent wake state  $-2 \leq V_c/v_h < V_{tr}/v_h$ . Here the flow pattern above the rotor disk is very similar to the turbulent

<sup>3</sup>The stream of air forced downwards by rotating blades

<sup>4</sup>In this paper we will assume that  $V_{tr}/v_h \in [-1.9, -1.6]$

wake of a bluff body [89]. In this region, when compared to the VRS, flow recirculation through the rotor has diminished and rotor vibrations have also decreased. But the rotor still experiences some roughness due to the (high) turbulence [89]. It is also in this region that equilibrium autorotation occurs. Note also that here too momentum theory is invalid.

- The windmill brake state  $V_c/v_h < -2$ . In this region the flow is again smooth with a definite upwards slipstream, and momentum theory is applicable, providing good rotor performance estimates [89].

### 3 The Vortex-Ring-State

A horizontal rotor creates a downward flow induced by the thrust generation. If the rotor moves along the direction of its induced flow, i.e. down, the downward induced flow will compete with the upward flow due to the descent motion. As a result, the smooth slipstream around the rotor disk is gradually destroyed. In particular, when the descent rate approaches the rotor induced velocity, the rotor enters its own wake, resulting in blade tip vortices recirculation. These vortices will then tend to pile up at the disk plane to create a so-called doughnut-shaped vortex ring [101, 100, 43]. Moreover, the onset and development of this so-called vortex ring state can be viewed as a spatial and temporal wake instability. By instability one means vortex rings as a result of wake recirculation in the plane of the rotor [101, 100]. Periodically however the character of this recirculation changes, as a partial vortex collapse causes flow asymmetry at the rotor disk. This phenomenon results in large fluctuations in rotor lift and torque [33]. More specifically these include high amplitude, low frequency blade flapping, low frequency vertical bounce of the helicopter, and substantial

loss of control effectiveness [101, 100].

#### 3.1 VRS: a hazardous flight condition

For a helicopter main rotor, VRS may occur for example in a descending flight, while for a helicopter tail rotor, VRS may occur during a sideways flight, or while in hover with a crosswind. For the case of a main rotor VRS condition, the symptoms are generally excessive vibrations, large unsteady blade loads, thrust/torque fluctuations, excessive loss of altitude, and loss of control effectiveness [125]. Hence flight in the VRS is a dangerous flight condition, especially if entered at low altitude. For the case of a tail rotor VRS, it is the vehicle yaw control (i.e. heading) that may become difficult or impaired.

For the 1982 - 1997 time frame, data from the U.S. NTSB<sup>5</sup>, U.S. Navy & Army, and the U.K. Air Accidents Investigation Branch show that 32 helicopter accidents were caused by flight into the VRS, most of them at altitudes less than 200 feet and at low airspeeds [141]. In April 2000, a Marine Corps V-22 Osprey tilt-rotor crashed in Arizona, resulting in the tragic loss of all 19 Marines on board. It was later determined that a contributing cause of that accident was flight into the VRS [49, 43].

It is nowadays well known that a significant number of VRS accidents were the result of accepting slight tailwinds or downwind approaches, hence reducing the actual horizontal air velocity<sup>6</sup>.

<sup>5</sup>National Transportation Safety Board

<sup>6</sup>This is often a point of concern for helicopter pilots, since the airspeed indication on most civil helicopters is somewhat ineffective below 35 to 40 knots [141]. This problem however can be solved by algorithmic methods, which use an internal model, available control inputs, and sensors measurements to infer airspeed and sideslip angle see [55, 60, 107, 71]

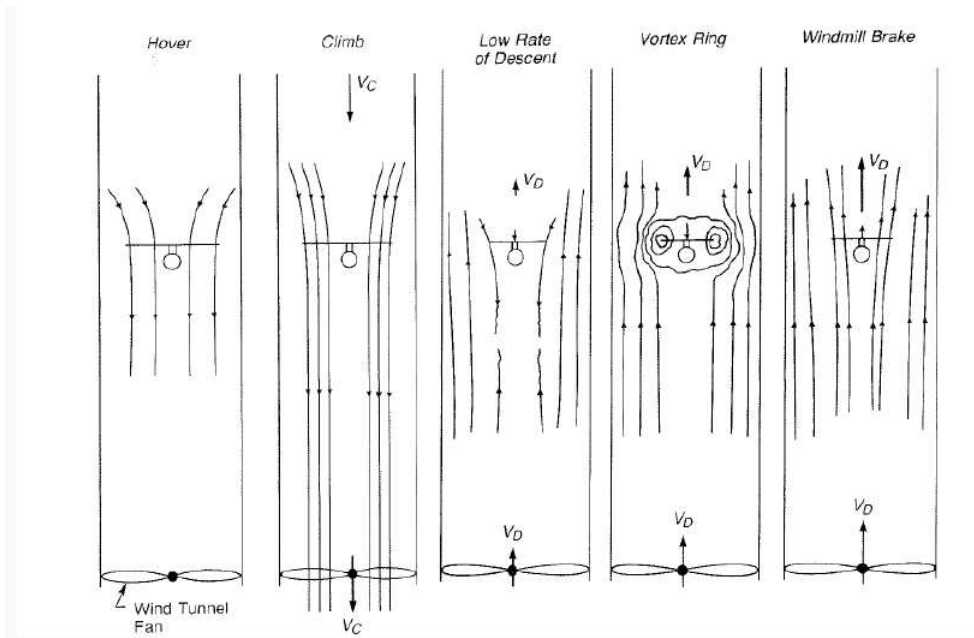


Figure 1: Helicopter axial flight (from [126])

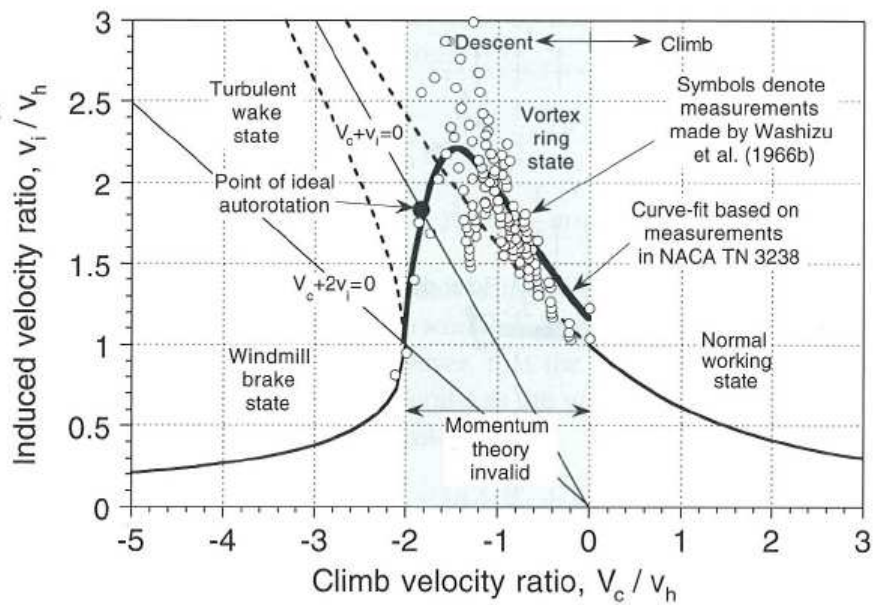


Figure 2: Axial flight: induced velocity variation as a function of vertical velocity (from [99])



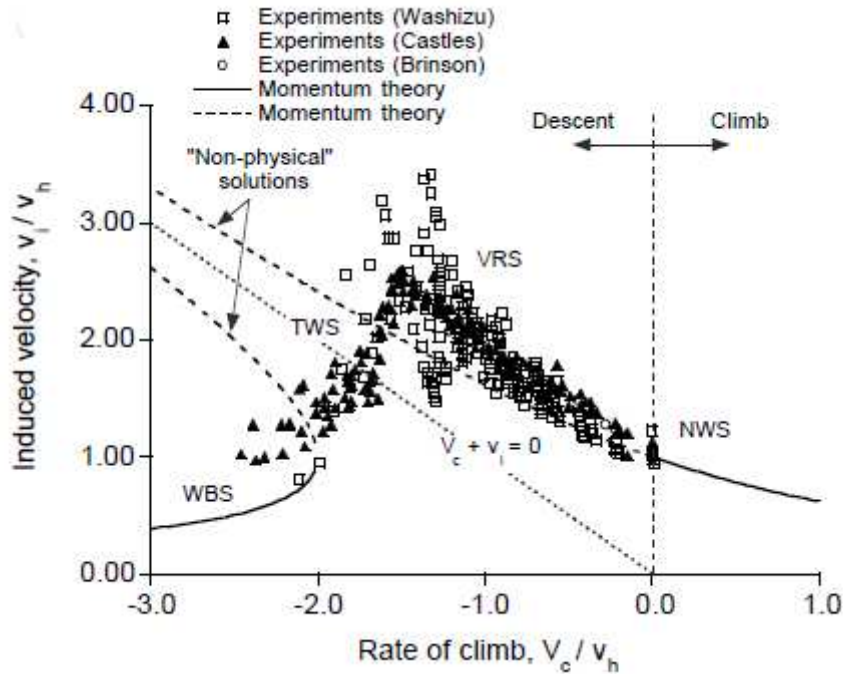


Figure 3: Axial flight: induced velocity scatter in descending flight through the VRS (from [100])

### 3.2 Flight conditions leading to the VRS

For a helicopter main rotor, VRS may occur in the following conditions

- In an axial descent with the rate of descent being about equal to the hover induced velocity
- At low speeds and steep descent angles
- When descending downwind into a landing area [115]
- In the final stages of a quick stop maneuver (such maneuvers are typically carried out close to the ground [115])
- When starting a power recovery after engine power off [61]
- When settling into the downwash of another aircraft

For a helicopter tail rotor, VRS may occur in the following conditions

- In sideways flight
- While hovering in a crosswind
- During a hover, turn over a spot [126]

### 3.3 The VRS region

Knowledge of the location of the VRS onset boundary, see Fig. 5, is important for safety procedures and operational reasons. One such example is when at low airspeeds it may be necessary for a helicopter to briefly transition through the VRS, in order to reach equilibrium autorotation [36].

It is also well known that the VRS onset boundary is significantly influenced by the specific criterion used to define it [47]. Indeed a boundary based on torque fluctuations

may be slightly different from the classic boundaries predicted using thrust fluctuations or blade flapping. A good review of various researched criteria, such as thrust or torque fluctuations can be found in [43, 90].

An experimental determination of the VRS boundary may be done by sequentially mapping out combinations of flight path angle, descent velocity, and forward velocity, where the rotor experiences high fluctuations in thrust, torque, and blade flapping [36].

### 3.3.1 Factors affecting the VRS region

It is known that the effect of a maneuver distorts the rotor wake, and hence may affect wake stability, and may potentially affect the VRS boundary. Acceleration or angular rate are indeed known to affect the onset and development of the VRS [108, 14]. For example for pull-ups or for other types of maneuvers that increase the rotor disk angle of attack, VRS onset conditions could be attained at a lower rate of descent and/or at higher forward speed, than predicted by the VRS boundary [101, 100]. Additional aspects that may also influence the VRS are rotor/fuselage aerodynamic interference [37], rotor disk angle of attack [100], blade stall [108], blade root cut out location, blade planform taper [108], and blade spanwise loading distribution<sup>7</sup>. On the other hand however, it appeared that the VRS region was insensitive to tail rotor interference [37].

Before concluding this section on the VRS region, we want to add one last comment on the issue of disk loading  $DL$ . This aspect was addressed through a series of flight tests, performed by ONERA almost a decade ago [86].

<sup>7</sup>Which is particularly affected by blade twist for low disk loading [36], with the disk loading  $DL$  being equal to thrust divided by rotor disk area

Fig. 4 shows the influence of vehicle mass increase (in other words disk loading increase) on the Dauphin helicopter VRS boundary region, where the blue line corresponds to an increase in vehicle mass. Note also that in this figure the velocities are not normalized by the induced velocity in hover. As the vehicle mass is increased we see that the size of the regime, where VRS is encountered, shifts downwards and grows dimensionally with the hover induced velocity  $v_i$ <sup>8</sup>. Conversely this also means that even though lightly loaded vehicles, such as helicopter UAVs<sup>9</sup>, will encounter the VRS for low vertical descent rates, a low forward speed should be sufficient for these UAVs to avoid the VRS.

### 3.4 Avoiding the VRS

As can be seen from Fig. 5, VRS is unlikely to occur if the speed along the flightpath is reasonably high. For example it was reported in [78] that a speed of 2 - 2.5 times the hover induced velocity safely avoids the VRS for any glide slope. The ONERA tests found that VRS effects were not observed beyond 1 time the hover induced velocity [87].

Further VRS is unlikely to occur if the glide slope is shallow. Limiting glide slopes to 20 - 30° avoids it at almost any forward speed [78]. This is confirmed by the 30° limit set by the U.S. Army [90]. Hence one should avoid vertical or steep descents whenever possible. If vertical descent is required, it should be attempted from the minimum allowable height above the ground, and at the minimum possible descent rate.

<sup>8</sup>Since  $v_i = \sqrt{DL/2\rho}$

<sup>9</sup>Which typically tend to have a lower disk loading than manned helicopters

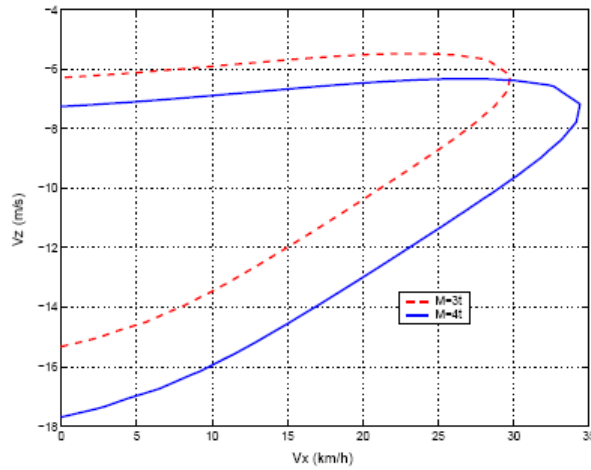


Figure 4: Helicopter mass influence on the Dauphin VRS domain (from [86])

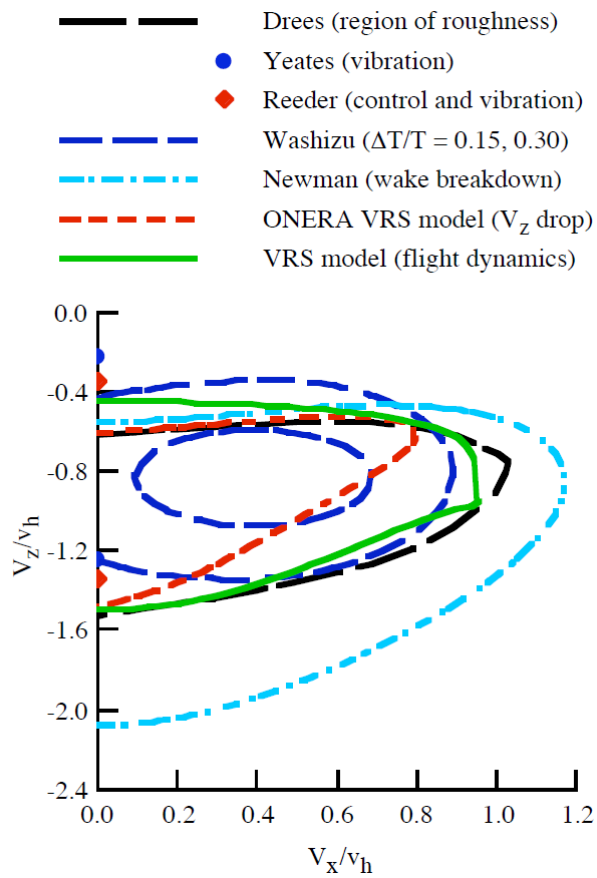


Figure 5: Helicopter VRS Boundaries (from [90])

### 3.5 VRS: the symptoms

For a helicopter main rotor, VRS may occur in a descending flight, and historically the VRS has also been associated with the so-called *power settling*, where more power is required to descend than to hover [126]. Indeed the power extracted from the airstream is less than the induced power<sup>10</sup> loss. In other words a substantial increase in the required power for equilibrium flight becomes necessary, basically to overcome the additional aerodynamic losses as the rotor descends into its own wake [102].

Aside from the power settling phenomenon, early signs of VRS flight may be given by thrust and/or torque fluctuations.

For thrust oscillations the following results were reported

- They are more severe than torque variations [19]
- They are more pronounced at oblique descent, with the angle of attack  $\alpha$  such that ( $\alpha = 60^\circ - 70^\circ$ ) rather than at vertical descent ( $\alpha = 90^\circ$ ) [19]
- The loss in thrust indicates the most turbulent region of the VRS  $-0.8 \leq V_c/v_h < -0.6$  [148]
- The period of the thrust fluctuations lies in the range 0.3 - 0.6 sec, for a model rotor tested in [148]
- Thrust oscillations appear to be independent of the disk loading [148]
- The range of variations of thrust oscillations lies in the order of 12 - 15 to 20 % of mean thrust, with the lower values reported in [86, 36], and the upper limit given in [100]

<sup>10</sup>Induced power is the power required to generate lift

For torque fluctuations, which may lead to directional (yaw) control problems, the following results were reported

- The increase in torque indicates the beginning of the VRS,  $V_c/v_h \simeq -0.4$  [148]
- The period of the torque fluctuation is about 3.7 sec, for a model rotor tested in [148]
- The torque fluctuation of a rotor with lower disk loading is larger than that of a rotor with higher disk loading [148]
- Compared with changes in rotor thrust, rotor torque variations are less significant [43]

### 3.6 Recovery from the VRS

First it should be noted that any recovery from the VRS is likely to result in a significant loss of altitude.

We provide now a few guidelines for a VRS recovery

- Exiting the VRS is best handled by a quick increase in forward or sideways airspeed through cyclic control, which effect is to sweep the recirculating wake away from the rotor disk [87, 101], and then apply collective pitch to cancel the rate of descent [90]
- The U.S. Navy NATOPS<sup>11</sup> states that the only solution for fully developed VRS is to enter autorotation to break the vortex ring and, when cyclic authority is regained, increase forward airspeed [90]
- Maneuvers suppress the effects of VRS. A rotor can maneuver but the vortex rings

<sup>11</sup>Naval Air Training and Operating Procedures Standardization

cannot, hence these will not stay in the rotor disk [31]

### 3.7 Aspects of rotor/blade design

We provide here a few guidelines related to rotor and blade design considerations

- There are no substantial differences in VRS inflow curves due to variations in rotor RPM and rotor radius [43]
- The effect of blade taper on VRS is weaker than the effect of blade twist [43]
- For moderate blade twist, blade stall has no effect on the descent rate for VRS onset. It is still possible however that stall could influence the subsequent development of the VRS [13]
- For moderate blade twist, the influence of twist on the behavior of the rotor is relatively minor [13]
- For high blade twist, the influence of twist on the behavior of the rotor is more pronounced. Such rotors with high blade twist seem to be more susceptible to VRS onset [101, 100]. But the appearance of blade stall on the inboard parts of a rotor, with highly twisted blades, was also shown to reduce the violence of the VRS [36]
- Thrust settling is associated with a considerable fall in blade loading but only at the outboard sections of the blade. The inboard sections of the blade play only a small role in the thrust settling phenomenon [13]

### 3.8 VRS: experimental investigations

We provide here a very brief review of published accounts, see [90] for a comprehensive

survey of flight and wind tunnel test data in the VRS, and [134] for recent flow visualizations.

Back in 1949, one of the first VRS wind tunnel test and flow visualization was performed in The Netherlands (Amsterdam), at what is now the NLR, by Dutch helicopter pioneer Jan Meyer Drees [54, 53].

In the 1950s and 1960s, several other researchers investigated the VRS. For instance the authors in [19] succeeded in measuring the thrust and torque oscillations of a rotor in the VRS. Fast forwarding to the 1990s, the authors in [148] performed a series of wind tunnel tests on a model helicopter, and stated that the turbulence associated with the VRS seemed to be the highest when the descent angle was between 60 - 75°, hence higher than for vertical descent. A few years later, the authors in [33] found that within the VRS, recirculation occurred across most of the disk plane, while a conical region of reverse flow existed at the disk center. They reported that when the VRS was fully developed, a symmetric, low frequency, stable limit cycle behavior was evident in the inflow dynamics, blade dynamics and rigid body dynamics.

In the past decade, the authors in [108] noted that there had been a disturbing tendency for rotor behavior to vary from one VRS test to another. Indeed vehicles with similar configurations had behaved differently during VRS flight tests. One of the results was that helicopter acceleration and angular rate may have a significant influence on the location of the VRS boundary, and that blade twist, blade root cut out, blade taper, and blade stall could all play a role in the development of the flow in the VRS.

As a final note, ONERA performed a se-

ries of flight tests [87, 137] on an instrumented Dauphin 6075. One of the striking results was that contrary to the common assumption, collective increase did not amplify VRS effects. The helicopter in VRS was generally insensitive to collective inputs. The authors also found that two VRS flights starting from close conditions could imply very different helicopter reactions. It was stipulated that the nature of the VRS turbulent flow could explain this chaotic behavior.

### 3.9 VRS: induced velocity models

In the last thirty years, several VRS induced velocity models for flight dynamics simulations have been formulated. We provide here a brief shortlist

- The Georgia Tech model by Chen and Prasad [46, 125, 44, 45, 43]. The authors developed an inflow model, based on [146], to account for the additional induced inflow at the rotor due to rotor-wake interaction in the VRS
- The Johnson models [88, 90]
- The ONERA model [86, 87]. Here an analytical criterion, based on [146], giving VRS limits and intensity was formulated. Further induced velocity and its fluctuations were also modeled through an additional pseudo-harmonic function, in which the computed spectrum was matched with the experimental one
- The Peters/He model [77, 120]. At first in [77]<sup>12</sup> the Peters/He finite state wake model [121, 122] was extended by deriving an empirical formula for the uniform induced flow in the VRS. Later in [120], the

<sup>12</sup>This approach is currently implemented in FLIGHTLAB

authors modified the mass flow parameters for both dynamic inflow and dynamic wake models to allow for a more realistic uniform inflow component in the VRS region

- Russian results were translated into English in the early 1970s by Shaydakov, and expressions for rotor induced velocity in the VRS can be found in [131]
- The Young model [149]
- The Wang/Perry model [143, 118]. Here a relation between the induced velocity of the rotor and the descent velocity of the helicopter was obtained by classical vortex theory, where basically the mean wake is modeled as a truncated vortex tube. The model however was limited to uniform inflow in vertical descent

Aside from induced velocity models, we refer also to the application of momentum theory to the prediction of VRS boundaries in [146, 119], and for minimum power requirements of an ideal rotor in descent in [78]. For an application of bifurcation theory to the problem of rotor aerodynamic instability in descending flight see [21].

### 3.10 VRS: wake models

Finally we conclude this VRS section by providing a brief review of published accounts relative to wake models. For a recent survey see [140].

The first results are related to the Vorticity Transport Model<sup>13</sup> of [34, 35] which was used to analyze a VRS rotor flow in [108, 13]. Some

<sup>13</sup>Basically a direct computational solution of the incompressible Navier-Stokes equations, expressed in vorticity velocity form, which is used to simulate the evolution of the wake of a helicopter rotor [34]

of the results have already been reported in this paper, in Sections 3.3.1 and 3.7. As a last comment, it was also reported that a better description for the wake dynamics encountered in VRS might be obtained by analogy with the low-Reynolds number vortex shedding from bluff bodies [108].

For free vortex wake models<sup>14</sup> capable of capturing the distortion of the wake geometry during maneuvers and VRS descent flight, see for example the research in [101, 100, 127], where some of the main conclusions have already been reported in the previous sections of this paper. Perhaps as a last additional comment we can refer to the hysteresis effect mentioned in [100], which states that reversing the combination of VRS airspeed and rate of descent back to the initial flight condition may not lead to the same time-history of the airloads.

## 4 Autorotation

Autorotation, in the case of a helicopter, is a flight condition in which no powerplant torque is applied to the main and tail rotors. During an autorotation, the main rotor is not driven by a running engine, but by air flowing through the rotor disk bottom-up, while the helicopter is descending [7]. For comparison, in autorotation about as much buoyancy is provided as a round parachute of the same rotor diameter [6]. An autorotative flight is thus entered when the engine fails on a single-engine helicopter, or when a tail rotor failure requires the pilot to shut down the engine. The power required to keep the rotor spinning is obtained from the vehicle's potential and kinetic energy. All helicopters are thus equipped with an overrunning clutch between the transmission and the engine, so that the rotor does not have to drive a

dead engine during an autorotative flight [126]. This flight condition is somewhat comparable to gliding for a fixed-wing aircraft, without an operating power plant.

### 4.1 Engine failure

For the 1990 - 1996 time frame, the U.S. NTSB assembled data over 1165 U.S. civilian helicopter accidents, which have been analyzed in [83]. For the purpose of this accidents analysis, helicopters had been split into four cost categories: low cost 0 - 600K\$, medium cost 600K\$ - 1.5M\$, high cost 1.5M\$ - 4M\$, and very high cost >4M\$. The data show that the loss of engine power was the most frequent (> 25%) first event<sup>15</sup> in helicopter accidents, for all but the most expensive helicopter category. For the most expensive helicopters, airframe and system failure/malfunction was the most common first event. Indeed in the very high cost category there is a much higher proportion of twin-engine vehicles, hence the lower rate of accidents due to engine failure. For the other three helicopter categories, and assuming that the data would still be representative for today's systems, one needs thus to consider engine failure as a probable and potentially hazardous event.

### 4.2 Autorotation: a risky maneuver

Now in case of an engine failure, i.e. an emergency situation, autorotation as mentioned earlier is the approved response for such an emergency.

Unfortunately autorotation is a risky maneuver. Autorotation on a manned helicopter requires a good deal of training, if disaster is to be avoided. Quick reaction and critically timed control inputs are indeed required for

<sup>14</sup>For an introduction see [99]

<sup>15</sup>The first event is the first anomalous occurrence that the NTSB codes as part of the accident sequence

a safe autorotative landing [16]. A delayed or improperly performed autorotation can turn an incident into an accident or fatality [76]. Autorotation is thus sometimes regarded as a *take what comes and pray* maneuver [20, 18].

For instance a 1980 statistics, see [123], showed that at least 27% of all emergency autorotations involving the AH-1, UH-1, OH-58, and OH-6 helicopters resulted in some degree of vehicle damage or personnel injury. Also mapping out the height-velocity diagram, see Fig. 6 and Section 4.5, through a series of flight tests had historically been an area of high risk [27]. An additional 1980 paper stated that there had been very few helicopters which had completed qualification testing, without an accident of some kind during height-velocity or autorotational landing maneuvers [27]. A further 1998 study [76] showed that helicopter autorotation accounted for 7% of 1852 helicopter accidents. Such a finding was found to be particularly disconcerting, since the autorotation maneuver is the approved response to an emergency, rather than an emergency itself [129].

In fact and due to safety concerns, both the U.S. Army and U.S. Air Force stopped performing autorotation training after studies showed that there were more injuries and aircraft damage from practicing autorotation, than from autorotations required by actual engine failures [129]. Idem on the civilian side, where in-flight autorotation training is a rare event [18]. Moreover, autorotation training in a simulator occurs infrequently, as even the best simulators poorly reproduce the cues required for an actual autorotation [18].

Hence the need for either having a fully automated autorotation system, or for having a semi-automatic system capable of assisting a pilot in performing the maneuver, by for in-

stance displaying control guidance cues on a cockpit display [17].

### 4.3 Detecting engine failure

For the case of an autorotative flight following main rotor engine failure, first the engine out event needs to be recognized.

A sudden reduction in engine torque if accompanied by either reduced collective pitch or accelerating rotor speed, would not indicate an engine failure. However a sudden reduction in torque, if accompanied by fixed collective stick and decelerating rotor speed, would be indicative of an engine failure [105]. Further a jerk is generally also felt on the yaw channel, since the tail rotor overcompensates the reduced main rotor torque. This said, the case where engine power is not lost suddenly but gradually may be more subtle or elusive to detect.

Once an engine failure has been detected, the pilot's (or computer) task during autorotative flight becomes mainly one of energy management [97].

### 4.4 Autorotation: the maneuver

From a performance standpoint, autorotation may be considered as a four phase maneuver [7, 6, 4, 5, 2]. We describe hereunder general maneuver guidelines for the case of a single main rotor, manned helicopter.

**The entry.** An autorotation maneuver depends on the flight condition before engine failure [97].

First the main rotor torque loss will require a tail rotor control change to reduce tail rotor thrust.



Then at low altitude and airspeed, below the so-called knee of the height-velocity curve ( $V_{cr}, h_{cr}$ ) see Fig. 6, the recommendation is to use increased collective to reduce the sink rate as the helicopter approaches 10 to 15 feet above the ground [97]. At about 10 feet above the ground the fuselage is leveled and collective is increased as the helicopter settles.

Now for the case of higher airspeeds, still below the airspeed at the knee, the collective may be reduced somewhat to regain or maintain rotor RPM, while the helicopter is decelerated using a cyclic flare [97]. In both these previous conditions, the rotor does not enter into a true condition of autorotation.

We consider now the case of higher altitude entries, i.e. above the altitude at the knee of the curve  $h_{cr}$ . Here the collective is generally reduced, as to prevent blade stall and rapid decay in rotor RPM, and the cyclic is moved forward to pitch the nose down in order to gain some forward airspeed. Attaining higher airspeed avoids entering the VRS, allows for buildup of rotor RPM and allows for minimum sink rate until the pilot initiates the flare maneuver [97]. It should also be noted that some flight conditions are considered more critical than others, especially when collective pitch and engine torque are high at the instant of power loss. For example this may be the case when loss of engine power occurs while the helicopter is in either one of these flying conditions: heavy weight, high altitude, hover, or vertical climb. In these situations collective should be lowered quickly to avoid a too high RPM speed decay [97].

This said, a too rapid lowering of collective pitch is also not advisable. Indeed this could potentially lead to extreme rotor blade flapping excursions and/or extreme rotor force increments [105]. Therefore the control

input should be that which provides the best compromise between achieving minimum rotor speed decay, acceptable rotor flapping excursions, and acceptable flight characteristics [105].

Now for the case of even higher altitude entries, a  $25^\circ$  nose-down pitch attitude could be a guideline, producing a rapid airspeed increase until minimum rate of descent is reached [65]. According to [65], shallower pitch attitudes delayed reaching the target airspeed and increased the altitude lost in the entry maneuver, while steeper nose down attitudes to quickly gain airspeed reduced the initial altitude loss but produced also higher rate of descents<sup>16</sup> which eventually resulted in excessive altitude loss.

**Steady autorotation.** This refers to the stabilized autorotation phase in which the helicopter descends at a constant rate, which may be chosen for minimum rate of descent<sup>17</sup> or maximum glide distance. In general the rotor RPM and rate of descent are controlled by collective setting, while airspeed is controlled by cyclic settings.

Regarding the main rotor, some stations on the rotor will absorb power from the air, while others will consume power, such that the net power at the rotor shaft is zero, or sufficiently negative to make up for losses in the tail rotor, transmissions, and accessories [126, 99]. In autorotation the helicopter needs a little less power than in powered flight, since the tail rotor does not require as much power.

<sup>16</sup>Note that high rate of descents and high airspeed complicate the flare maneuver since more energy needs to be dissipated

<sup>17</sup>Minimum rate of descent in power-off autorotation in forward flight occurs at the minimum power speed [89], the so-called *bucket speed*

Concerning the rate of descent, an increase in solidity<sup>18</sup> would result in a decrease of the rate of descent [51, 63], while an increase in bank angle or sideslip angle would result in an increase of the rate of descent [61].

Additionally, main rotor collective plays a crucial role in steady autorotation. On the one hand, as the air starts flowing up through the rotor system, the main rotor RPM will start to increase, and hence may get too high. In this case, an increase in collective pitch is required to lower main rotor RPM. On the other hand, main rotor collective pitch angle should be kept low enough to prevent rotor stall. Indeed the angle of attack over the inboard stations of the rotor blades is generally high, hence corresponding to a stalled region. Therefore collective pitch should be kept low enough to prevent stall propagating out from the blade root region, which will tend to quickly decrease RPM, because of the high profile drag associated with stall [99].

**Flare for landing.** The approach to the landing area should preferably be done into the wind<sup>19</sup>. The purpose of the flare is to reduce the sink rate, reduce forward airspeed, maintain or increase rotor RPM, and level the attitude for a proper landing, i.e. tail rotor clearance. This is generally done in the following way [63]: by first moving cyclic to the rear as to reduce the sink rate and reduce forward airspeed. The next step is to apply cyclic forward in order to level the attitude, while increasing collective. The increase in collective is generally done to prevent rotor overspeed, and reduce the sink rate even further.

<sup>18</sup>Relative blade area

<sup>19</sup>Note that wind shear and direction changes near the ground may suddenly reduce an apparent safe condition to an unsafe one [61]

It should be noted that there is an optimal altitude above ground at which the flare should be initiated, and this altitude becomes increasingly critical as vehicle weight is increased<sup>20</sup>. This optimal altitude depends on many factors, including descent rate, airspeed, headwind component, and the bandwidth of the control system.

As a final note on flare, one should be aware that the helicopter flare capability is even more important for power-off landings than the steady-state descent rate [89]. The flare capability depends among others on a high rotor kinetic energy [133], which requires a high rotor speed and/or a large blade moment of inertia. Additionally the flare capability is also improved as disk loading decreases [133], and further depends also on blade stall limits. In general blade stall margin should be high<sup>21</sup>, both for good flare characteristics, and for a minimal loss of rotor speed before lowering collective just after power failure [89, 142].

**Touchdown.** Here for obvious reasons, maximum rate of decent and airspeeds should not be exceeded. Helicopter horizontal deceleration is further achieved through the friction force between the ground and the skids/wheels.

#### 4.5 The Height-Velocity (H-V) diagram

The capability of a helicopter to perform a safe autorotative landing after a power failure is limited by the structural and aerodynamic design of the helicopter, for certain combinations of altitude above ground and airspeed [116]. In fact power failure within the dangerous or

<sup>20</sup>Note that an increase in weight may also result in rotor overspeed

<sup>21</sup>Even though a transient flare maneuver will result in a delayed blade stall and lift overshoot, or so-called *dynamic stall*

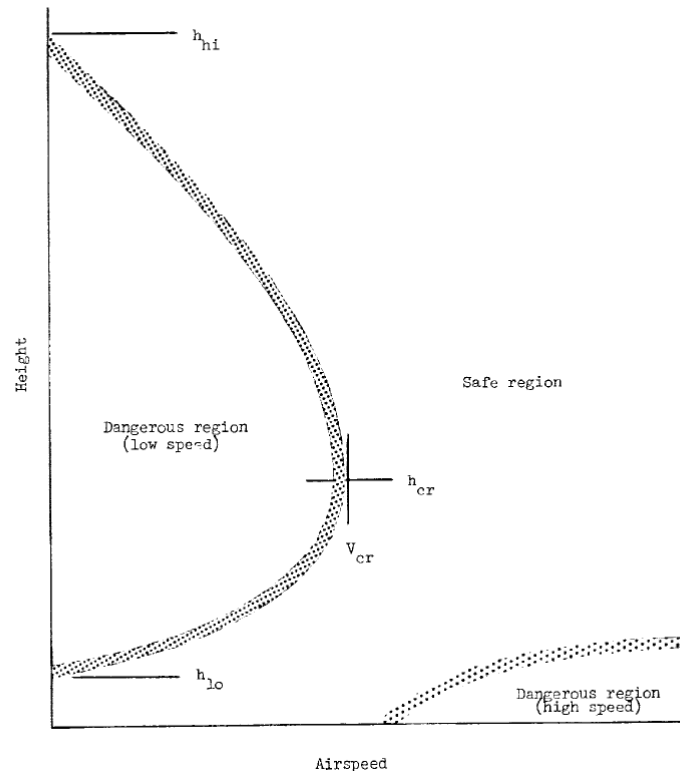


Figure 6: Typical Height-Velocity diagram (from [116])

unsafe regions defined by these combinations of geometric height and airspeed may result in high risk of severe damage to the aircraft and/or injury to its occupants. These limiting combinations of airspeed and height are often expressed as the height-velocity diagram<sup>22</sup>, see Fig. 6.

Knowledge of these dangerous regions is important for safety procedures and operational reasons. Ideally one would like to eliminate these unsafe regions altogether, or at least reduce their size. For example, eliminating height-velocity restrictions had already been demonstrated with the *Kolibrie* helicopter, built by the Nederlandse Helikopter Industrie (NHI) in the late 1950s. It was designed by Dutch helicopter engineers and pioneers Jan M. Drees and Gerard F. Verhage. The

<sup>22</sup>Also called the *deadmans zone*

helicopter was ram-jet powered, and these were positioned at the blade tips. This resulted in very high main rotor rotational inertia. Later in the U.S.A. the concept of the so-called High Energy Rotor (HER) [147, 52] was investigated, using blades with high rotational inertia. The goal of the HER was to eliminate the unsafe regions, but also to allow for less demanding autorotation maneuvers, and finally to use the rotor kinetic energy as a source of transient power for better maneuverability.

Now operations in these unsafe regions are not uncommon, and may occur during specific missions, such as cargo sling, hoist<sup>23</sup> operations, or aerial photography [23]. Consequently, pilots operating in these unsafe

<sup>23</sup>The mechanism by which external loads may be raised or lowered vertically

regions are exposed to a higher level of risk, because the potential to recover from an emergency, such as power and drive train failure, is significantly reduced.

Height-velocity investigations can be traced back to the late 1950s and early 1960s, see [92, 84, 73]. In [116] flight-test data was used to derive semi-empirical functions of a generalized non dimensional height-velocity diagram, independent of density altitude and gross weight variations. In the late 1970s it was pointed out in [23] that high rotor inertia, low disk loading, and a high maximum thrust coefficient could reduce the size of these unsafe zones. Finally more recent height-velocity flight tests can be found in [136] for the Bell 430, and in [58] for three Bell 407 main rotor configurations.

#### 4.6 Factors affecting autorotation

We provide here a brief review of published accounts related to autorotative experimental investigations and analytical modeling. For additional topics such as autogyro rotor dynamics at high advance ratio see [128], and for a detailed review of passive and active autorotational assist concepts, such as auxiliary energy devices, tip jets, flywheel, multiple engines, compressed gas, hydraulic turbine, batteries, electric motor, magnetic rotor head, extra tip weights, rotor overspeed, parachute, rockets, improved landing gear, and air bag, see [145, 132, 32].

Autorotation investigations can be traced back to the 1920s in [70, 103]. Later in the 1940s it was found in [111, 110] that for steady autorotative vertical descent, the approximation of a constant induced velocity lost its significance once the blades stalled, at higher pitch angle. It was already stated in these studies that blade stall could be avoided by reducing blade pitch rapidly, and by using

blades with large moment of inertia.

For autorotative flight tests and performance investigations in terms of rate of descent, and lift to drag ratio see [68, 138], for the effect of collective pitch see [117], for the effect of weight see [116, 144, 65, 3], for the effect of Reynolds number see [109], for the effect of air density see [116, 126], for the effect of flight path angle see [67], and for a review of autorotation indices see [123].

We elaborate here a little further on the effect of weight. It is known that an increase in vehicle weight does not appreciably affect touchdown rate of descent and ground run distance [144, 3]. However at heavy weight, frequent collective control adjustments are required to prevent overspeeding of the main rotor during the flare, hence the timing of the collective application becomes more critical at heavy weight.

Regarding aeromechanical stability during autorotation, it is known that dynamic inflow significantly improves body damping correlation with experimental results, in roll and pitch mode [66]. Further there is an intimate relationship between main rotor RPM and the low frequency rigid-body modes, such as pitching-moment [79]. Also the roll attitude is reversed in autorotation, the absence of the tail rotor force means that the main rotor in-plane sideforce<sup>24</sup> can now dominate, resulting in an opposite roll attitude [80]. There is also a reduced speed stability in autorotation [80]. Finally a high autorotative rate of descent in a fairly level attitude gives a large positive tail plane angle of attack, producing a considerable nose down moment, hence requiring a more aft cyclic position [80].

<sup>24</sup>This sideforce is due to the nonuniform inflow along the rotor disk

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Before finishing our discussion on autorotation, we address one last aspect related to blade design. It was already found sixty years ago that twist had a significant effect on inflow during descent [42]. Recently in [48] the downwash and upwash regions along blade span were analyzed and it was shown that, for the case of vertical autorotation, a large positive blade linear twist leads to the highest autorotation efficiency, i.e. rotor thrust being maximum for a given vertical flight velocity.

## 5 Optimal autorotation

Finally the purpose of our paper is to provide a literature review relative to the optimal, and possibly automatic, autorotation problem, and its solution through optimal control. The reviewed research covers the case where an autorotative landing of a single-engine helicopter is formulated as a nonlinear, constrained optimal control problem.

### 5.1 Automatic autorotation

It is first relevant to point out that currently in service manned helicopters do not autorotate automatically. This said, the concept of having an avionics system performing an automatic autorotation is not new. In 1970 such a system was already described for high powered, high speed helicopters in [105]. Its role was to detect engine failure and to automatically lower collective stick in a timely manner, before rotor stall was encountered. The concept however did not go as far as to describe automatic landing of a helicopter after power failure.

It is also interesting to note that an automatic autorotation system could potentially represent an alternative to multiple engine helicopter configurations [105], as a safeguard

against catastrophic engine failure. By eliminating the need for multi-engine helicopters, the availability of such a system could obviously translate into substantial cost savings, especially since helicopter purchase price is even more sensitive to installed power than to empty weight [75].

We move now our discussion towards the field of optimal control.

### 5.2 Optimal control

First a helicopter can be seen as a dynamical system, which is at any given time defined by its state. The system accepts inputs, over which it has no direct command but to transform them into outputs. When one or more of the systems output variables need to follow some desired value over time, i.e. a reference signal, a controller may manipulate the systems inputs to obtain the desired effect on the output [64]. A further extension of this concept is provided by optimal control, in which the controller determines the input signals in order to minimize some performance criterion, while satisfying the system physical constraints [124, 93, 38].

As with most nonlinear control problems, the resulting formulation does not have a closed-form solution, hence not solvable analytically but through numerical algorithms [74, 57, 81, 24, 82]. Numerical algorithms for nonlinear optimal control can be separated into two main categories: indirect and direct methods [130].

In the early years of optimal control (1950s-1980s) the favored approach for solving general optimal control problems was that of indirect methods. In an indirect method, the calculus of variations [124] is employed to obtain the first-order optimality conditions. These

conditions result in an infinite dimensional non-linear two-point or multi-point boundary value problem. To solve this problem, a suitable numerical method is used to render it of finite dimension. Examples of numerical methods include shooting methods, finite element discretization, and gradient methods such as the Sequential Gradient Restoration Algorithm (SGRA). Indirect methods are known to show a fast numerical convergence in the neighborhood of the optimal solution, and to deliver highly accurate solutions [72]. However they require the derivation of the optimal control equations, a potentially complicated task for complex systems. An additional disadvantage resides in the necessity and difficulty to provide suitable starting guesses, for all variables including the adjoint variables.

On the other hand, direct methods have risen to prominence in numerical optimal control since the late 1980s [1]. In a direct method, discretization is followed by optimization. The discretization can be done in a number of ways, such as collocation, control discretization, shooting, direct transcription, and pseudospectral. The cost function, constraints and boundary conditions, are all expressed in terms of the discrete values of the states and controls. This in turn defines a finite-dimensional Non-Linear-Programming (NLP) problem [22]. The obtained NLP problem is efficiently solved by Interior Point (IP) methods [91], or by Sequential-Quadratic-Programming (SQP) [112]. An additional feature of direct methods is that they enjoy large convergence radius, allowing for a converged optimal solution from a large range of different initial guesses [72].

We provide next a literature survey relative to optimal autorotation through nonlinear optimal control.

### 5.3 Optimal autorotation through indirect methods

One of the first accounts of optimal autorotative flight dates back to the early 1970s [94]. The helicopter model was a 2-D<sup>25</sup> quasi-steady point mass approximation, which was subsequently linearized about a trimmed flight condition. Further unsteady aerodynamics effects, such as blade dynamics, were neglected. Also an instantaneous response in vehicle pitch was assumed, and rotor RPM was allowed to vary. Additionally, and to allow for simplicity, thrust inclination and its magnitude were taken as control inputs, instead of the pilot controls. The constraints included bounds on thrust tilt angle, angle of attack, rotor RPM, and rate of descent.

The first application of nonlinear optimal control to the problem of optimal autorotative flight path was presented in [88]. The helicopter model was a 2-D nonlinear model, see [69]. Here the 1-D inflow model was based on momentum theory, with a time lag, including a correction for ground effect, and an empirical model for the induced flow in the VRS<sup>26</sup>. The cost function of the optimal control problem was a weighted sum of the squared horizontal and vertical velocities at touchdown. To allow for simplicity, the force coefficients  $C_X$  and  $C_T$  were taken as control inputs, instead of the pilot controls. A steepest descent algorithm was implemented to solve for a two-point boundary problem. The method showed that the optimal control solution for descent from hover, after power loss, was a purely vertical flight<sup>27</sup>.

<sup>25</sup>Hence in the vertical plane

<sup>26</sup>It was pointed out in [142] that the basis for this inflow model was experimental data from helicopter rotors with blades having low twist

<sup>27</sup>However in practice some forward speed is highly advisable as to avoid the VRS

A similar model and cost objective were used in [97, 96], with this time the additional inclusion of inequality constraints on the control inputs to represent the limitation of the rotor thrust coefficient, and a path inequality constraint on vehicle sink-rate during descent. The optimal problem was solved with the SGRA algorithm. In [95] the model was modified to include bounds on collective pitch, and rotor angular speed, which are more closely related to physical limits. It was also shown that a substantial reduction in the height-velocity restriction zone could be made using nonlinear optimal control.

Further attempts to find and minimize the height-velocity zone were presented in [114, 113]. Here a more detailed 2-D nonlinear model was used. In particular it included quasisteady Tip-Path-Plane (TPP) flapping motion, and an empirical inflow model valid in the VRS, although different from the one presented in [88, 97]. The optimal problem included bounds on control inputs and states, and was solved with the SGRA algorithm. In particular these studies found that pilot reaction time to engine failure, location of the landing area, and wind speed had a significant impact on the success of the emergency maneuver. Additionally the knee point  $(V_{cr}, h_{cr})$  in Fig. 6 had a higher velocity and a lower height when the helicopter was climbing rather than descending. In climb mode the power requirement was obviously higher, therefore at the instant of power failure the decay of rotor speed would be more rapid than if the helicopter was in a descent mode.

A recent contribution was presented in [62], where partial power recovery to a flyaway condition, partial power landing, and autorotation were analyzed, based on a 2-D nonlinear model [69, 88]. The optimal problem was also solved with the SGRA algorithm. The optimization

problem was extended here with the inclusion of bounds on control input rates, and the cost functional included also an integral part, basically a running cost over time, to penalize deviations from nominal values on main rotor RPM, and helicopter forward and vertical velocities.

#### 5.4 Optimal autorotation through direct methods

The first application of a direct method to solve a nonlinear autorotative optimal control problem was presented in [39], for a 2-D nonlinear model of a tiltrotor aircraft. The direct collocation method was used to discretize the problem, which resulted in a large-scale NLP problem. The numerical optimizer NPSOL [8] was then used to solve the problem.

The first application in the case of a helicopter was presented in [40]. The model was again a 2-D nonlinear model, similar to the one of [88], with wind and RPM degrees of freedom, but excluding rotor dynamics, and having the thrust coefficient and TPP angle as control inputs. Constraints included also bounds on control input rates. Here too the collocation method was selected, coupled this time to the SNOPT numerical optimizer [9]. A similar optimization problem was also investigated in [20], and see also [85, 20, 41] for the derivation of height-velocity diagrams.

Later the results of [40, 20] were extended in [16, 17] by including a flare law, that would take over from the optimal guidance at a pre-determined altitude near the ground, and flare the helicopter based on a more conventional compensatory control law. The system was also used to provide guidance cues on a cockpit pilot display, and was flight tested on a Bell 206 flight simulator [17].

The first application to solve a 3-D non-

linear autorotative optimal control problem was presented in [25, 26]. The model included the usual rigid body motion kinematics, with additional degrees of freedom in main rotor RPM, and available shaft power. Also Glauert's formula was used for the induced inflow model, and quasisteady blade flapping was included. However the model was not published in English. The functional cost included weights which were made altitude dependent. It is assumed that a Nelder-Mead<sup>28</sup> [104] numerical algorithm was used to solve the optimal control problem.

In [50] the use of an on-line, nonlinear, model-predictive controller augmented by a recurrent neural network was presented. The helicopter model was similar to the one presented in [88, 97], and the optimal problem was solved by a series of recurrent neural networks. However an investigation of the convergence characteristics of the neural network, i.e. whether convergence can be guaranteed and under what conditions, is still necessary.

We conclude here by mentioning extensive results for general helicopter trajectory optimization in 2-D in [28], and in 3-D in [29, 30].

### 5.5 Optimal autorotation through robot learning

Machine and robot learning represent a branch of statistics and computer science, which explores algorithms and associated architectures that learn from observed facts [106, 135, 15]. Additionally, and ideally, the robots need to be able to adapt to future real-world changes. The algorithms of machine learning are organized by a taxonomy based

<sup>28</sup>Based on evaluating a function at the vertices of a simplex, then iteratively shrinking the simplex as better points are found until some desired bound is obtained

on the knowledge of the provided outcome [56]. The three main classes of concern are supervised learning, unsupervised learning and reinforcement learning<sup>29</sup> [56].

In 2004 a group at Chiba University (Japan) [59] investigated the autorotation flight of a R/C<sup>30</sup> helicopter, based on a coupled Proportional Integral neural network controller. It is however unclear whether the group successfully flight tested an automatic autorotation system, since results have only been published in Japanese.

The first reported automatic autorotation was performed at Stanford University in 2008, on a R/C helicopter [12, 11, 10], using an apprenticeship learning based approach. The procedure included the collection of flight data from an expert pilot. The data was then used to build a dynamics model of the helicopter in autorotation. An autorotation trajectory was further split into three flight phases: glide, flare, and landing. The model included the usual rigid body motion kinematics, with an additional degree of freedom in main rotor RPM. Finally Differential Dynamic Programming (DDP), an extension of the Linear Quadratic Regulator (LQR) formalism for non-linear systems, was used to derive a feedback controller.

Further work in automatic autorotation through reinforcement learning can be found in [98], where the algorithm was evaluated by simulations based on a point-mass model of a modified OH-58A helicopter, similar to the 2-D model presented in [88].

We conclude here by mentioning results for a pilot support system utilizing fuzzy system

<sup>29</sup>Also known as neuro-dynamic programming

<sup>30</sup>Remotely Controlled



techniques for helicopter autorotation in [139].

## 6 Conclusion

The primary purpose of this paper was to present an updated, albeit brief, representative snapshot of the VRS, autorotation and optimal autorotation. The main focus was from a common qualitative approach, with emphasis on concepts rather than on details. This survey was essentially tailored for researchers interested in designing control systems, for helicopter flight in the VRS and autorotation, such as automatic VRS avoidance, automatic recovery from VRS flight, and automatic autorotation.

In conclusion as this review demonstrates, there was and still is considerable activity in research related to the VRS and autorotation. Future helicopter developments in these aforementioned areas will undoubtedly focus on methods tailored towards advanced computational fluid dynamics and wake modeling, and state of the art control systems.

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