



## Executive summary

# Testing of a Freeze-proof Condenser for the Tracker Thermal Control System on AMS-02

### Problem area

The Tracker Thermal Control System (TTCS) is part of the Alpha Magnetic Spectrometer (AMS) planned aboard the truss of the International Space Station (ISS). The TTCS uses CO<sub>2</sub> as a thermal working fluid and rejects heat dissipated by AMS at two radiators. In case of an accidental power-down of the AMS02 experiment, resulting in a loss of radiator heater control, the Tracker radiators and connected TTCS-condensers may cool down as low as -120°C, which is well below the CO<sub>2</sub>-freezing point (-56 °C @ 3MPa). During uncontrolled radiator heat-up and thawing of the solid CO<sub>2</sub>, liquid CO<sub>2</sub> can be trapped in between solid parts resulting in very high pressures. This in turn demands for a very special condenser design.

### Description of work

The paper describes the theory we have on the freezing phenomenon and the subsequent verification tests that were carried out. Furthermore a condenser design, including stress calculations, is described capable to withstand a worst case freezing scenario.

### Results and conclusions

The tests confirmed our theory. It is concluded that the pressure build up during thawing -while condenser in and outlets are blocked- follows the CO<sub>2</sub> melting line found in the CO<sub>2</sub> 3-phase diagram. Therefore the condenser Maximum Design Pressure directly follows from this diagram once the maximum unloaded condenser temperature is known.

### Applicability

Wherever freezing may occur in tubes; valid for CO<sub>2</sub> and probably most other working fluids other than water.

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**Testing of a Freeze-proof Condenser for the Tracker Thermal Control System on AMS-02**

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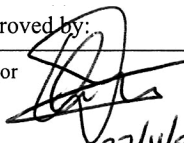
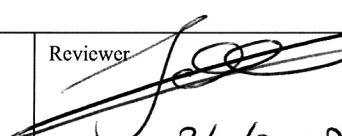
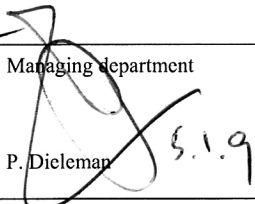
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## Summary

The paper describes freezing and pressure tests required to develop a freeze-proof condenser for the Tracker Thermal Control System (TTCS). The TTCS is a mechanically pumped two-phase carbon-dioxide loop dedicated to control the temperature of the Tracker electronics. The TTCS is part of the Alpha Magnetic Spectrometer planned aboard the truss of the International Space Station (ISS). The TTCS collects the heat at two evaporators and rejects it at two radiators. In case of an accidental power-down of the AMS02 experiment, resulting in a loss of radiator heater control, the Tracker radiators and the connected TTCS-condensers may cool down as low as  $-120\text{ }^{\circ}\text{C}$ , which is well below the  $\text{CO}_2$ -freezing point ( $-56\text{ }^{\circ}\text{C}@3\text{MPa}$ ). During uncontrolled radiator heat-up and thawing of the solid  $\text{CO}_2$ , liquid  $\text{CO}_2$  can be trapped in between solid parts resulting in high pressures. To withstand these high pressures, a high-pressure resistant condenser has been developed. In order to verify the design it was needed to determine the maximum pressure inside the condenser during thawing. The pressure determination is performed by a dedicated test set-up with strain gauges.

The paper describes the test and the implications on the condenser design. After an introduction on AMS, a description of the test set-up is given, followed by details on the calibration of the pressure sensors. Subsequently the test and test results are described.

It is concluded that the pressure build up during thawing follows the  $\text{CO}_2$  melting line found in the  $\text{CO}_2$  3-phase diagram. Therefore the condenser maximum design pressure directly follows from this diagram when the maximum unloaded condenser temperature is known. From thermal analyses this temperature was found to be  $-5\text{ }^{\circ}\text{C}$  leading to a condenser maximum design pressure of 300 MPa. The condenser, comprising small diameter Inconel 718 tubing attached to a grooved aluminium plate ( $d_{\text{in}} = 1.0\text{mm}$ ,  $d_{\text{out}} = 3.0\text{ mm}$ ) is shown -by accepted stress calculation methods- to withstand this pressure within applicable safety margins.

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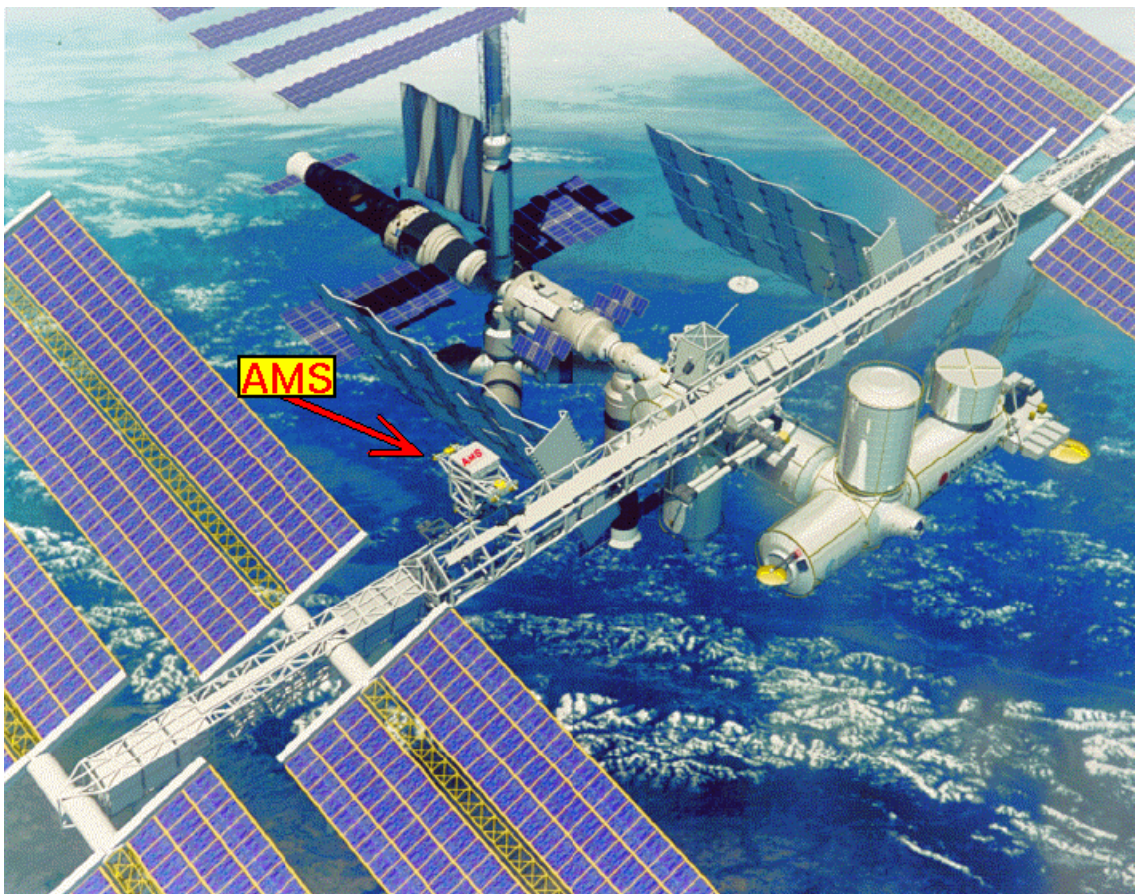
## **Definitions, acronyms, abbreviations**

AMS-02	Alpha Magnetic Spectrometer
CO <sub>2</sub>	Carbon dioxide
CTE	Coefficient of Thermal Expansion
ISS	International Space Station
MDP	Maximum Design Pressure
NLR	National Aerospace Laboratory of the Netherlands
TTCS	Tracker Thermal Control System

## 1 Introduction

### 1.1 Scope

AMS-02 is a high energy particle physics experiment in space to be mounted on the International Space Station (ISS) for a three year mission. The main physics goals in the astroparticle domain are the anti-matter and the dark matter searches. An artist impression of its location on ISS is shown in *Figure-1*.



*Figure-1: AMS-02 on ISS, artist impression*

A precursor version of the detector (AMS-01) operated successfully in a 10-days NASA Shuttle flight in June 1998. From a thermal perspective, AMS-01 was far less demanding, as it could take advantage of its huge permanent magnet to be used as a thermal heat sink. Moreover, the space shuttle flight was dedicated for AMS-01 and the shuttle orientation could be changed in a thermally convenient manner. The construction of AMS-02 is based on the experience gained from AMS-01. However, as it lacks the huge thermal mass -the permanent magnet is exchanged by a superconducting coil magnet and a helium vessel- relatively complex active cooling systems are required.

## 1.2 Tracker thermal control system

The AMS-02 Tracker Thermal Control System (TTCS) is a mechanically pumped two-phase carbon dioxide cooling loop. The main objective is to provide accurate temperature control of AMS Tracker front-end electronics. An additional objective is to prove and qualify a two-phase pumped cooling system in orbit and collect operational data in micro-g environment over a period of three years. The TTCS has to cope with a long distance distributed heat load, in a very limited space. The solution is to have two evaporator tubes in parallel, each approximately 9 meters long, with an inner diameter of 2.6mm. The AMS Tracker electronics dissipated heat is transported from the evaporators to two condensers in parallel, connected to heat pipe radiators. The liquid is transported back to the evaporators by means of a mechanical pump. One radiator is located at the top WAKE (anti-flight direction) side and the other one at the RAM (flight direction) side of the AMS instrument. The tracker hybrid electronics are situated at the periphery of the tracker silicon planes and are located inside the cryogenic magnet, as seen in *Figure-2*. A total of 144 Watt is produced at 192 locations and an additional 6-10 Watt cooling capacity is required for additional electronics and a Star Tracker, which is also attached to the loop. The temperature requirements for the silicon waver and the front-end electronics are: operating temperature  $-10/+25^{\circ}\text{C}$ , temperature stability  $3^{\circ}\text{C}$  per orbit. The main reason for choosing  $\text{CO}_2$  as a working fluid is its low pressure drop when it flows through the relatively long evaporators and small diameter condenser tubing and accompanying long feed and return lines. Further details on the TTCS can be found in [Ref. 1, 2, and 3].

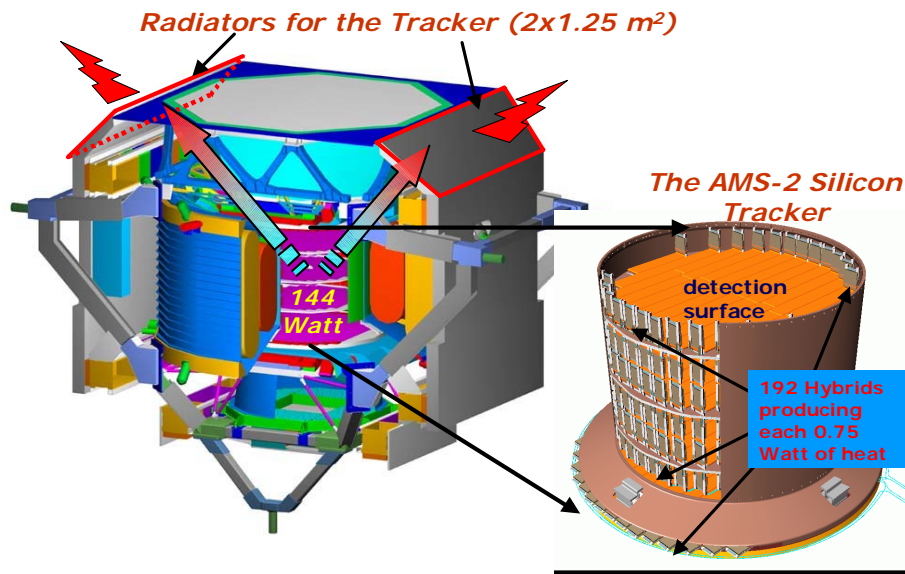


Figure-2: AMS-02 schematic overview

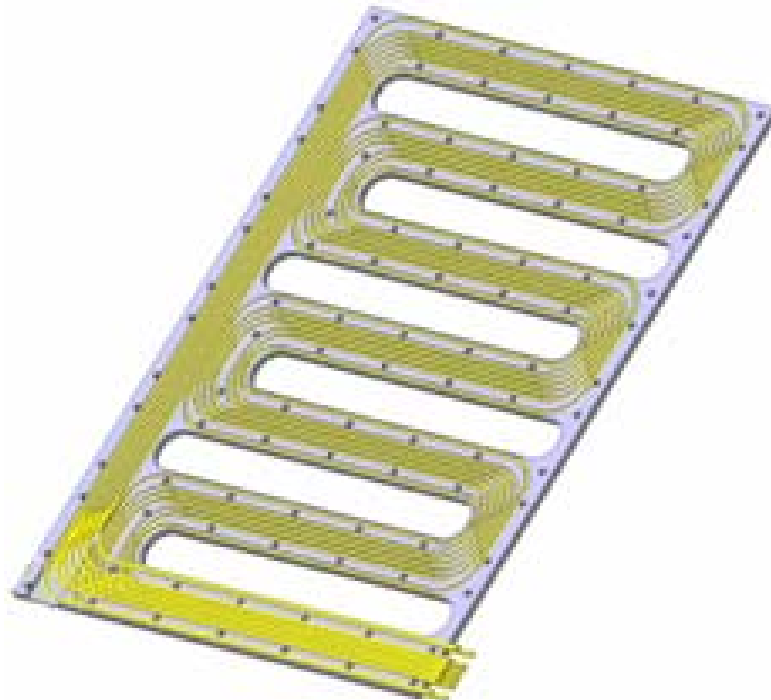


## 2 Problem statement and our theory on condenser freezing

The TTCS rejects its collected heat at two radiators. In case of an accidental power-down of the AMS02 experiment, resulting in a loss of radiator heater control, the Tracker radiators and the connected TTCS-condensers may cool down as low as  $-120^{\circ}\text{C}$ , which is well below the  $\text{CO}_2$ -freezing point ( $-56^{\circ}\text{C}@3\text{MPa}$ ). During uncontrolled radiator heat-up and thawing of the solid  $\text{CO}_2$ , liquid  $\text{CO}_2$  can be trapped in between solid parts resulting in high pressures.

To withstand these high pressures, a high-pressure resistant condenser has been developed. These condensers consist of 7 small diameter tubes in parallel, attached to a grooved aluminum plate using an epoxy as shown in *Figure-3*. Each condenser is to be bolted to a radiator and in an extreme case it may cool down to  $-120^{\circ}\text{C}$  during an accidental AMS power down.

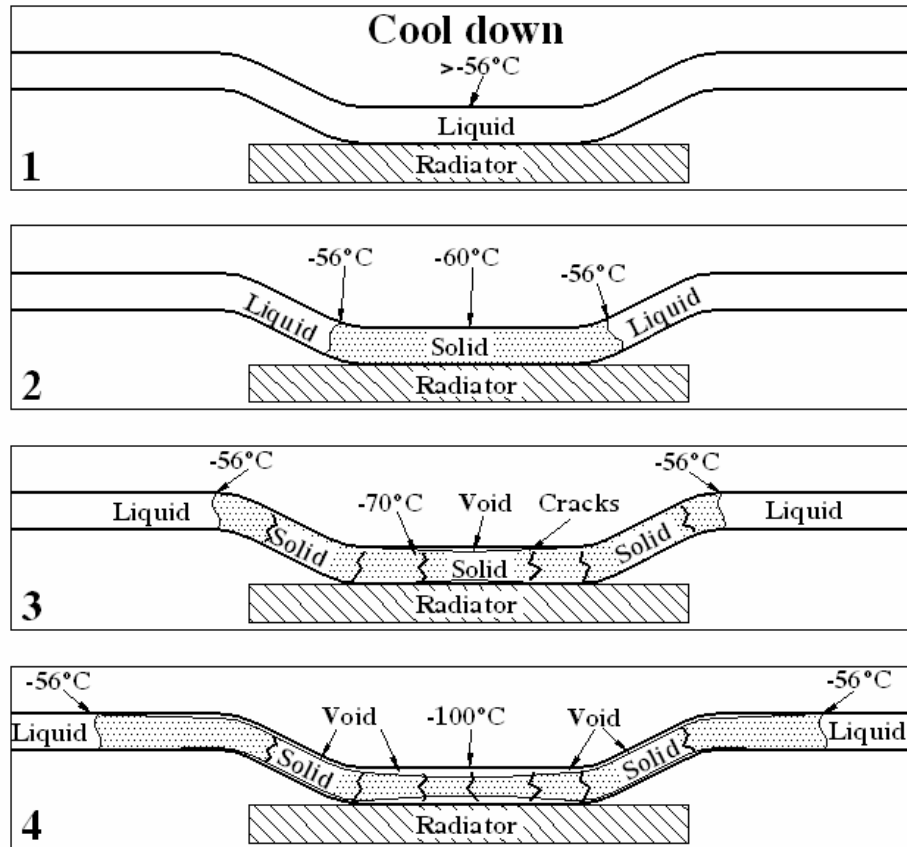
The condenser's 7 feed and 7 return lines –being just the condenser tube ends- end up in manifolds, located approximately 0.8m away from the radiator.



*Figure-3: Condenser lay-out*

The manifold temperature is calculated to be always higher than the  $\text{CO}_2$  freezing point. The temperature distribution along the feed & return lines will gradually change from condenser temperature to manifold temperature.

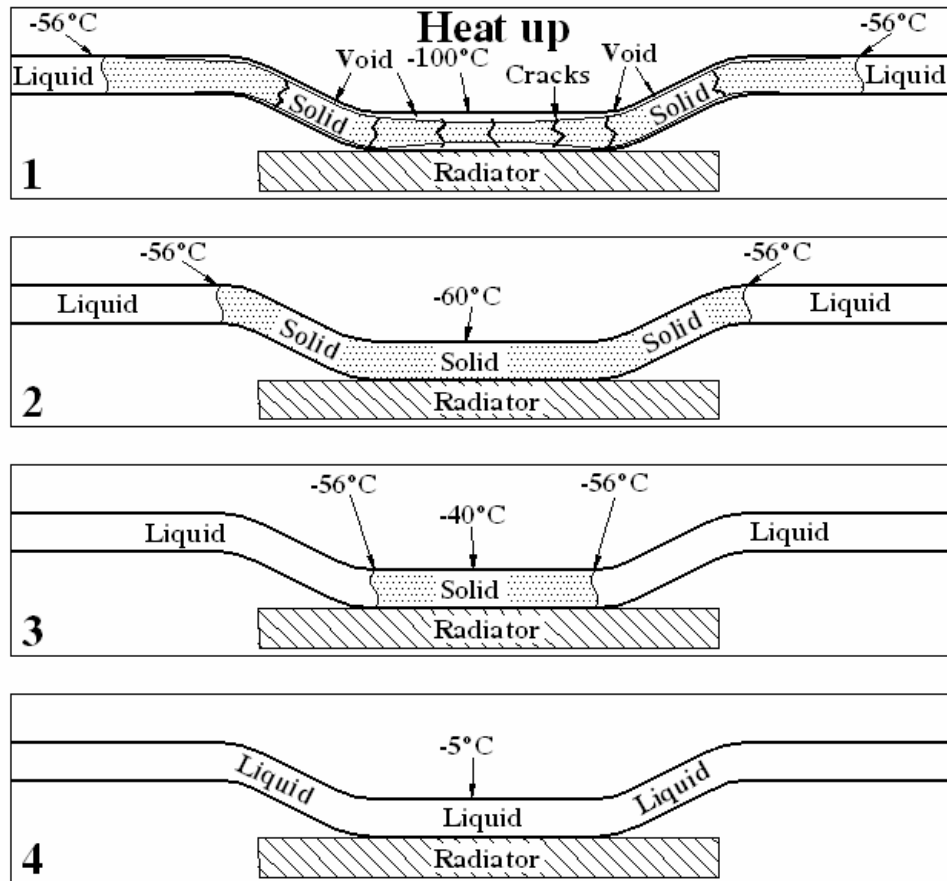
A cool down sequence will be as follows, see also *Figure-4*:



*Figure-4: Condenser cool down sequence*

1. Initial state: radiator, condenser and feed & return lines are well above freezing point ( $>-56^{\circ}\text{C}$ )
2. Radiator temperature falls below freezing point ( $<-56^{\circ}\text{C}$ ), condenser and feed & return lines start to freeze
3. Freezing front moves along feed & return lines, away from the condenser, meanwhile blocking liquid that otherwise would possibly enter the voids in the condenser. As the tube diameter is very small (only 1 mm) liquid blocking is considered to be close to the freezing front.
4. Radiator temperatures decreases and freezing front moves further away from the condenser.

A heat up (thaw) sequence will be as follows, see also *Figure-5*:



*Figure-5: Condenser heat up sequence*

1. Initial state: radiator, condenser and part of feed & return lines are frozen ( $<-56^{\circ}\text{C}$ )
2. Radiator and condenser temperatures are increasing, solid  $\text{CO}_2$  expands to its original value, filling the voids
3. Condenser exits are blocked; condenser is above  $-56^{\circ}\text{C}$ , condenser pressure increases.  $\text{CO}_2$  is still (partly) frozen, because of the high pressure
4. When feed & return lines thawing finally frees up the condenser exits, pressure rapidly decreases, causing the condenser content to melt.

The high pressure build-up occurs in step 3 of the heat up sequence, where the feed & return lines are blocked by solid  $\text{CO}_2$  while the condenser temperature is rising. As a pressure build-up will change the melting temperature of  $\text{CO}_2$ , frozen  $\text{CO}_2$  exists above  $-56^{\circ}\text{C}$ . The higher the condenser temperature, with blocked feed & return lines, the higher the internal pressure.

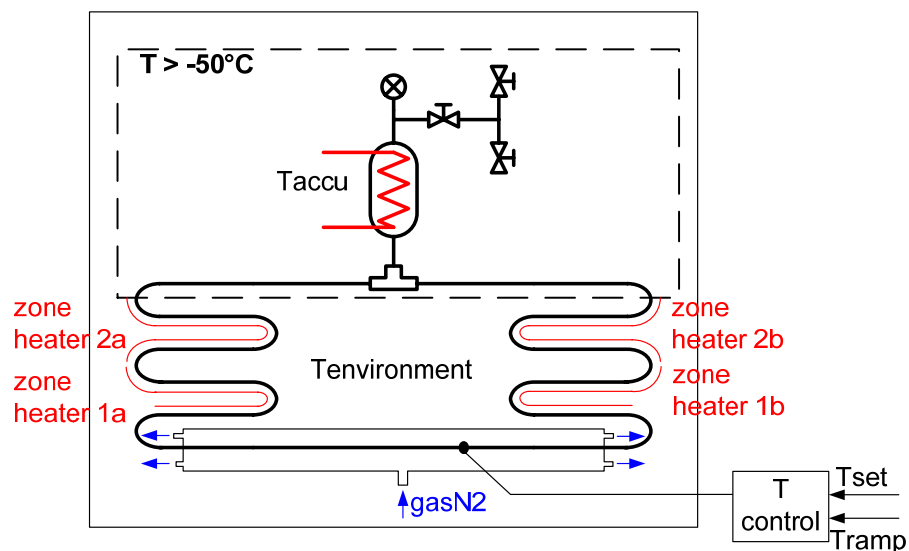
### 3 Testing

#### 3.1 Test objective

The test objective is to verify our understanding of the CO<sub>2</sub> freezing phenomenon as described in the previous chapter, by test, and to subsequently determine the condenser's maximum design pressure, MDP.

#### 3.2 Test set-up description

A downscaled CO<sub>2</sub>-loop has been built and tested in an environment where temperatures could be set well below the freezing point of CO<sub>2</sub>. The mini-loop, schematically shown in *Figure-6*, comprised the following components: a temperature controlled accumulator, a fill/drain port, thermocouples, zone heaters, several meters flight representative tubing and a measurement section where 8 strain gauges were attached to the condenser tube. As inconel 718 was not available at the time, cold drawn stainless steel 316 Ti was used instead ( $d_{in}=1\text{mm}$ ,  $d_{out}=3\text{mm}$ ).



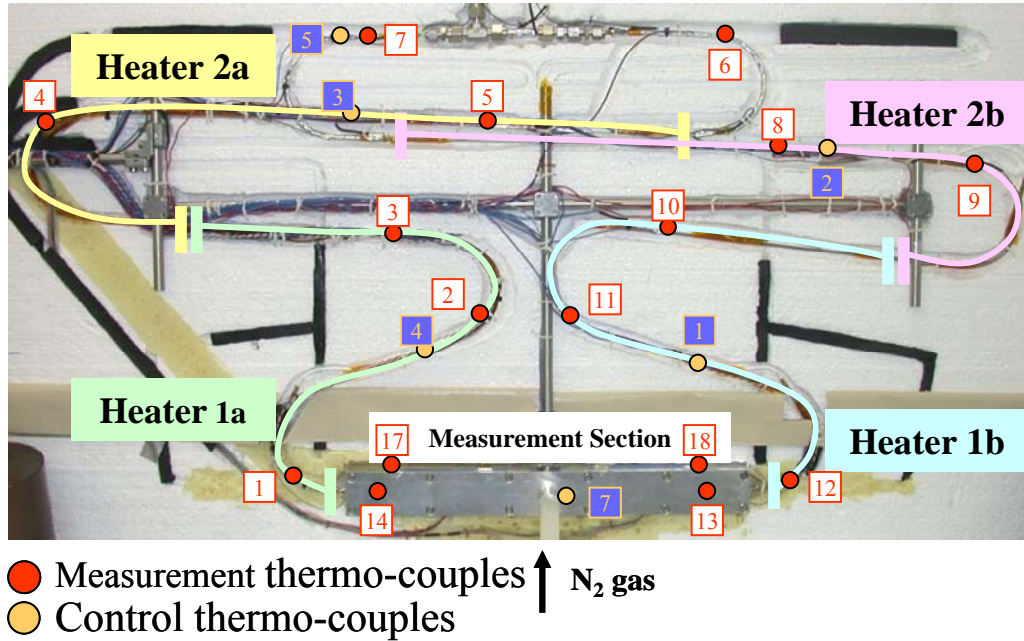
*Figure-6: Test set-up schematic (here without viewing glass)*

From a mechanical (pressure and stress) point of view, it was considered not necessary to embed the condenser tube in a grooved aluminum plate. This is a justifiable approach, because the internal pressure will reach its maximum at the maximum condenser temperature with blocked feed & return lines. The maximum unloaded condenser temperature was calculated, using the thermal modelling tool 'SINDA', to be  $-5^{\circ}\text{C}$ . This is relatively close to the epoxy cure temperature of  $+20^{\circ}\text{C}$  and because of its high flexibility, additional stresses due to bonding and differences in CTE are considered negligible.

The condenser section was equipped with strain gauges and put inside an aluminum enclosure which could be flushed with temperature controlled gaseous nitrogen.

Zone wire heaters were soldered to the feed & return lines to be able to heat these lines in a controlled manner. Several monitoring thermo-couples were attached and seven control thermo-

couples were used to provide direct feedback to the temperature control equipment. The location of these couples together with the zone heaters are shown in *Figure-7*.

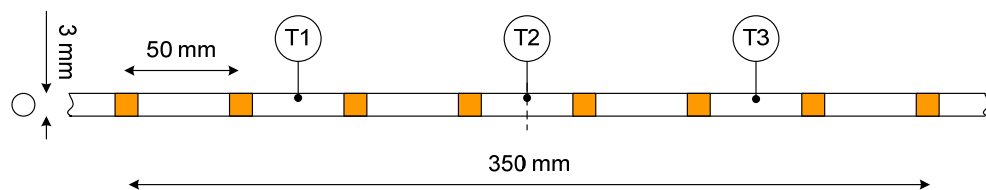


*Figure-7: Thermocouples and zone heater locations*

The mini-loop was insulated using expanded poly styrene and put into a climate chamber.

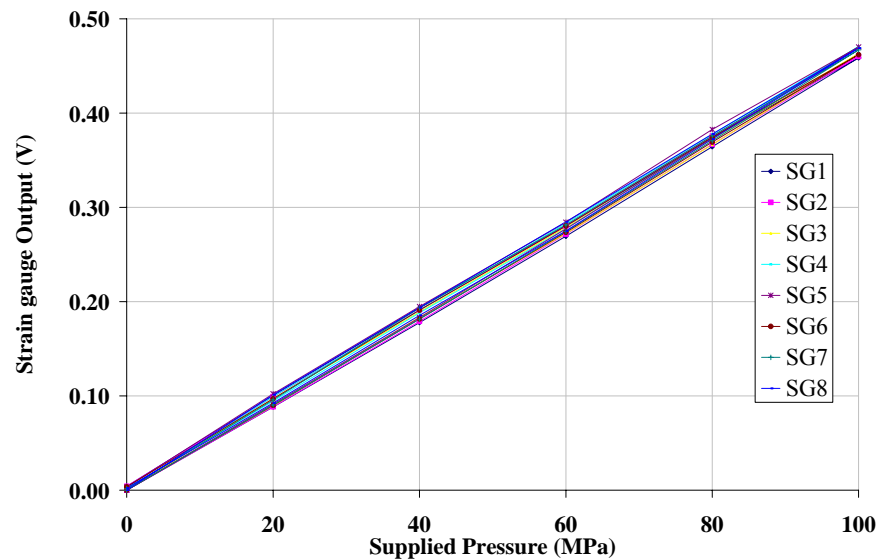
### 3.3 Set-up calibration

The tube inside the measurement section was equipped with 8 strain gauge bridges, as shown schematically in *Figure-8*. Each bridge consisted of two EA-06-050TG-350 strain gauges with measuring grids in two orthogonal directions. The grids were connected in a Wheatstone bridge: two grids in the tangential direction and two grids in the longitudinal direction of the tube. The bridges were covered with a protective coating and compensated for zero-shift with temperature. Due to the strong curvature of the tube the bridges appeared to have large initial zero balances and the remaining zero shift with temperature was very non-linear. With an excitation of 5 Volt and the given dimensions and material of the tube the theoretical sensitivity of the bridges is 0.433 mV/100 MPa. The actual sensitivities have been calibrated at three different temperature levels: -18°C, -54°C and -68°C. Also, the actual zero shifts with temperature have been determined for the temperature range -120°C ↔ 30°C. These zero shifts appeared to be quite large and it was decided to concentrate on the delta strain gauge output, just at the point where the high internal pressure build-up is relieved when the feed & return lines thaw.



*Figure-8: Condenser section equipped with strain gauges*

The output voltage of all eight strain gauges were measured while applying pressures of subsequently 0, 20, 40, 60, 80, 100, 80, 60, 40, 20, 0 MPa. This was repeated at three different temperatures;  $-18^{\circ}\text{C}$ ,  $-54^{\circ}\text{C}$  and  $-68^{\circ}\text{C}$ . The maximum applied pressure (100 MPa) was limited by the hand pump; the tube was calculated to remain elastic up to 250 MPa. Typical measurement results are plotted in *Figure-9*. The output appeared to be linear as expected, and within elastic limits of the tube, during the tests the outputs above 100 MPa were obtained by extrapolation.



*Figure-9: Strain gauge calibration curves*

## 4 Test execution

The loop was evacuated and filled with CO<sub>2</sub> with the liquid level approximately halfway the accumulator, checked by a viewing glass mounted in parallel. While being pressurized by controlling the accumulator temperature, the condenser representative part of the loop was cooled down to -120 °C, which is well below the freezing point of CO<sub>2</sub>, by a representative calculated temperature slope. After a dwell time of several hours the condenser temperature was controlled with a preset slope to a temperature above the freezing point, while the strain gauges were being monitored (blue line in *Figure-10*). Subsequently the condenser feed & return line zone heaters were switched on, one after another, until the solid CO<sub>2</sub> blockage was removed, clearly visible as a sudden strain gauge output drop, indicating a pressure relief. This test was repeated for a condenser maximum heat-up temperature of -48.5, -40, -30 and -20°C. The accumulator temperature was kept constant, while the climate chamber temperature was kept at -70 °C during the freezing and thawing process.

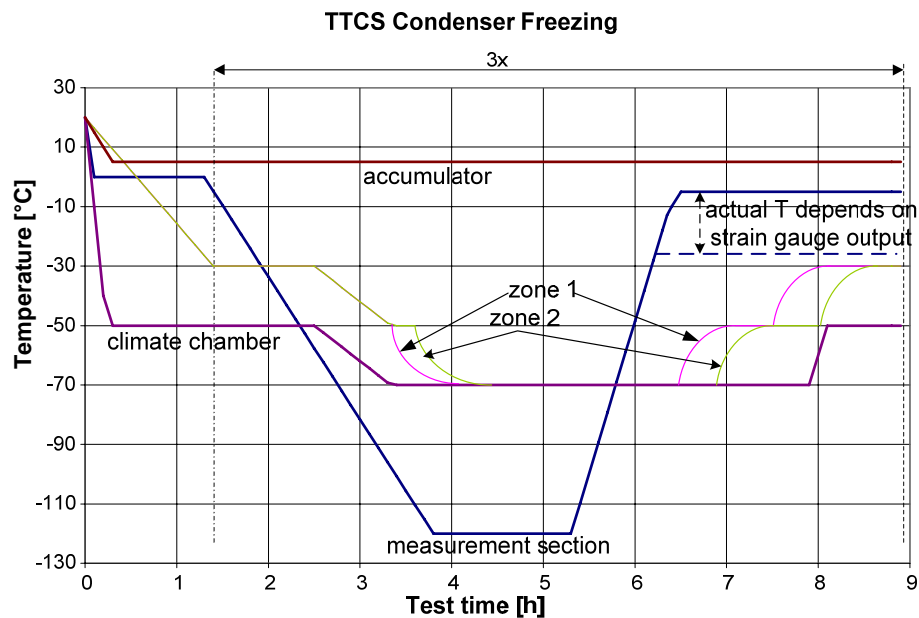


Figure-10: Test execution; required temperature profiles

## 5 Test results

### 5.1 Method discussion

During the calibration tests and during pre-testing it became clear that the strain gauges zero-offsets were highly sensitive to temperature and temperature slopes. As it appeared to be difficult to reliably compensate for these zero-offsets, it was decided to perform the test step-wise by repeating it for various maximum blocked condenser heat up temperatures. The advantage of this method is that the maximum occurring pressure and a known “zero” pressure are measured after one another in a split second, during the pressure relief as shown in *Figure-11*.

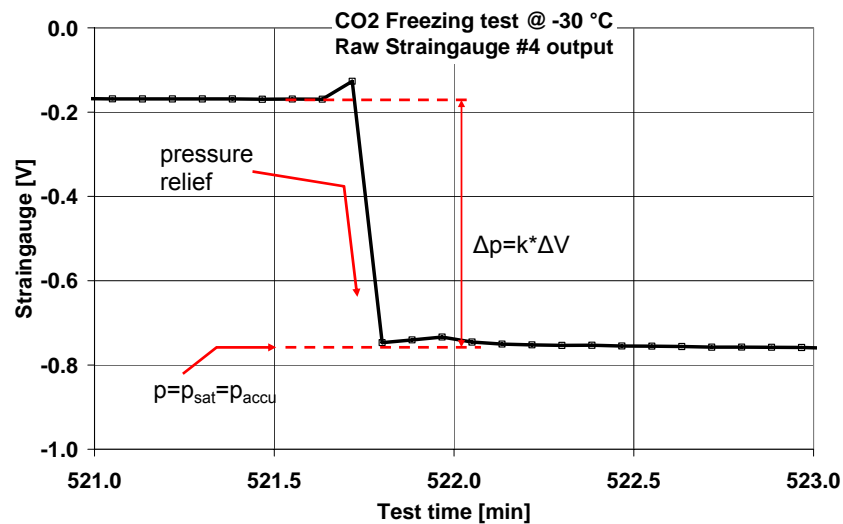
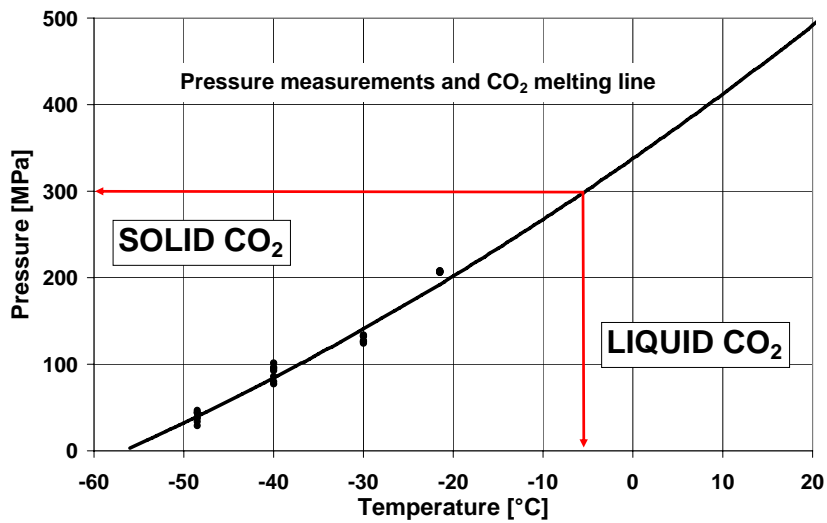


Figure-11: Typical strain gauge output during pressure relief, readings taken every 5 seconds



Here  $\Delta V_{SG4}=0.6$  Volt;  $k_{SG4,-30}=215.2$  MPa/V;  $p_{accu}=3.52$  MPa resulting in  $p=k*\Delta V+p_{accu} = 215.2*0.6+3.52 = 132.6$  MPa.

The procedure was repeated for 8 strain gauges and 4 temperatures and plotted along the CO<sub>2</sub> melting line, as shown in *Figure-12*. It is clear that the pressure build-up in the capillary tube follows this melting line. Using a thermal model (SINDA), the maximum condenser temperature was calculated to be -5 °C, corresponding to a pressure of 300 MPa, see melting line.



*Figure-12: Measured data plotted along CO<sub>2</sub> melting line*

Based on this pressure the required material and tube dimensions can be determined.

## 5.2 Stress calculations

From [*Appendix A, equation (5)*], it can be found that a tube with  $r_i$  of 0.5 mm and a wall thickness of 1mm and an internal pressure of 300 MPa needs to sustain a von Mises stress of 586 MPa. Using a safety factor of 1.5 for yield [*Ref. 7, p39*], [*Ref. 8, p119*], the required material yield stress must be higher than  $1.5 \times 586 = 879$  MPa. This is feasible for Inconel 718, see *Table-1*. An Inconel 718 tube with above dimensions will burst at 1564 MPa [*Appendix A equation (11)*]; using a safety factor of 4.0 [*Ref. 7, p39*], [*Ref. 8, p119*],  $1564/4.0 = 391$  MPa is allowed. As a maximum of 300 MPa will occur while thawing, the envisaged tube easily complies the requirement. The material ‘Inconel 718’ was chosen as a likely candidate as it was used for similar reasons as part of the radiator of the International Space Station [*Ref. 6*].

	8190 kg/m <sup>3</sup>
Modulus of elasticity	200 Gpa
Yield stress	1034 MPa [Ref.6]
Ultimate tensile stress	1280 MPa [Ref. 6]
Coefficient of thermal expansion	13.0 $\mu\text{m/m } ^\circ\text{C}$ [Ref. 4]
Thermal conductivity	11.4 W/m.K [Ref. 4]
Specific heat	435 J/kg.K [Ref. 4]

Table-1: Inconel 718 properties

## 6 Conclusions

From the tests it is concluded that the pressure build up during thawing -while condenser in and outlets are blocked- follows the CO<sub>2</sub> melting line found in the CO<sub>2</sub> 3-phase diagram. Therefore the condenser Maximum Design Pressure directly follows from this diagram once the maximum unloaded condenser temperature is known. From thermal analysis calculations this temperature was found to be -5 °C leading to a condenser MDP of 300 MPa.

A condenser comprising small diameter Inconel 718 tubing ( $d_{in} = 1.0\text{mm}$ ,  $d_{out} = 3.0\text{mm}$ ) is shown -by accepted stress calculation methods- to withstand this pressure using a safety factor of 1.5 for yield and 4.0 for burst.

As no significant pressure rise was measured during the -120 °C to -56 °C temperature slope it is concluded that during the solidification process the entrance and exit are blocked early in the freezing process, preventing additional liquid CO<sub>2</sub> from entering the condenser. In other words, if such blockage would not occur and thus allowing liquid to fill the cracks and voids during cool down, this would have resulted in an immediate pressure increase as soon as the temperature started to rise from -120 °C. The measured pressure would have been higher than that according to the CO<sub>2</sub> melting line, which –as we showed- clearly is not the case.

The MDP measurement with strain gauges showed to be successful and can be used to support freezing issues for other fluids.

## References

- [1] A.A. Woering (NLR), A. Pauw (NLR), A.W.G. de Vries (NLR), A.A.M. Delil (NLR), B. Verlaat (NIKHEF): Thermal Modeling Issues Concerning the Mechanically Pumped Two-Phase CO<sub>2</sub> Cooling for the Ams-2 Tracker. SAE technical paper 2002-01-2466, Int. Conf. On Environmental Systems, San Antonio, TX, USA, July 2002.
- [2] J. van Es, G. van Donk, A. Pauw, B. Verlaat, C.A.M. Rens, J. Jaarsma, M.P.A.M. Brouwer, “AMS02 Tracker Thermal Control System (TTCS) design, model and breadboard results”, 34th Int. Conf. on Environmental Systems, Colorado Springs, USA, 2004.
- [3] A.A.M. Delil, “Two-Phase Developments for the International Space Station ISS: - AMS-2 TTCS, a Mechanically Pumped Two-Phase CO<sub>2</sub> Cooling Loop for the Alpha Magnetic Spectrometer Tracker Experiment - CIMEX-3, Versatile Two-Phase Loop for the Fluid Science Laboratory”, 12th Int. Heat Pipe Conference, Moscow, Russia, May 2002.
- [4] Bibus Metals: Datasheet Inconel Alloy 718. W.Nr. 2.4668.
- [5] Young, Warren C. (1989): Roark’s Formulas for Stress and Strain. 6th Edition. McGraw Hill Book Company.
- [6] Robert L. Broeren, R. John Duschatko, International Space Station Alpha Design-To-Freeze Radiators. SAE technical paper series 951652.
- [7] MIL-STD-1522A 1984 Standard General Requirements for Safe design and Operation of Pressurized Missile and Space Systems.
- [8] MIL-HDBK-340 1985 Application guidelines for MIL-STD-1540B; test requirements for space vehicles.

## Appendix A Stress equations

For a thick walled cylinder or tube with an inner radius  $r_i$  and outer radius  $r_o$ , under an internal pressure  $p_i$ , the maximum tangential stress is given by:

$$\sigma_t = p_i \left( \frac{r_o^2 + r_i^2}{r_o^2 - r_i^2} \right) \quad (1)$$

The maximum radial stress by:

$$\sigma_r = -p_i \quad (2)$$

The longitudinal stress by:

$$\sigma_l = p_i \left( \frac{r_i^2}{r_o^2 - r_i^2} \right) \quad (3)$$

The longitudinal stress is zero if the ends of the tube are open. As a measure of the limiting stress, the Von Mises stress is taken. It is defined as (for  $\sigma_l = 0$ ):

$$\sigma_{VM} = \sqrt{\sigma_r^2 + \sigma_t^2 - \sigma_r \sigma_t} \quad (4)$$

Substituting the expressions for  $\sigma_r$  and  $\sigma_t$  yields:

$$\sigma_{VM} = p_i \sqrt{\alpha^2 + \alpha + 1} \quad (5)$$

Where:

$$\alpha = \frac{r_o^2 + r_i^2}{r_o^2 - r_i^2} \quad (6)$$

Note that the relation between radii and wall thickness is given by:

$$r_o = r_i + t \quad (7)$$

### Calculation for yield limit:

For a tube with  $r_i = 0.5 \cdot 10^{-3} \text{m}$  and  $r_o = 1.5 \cdot 10^{-3} \text{m}$ , using eq. (6),  $\alpha$  equals to 1.25. Using eq. (5), for an internal pressure  $p_i = 300 \text{Mpa}$ , the Von Mises stress is calculated to be 586 MPa. The tube needs to sustain this stress with a safety factor of 1.5 without yielding. The material yield stress therefore must be lower than  $1.5 \cdot 586 \text{MPa}$ , which is the case for Inconel 718 after precipitation hardening.

Using a safety factor of 4.0 on the ultimate strength (which is the limiting value), the maximum allowable Von Mises stress becomes:



$$\sigma_{VM} = 0.25\sigma_u \quad (8)$$

For Inconel 718 this would result in a maximum allowable Von Mises stress of  $0.25 \times 1280 = 320$  MPa, which is lower than the previously calculated 586 MPa and therefore would not comply the requirement with respect to burst.

However, as a thick walled cylinder is pressurized, the bore material, which is the most highly stressed part of the cylinder, begins to yield. With further increase of the pressure the yield surface begins to propagate. At some stage, when more and more of the cylinder is entering the plastic regime, the bore material starts to strain hardening. When the pressure is removed, it leaves residual compression in the inner part and residual tension in the outer part. Therefore the cylinder can withstand higher pressures than would follow from the standard pressure vessel formulas, which are in fact only valid for isotropic material, resulting in conservative estimates of the wall thickness necessary to withstand high pressures.

The technique of applying a high initial pressure is frequently used in the manufacturing of gun barrels and cannons and is called autofrettage. The expression for the bursting pressure,  $p_u$ , is a function of the ultimate tensile strength and the radii of the tube:

$$p_u = 2\sigma_u \frac{r_o - r_i}{r_o + r_i} \quad (9)$$

commonly known as the mean diameter formula. It is essentially empirical but agrees reasonably well with experiments for both thin and thick cylindrical tubes.

For very thick tubes the formula:

$$p_u = \sigma_u \ln \frac{r_o}{r_i} \quad (10)$$

is preferable. Greater accuracy can be obtained by using with this formula a multiplying factor that takes into account the strain hardening properties of the material [Ref. 5]:

$$p_u = \frac{2\sigma_y}{\sqrt{3}} \left(2 - \frac{\sigma_y}{\sigma_u}\right) \ln \frac{r_o}{r_i} \quad (11)$$

For a tube with  $r_i = 0.5 \times 10^{-3}$  m and  $r_o = 1.5 \times 10^{-3}$  m, and for precipitation hardened Inconel 718 with  $\sigma_y = 1034$  MPa,  $\sigma_u = 1280$  MPa the tube will burst at  $p_u = 1564$  Mpa. With a safety factor of 4, the maximum allowed internal pressure with respect to the burst requirement therefore will be  $1564/4 = 391$  Mpa. For our condenser the Maximum Design Pressure was found to be 300 MPa, which is lower than the allowed 391 MPa and therefore meets the requirement.