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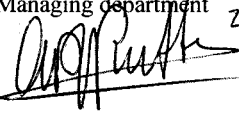
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Estimating rare event probabilities in large scale stochastic hybrid systems by sequential Monte Carlo simulation

Henk A.P. Blom, Jaroslav Krystul and G.J. (Bert) Bakker

Abstract— We study the problem of estimating small reachability probabilities for large scale stochastic hybrid processes through Sequential Monte Carlo (SMC) simulation. Recently, [Cerou et al., 2002, 2005] developed an SMC approach for diffusion processes, and referred to the resulting SMC algorithm as an Interacting Particle System (IPS). In [Krystul&Blom, 2004, 2005] it was shown that this IPS approach works very well for a diffusion example, but has its limits when applied to a switching diffusion with large differences in discrete state (mode) probabilities or with rare mode switching. In order to cope with these problems, in [Krystul&Blom, 2004, 2005, 2006] the IPS approach has been extended to Hybrid IPS (HIPS) versions. Unfortunately, these HIPS versions may need impractically many particles when the space of the discrete state component is very large. Such situation typically occurs when the stochastic process considered is highly distributed and incorporates many local discrete valued switching processes. Then the vector of local discrete valued components has a state space the size of which is exponentially large. The aim of the current work is formulate the estimation of extremely small rare event probabilities in stochastic hybrid systems with a large state space for the discrete valued process component into one of a hierarchical estimation process, and to use this for the derivation of a Hierarchical HIPS version. The effectiveness of the approach is illustrated for evaluating the risk of collision between two aircraft in a scenario of the future.

Index Terms— Air transportation, Collision processes, Monte Carlo methods, Risk analysis, Safety, Sequential estimation, Stochastic systems

I. INTRODUCTION

The aim of the current paper is to study the problem of estimating small reachability probabilities for large scale stochastic hybrid processes through Sequential Monte Carlo (SMC) simulation. In particular, we are interested in estimating of collision risk in free flight by addressing this within the framework of stochastic hybrid systems. For

applications to other safety critical industries, e.g. nuclear, chemical, such approach has been well developed (e.g. Labeau et al., 2000) within the class of Piecewise Deterministic Markov processes (Davis, 1984, 1993). For risk evaluation of air traffic operations, however it is needed to include Brownian motion in the models to represent the effect of random wind disturbances on aircraft trajectories. This is supported by the class of generalized stochastic hybrid systems (Pola et al., 2003).

This paper considers the problem of estimating the probability that a generalized stochastic hybrid system reaches a particular small sub-set of the state space and within some time horizon. Numerical analysis of such reachability problems is known to be demanding. An alternative is to accomplish this by Monte Carlo simulation. The advantage of Monte Carlo (MC) methods for reachability probability estimation is that they do not require specific assumptions on the system under consideration. However, obtaining accurate estimates of rare event probabilities, say about 10^{-9} per flying hour requires to simulate at least 10^{11} flying hours, which is very time consuming.

In order to assess collision risk of free flight operation, the idea is to perform many Monte Carlo simulations with a model of the free flight operation and, while doing so, to estimate the collision risk by counting the number of collisions and divide this by the number of simulated flight hours. Though this idea is simple, to make it work in practice we need an effective way of speeding up the Monte Carlo simulation. This paper describes the way we are doing this by extending the Interacting Particle System (IPS) approach of Cérou et al. (2002, 2005) to collision risk assessment for a strong Markov process model of AMFF operations.

Recently [Cerou et al., 2002, 2005] developed a sequential MC algorithm for estimating such small reachability probabilities of strong Markov processes within some guaranteed level of precision. The key idea behind this approach is to express the small probability to be estimated as the product of a certain number of larger probabilities, which can be efficiently estimated by the Monte Carlo approach. This can be achieved by introducing sets of intermediate states that are visited one set after the other, in an ordered sequence,

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before reaching the final set of states of interest. The reachability probability of interest is then given by the product of the conditional probabilities of reaching a set of intermediate states given that the previous set of intermediate states have been reached. Each conditional probability is estimated by simulating in parallel several copies of the system, i.e. each copy is considered as a particle following the trajectory generated through the system dynamics. To guarantee the precision, the MC simulated process must have the strong Markov property.

The paper is organized as follows. Section II formulates the problem considered. Section III develops a factorization of the risk. Section IV characterizes each of the risk factors. Section V uses this to develop the sequential MC simulation approach. Section VI applies this approach towards estimating the collision risk of demanding free flight scenario. Section VII draws conclusions.

II. MONTE CARLO SIMULATION OF COLLISION RISK

Throughout this and the next sections, all stochastic processes are defined on a complete stochastic basis $(\Omega, \mathcal{F}, \mathbb{F}, \mathbb{P}, \mathbb{T})$ with $(\Omega, \mathcal{F}, \mathbb{P})$ a complete probability space, and \mathbb{F} is an increasing sequence of sub- σ -algebra's on the positive time line $\mathbb{T}=\mathbb{R}_+$, i.e. $\mathbb{F} \triangleq \{\mathcal{F}_t, t \in \mathbb{T}\}$, \mathcal{J} containing all P-null sets of \mathcal{F} and $\mathcal{J} \subset \mathcal{F}_s \subset \mathcal{F}_t \subset \mathcal{F}$ for every $s < t$.

We assume that air traffic operations are represented by a generalised stochastic hybrid process $\{x_t, \theta_t\}$ which satisfies the strong Markov property [Bujorianu & Lygeros, 2005]. For an N -aircraft free flight traffic scenario the stochastic hybrid process $\{x_t, \theta_t\}$ consists of components $x_t \triangleq \text{Col}\{x_t^0, x_t^1, \dots, x_t^N\}$ and $\theta_t \triangleq \text{Col}\{\theta_t^0, \theta_t^1, \dots, \theta_t^N\}$, x_t^i assumes values from \mathbb{R}^{n_i} , and θ_t^i assumes values from a finite set (M^i) .

Physically, $\{x_t^i, \theta_t^i\}$, $i=1, \dots, N$, is the hybrid state process related to the i -th aircraft, and $\{x_t^0, \theta_t^0\}$ is the non-aircraft related hybrid state process. The process $\{x_t, \theta_t\}$ is $\mathbb{R}^n \times \mathbb{M}$ -

valued with $n = \sum_{i=0}^N n_i$ and $M = \bigotimes_{i=0}^N M_i$.

In order to model collisions between aircraft, we introduce mappings from the Euclidean valued process $\{x_t\}$ into the relative position and velocity between a pair of two aircraft (i, j) . The relative horizontal position is obtained through the mapping $y^{ij}(x_t)$, the relative horizontal velocity is obtained through the mapping $v^{ij}(x_t)$. The relative vertical position is

obtained through the mapping $z^{ij}(x_t)$, and vertical rate of climb/descent is obtained through the mapping $r^{ij}(x_t)$. The relation between these position and velocity mappings satisfies the following two equations:

$$dy^{ij}(x_t) = v^{ij}(x_t) dt \quad (1)$$

$$dz^{ij}(x_t) = r^{ij}(x_t) dt \quad (2)$$

A collision between aircraft (i, j) means that the process $\{y^{ij}(x_t), z^{ij}(x_t)\}$ hits the boundary of an area where the distance between aircraft i and j is smaller than their physical size. Under the assumption that the length of an aircraft equals the width of an aircraft, and that the volume of an aircraft is represented by a cylinder the orientation of which does not change in time, then aircraft (i, j) have zero separation if $x_t \in D^{ij}$ with:

$$D^{ij} = \{x \in \mathbb{R}^n; |y^{ij}(x)| \leq (l_i + l_j)/2 \text{ AND } |z^{ij}(x)| \leq (s_i + s_j)/2\}, \quad i \neq j \quad (3)$$

where l_j and s_j are length and height of aircraft j . For simplicity, we assume that all aircraft have the same size, by which (3) becomes:

$$D^{ij} = \{x \in \mathbb{R}^n; |y^{ij}(x)| \leq l \text{ AND } |z^{ij}(x)| \leq s\}, \quad i \neq j \quad (4)$$

Although all aircraft have the same size, notice that in (4), D^{ij} still depends of (i, j) . If x_t hits D^{ij} at time τ^{ij} , then we say a collision event between aircraft (i, j) occurs at moment τ^{ij} , i.e.

$$\tau^{ij} = \inf\{t > 0; x_t \in D^{ij}\}, \quad i \neq j \quad (5)$$

The first moment τ^i of collision with any of the other aircraft, i.e.

$$\tau^i = \inf_{j \neq i} \{\tau^{ij}\} = \inf_{j \neq i} \{t > 0; x_t \in D^{ij}\} = \inf\{t > 0; x_t \in D^i\} \quad (6)$$

with $D^i = \bigcup_{j \neq i} D^{ij}$

From this moment τ^i on, we assume that the differential equations for $\{x_t^i, \theta_t^i\}$ stop evolving.

An unbiased estimation procedure of the risk would be to simulate many times aircraft i amidst other aircraft over a period of length T and count all cases in which the realization of the moment τ^i is smaller than T . An estimator for the collision risk of aircraft i per unit T of time then is the fraction

of simulations for which $\tau^i < T$.

III. RISK FACTORIZATION USING MULTIPLE CONFLICT LEVELS

Cérou et al. (2002) have developed a novel way of speeding up Monte Carlo simulation to estimate the probability that an \mathbb{R}^n -valued strong Markov process x_t hits a given “small” subset $D \in \mathbb{R}^n$ within a given time period $(0, T)$. This method essentially consists of taking advantage of an appropriately nested sequence of closed subsets of \mathbb{R}^n : $D = D_m \subset D_{m-1} \subset \dots \subset D_1$, and then start simulation from outside D_1 , and subsequently simulate from D_1 to D_2 , from D_2 to D_3 , ..., and finally from D_{m-1} to D_m . In order to apply this approach to the free flight operational concept considered we identified the following approach in defining a sequence of nested subsets.

Prior to a collision of aircraft i with aircraft j a sequence of conflicts ranging from long term to short term always happened. In order to incorporate this explicitly in the MC simulation, we formalize this sequence of conflict levels through a sequence of closed subsets of \mathbb{R}^n :

$$D^{ij} = D_m^{ij} \subset D_{m-1}^{ij} \subset \dots \subset D_1^{ij} \text{ with for } k = 1, \dots, m:$$

$$D_k^{ij} = \{x \in \mathbb{R}^n; |y^{ij}(x) + \Delta v^{ij}(x)| \leq d_k \text{ AND} \\ |z^{ij}(x) + \Delta r^{ij}(x)| \leq h_k, \text{ for some } \Delta \in [0, T_k]\}, i \neq j \quad (7)$$

with d_k , h_k and T_k the parameters of the conflict definition at level k , and with $d_m = l$, $h_m = s$ and $T_m = 0$, and with $d_{k+1} \geq d_k$, $h_{k+1} \geq h_k$ and $T_{k+1} \geq T_k$. If x_t hits D_k^{ij} at time τ_k^{ij} , then we say the first level k conflict event between aircraft (i, j) occurs at moment τ_k^{ij} , i.e.

$$\tau_k^{ij} = \inf\{t > 0; x_t \in D_k^{ij}\} \quad (8)$$

Similarly as we did for reaching the collision level by aircraft i , we consider the first moment τ_k^i that aircraft i reaches conflict level k with any of the other aircraft, i.e.

$$\tau_k^i = \inf_{j \neq i} \{\tau_k^{ij}\} = \inf_{j \neq i} \{t > 0; x_t \in D_k^{ij}\} = \inf_{j \neq i} \{t > 0; x_t \in D_k^i\} \quad (9)$$

$$\text{with } D_k^i \triangleq \bigcup_{j \neq i} D_k^{ij}$$

Following the approach of Cérou et al. (2002), next we define $\{0, 1\}$ -valued random variables $\{\mathcal{X}_k^i, k = 1, \dots, m\}$ as follows:

$$\mathcal{X}_k^i = 1, \text{ if } \tau_k^i < T \text{ or } k = 0 \\ = 0, \text{ else}$$

By using this \mathcal{X}_k^i definition we can write the probability of collision of aircraft i with any of the other aircraft as a product of conditional probabilities of reaching the next conflict level given the current conflict level has been reached:

$$\mathbb{P}(\tau_m^i < T) = \mathbb{E}[\mathcal{X}_m^i] = \mathbb{E}\left[\prod_{k=1}^m \mathcal{X}_k^i\right] = \prod_{k=1}^m \mathbb{E}[\mathcal{X}_k^i | \mathcal{X}_{k-1}^i = 1] \\ = \prod_{k=1}^m \mathbb{P}(\tau_k^i < T | \tau_{k-1}^i < T) = \prod_{k=1}^m \gamma_k^i \quad (10)$$

$$\text{with } \gamma_k^i \triangleq \mathbb{P}(\tau_k^i < T | \tau_{k-1}^i < T)$$

With this, the problem can be seen as one to estimate the conditional probabilities γ_k^i in such a way that the product of these estimators is unbiased. Because of the multiplication of the various individual γ_k^i estimators, which depend on each other, in general such a product may be heavily biased. The key novelty of Cérou et al. (2002) was to show that such a product may be evaluated in an unbiased way when $\{x_t\}$ makes part of a larger stochastic process that satisfies the strong Markov property. This approach is explained next.

IV. CHARACTERIZATION OF THE RISK FACTORS

Let us denote $E' = \mathbb{R}^{n+1} \times M$, and let \mathcal{E}' be the Borel σ -algebra of E' . For any $B \in \mathcal{E}'$, $\pi_k^i(B)$ denotes the conditional probability of $\xi_k \triangleq (\tau_k, x_{\tau_k}, \theta_{\tau_k}) \in B$ given $\mathcal{X}_l^i = 1$ for $1 \leq l \leq k$.

Define $\bar{D}_k^i = (0, T) \times D_k^i \times M$, $k = 1, \dots, m$. Then the estimation of the probability for ξ_k to arrive at the k -th nested Borel set \bar{D}_k^i is characterized through the following recursive sequence of transformations

$$\pi_{k-1}^i(\cdot) \xrightarrow{\text{prediction}} p_k^i(\cdot) \xrightarrow{\text{conditioning}} \pi_k^i(\cdot), \\ \downarrow \\ \gamma_k^i$$

where $p_k^i(B)$ is the conditional probability of $\xi_k \in B$ given $\mathcal{X}_l^i = 1$ for $0 \leq l \leq k-1$. Because $\{x_t, \theta_t\}$ is a strong Markov process, $\{\xi_k\}$ is a Markov sequence, the prediction of which satisfies:

$$p_k^i(B) = \int_{E'} p_{\xi_k | \xi_{k-1}}(B | \xi) \pi_{k-1}^i(d\xi) \text{ for all } B \in \mathcal{E}' \quad (11)$$

Next we characterize the conditional probability of reaching the next level:

$$\begin{aligned}\gamma_k^i &= \mathbb{P}(\tau_k^i < T \mid \tau_{k-1}^i < T) \\ &= \mathbb{E}[\chi_k^i \mid \chi_{k-1}^i = 1] \\ &= \int_{E'} 1_{\{\xi \in \bar{D}_k^i\}} p_k^i(d\xi).\end{aligned}\quad (12)$$

And the conditioning satisfies:

$$\pi_k^i(B) = \frac{\int_B 1_{\{\xi \in \bar{D}_k^i\}} p_k^i(d\xi)}{\int_{E'} 1_{\{\xi \in \bar{D}_k^i\}} p_k^i(d\xi')} \quad \text{for all } B \in \mathcal{E}'. \quad (13)$$

With this, each of the m terms γ_k^i in (10) is characterized as a solution of a sequence of “filtering” kind of equations (11)-(13). An important difference with “filtering” equations is however that (11)-(13) are ordinary integral equations, i.e. they have no stochastic term entering them.

V. INTERACTING PARTICLE SYSTEM BASED RISK ESTIMATION

Based on this theory, an advanced Interacting Particle System (IPS) simulation algorithm is explained next for a multi aircraft scenario the operation of which is based on the AMFF operational concept [KleinObbink, 2003]. The transformations (11)-(13) lead to the IPS algorithm to estimate $\mathbb{P}(\tau_m < T)$. By $\gamma_k^{N_p}$, $p_k^{N_p}$ and $\pi_k^{N_p}$ we denote the numerical approximations of γ_k , p_k and π_k respectively (to simplify expressions we drop index i). When simulating from D_{k-1} to D_k , a fraction $\gamma_k^{N_p}$ of the Monte Carlo simulated trajectories only will reach D_k within the time period $(0, T)$.

In [Krystul & Blom, 2005, 2006] two versions of IPS algorithm are presented. In these two versions N_p particles are used per each mode $\theta \in M$ in total $N_p \times N$ particles, here $N = |M|$ is the number of elements in M . The resampling in these algorithms is done separately for each θ -mode (i.e. stratified sampling with strata corresponding to θ -modes). If N is big then the algorithm becomes inefficient and slow, since a huge number of particles must be treated. In this section we present a version of IPS algorithm which aims to handle this problem. The idea is to introduce new “aggregated” mode process $\{\kappa_t\}$:

$$\kappa_t(\omega) = F(\theta_t(\omega)), \quad t \in R_+,$$

Where $F: M \rightarrow M_\kappa$ and $|M_\kappa| < |M|$. An element $\kappa \in M_\kappa$ corresponds to a set of θ -modes: $F^{-1}(\kappa) \subset M$. In our new algorithm the resampling is done conditionally on κ -modes, (i.e. stratified sampling with strata corresponding to κ -modes).

By $\varphi_t(\kappa)$ we will denote an approximation of κ -mode probability $p_{\kappa_t^h}(\kappa)$ (i.e. total weight of particles in κ -mode) and $\varphi_0(\kappa) = p_{\kappa_0^h}(\kappa)$. By $\Lambda = (\lambda_{ij}(x))_{i,j=1}^N$ we denote the infinitesimal switching rate matrix of the discrete valued component $\{\theta_t\}$. The particle is defined as a triplet (x, θ, ω) , $\omega \in [0, 1]$, $x \in \mathbb{R}^n$ and $\theta \in \mathbb{M}$. The first component of x counts the time.

Advanced IPS Step 0. Initial setup

- Choose an appropriate nested sequence of closed subsets D_j , ($j = 1, \dots, m$), of \mathbb{R}^{n-1} such that $D = D_m \subset D_{m-1} \subset \dots \subset D_1$, and define $\bar{D}_k \triangleq (0, T) \times D_k \times \mathbb{M}$, $k = 1, \dots, m$.

- Choose a small $\varepsilon > 0$ and compute

$$r = \max_{\substack{i=1, \dots, N \\ x \in \mathbb{R}^n}} \left| \sum_{\substack{j=1 \\ j \neq i}}^N \lambda_{ij}(x) \right| + \varepsilon.$$

- Choose a discretization step $h = \frac{T}{J}$.

Advanced IPS Step 1. Initial sampling; $k = 0$.

- At time $t = 0$ we start with a set of N_p particles for each mode $\kappa \in \mathbb{M}_\kappa = \{e_1, \dots, e_S\}$:

$$\{x_0^{\kappa,i}, \theta_0^{\kappa,i}, \omega_0^{\kappa,i}\}_{i=1}^{N_p}, \quad \kappa \in \mathbb{M}_\kappa,$$

where $\theta_0^{\kappa,i}$ are independently drawn from $p_{\theta_0 | \kappa_0}(\cdot | \kappa)$, $x_0^{\kappa,i} \in \mathbb{R}^{n-1}$ are independently drawn from

$p_{x_0 | \theta_0}(\cdot | \theta_0^{\kappa,i})$ with the first component of $x_0^{\kappa,i}$ equal to zero, and the initial weights satisfy

$$\omega_0^{\kappa,i} = \frac{p_{\kappa_0}(\kappa)}{N_p}, \quad i = 1, \dots, N_p, \quad \kappa \in \mathbb{M}_\kappa.$$

- Then

$$\pi_0^{N_p}(B, \theta) = \sum_k \sum_{i=1}^{N_p} \omega_0^{\kappa,i} \delta_{\{x_0^{\kappa,i}, \theta_0^{\kappa,i}\}}(B, \theta),$$

$$B \in \mathcal{B}(\mathbb{R}^n), \quad \theta \in \mathbb{M}$$

and

$$\gamma_0^{N_p} = 1.$$

Iteration: for $k = 1, \dots, m$ over step 2 (prediction) and step 3 (assessment)

Advanced IPS Step 2. Prediction: $\pi_{k-1}^{N_p} \longrightarrow p_k^{N_p}$;

Start with empty sets S_k^κ , $\kappa \in \mathbb{M}_\kappa$, to store \bar{D}_k arrived particles.

For $j = 1, \dots, J$, with $h = \frac{T}{J}$ and $\hat{t}_j := T_{k-1} + h \cdot j$

Substep 2.j.a (Interaction based κ resampling for each particle):

Do for each particle in $\{x_{\hat{t}_{j-1}}^{\eta,i}, \theta_{\hat{t}_{j-1}}^{\eta,i}, \omega_{\hat{t}_{j-1}}^{\eta,i}\}$ for which

$\omega_{\hat{t}_{j-1}}^{\eta,i} \neq 0$:

- Sample a $\kappa_{\hat{t}_j}^{\eta,i} \in \mathbb{M}_\kappa$ from

$$\tilde{P}_{\kappa_{\hat{t}_j}^h | X_{\hat{t}_{j-1}}^h, \theta_{\hat{t}_{j-1}}^h, \kappa_{\hat{t}_{j-1}}^h}(\kappa | x_{\hat{t}_{j-1}}^{\eta,i}, \theta_{\hat{t}_{j-1}}^{\eta,i}, \eta) \text{ with}$$

$$\tilde{P}_{\kappa_{\hat{t}_j}^h | X_{\hat{t}_{j-1}}^h, \theta_{\hat{t}_{j-1}}^h, \kappa_{\hat{t}_{j-1}}^h}(\kappa | x_{\hat{t}_{j-1}}^{\eta,i}, \theta_{\hat{t}_{j-1}}^{\eta,i}, \eta) \triangleq \begin{cases} 1 - \alpha, & \text{if } \kappa = \eta \\ \alpha, & \text{else} \end{cases}$$

and α some fixed small probability.

- For all $\kappa, \eta \in \mathbb{M}_\kappa$, $i = 1, \dots, N_p$ evaluate the transition probabilities

$$P_{\kappa_{\hat{t}_j}^h | X_{\hat{t}_{j-1}}^h, \theta_{\hat{t}_{j-1}}^h, \kappa_{\hat{t}_{j-1}}^h}(\kappa | x_{\hat{t}_{j-1}}^{\eta,i}, \theta_{\hat{t}_{j-1}}^{\eta,i}, \eta) \approx$$

$$\approx \sum_{\theta \in F^{-1}(\kappa)} [I_{\theta_{\hat{t}_{j-1}}^{\eta,i}} + (P_{\theta_{\hat{t}_{j-1}}^{\eta,i}}^*(x_{\hat{t}_{j-1}}^{\eta,i}))rh]e^{-rh},$$

where $P^*(\zeta) = \frac{1}{r} \tilde{\Lambda}(\zeta) + I$, with $\tilde{\Lambda}(\cdot)$ defined by

$$\tilde{\Lambda}(x) = \begin{cases} \Lambda(x) & \text{if } x_1 \in [0, T), \\ O^{N \times N} & \text{if } x_1 \notin [0, T). \end{cases}$$

- Adapt the weights:

$$\tilde{\omega}_{\hat{t}_{j-1}}^{\eta,i} = \begin{cases} \omega_{\hat{t}_{j-1}}^{\eta,i} \frac{P_{\kappa_{\hat{t}_j}^h | X_{\hat{t}_{j-1}}^h, \theta_{\hat{t}_{j-1}}^h, \kappa_{\hat{t}_{j-1}}^h}(\kappa_{\hat{t}_j}^{\eta,i} | x_{\hat{t}_{j-1}}^{\eta,i}, \theta_{\hat{t}_{j-1}}^{\eta,i}, \eta)}{1 - \alpha} & \text{if } \kappa = \eta \\ \omega_{\hat{t}_{j-1}}^{\eta,i} \frac{P_{\kappa_{\hat{t}_j}^h | X_{\hat{t}_{j-1}}^h, \theta_{\hat{t}_{j-1}}^h, \kappa_{\hat{t}_{j-1}}^h}(\kappa_{\hat{t}_j}^{\eta,i} | x_{\hat{t}_{j-1}}^{\eta,i}, \theta_{\hat{t}_{j-1}}^{\eta,i}, \eta)}{\alpha} & \text{if } \kappa \neq \eta \end{cases}$$

- Select per $\kappa \in \mathbb{M}_\kappa$ all particles $\{x_{\hat{t}_{j-1}}^{\eta,i}, \theta_{\hat{t}_{j-1}}^{\eta,i}, \tilde{\omega}_{\hat{t}_{j-1}}^{\eta,i}, \kappa_{\hat{t}_j}^{\eta,i}\}$, $\eta \in \mathbb{M}_\kappa$, $i = 1, \dots, N_p$, for which $\kappa_{\hat{t}_j}^{\eta,i} = \kappa$, and renumber the indices of these particles such that the $\kappa_{\hat{t}_j}^{\eta,i}$ value is recognizable from the first index of a new set of particles. This yields for each $\kappa \in \mathbb{M}_\kappa$ the following new set of particles $\{\bar{x}_{\hat{t}_{j-1}}^{\kappa,i}, \bar{\theta}_{\hat{t}_{j-1}}^{\kappa,i}, \bar{\omega}_{\hat{t}_{j-1}}^{\kappa,i}\}_{i=1}^{N_{\hat{t}_j}^\kappa}$ if $N_{\hat{t}_j}^\kappa \neq 0$, and an empty set $\{\}$ if $N_{\hat{t}_j}^\kappa = 0$.

Substep 2.j.b (Prediction):

- Determine the new set of particles

$\{x_{\hat{t}_j}^{\kappa,i}, \theta_{\hat{t}_j}^{\kappa,i}, \omega_{\hat{t}_j}^{\kappa,i}\}_{i=1}^{N_{\hat{t}_j}^\kappa}$, $\kappa \in \mathbb{M}_\kappa$ by evaluating for each particle a new value $x_{\hat{t}_j}^{\kappa,i}$ according to Euler discretization scheme:

$$x_{\hat{t}_j}^{\kappa,i} = \bar{x}_{\hat{t}_{j-1}}^{\kappa,i} + a(\bar{x}_{\hat{t}_{j-1}}^{\kappa,i}, \bar{\theta}_{\hat{t}_{j-1}}^{\kappa,i})h + b(\bar{x}_{\hat{t}_{j-1}}^{\kappa,i}, \bar{\theta}_{\hat{t}_{j-1}}^{\kappa,i})(W_{\hat{t}_j}^i - W_{\hat{t}_{j-1}}^i)$$

and new value $\theta_{\hat{t}_j}^{\kappa,i}$ by independent sampling with replacement from:

$$P_{\theta_{\hat{t}_j}^{\kappa,i} | \kappa_{\hat{t}_j}^h, X_{\hat{t}_{j-1}}^h, \theta_{\hat{t}_{j-1}}^h}(\theta | \kappa, \bar{x}_{\hat{t}_{j-1}}^{\kappa,i}, \bar{\theta}_{\hat{t}_{j-1}}^{\kappa,i}) \approx$$

$$1_\kappa(F(\theta)) [I_{\bar{\theta}_{\hat{t}_{j-1}}^{\kappa,i} \theta} + (P_{\bar{\theta}_{\hat{t}_{j-1}}^{\kappa,i} \theta}^*(\bar{x}_{\hat{t}_{j-1}}^{\kappa,i}))rh] e^{-rh}$$

$$\approx \frac{1_\kappa(F(\theta)) [I_{\bar{\theta}_{\hat{t}_{j-1}}^{\kappa,i} \theta} + (P_{\bar{\theta}_{\hat{t}_{j-1}}^{\kappa,i} \theta}^*(\bar{x}_{\hat{t}_{j-1}}^{\kappa,i}))rh] e^{-rh}}{\sum_{\theta \in \mathbb{M}} 1_\kappa(F(\theta)) [I_{\bar{\theta}_{\hat{t}_{j-1}}^{\kappa,i} \theta} + (P_{\bar{\theta}_{\hat{t}_{j-1}}^{\kappa,i} \theta}^*(\bar{x}_{\hat{t}_{j-1}}^{\kappa,i}))rh] e^{-rh}}$$

- The weights are not changed: $\omega_{\hat{t}_j}^{\kappa,i} = \bar{\omega}_{\hat{t}_{j-1}}^{\kappa,i}$.

Substep 2.j.c (Store and neutralize arrived particles):

- If $x_{\hat{t}_j}^{\kappa,i} \in \bar{D}_\kappa$, then a copy of the particle $\{x_{\hat{t}_j}^{\kappa,i}, \theta_{\hat{t}_j}^{\kappa,i}, \omega_{\hat{t}_j}^{\kappa,i}\}$ is stored in the set S_k^κ , and then in the original particle we set $\omega_{\hat{t}_j}^{\kappa,i} = 0$.

Advanced IPS Step 3. Assess arrived particles;

- Renumbering the particles in S_k^κ yields a set of particles $\{\hat{x}_k^{\kappa,i}, \hat{\theta}_k^{\kappa,i}, \hat{\omega}_k^{\kappa,i}\}_{i=1}^{N_{S_k^\kappa}^\kappa}$ with $N_{S_k^\kappa}^\kappa$ the number of particles in S_k^κ .
- Approximation of γ_k : $\gamma_k \approx \gamma_k^{N_p} = \sum_{\kappa \in \mathbb{M}_\kappa} \sum_{i=1}^{N_{S_k^\kappa}^\kappa} \hat{\omega}_k^{\kappa,i}$. If $N_{S_k^\kappa} = 0$ for all $\kappa \in \mathbb{M}_\kappa$, then the algorithm stops with estimate $P_{hit}(0, T) \approx 0$.

Advanced IPS Step 4. Resampling: $p_k^{N_p} \longrightarrow \pi_k^{N_p}$;

- Resample N_p particles per mode $\kappa \in \mathbb{M}_\kappa$ from S_k^κ according the following scheme:
 - If $N_{S_k^\kappa} > N_p$ then draw N_p particles $\{x_k^{\kappa,i}, \theta_k^{\kappa,i}, \omega_k^{\kappa,i}\}$ independently from the empirical measure $\pi_k^{\kappa, N_p} = \sum_{i=1}^{N_{S_k^\kappa}^\kappa} \hat{\omega}_k^{\kappa,i} \delta_{\{x_k^{\kappa,i}, \theta_k^{\kappa,i}\}}$ and set $\omega_k^{\kappa,i} = \frac{\sum_{i=1}^{N_{S_k^\kappa}^\kappa} \hat{\omega}_k^{\kappa,i}}{\gamma_k^{N_p} \cdot N_p}$.
 - If $\frac{1}{2} N_p \leq N_{S_k^\kappa} \leq N_p$ then

1. Copy the $N_{S_k^*}$ particles, i.e.

$$\{x_k^{\kappa,i}, \theta_k^{\kappa,i}, \omega_k^{\kappa,i}\} = \{\hat{x}_k^{\kappa,i}, \hat{\theta}_k^{\kappa,i}, \hat{\omega}_k^{\kappa,i}\} \text{ and set}$$

$$\omega_k^{\kappa,i} = \hat{\omega}_k^{\kappa,i} \frac{N_{S_k^*}}{\gamma_k^{N_p} \cdot N_p} \text{ for } i = 1, \dots, N_{S_k^*},$$

2. Draw $N_p - N_{S_k^*}$ particles $\{x_k^{\kappa,i}, \theta_k^{\kappa,i}, \omega_k^{\kappa,i}\}$ independently from the empirical measure

$$\pi_k^{\kappa, N_p} = \sum_{i=1}^{N_{S_k^*}} \hat{\omega}_k^{\kappa,i} \delta_{\{x_k^{\kappa,i}, \theta_k^{\kappa,i}\}} \text{ and set } \omega_k^{\kappa,i} = \frac{\sum_{i=1}^{N_{S_k^*}} \hat{\omega}_k^{\kappa,i}}{\gamma_k^{N_p} \cdot N_p}.$$

- If $0 < N_{S_k^*} < \frac{1}{2} N_p$ then

1. Copy the $N_{S_k^*}$ particles, i.e.

$$\{x_k^{\kappa,i}, \theta_k^{\kappa,i}, \omega_k^{\kappa,i}\} = \{\hat{x}_k^{\kappa,i}, \hat{\theta}_k^{\kappa,i}, \hat{\omega}_k^{\kappa,i}\} \text{ and set}$$

$$\omega_k^{\kappa,i} = \hat{\omega}_k^{\kappa,i} \frac{1}{2 \cdot \gamma_k^{N_p}} \text{ for } i = 1, \dots, N_{S_k^*},$$

2. Draw $N_p - N_{S_k^*}$ particles $\{x_k^{\kappa,i}, \theta_k^{\kappa,i}, \omega_k^{\kappa,i}\}$ independently from the empirical measure

$$\pi_k^{\kappa, N_p} = \sum_{i=1}^{N_{S_k^*}} \hat{\omega}_k^{\kappa,i} \delta_{\{x_k^{\kappa,i}, \theta_k^{\kappa,i}\}} \text{ and set}$$

$$\omega_k^{\kappa,i} = \frac{\sum_{i=1}^{N_{S_k^*}} \hat{\omega}_k^{\kappa,i}}{2 \cdot \gamma_k^{N_p} \cdot (N_p - N_{S_k^*})}$$

- If $N_{S_k^*} = 0$ then make dummy particles

$$\{x_k^{\kappa,i}, \theta_k^{\kappa,i}, \omega_k^{\kappa,i}\} \text{ with } \omega_k^{\kappa,i} = 0$$

- The new set of particles per mode κ is

$$\{x_k^{\kappa,i}, \theta_k^{\kappa,i}, \omega_k^{\kappa,i}\}_{i=1}^{N_p}, \kappa \in \mathbb{M}_\kappa.$$

- If $k < m$ then (to account for empty sets S_k^*) do one interaction based resample **Substep 2.j.a*** (specified below) and prediction **Substep 2.j.b.** and then repeat steps 2, 3 for $k := k + 1$.

- If $k = m$, then stop with an estimate

$$P_{hit}(0, T) \approx \prod_{k=1}^m \gamma_k^{N_p}.$$

Description of Substep 2.j.a* (Interaction based resampling)

- For all $\kappa, \eta \in \mathbb{M}_\kappa$, $i = 1, \dots, N_p$ evaluate the transition probabilities

$$P_{\kappa_i^h | X_{i-1}^h, \theta_{i-1}^h, \kappa_{i-1}^h}(\kappa | x_{i-1}^{\eta,i}, \theta_{i-1}^{\eta,i}, \eta) \approx \approx \sum_{\theta \in F^{-1}(\kappa)} [I_{\theta_{i-1}^{\eta,i}} + (P_{\theta_{i-1}^{\eta,i}}^*(x_{i-1}^{\eta,i}))rh]e^{-rh},$$

where $P^*(\zeta) = \frac{1}{r} \tilde{\Lambda}(\zeta) + I$, with $\tilde{\Lambda}(\cdot)$ defined by

$$\tilde{\Lambda}(x) = \begin{cases} \Lambda(x) & \text{if } x_1 \in [0, T), \\ O^{N \times N} & \text{if } x_1 \notin [0, T). \end{cases}$$

Evaluate probabilities of modes:

$$P_{\kappa_i^h}(\kappa) \approx \varphi_{i_j}(\kappa) = = \sum_{\eta \in \mathbb{M}_\kappa} \sum_{i=1}^{N_p} P_{\kappa_i^h | X_{i-1}^h, \theta_{i-1}^h, \kappa_{i-1}^h}(\kappa | x_{i-1}^{\eta,i}, \theta_{i-1}^{\eta,i}, \eta) \omega_{i-1}^{\eta,i}$$

- For each $\kappa \in \mathbb{M}_\kappa$ independently draw N_p random pairs $(\bar{x}_{i-1}^{\kappa,i}, \bar{\theta}_{i-1}^{\kappa,i})$, $i = 1, \dots, N_p$ from the following particle spanned unnormalized joint measure:

$$\bar{P}_{X_{i-1}^h, \theta_{i-1}^h | \kappa_i^h}(B, \theta | \kappa) = P_{X_{i-1}^h, \theta_{i-1}^h | \kappa_i^h}(B, \theta, \kappa) \approx \approx \sum_{\eta \in \mathbb{M}_\kappa} \sum_{i=1}^{N_p} P_{\kappa_i^h | X_{i-1}^h, \theta_{i-1}^h, \kappa_{i-1}^h}(\kappa | x_{i-1}^{\eta,i}, \theta_{i-1}^{\eta,i}, \eta) \omega_{i-1}^{\eta,i} \delta_{\{x_{i-1}^{\eta,i}, \theta_{i-1}^{\eta,i}\}}(B, \theta)$$

- This yields for each $\kappa \in \mathbb{M}_\kappa$ the following set of particles $\{\bar{x}_{i-1}^{\kappa,i}, \bar{\theta}_{i-1}^{\kappa,i}, \bar{\omega}_{i-1}^{\kappa,i}\}_{i=1}^{N_p}$ with $\bar{\omega}_{i-1}^{\kappa,i} = \varphi_{i_j}(\kappa)/N_p$.

In order to apply this new IPS algorithm towards the assessment of collision risk of free flight, we need to develop a MC simulator of these operations the simulated trajectories of which are known to constitute a version of a generalized stochastic hybrid process that is strong Markov. [Bujorianu&Lygeros, 2005]. [Everdij&Blom, 2005, 2006] have developed a Stochastically and Dynamically Colored Petri Net (SDCPN) formalism that accomplishes this.

VI. IPS BASED ESTIMATION OF AMFF COLLISION RISK

In Everdij et al. (2006) it is shown how the SDCPN formalism has been used to develop a MC simulation model of the AMFF operation. The IPS algorithm developed above is now applied to this AMFF MC simulation model. The scenario has two aircraft, the flight plans of which cause the aircraft to be on a head on collision course.

A. Parameterization of the IPS simulations

The main safety critical parameter settings of the free flight enabling technical systems (GNSS, ADS-B and ASAS) are given in the following table.

Model Parameter	Probability
Global GNSS down	1.0×10^{-5}
Global ADS-B down ¹	1.0×10^{-6}
Aircraft ADS-B Receiver down	5.0×10^{-5}
Aircraft ADS-B Transmitter down	5.0×10^{-5}
Aircraft ASAS System mode corrupted	5.0×10^{-5}
Aircraft ASAS System mode failure	5.0×10^{-5}

The IPS conflict levels are defined by parameter values for lateral conflict distance d_k , conflict height h_k and time to conflict T_k . These values have been determined through two steps. The first was to let an operational expert make a best guess of proper parameter values. Next, during initial simulations with the IPS some fine-tuning of the number of levels and of parameter values per level has been done. The resulting values are given in the next table.

k	1	2	3	4	5	6	7	8
d_k (Nm)	4.5	4.5	4.5	4.5	2.5	1.25	0.5	0.054
h_k (ft)	900	900	900	900	900	500	250	131
T_k (min)	8	2.5	1.5	0	0	0	0	0

B. Two head-on flying aircraft

In this simulation two aircraft start at the same flight level, some 250 km away from each other, and fly on opposite direction flight plans head-on with a ground speed of 240 m/s.

By running ten times the classical IPS algorithm [Cerou et al., 2002, 2005] the collision risk is estimated ten times. The number of particles per IPS simulation run is 12,000. The total simulation time took about 5 hours on two machines, and the load of computer memory per machine was about 0.5 Gigabyte. For the first four IPS runs, the estimated fractions $\gamma_k^{N_p}$ are given in the table below for each of the conflict levels, $k = 1, \dots, 8$.

Level	1 st IPS	2 nd IPS	3 rd IPS	4 th IPS
1	1.0000	1.0000	1.0000	1.0000
2	0.0003	0.0007	0.0007	0.0007
3	0.0000	0.0000	0.0036	0.0148
4	0.0000	0.0000	0.0116	0.0003
5	0.0000	0.0000	0.0046	0.0000
6	0.0000	0.0000	0.0000	0.0000
7	0.0000	0.0000	0.0000	0.0000
8	0.0000	0.0000	0.0000	0.0000
Product of fractions	0.0000	0.0000	0.0000	0.0000

¹ Global ADS-B down refers to frequency congestion/overload of the data transfer technology used for ADS-B.

Apparently, none of the particles reaches level sets of number 6 or higher. Hence, the estimated collision probability is zero. Obviously, this is a very unreliable estimate of the collision risk. In other words, the IPS approach does not work well for this case.

In order to estimate the collision risk for this case we used our new more advanced version of IPS algorithm (section V), the results of which are given in the following table.

Level	1 st Adv IPS	2 nd Adv IPS	3 rd Adv IPS	4 th Adv IPS
1	0.9998	1.0000	1.0000	1.0000
2	0.3076	0.2848	0.2911	0.2986
3	0.0575	0.0483	0.0576	0.0561
4	0.0824	0.0625	0.0593	0.0337
5	0.0275	0.0184	0.0244	0.0163
6	0.0641	0.4296	0.2579	0.0444
7	0.1163	0.0427	0.0398	0.8953
8	0.6180	0.5792	0.5809	0.5013
Product of fractions	1.84×10^{-7}	1.68×10^{-7}	1.45×10^{-7}	1.84×10^{-7}

The estimated mean probability of collision between the two aircraft equals 1.67×10^{-7} . The minimum and maximum values stay within 25% of the mean value, which shows that the estimated value is quite accurate. It is remarkable to see that the variation in the fractions per level is significantly larger than the variation in the estimated collision probability product of the fractions. Apparently, the dependency between the fractions $\gamma_k^{N_p}$ reduces the variation in the multiplication of these fractions. This is a convincing illustration of the power of IPS based algorithms for a complex hybrid state strong Markov process.

VII. CONCLUDING REMARKS

This paper studied collision risk estimation of a free flight operation through a sequential Monte Carlo simulation. Sequential MC simulation method of [Cerou et al., 2002, 2005] has been extended for application to collision risk estimation in air traffic, and has subsequently been applied to a model of free flight.

The results obtained clearly show that our new IPS based collision risk estimation method allows to speed up Monte Carlo simulation by orders of magnitude for a much more complex simulation model than what was possible before (e.g. Blom et al., 2003a; Everdij et al., 2006). The main value of having performed this collision risk estimation for an initial simulation model of AMFF is that this provides valuable feedback to the design team and allows them to learn from Monte Carlo simulation results they have never seen before. The designers can use it for adapting the AMFF design such that it can better bring into account future high traffic levels.

In its current form the sequential MC simulation approach works well, but at the same time poses very high requirements

on the availability of dynamic computer memory and simulation time. The good message is that in literature on sequential MC simulation (e.g. Doucet et al., 2001; Glassermann, 2003; DelMoral, 2004) complementary directions have been developed which remain to be explored for application to free flight collision risk estimation. These potential improvements of the sequential MC simulation approach will be studied in follow-up research.

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