A method for predicting the rolling resistance of aircraft tires in dry snow

G.W.H. van Es
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Summary

This paper describes a method for predicting the rolling resistance of aircraft tires rolling in dry snow. Knowledge about the rolling resistance on a snow-covered surface is required when complying with aircraft certification and operational rules, which account for runway surface conditions. In addition to the rules, the Joint Aviation Authorities (JAA) have issued Advisory Material Joint AMJ 25X1591, a document providing information, guidelines, and recommendations for calculating the rolling resistance of aircraft tires in dry snow. The analytical method presented in AMJ 25X1591 gives unsatisfactory results when compared to experimental data. In this paper a new method is presented for predicting the rolling resistance due to snow. This new method and the AMJ method are validated by comparing the results of both methods for single tires and full-scale aircraft with available experiments. In general, a much better agreement with experimental data is found for the new method than for the AMJ method.
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(38 pages in total)
Nomenclature

\( b \) = effective tire width
\( D_c \) = rolling resistance due to compression
\( D_d \) = rolling resistance due to motion
\( D_r \) = rolling resistance on a dry hard surface
\( h_o \) = initial snow depth
\( h_f \) = final snow depth after compression
\( \ell_f \) = tire footprint length
\( p \) = pressure
\( R \) = tire radius
\( r \) = void ratio
\( u_o \) = dimensionless initial snow depth
\( u_f \) = dimensionless final snow depth
\( V_g \) = ground speed
\( w \) = maximum tire width
\( \delta \) = static vertical tire deflection
\( \lambda \) = grain structure index
\( \rho \) = snow density
\( \rho_o \) = initial snow density
\( \rho_f \) = final snow density
\( \rho_i \) = density of ice
\( \sigma \) = unconfined compressive strength of snow
\( \sigma_i \) = compressive strength of ice
1 Introduction

When taking off from a runway covered with slush, standing water or snow, the required take-off ground-run distance will be longer than on a dry runway. This is the result of the fact that slush, standing water and snow on the runway will generate a rolling resistance that increases the total resistance on the aircraft during the ground-run. For this reason the Joint Aviation Authorities (JAA) have established aircraft certification and operational rules accounting for runway surface conditions (see JAR 25X1591). In addition to the rules, the JAA have issued Advisory Material Joint AMJ 25X1591, a document providing information, guidelines, recommendations and acceptable means of compliance concerning take-offs and landings on wet and contaminated runways. A study conducted by the National Aerospace Laboratory NLR (Ref. 1), has shown that there is a four-fold increase in the probability of an accident for aircraft operating on wet and contaminated runways. This indicates the importance of certification and operational rules for wet and contaminated runway operations.

AMJ 25X1591 considers a number of contaminates like water, slush, ice, wet snow and dry snow. The current version of AMJ 25X1591 (Change 14, 27 May 1994) assumes that the rolling resistance caused by dry snow on the runway, can be calculated using the models originally developed for water and slush covered runways. Since snow is compressible and water and slush are not, this assumption is incorrect from a physical point of view. This is also clearly illustrated in Ref. 2, where experimentally determined rolling resistance in dry snow for a Falcon 20 aircraft shows quite different results from rolling resistance calculated according to AMJ 25X1591.

Currently a number of models are available to calculate the rolling resistance of tires in dry snow (See Ref. 3). Many of these models have been developed to analyse the mobility of military vehicles. These mobility models vary from purely empirical to analytical. From the study presented in Ref. 3 it becomes clear that these mobility models are not suited for aircraft tires. A more suitable model is developed by Lidstrom (Ref. 4). This model gives better results for aeronautical use than the mobility models (See Ref. 3). However, Lidstrom’s model can only be used for low snow densities and high tire pressures. Furthermore, a number of approximations and assumptions are made which reduce the accuracy of the model.

The objective of this paper is to present an accurate method for predicting the rolling resistance of aircraft tires in dry snow which can be used for all practical snow densities. This method should be considered instead of the current method recommended by the JAA. The paper is organised as follows. Basic snow characteristics are outlined first. A theoretical method for predicting the rolling resistance of a tire in dry snow is then described. The presented method is then compared with experimental data from tests employing single tires and full-scale aircraft.
2 Basic snow characteristics

Snow is a porous, permeable aggregate of ice grains, which can be single crystals, or close groupings of several crystals. Air and water vapor surround the crystals. Snow is a complicated material because of the variety of forms it can take and because it is thermodynamically unstable, which means that the physical properties of the snow can change with time (known as snow metamorphism). A large number of different types of snow are known. A possible classification of these types including typical densities, is given as follows:

- New snow (50-200 kg/m³)
- Powder snow (200-450 kg/m³)
- Compacted snow (450-700 kg/m³)
- Wet snow (300-700 kg/m³)

All these types of snow are compressible. This means that the volume changes when the snow is loaded. This volume change becomes evident as surface compaction. Density is probably the most useful single parameter of snow properties because e.g. structural characteristics show some correlation with density. The compression strength of snow, and thus its bearing capacity, also appears to be strongly influenced by its density. Below a density of 400 kg/m³ the bearing capacity of snow is usually very low resulting in large volume changes of the snow when it is loaded. At higher densities higher loads are required to compress the snow. Whenever snow is subjected to loads e.g. by moving the snow with a bulldozer or running a tire over it, the snow becomes stronger than natural snow of the same density. This is a result of bond growth between the ice crystals. As already pointed out, snow is metamorphic. Environmental influences like wind, temperature fluctuations, and rain can change the characteristics of snow within days and even within hours.

The behaviour of undisturbed snow under loading has been studied by the US ARMY Cold Regions Research and Engineering Laboratory CRREL (See e.g. Ref. 5 and 6). Results of such studies show that the tire pressure has a significant influence on the final snow density after loading. Table 1 gives an overview of the final snow densities used in the CRREL snow mobility models as a function of the tire pressure. Note that the values listed in table 1 refer to undisturbed snow. An adjustment to the original table was made for tires having a pressure of less than 150 kPa, based on experimental data for these tires. Most aircraft tires have pressures in the range of 560-1400 kPa. For such tire pressures the final snow density will be 600 or 650 kg/m³ according to table 1.
3 Description of the method

3.1 Introduction
When a tire moves over a surface covered with dry snow, it will deform the snow. The work
needed for this deformation results in forces acting on the tire opposite to the direction of
motion. In general the resistance of dry snow results from two forces (Ref. 3):
• compression resistance $D_c$
• vertical displacement resistance $D_d$
Theoretical methods for predicting both these forces will be discussed.

3.2 Resistance due to compression
When a tire rolls over a snow-covered surface, compaction of the snow occurs. The work
needed to compact the snow layer from an initial snow volume $V_1$ to a final snow volume $V_2$ is
given by (Ref. 3):
$$W = \int_{V_1}^{V_2} p \, dV$$
With $p$ the pressure exerted on the snow. Because the rolling resistance in snow due to
compression is the work needed to compress the snow per distance covered, Eq.1 can be
rewritten into (using the relation $dV=b.\,ds.\,dh$, with $ds$ distance travelled)$^3,4$:
$$D_c = \int_{h_o}^{h_f} b \, \sigma \, dh$$
In which $\sigma$ is the unconfined compressive strength of snow, $b$ is the effective tire width at the
point of contact between tire and snow surface, $h_o$ is the initial snow depth, and $h_f$ is the snow
depth after compression. The effective tire width can be obtained through a geometrical analysis
of the tire section in combination with the snow surface. This width is given by (Ref. 3):
$$b=2w\sqrt{\frac{\delta + h_o}{w}} - \left(\frac{\delta + h_o}{w}\right)^2$$
With: $\delta$ = vertical tire deflection (static load)
$w$ = maximum tire width
For $(\delta + h_o)/w \geq 0.5$ the effective tire width can be regarded as equal to the maximum tire width
$w$.
The bearing capacity of natural snow is usually very low, especially for snow with a density of
less than 400 kg/m$^3$. This results in large snow deformations in the form of compaction. This
deformation continues until there is equilibrium between the snow’s bearing capacity and the load placed on the tire. Strength studies of snow have shown that a relation exists between the initial snow density and its unconfined compressive strength. Ref. 8 gives the following empirical equation for the unconfined compressive strength of snow:

\[ \sigma = \sigma_i e^{-\lambda r^2} \]  

(4)

Where \( \sigma_i \) is the compressive strength of ice (=1.10\(^6\) N/m\(^2\)), \( \lambda \) is a grain structure index, and \( r \) is the void ratio defined as the ratio of void volume to the volume of solid ice grains. The void ratio is given by:

\[ r = \frac{\rho_i}{\rho} - 1 \]  

(5)

In which \( \rho_i \) is the density of ice (=920 kg/m\(^3\)) and \( \rho \) the actual snow density. The grain structure index \( \lambda \) is an empirical constant, which varies between 1 and 2 (Ref. 8). For natural snow this index is equal to 1.5 (Ref. 3).

Combining Eq. 2 and 4 results in:

\[ \int_{h_o}^{h_f} e^{-\lambda r^2} dh \]  

By introducing the relation \( u = \sqrt{\lambda} \left( h/h_i - 1 \right) = \sqrt{\lambda} \left( \rho_i/\rho - 1 \right) \), Eq. 6 can be written as:

\[ D_c = \frac{b \sigma_i}{\rho_i \sqrt{\lambda}} \int_{u_o}^{u_f} e^{-u^2} du \]  

(7)

In which:

\[ u_o = \sqrt{\lambda} \left( \frac{\rho_i}{\rho_o} - 1 \right) \] and

\[ u_f = \sqrt{\lambda} \left( \frac{\rho_i}{\rho_f} - 1 \right) \]

Note that \( \rho_o \) is the initial snow density and \( \rho_f \) is the final snow density after compression by the tire. The integral in Eq. 7 is equal to the standard error function except for the missing term \( 2/\sqrt{\pi} \). Numerical solutions of the integral in Eq. 7 are presented in figure 1 for the five final snow densities of table 1. The resistance due to snow compression can be obtained from Eq. 7 in combination with figure 1.

Figure 2 gives an example of the resistance due to compression of a single tire of type VII with a diameter of 0.37 m, a width of 0.14 m, tire pressure of 937 kPa, and with a normal load of 4400 Newton. Up to a snow density of about 350 kg/m\(^3\) the resistance due to compression increases linearly with increasing initial snow density. Around a snow density of 420 kg/m\(^3\) a maximum occurs in the resistance due to compression. For higher snow densities the resistance decreases because the bearing capacity of the snow increases, resulting in less deformation of
the snow. For the same tire the variation of the grain structure index $\lambda$ is analysed. For a snow depth of 80 mm, $D_c$ is calculated as function of initial snow density for three values of $\lambda$. The results are shown in figure 3. A higher grain structure index indicates stronger snow which results in a lower rolling resistance due to compression. Furthermore, the maximum drag due to compression occurs at a higher initial snow density when the grain structure index is increased.

### 3.3 Resistance due to motion

When a tire moves over a surface covered with snow, the snow particles that are being compressed have to be given enough dynamic energy to move them in vertical direction. The energy needed for this will result in a force acting on the tire opposing the forward movement of the tire. This force can be obtained by considering the kinetic energy of a snow particle being compressed. The resulting force acting on the tire opposite to the direction of motion is given by (Ref. 3 and 4):

$$D_d = \frac{b}{2} \int \rho V_g^2 \sin^2 \alpha \, dh$$

Where $V_g$ is the ground speed and $\alpha$ is defined in figure 4.

With the help of figure 4 and standard trigonometric relations Eq. 8 can be solved. The result is:

$$D_d = \frac{b}{2} h_o \rho_o V_g^2 \left[ \left( 1 - \cos^2 \alpha \right) - \frac{2}{R^2} h_f \cos \alpha \right] \left( \frac{h_o}{h_f} \right) +$$

$$+ \left( h_o - h_f \right) \left( \frac{2}{R^2} \cos \alpha \right) + \frac{2 h_f}{R^2} - \frac{1}{2R^2} \left( h_o^2 - h_f^2 \right)$$

Where $R$ is the tire radius. The final snow depth is given by the relation $h_f = h_o \rho_o/\rho_f$ (preservation of mass) assuming that all compacting in front of the tire occurs in the vertical direction only.

The dynamic resistance $D_d$ appears to be directly influenced by the final snow depth and therefore by the initial snow density according to the relation $h_f = h_o \rho_o/\rho_f$. To illustrate this effect the dynamic snow resistance is defined by $D_d = k V_g^2$. Figure 5 gives an example of $k$ as a function of initial snow density and snow depth for a single tire of type VII, with a diameter of 0.37 m, a tire width of 0.14 m, tire pressure of 937 kPa, and a normal load of 4400 Newton. It becomes clear that $k$ in this example has a maximum around an initial snow density $\rho_o$ of 200 kg/m$^3$. Below this density the influence of the low density of the snow becomes a dominant factor (see Eq. 9). Above 200 kg/m$^3$ the effect of the reduced sinkage ($= h_o - h_f$) becomes the dominant factor (see Eq. 9). This example shows that at low densities of approximately 150-300...
kg/m$^3$, the ground speed has a strong influence on the rolling resistance. In general the maximum dynamic resistance is a complex function of tire radius, final snow depth, tire foot print length, and initial snow depth. An analysis of a number of aircraft configurations showed that in general the maximum dynamic resistance occurs at a snow density of around 200 kg/m$^3$. 
4 Calculation of the rolling resistance of a complete aircraft rolling in dry snow

4.1 Basic approach

The total rolling resistance of an aircraft rolling along a snow-covered runway is given by:

\[ D_{\text{rolling}} = D_r + D_c + D_d \]  \hspace{1cm} (10)

In which \( D_r \) is the rolling resistance on a dry hard surface. The equations presented in this paper for \( D_c \) and \( D_d \) are for single tires. A complete aircraft has at least 3 tires, one on each main landing gear and one on the nose landing gear. To obtain the total aircraft rolling resistance due to snow, the resistance \( D_c \) and \( D_d \) for each single tire have to be calculated and summed.

The rolling resistance on dual tire landing gears (found on both nose and main gears) is simply the resistance of both single tires added together. The interference effects between both tires as found on dual tire configurations running through slush or water, is not likely to be present when rolling over a snow covered surface. The rolling resistance originates from the vertical compaction of the snow layer. Although there is some deformation perpendicular to the tire motion direction present, this deformation occurs mainly at or below the bottom of the rut (Ref. 9) and therefore does not affect the deformation in front of the adjacent tire. Hence interference effects can be ignored.

Another multiple-tire configuration is the bogie landing gear. After the initial compression of the snow by the leading tires, the snow in the rut becomes stronger and a higher pressure must be applied to compress the snow further (Ref. 4). Therefore, the trailing tires will have a lower rolling resistance than the leading tires. Experimental data presented in Ref. 10 for tires with an inflation pressure of 179 kPa, show that the resistance of the trailing tire can be around 20\% of the resistance of the leading tire. However, as shown in Ref. 6, aircraft tires which have an inflation pressure in the range of 560-1400 kPa, compact the snow such that further compaction of a significant level is only possible if the pressure of the trailing tire is at least more than 2100 kPa. The compaction of the snow in the rut by the trailing tires, which have an inflation pressure of 560-1400 kPa, will be minimal (Ref. 6). Therefore the rolling resistance of the trailing tires can be neglected when compared to the rolling resistance of the leading tires. Hence, the resistance on a bogie landing gear is equal to that of a dual tire configuration.

All other multiple-tire configurations can be treated in the same manner as described above. Unpublished experimental results have shown that the snow sprays coming from the tires are limited to small portions, which hardly strike the airframe. The speed and the density of the snow spray are much less than for instance water spray. Therefore, the resistance due to snow impingement on the airframe can be neglected.

In order to calculate the rolling resistance the static tire deflection \( \delta \) and tire footprint length have to be known. In the absence of experimental data empirical equations presented in Ref. 11 and 12 can be used.
4.2 Limitations of the presented method

For all calculations the following limitations apply:

- The snow depth must not exceed the tire radius. If so, there will be additional resistance due to "bulldozing" which is not considered in the presented method.

- Great care must be taken when comparing the results of the presented method with results obtained on surfaces with processed snow (e.g. snow that has been blown back onto the runway by snow removal equipment). It is known that processed snow behaves in a significantly different manner than natural snow (Ref. 13). This will result in different values for the final snow densities than the snow densities presented in table 1. Also the grain structure index $\lambda$ for processed snow will be different than for natural snow.

- The method only applies to dry snow.
5 Comparison with test data

5.1 Single tire test data
Test data of single aircraft tires rolling in dry snow are scarce. A literature search revealed only a very limited number of sources which could be used. Unfortunately, most data cannot be used for analysis due to the fact that processed snow was used during the tests and no natural snow. Two data sources were found which could be used. First single tire test data from Ref. 14 are used for comparison with the presented method. Ref. 14 provides test data obtained from a modified BV friction test vehicle. Tests were conducted with two different tires (radius 0.35 m, width between 0.12-0.254 m and inflation pressures between 165-550 kPa) at a limited number of ground speeds which varied between 27-43 kts. The natural snow used in these tests had a low density which did not exceed 130 kg/m³. The snow depth varied between 10 and 90 mm. Ref. 14 also presents data of several runs made on a dry, snow-free surface. These results are subtracted from the resistance values measured on a snow-covered surface to obtain the rolling resistance due to snow. These results are compared with the present method and the AMJ method. The method of JAA AMJ 25X1591 is presented in an appendix to this paper. The results are shown in figure 6. The theoretical and experimental results for the presented method correlate better than the AMJ method. The standard deviation for the presented method is 31% compared to 50% of the AMJ method.

In addition to the data obtained from Ref. 14, resistance data for single tires rolling in natural snow presented in Ref. 10 are used. The tires used had a diameter of around 0.74 m, a width of 0.27 m, and an inflation pressure of 179 kPa. All tests reported in Ref. 10 were conducted at ground speeds of less than 6 kts. The natural snow used in these tests was of low density and did not exceed 250 kg/m³. The snow depth varied between 100 and 360 mm. The comparison between experimental results and theory is shown in figure 7. The theoretical and experimental results for the presented method appear to correlate much better than the AMJ method. The standard deviation for the presented method is 36% compared to 98% of the AMJ method. The AMJ method completely underestimates the resistance.

Although in these two cases the presented method has a reasonable high standard deviation, it can still be regarded as useful when considering the number of variables involved and the inaccuracies of e.g. the measurements of snow depth, snow density and the resistance itself.

5.2 Full scale test data
Full scale test data of aircraft rolling on snow covered runways are available for a number of aircraft. Most of these tests were conducted in combination with braking friction tests. Unfortunately, a large number of the tests were not conducted in natural snow. Snow was either
blown onto the runway or processed in another way before it was put on the runway surface. Examples of tests conducted in processed snow are given in references 15, 16 and 17. These test results cannot be used to validate the presented method. Fortunately, there are tests conducted in natural snow. Tests in natural snow have been conducted by the National Aerospace Laboratory NLR, SAAB AB, and Dassault Aviation using a Citation II, a SAAB 2000 and a Falcon 2000 respectively. These tests were conducted as part of a project known as "CONTAMRUNWAY". This project was funded by the European Commission under the transport RTD programme, 4th framework. The National Research Council Canada NRC has conducted tests in natural snow using a Falcon 20. This work is part of the international Joint Winter Runway Friction Measurement Program.

The test results obtained in the “CONTAMRUNWAY” project and the Joint Winter Runway Friction Measurement Program will be compared with theoretical results of the method presented in this paper and with the method of JAA AMJ 25X1591 in the following sections.

### 5.2.1 Comparison with the Citation II test data

The National Aerospace Laboratory NLR has conducted tests with a Citation II on a snow-covered runway of Skavsta airport, in Sweden (See Ref. 18 for details). The results of these tests will be compared with the method presented in this paper. Figure 8 shows the comparison between experimental results, the JAR AMJ 25X1591 method (noted as AMJ), and with the presented method. Correlation between the experimental data and presented method appears to be good. The differences all lie within the overall accuracy of the experimental data. Note that the error bars in figure 8 were calculated considering the data reduction method and inaccuracies of the measured variables. The AMJ method does not correlate well with the experimental data. At low speeds the AMJ method gives almost no resistance whereas the experimental data and the present method do show a considerable amount of resistance at these low speeds.

### 5.2.2 Comparison with the SAAB 2000 test data

SAAB AG has conducted tests with a SAAB 2000 on a snow-covered runway of Linköping airport, in Sweden (See Ref. 19 for details). The results of these tests will be compared with the method presented in this paper. Figure 9 shows the comparison between experimental results, the JAR AMJ 25X1591 method, and with the presented method. The present method predicts a higher rolling resistance than measured. The resistance due to compression $D_c$ is overestimated whereas the variation of rolling resistance with ground speed is similar to the experimental found variation. The snow in the tests was very homogenous and constant in depth. It is therefore unlikely that the difference between present method and experimental data is caused
by variations in snow depth and/or density. During the tests the acceleration was measured. This acceleration was then compared to the calculated acceleration on a dry runway. The difference between both accelerations is then multiplied with the mass of the aircraft to obtain the rolling resistance due to snow. Ref. 19 does not provide details about the accuracy of this approach. In general the AMJ method underestimates the rolling resistance due to snow. At low speeds the AMJ method gives almost no resistance whereas the experimental data and the present method do show a considerable amount of resistance at these low speeds.

5.2.3 Comparison with the Falcon 2000 test data
Dassault Aviation has conducted tests with a Falcon 2000 on a snow-covered runway of Ivalo airport, in Finland (See Ref. 20 for details). The results of these tests will be compared with the method presented in this paper. Figure 10 shows the comparison between experimental results, the JAR AMJ 25X1591 method, and with the presented method. In figure 10 the average rolling resistance due to snow for each test run is presented. To indicate the variation in the experimental derived rolling resistance, the standard deviation of each data point is also plotted in figure 10. Correlation between the experimental data and presented method appears to be good. However, the two data points at low ground speeds are above the present method. The standard deviation of one data point at these low ground speeds is significant, which indicates some uncertainty for this result. The standard deviation of the other data point is much less, so for this result it is unclear why the present method underestimates the rolling resistance at low speeds. The AMJ method does not correlate well with the experimental data. At low speeds the AMJ method gives almost no resistance whereas the experimental data and the present method do show a considerable amount of resistance at these low speeds.

Besides results for all tires in the snow also results for the nose gear only in snow are presented in Ref. 20. Figure 11 shows the results of these tests compared with results of the present method and the AMJ method. Only results at high ground speeds are available. The present method correlates reasonable well with the experimental data. This confirms that the interference effects between both tires can be ignored and that the rolling resistance due to snow of a dual tire configuration is simply the resistance of both tires added together. The AMJ method underestimates the rolling resistance of the nose gear only.

5.2.4 Comparison with the Falcon 20 test data
The Falcon 20 of the Canadian Institute for Aerospace Research (IAR-NRC) was tested on a number of contaminated runways, including snow-covered runways. Some results of these tests are presented in Ref. 2, 15 and 19. Most of the tests were conducted in snow that was obtained by blowing snow from the runway infields onto the test sections. Only a limited number of tests
were conducted in natural snow (See Ref. 21). The results of the tests conducted in natural snow will be compared with the method presented in this report.

Figure 12 and figure 13 show the comparison between experimental results, the JAR AMJ 25X:1591 method, and the presented method. In Ref. 21 the average rolling resistance for each run is presented. To indicate the variation in the experimental derived rolling resistance, the standard deviation of each data point is also plotted in figure 12 and figure 13. There is a similar variation of the rolling resistance with ground speed as for the Citation II tests. At low ground speeds a considerable amount of resistance is present as for the Citation II results. The presented method tends to overestimate the rolling resistance. This difference between test data and the results of the presented method can be explained as follows. During the tests conducted by IAR-NRC, no engine parameters were recorded. It is assumed that all tests are conducted with idle thrust. In Ref. 21 Newton’s second law is applied to an aircraft moving on the runway. The measured accelerations and ground speeds are combined with the data for the lift, drag, thrust and other basic characteristics of the aircraft, to calculate the rolling resistance due to snow. A simple equation is used to calculate the idle thrust as a function of air speed only. This simple approach for calculating the idle thrust can introduce inaccuracies in the derived rolling resistance. For instance if there was actually more thrust than in the idle setting during the tests, the derived rolling resistance would be too low. Tests conducted with idle thrust settings can also be influenced by considerable effective lags in the engine cycle during the transition from full take-off thrust to idle thrust. Bias will be introduced into the thrust calculation when these lags are not accounted for. If these lags in engine cycle were present in the IAR-NRC tests, the derived rolling resistance due to snow would be too low. Note that the lags in engine cycle were accounted for when analysing the data of the tests with the Citation II, which were also conducted with idle thrust settings (Ref. 18). The tests presented in figure 13 were conducted with almost identical snow conditions (snow density and depth) as the Citation II tests presented in figure 8. The nose and main gear tires of the Falcon 20 and the Citation II are similar in size. The Falcon 20 has a dual tire configuration on both nose and main gear whereas the Citation II has single tires on both the nose and main gear. The rolling resistance due to snow compression for the Falcon under the conditions noted in figure 13 should therefore be roughly twice as high as the rolling resistance of the Citation II presented in figure 8. However, the rolling resistance of the Falcon 20 is of the same order as that of the Citation II. The present method does predict a rolling resistance that is almost twice as high as that for the Citation II. During the tests with the Citation II engine parameters, needed to calculate the thrust using an engine database, were recorded. This approach results in a much more accurate and reliable estimation of the thrust during the tests than the simple method used by IAR-NRC. With the assumption that the results obtained with Citation II are accurate, it can be concluded that the rolling resistance due to snow obtained by IAR-NRC with the Falcon 20 is likely to be too low. However, a more detailed analysis of the IAR-NRC results should be conducted to confirm these conclusions.
The AMJ method does not correlate well with the experimental data obtained with the Falcon 20. At low speeds the AMJ method gives almost no resistance whereas the experimental data and the present method do show a considerable amount of resistance at these low speeds. The AMJ method also shows a stronger variation of the rolling resistance with the ground speed than the experimental data.
6 Conclusions and recommendations

A new method for predicting the rolling resistance of a complete aircraft in dry snow is presented in this paper. It is concluded from a comparison with test data that the presented method is better than the method suggested in the current JAA AMJ 25X1591 (Change 14, 27 May 1994), which is used for certification. It is therefore recommended to consider the use of the presented method for the prediction of the rolling resistance due to dry snow, instead of the JAA AMJ 25X1591 method. However, further validation of the presented method is recommended. Especially larger aircraft with bogie landing gears should be analysed.
7 Acknowledgements

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8 References


Appendix: Calculation method of JAR AMJ 25X1591 (Change 14, 27 May 1994)

According to JAR AMJ 25X1591, calculations should take account of landing gear displacement drag as follows.

**Basic Tire Drag**
The drag on a single tire is given by
\[ D = \frac{1}{2} \rho V^2 b d C_D \]
Where \( \rho \) is the density of the precipitation, \( d \) is the depth of precipitation and \( b \) is the tire width at the surface and may be found from Eq. 3. The value of \( C_D \) may be taken as 0.75 for an isolated tyre.

**Multiple Wheels**
A typical dual wheel trailing arm arrangement shows a drag 1.6 times the single wheel drag (including interference) whereas the factor can be up to twice the single wheel drag if the wheels are in front of the main leg. For a typical four wheel bogie layout the drag was 3.35 times the single wheel drag (again including interference).

Note: The JAR AMJ 25X1591 uses the term “drag”, whereas in this paper the term “resistance” is used instead of “drag”.

Table 1: Final snow densities used in the CRREL shallow snow mobility model (Ref. 7).

<table>
<thead>
<tr>
<th>Tire pressure kPa (psi)</th>
<th>Final Snow density kg/m³</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;150 (&lt;22)</td>
<td>450</td>
</tr>
<tr>
<td>150-211 (22-31)</td>
<td>500</td>
</tr>
<tr>
<td>211-350 (31-50)</td>
<td>550</td>
</tr>
<tr>
<td>351-700 (51-101)</td>
<td>600</td>
</tr>
<tr>
<td>&gt; 700 (&gt;101)</td>
<td>650</td>
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Fig. 1: Solution of the integral in Eq. 7 for several final snow densities as function of initial snow density (λ=1.5).
Fig. 2: $D_c$ as function of initial snow density and snow depth.
$D_c$ (Newton)

Initial snow density (kg/m$^3$)

$\lambda = 2.0$

$\lambda = 1.5$

$\lambda = 1.0$

Fig. 3: Influence of grain structure index on $D_c$. 
Fig. 4: Geometry of a tire rolling in snow.
Fig. 5: $k$ as function of initial snow density and snow depth for a single tire.
Fig. 6: Correlation of experimental data and theoretical results for a single tire (data Ref. 14).
Fig. 7: Correlation of experimental data and theoretical results for a single tire (data Ref. 10).
Fig. 8: Citation II rolling resistance on a snow covered surface (snow depth = 40 mm, snow density = 120 kg/m$^3$).
Fig. 9: SAAB 2000 rolling resistance on a snow covered surface (snow depth =87.5 mm, snow density=109 kg/m^3).
Fig. 10: Falcon 2000 rolling resistance on a snow covered surface (snow depth =100 mm, snow density=110 kg/m³).
Fig. 11: Falcon 2000 nose gear rolling resistance on a snow covered surface (snow depth = 100 mm, snow density = 110 kg/m³).
Fig. 12: Falcon 20 rolling resistance on a snow covered surface (snow depth = 58 mm, snow density = 280 kg/m³).
Fig. 13: Falcon 20 rolling resistance on a snow covered surface (snow depth =40 mm, snow density=130 kg/m³).