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Sound diffraction by the splitter in a turbofan rotor-stator gap swirling flow

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Summary

Rotor-stator interaction noise is one of the main noise sources in modern turbofan engines. Propagation of sound waves in the rotor-stator gap is affected by the mean swirling flow and by diffraction by the splitter. In a previous study on uniform flow propagation it was shown that the effect of diffraction is to redistribute the sound energy over the radial modes. A mean swirling flow alters the axial wave number and the mode shape of the radial modes. The combined effects of diffraction and mean swirling flow can be calculated by means of a generalized matching technique. This technique and some results for a relevant turbofan configuration are presented.

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List of symbols

A	modal coëfficient
h	hub-to-tip ratio
m	circumferential harmonic
$N1, N2, N3$	number of modes in region I, II, and III, respectively
p	pressure (perturbation)
r, θ, x	cylindrical co-ordinates
s	radial splitter position
t	time co-ordinate
u_r, u_θ, u_x	perturbed velocity components
V_θ, V_x	mean velocity components
$\vec{v}$	velocity vector
α	axial wave number
γ	specific heat ratio (= 1.4)
ρ	density (perturbation)
Ω	rotational shaft speed
ω	frequency

subscript:

0	mean flow property
$j = 1, 2, 3$	in region I, II, or III
$\mu = 1, 2, 3, \ldots$	radial mode number

superscript:

+	downstream propagating
-	upstream propagating

SOUND DIFFRACTION BY THE SPLITTER IN A TURBOFAN ROTOR-STATOR GAP SWIRLING FLOW

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Abstract

Rotor-stator interaction noise is one of the main noise sources for modern turbofan engines. Propagation of sound waves in the rotor-stator gap is affected by the mean swirling flow and by diffraction by the splitter. In a previous study on uniform flow propagation it was shown that the effect of diffraction is to redistribute the sound energy over the radial modes. A mean swirling flow alters the axial wave number and the mode shape of the radial modes. The combined effect of diffraction and mean swirling flow can be calculated by means of a generalized matching technique. This technique and some results for a relevant turbofan configuration are presented.

INTRODUCTION

For an adequate analysis of the environmental problem of aircraft noise, a physical model for each of the noise components is needed. One of the main noise sources for modern turbofan engines is rotor-stator interaction noise. Downstream of the fan the duct of a turbofan engine splits into a by-pass duct and an engine duct. Since the fan rotates, swirl is added to the airflow in the ducts. In order to take this swirl out of the airflow and recover its energy, stators are placed in the engine duct and in the by-pass duct. Rotor wakes interact with the stator vanes and thus generate sound.

In order to calculate the radiated rotor-stator interaction noise, viscous wakes coming from the fan blades can be modelled analytically (Ref. 1). Interaction of the wakes with the stator vanes and interaction of generated sound waves with blade rows can be calculated by means of the NLR lifting surface model (Refs. 2, 3). Up to now, the propagation of sound waves between rotor and stator, including diffraction by the splitter, was based on a uniform flow assumption (Ref. 4). In this paper a method is described that also includes the effect of a non-uniform, swirling flow on the propagation of sound. With this method more accurate input can be generated for rotor blockage calculations (Ref. 3) and liner optimisation studies (Ref. 5).

The diffraction of acoustic waves by the splitter in a turbofan rotor-stator gap is investigated using a generalized mode matching technique. The traditional mode matching technique is well known in the electromagnetic theory of waveguides (Ref. 6) and it is also applied in acoustics (Ref. 7). Results of our generalized matching technique are validated against an analytical Wiener-Hopf solution for uniform flows that was obtained by Nijboer & Sijtsma (Ref. 4).

GEOMETRY AND FLOW CONDITIONS

Geometry

The diffraction problem is considered in an infinitely long, straight, cylindrical duct having a constant hub radius. All length scales are made dimensionless by the (constant) duct radius. The hub position is given by the hub to tip ratio h. For $x > 0$ the duct is split by an infinitely thin splitter. The (constant) radial position of the splitter is given by s. Hence, the geometry can be considered to be three half-infinite ducts: $x < 0$, $h \le r \le 1$ (region I), $x > 0$, $h \le r \le s$ (region II), and $x > 0$, $s \le r \le 1$ (region III), as shown in figure 1.

Mean Flow

The mean flow is assumed to satisfy the Euler equations. All variables are scaled by the mean density at the duct outer radius (r = 1), and the speed of sound at the duct outer radius. The mean density, ρ, the mean velocity, $\vec{v}$, and the mean pressure p take the form

$$\rho = \rho_0(r),\ \vec{v} = V_\theta(r)\vec{e}_\theta + V_x(r)\vec{e}_x,\ p = p_0(r) = \frac{1}{\gamma} - \int_r^1 \frac{\rho_0 V_\theta^2}{\tilde{r}} \mathrm{d}\tilde{r}\ . \tag{1}$$

Here γ denotes the specific heat ratio and is taken to be 1.4. The radial profile for the mean density and for the mean velocity components is free to choose. Note that the mean flow profile (1) is valid in all three regions.

Perturbations

In order to describe disturbances superimposed on the mean flow, the Euler equations are linearised. We assume that the disturbances have an exponential dependence like

$$f(x,r,\theta,t) = f(r)\exp(i\alpha x + im\theta + i\omega t)\ . \tag{2}$$

In each of the regions I, II, and III the linearised Euler equations reduce to a set of eigenvalue equations for the eigenvalue α. With each eigenvalue α corresponds a vector of eigenfunctions $\{\rho(r), u_r(r), u_\theta(r), u_x(r), p(r)\}$. In uniform flow the eigenvalues and eigenfunctions can be determined analytically in each of the three regions. In a non-uniform flow they can be calculated by means of the SWIFDA code (SWIrling Flow Duct Acoustics) that is described by Nijboer (Ref. 8).

The equations admit a solution having a square root singularity at the leading edge of the splitter. This type of singularity is well known for uniform flows (Ref. 9). When an axial flow is present, all variables except the circumferential velocity exhibit a square root singularity. The only effect of the mean swirl on the singularity is to change the local speed of sound.

Values For Example Calculations

For the examples presented in this paper a realistic turbofan cutback flight condition is chosen.

The turbofan rotor-stator gap is described by a hub to tip ratio h = 0.6 and a splitter position s = 0.752. Both for the uniform flow and swirling flow cases

$$\rho_0(r) = 1, \quad V_x(r) = 0.45. \tag{3}$$

In the uniform flow case there is no swirl. In the swirling flow case a realistic profile for the swirl is

$$V_\theta(r) = 0.225/r + 0.1r. \tag{4}$$

Consider an engine having 26 rotor blades and 65 stator vanes. The fan has a rotational shaft speed Ω = 1.15. For this situation 2 times Blade Passing Frequency corresponds to m = 13 and ω = 60.

EFFECT OF MEAN SWIRLING FLOW

In the rotor-stator gap there can be acoustic perturbations propagating both upstream and downstream and vorticity and entropy perturbations that propagate downstream. Due to the non-uniformity of the mean flow the acoustic type disturbances and the vorticity and entropy type disturbances can not be treated as uncoupled. The acoustic perturbations are pressure dominated, but, due to non-uniformity of the mean flow, have small vorticity component. The vorticity and entropy perturbations have a small pressure component due to the non-uniformity of the mean flow. Also, the acoustic type eigenfunctions do not form a complete orthogonal set for the pressure disturbance anymore.

Due to a mean swirling flow the axial wave numbers and the corresponding eigenfunctions change for all modes. Compared to uniform flow, the number of cut-on modes can be different in a swirling flow. For example, in region I, for the discussed values for mean flow and perturbations, in uniform flow 9 upstream and 9 downstream modes are cut-on, whereas in swirling flow 10 upstream and 10 downstream modes are cut-on. In region II 4 upstream and downstream modes are cut-on, both for uniform and swirling flow. In region III this number is 6, again both for uniform and swirling flow.

The effect of swirl on the eigenfunctions is most profound for the lowest order radial modes. For non-axisymmetric ($m \neq 0$) perturbations in a uniform flow these modes are localized near the highest radial position. For certain swirl profiles and circumferential mode numbers, however, this may change towards localization near the lowest radial position. It is anticipated that for these modes the effect of swirl on diffraction will be large.

DIFFRACTION BY MODE MATCHING

For the diffraction of sound waves we assume that in the matching plane $x = 0$ there is no coupling between acoustic type waves and vorticity or entropy type waves. In uniform flows this is a valid assumption. In non-uniform flow this is at least a good approximation. In each of the three regions the solution will therefore be decomposed in an infinite series of acoustic type eigensolutions.

In order to calculate the effect of diffraction for non-uniform flow a generalized mode matching technique is developed. The idea behind the matching technique is quite simple: given a sound field that propagates towards $x = 0$ from the three regions, match waves that are outgoing from $x = 0$, such that the total field is continuous at $x = 0$.

In order to describe the sound propagation the duct modes in the three different regions are calculated. The traditional way for the matching technique (in uniform flow) is then to make use of the orthogonality properties of the pressure eigenfunctions and construct a scattering matrix (Refs. 6, 7). Due to the swirling flow, however, the eigensolutions are not orthogonal. Therefore, the mode matching technique has to be generalized by using a least-squares approach on pressure and velocity eigenfunctions.

In order to have a completely continuous solution infinitely many modes are required for the response field. However, it is only possible to include a finite number. At $x = 0$ this yields

$$\begin{aligned}
&\sum_{\mu=1}^{\mathrm{N1}} A_{1\mu}^{-} p_{1\mu}^{-}(r) - \sum_{\mu=1}^{\mathrm{N2}} A_{2\mu}^{+} p_{2\mu}^{+}(r) = \sum_{\mu=1}^{\infty} A_{1\mu}^{+} p_{1\mu}^{+}(r) - \sum_{\mu=1}^{\infty} A_{2\mu}^{-} p_{2\mu}^{-}(r), \quad r < s, \\
&\sum_{\mu=1}^{\mathrm{N1}} A_{1\mu}^{-} p_{1\mu}^{-}(r) - \sum_{\mu=1}^{\mathrm{N3}} A_{3\mu}^{+} p_{3\mu}^{+}(r) = \sum_{\mu=1}^{\infty} A_{1\mu}^{+} p_{1\mu}^{+}(r) - \sum_{\mu=1}^{\infty} A_{3\mu}^{-} p_{3\mu}^{-}(r), \quad r > s,
\end{aligned} \tag{5}$$

where $p^{\pm}_{j\mu}$ are the downstream (+) and upstream (–) propagating pressure eigensolutions. Here $j = 1, 2, 3$ corresponds to region I, II, and III and μ denotes radial order. The left-hand-side of equation (5) consists of the unknown outgoing waves; the right-hand-side consists of known incoming waves. For the other variables similar equations hold. The numbers N1, N2, and N3 must be chosen in such a way that the approximate solution converges towards the physical solution. For this

$$(\mathrm{N1}-1)=(\mathrm{N2}-1)+(\mathrm{N3}-1), \quad (\mathrm{N2}-1)/(\mathrm{N3}-1)=(s-h)/(1-s), \tag{6}$$

should be used (Refs. 6, 7). For the examples presented below N1 = 201, N2 = 77, and N3 = 125 were used.

The eigensolutions are calculated on a uniform grid and the matching is performed point wise on these grid points. This means that the number of equations used for the three regions has the same ratio as the number of modes used. For convenience the radial position of the splitter, *s*, is chosen to coincide with one of the grid points.

Due to the singular behavior of the solution at the leading edge of the splitter the convergence of this matching technique will be relatively slow and a lot of modes are needed. Not only choosing the pressure and the axial velocity as matching variables (as is done traditionally), but also the circumferential velocity can improve the performance of the matching technique. The latter variable does not exhibit singular behavior. In order to diminish the effect of the square-root singularity on the convergence even further a weight function is introduced. This weight function is applied to the pressure and axial velocity components and weighs the splitter position less severely than the other positions by canceling the square-root behavior. For the circumferential velocity a constant weight function is used.

In this way at every grid point three linear equations are obtained for the unknown amplitudes. This generates a large set of equations, which is solved for the unknown amplitudes in the least-squares sense. The results of this technique are presented below.

Validation For A Uniform Flow

The effect of diffraction in a uniform flow can be seen in figure 2 where the real part of the pressure is shown after diffraction. The axial domain is chosen to range from $x = -0.2$ to $x = 0.1$, which is typical for a turbofan rotor-stator gap, where the trailing edge of the rotor is at $x = -0.2$ and the leading edge of the stator rows is at $x = +0.1$. The incoming field consisted of one radial harmonic (μ) with unit amplitude.

Results of the matching technique are compared to the analytical results of the Wiener-Hopf technique for uniform flows. Due to the singular behavior of the solution at the leading edge of the splitter, the solution in the plane $x = 0$ is less accurate when using the matching technique. However, the error is predominantly in the high order radial harmonics, which are cut-off.

Therefore, away from the $x = 0$ plane the accuracy of the matching solution is far better than in the plane $x = 0$.

In figure 3 the pressure solution at x = -0.2 and at x = 0.1 is shown. This is done for diffraction of the first radial harmonic coming from region III (figure 2a). In each plot two symbols are shown, one for the Wiener-Hopf solution (○) and one for the matching solution (+). Both symbols are on top of each other. Hence, the presented matching technique is an accurate technique for calculating diffraction problems. Moreover, it can be extended to non-uniform mean flows.

Application To A Swirling Flow

In figure 4 the solution after diffraction is shown for a mean swirling flow. The same cases are presented as for the uniform flow case of figure 2. That is, the same radial order eigenfunctions are used as input, although the eigenfunction shape is different. The most dramatic effect is for the $\mu = 1$ mode coming from region III (Fig. 2a vs. 4a). Due to the fact that the shape of the eigenfunction has changed from localization near the outer radius in uniform flow towards localization near the splitter in swirling flow, the splitter plate almost acts as a point source in the swirling flow case, whereas in uniform flow the wave propagates almost unchanged into region I. Also, in swirling flow the $\mu = 1$ mode coming from region II propagates almost unchanged into region I (Fig. 2c vs. 4c). In uniform flow this wave is much more diffracted. This is again due to the change in eigenfunction shape.

For the $\mu = 6$ mode coming from region III one can clearly see the change in axial wave number (Fig. 2b vs. 4b). Also the propagation of the $\mu = 10$ mode in region I for the swirling flow case is visible, as opposed to the $\mu = 9$ mode being the highest cut-on mode for uniform flow in this region. The $\mu = 4$ mode coming from region II also shows the change in axial wave number (Fig. 2d vs. 4d). Moreover, the amplitude of the solution in region I is lower in uniform flow than in swirling flow.

When comparing figure 2 with figure 4 it can be concluded that both diffraction and swirl have an important effect on the propagation of sound in the rotor-stator gap.

CONCLUSIONS

Propagation of sound in the rotor-stator gap of a turbofan engine can be significantly affected by diffraction by the splitter and by the mean swirling flow. In this paper a method is presented for the calculation of both effects. The method is based on matching unknown outgoing waves to known incoming waves by means of a least square fit technique. The waves themselves are obtained from the NLR SWIFDA code for the calculation of eigensolutions in a non-uniform swirling flow.

The matching technique can be considered to be an extension of the traditional scattering matrix technique for uniform flows. The technique was validated against an analytical solution for uniform flows that was obtained by means of a Wiener-Hopf method. Results from both techniques were in perfect agreement.

Results for uniform flow show that the effect of diffraction can be quite large. Differences up to a couple of decibels per mode can arise in the sound pressure level that is transmitted through the rotor (Ref. 4). The reason being that due to diffraction the energy is redistributed over the radial modes and, hence, the rotor effectively blocks a different sound field. Swirl alters the axial wave numbers and the shape of the eigenfunctions. For some conditions this change can be quite large. Because of this, the energy of the incoming sound field and of the outgoing response field is distributed differently over the radial modes compared to a uniform flow. Using the matching technique this effect can be calculated and accurate input for rotor blockage calculations or liner optimization studies can be generated.

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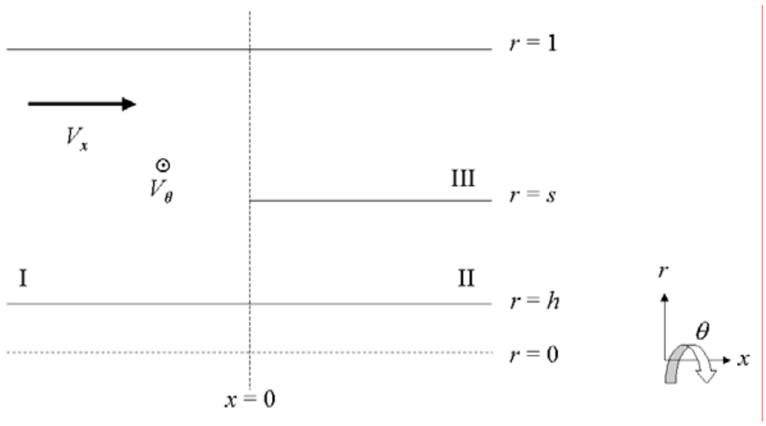


Figure 1 Geometry

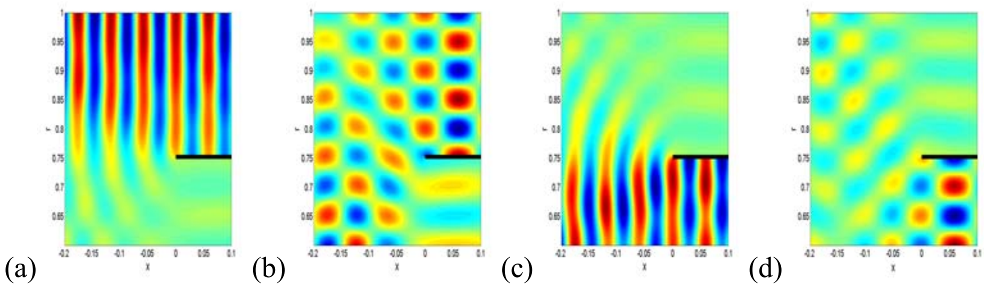


Figure 2 Real part of the pressure component after diffraction in uniform flow. Incoming waves from region III $\mu = 1$ (a), $\mu = 6$ (b), and from region II $\mu = 1$ (c), $\mu = 4$ (d).

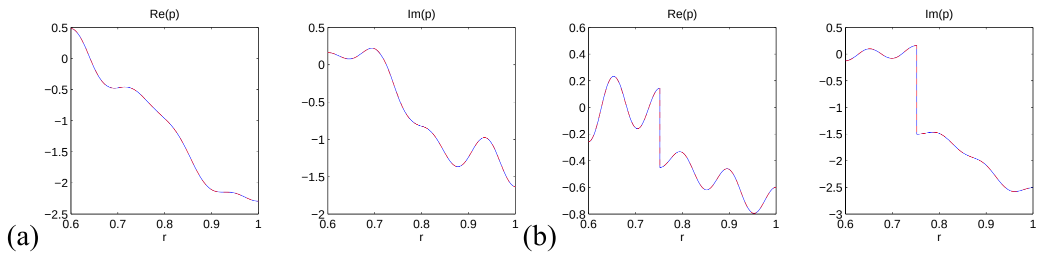
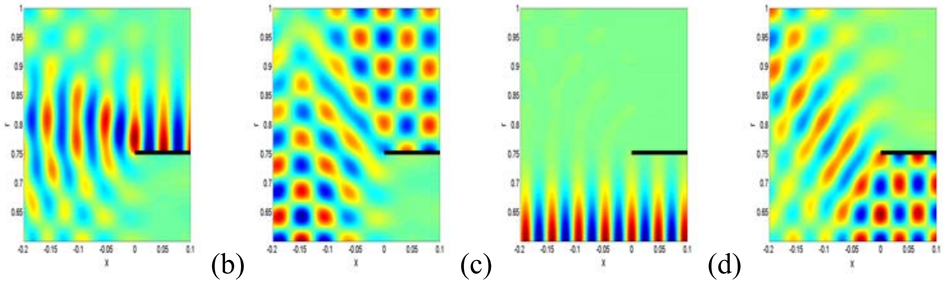


Figure 3 Pressure solution at x = -0.2(a) and at x = 0.1 (b) for diffraction of the first radial mode ($\mu = 1$) from region III. Solution by Wiener-Hopf technique (blue solid line) and Matching technique (red dashed line).



(a) (b) (c) (d)

Figure 4 Real part of the pressure component after diffraction in swirling flow. Incoming waves from region III μ = 1 (a), μ = 6 (b), and from region II μ = 1 (c), μ = 4 (d).