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Summary

This report presents results of a study into PSD gust design load calculation methods. A Stochastic Simulation procedure based on the probability of exceeding the design level is developed. The probability of design level exceedance is equal for linear and nonlinear aircraft systems, so that the method produces equivalent design conditions for linear and nonlinear systems. The Stochastic Simulation procedure is defined such, that estimations for the attained accuracy can be given.

The results with Stochastic Simulation are compared to Deterministic PSD methods that have been studied in previous phases of this project. The MFB and the IDPSD methods produce results that approach the Stochastic Simulation results in some way, however there are still significant differences. The SG method results deviate considerably from the results of the other Deterministic methods as well as from Stochastic Simulation results.

It is felt that the NLR Stochastic Simulation procedure is a good representation of the PSD Continuous Turbulence concept that is applicable equivalently to linear and nonlinear aircraft systems.

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List of symbols and abbreviations

$\bar{A}$	weighted average aircraft response factor for turbulence	
$b_{1,2}$	turbulence intensity parameters	
f	frequency	[Hz]
G	von Karman gust filter transfer function	$[s^{½}]$
H	transfer function	
I_y	aircraft inertia around lateral axis	$[kgm^2]$
j	$\sqrt{-1}$	
k	impulse strength	
K	feedback gain	
K_{eq}	multiplication factor to feedback gain in IDPSD method	
L	turbulence scale length	[m]
M_b	wing root bending moment	[Nm]
M_t	wing root torsion moment	[Nm]
$m_{a/c}$	aircraft mass	[kg]
N(0)	number of positive zero-level crossings per second	$[s^{-1}]$
Δn	load factor	
$P_{1,2}$	parameters in the probability density function of turbulence	
s	"first system" output	
t	time	[s]
tp	probability parameter, defined by $tp=x/\sigma_x$ with $x=N(0;\sigma_x)$	
T	Deterministic gust length	[s]
T_g	length of Stochastic gust patch	[s]
U_σ	PSD design gust velocity	$[ms^{-1}]$
V	aircraft speed (TAS)	$[ms^{-1}]$
w	gust speed	$[ms^{-1}]$
W	gust speed signal in frequency domain	[m]
y	an output (aircraft load) of an aircraft system	
	centre of gravity acceleration of Noback model	$[m/s^2]$
z	an output of an aircraft system	
	centre of gravity acceleration by controller action of Noback model	$[m/s^2]$
ρ	correlation coefficient	
σ_w	turbulence intensity	$[ms^{-1}]$
σ_{wr}	representative turbulence intensity: the turbulence intensity that gives the largest contribution to the probability of design level exceedance	$[ms^{-1}]$
Φ	Power Spectral Density	
Φ^n_{ww}	normalized von Karman turbulence spectrum	[s]

φ	phase angle
$[\ ]^*$	complex conjugate

Abbreviations

cor	correlated
des	design
IDPSD	Indirect Deterministic Power Spectral Density method
LAS	Load Alleviation System
max	maximum
MFB	Matched Filter Based method
nocon	for the open loop system
nolim	for the closed loop system with linear (unlimited) load alleviation
nonlin	for the closed loop system with nonlinear (limited) load alleviation
PSD	Power Spectral Density
SG	Spectral Gust method

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1 Introduction

Under contract with the Netherlands Department of Civil Aviation RLD, NLR has been carrying out studies into PSD gust load calculation methods for application to nonlinear aircraft. The results of this investigation in 1995 are given in reference 6. The present report describes the work done in 1996 and 1997; a Stochastic Simulation procedure has been developed and analysed, that defines design levels by the proportion of time that the level is exceeded. Such a definition makes it possible to apply the procedure to aircraft models with nonlinear control systems.

Some Deterministic PSD methods exist that aim to comply with the Continuous Turbulence (PSD) airworthiness requirement. These Deterministic methods have been studied in the previous phases of this RLD project. In the present investigation, the Deterministic methods Matched Filter Based 1-Dimensional Search (MFB), Indirect Deterministic PSD (IDPSD), and Spectral Gust (SG) are applied to three aircraft models, comparing the results with the Stochastic Simulation procedure. Appendix C discusses the Deterministic PSD methods investigated. The three aircraft models are described in appendix B, and they are the following models:

- A model with two degrees of freedom (pitch and plunge) of a large aircraft, provided with a gust load alleviation system that controls the centre of gravity acceleration by symmetric aileron deflection (called "Noback model" here).
- A model with Fokker-100-like characteristics, consisting of two rigid (pitch and plunge) and ten flexible symmetric degrees of freedom, has been equipped with a Load Alleviation System (LAS) that feeds back the aircraft centre of gravity acceleration to a symmetric aileron deflection in order to reduce the wing root bending moment when a gust is encountered.
- A model of the A310 aircraft that has been distributed among the Gust Specialists to serve as a universal reference model. This symmetric model consists of two rigid and three elastic modes and has a Load Alleviation System using symmetric aileron and spoiler deflection.

These aircraft models are linear, and nonlinearities are only introduced by limited controller action. Calculations have been carried out for the linear systems as well as for the nonlinear systems.

Chapter 2 describes the Continuous Turbulence concept and how the present JAR/FAR Continuous Turbulence Design Envelope Analysis airworthiness requirement (Ref. 9 and 10) for nonlinear systems has been interpreted in this report. Design levels are defined in such a way that the amount of time that the design level is exceeded is the same for linear and non-linear aircraft systems (sometimes referred to as PEC, Probability of Exceedance Criteria). In the line of this interpretation, a Stochastic Simulation procedure is discussed that can be applied to nonlinear systems. As the method is stochastic, attention is paid to possible scatter in the results, and estimations for the attained accuracy are formulated.

Results of Stochastic Simulation are compared to results of the Deterministic PSD methods in chapter 3, and the implications of differences in the results are discussed.

Conclusions are given in chapter 4.

2 The Continuous Turbulence concept and Stochastic Simulation

2.1 The PSD turbulence model

In the Continuous Turbulence, or Power Spectral Density (PSD), approach as developed in reference 3, turbulence is regarded as a continuous random process. Its Gaussian statistical properties are considered to vary only slowly, so that the process is stationary over short periods of time (up to e.g. 10 minutes). In this way, the various relations of output to input developed for a stationary Gaussian process still apply for these periods.

The stochastic turbulence process w(t) describing the occurring turbulence velocities as function of time has a mean velocity of zero. The turbulence standard deviation or intensity σ_w is assumed to vary slowly, and can also be described as a stochastic variable with a Gaussian distribution. The probability density function of σ_w is specified by the following expression:

$$p(\sigma_w) = \frac{P_1}{b_1}\sqrt{\frac{2}{\pi}}\, e^{\left(\frac{-\sigma_w^2}{2b_1^2}\right)} + \frac{P_2}{b_2}\sqrt{\frac{2}{\pi}}\, e^{\left(\frac{-\sigma_w^2}{2b_2^2}\right)} \quad , \quad \sigma_w > 0. \tag{1}$$

The quantities P_1, P_2, b_1, and b_2 are parameters that depend on altitude.

Analyses of measured turbulence patches (Noback, Ref. 4) indicate that the normalized von Karman spectrum $\Phi^n{}_{ww}$ gives a fairly good description of the power spectrum shape of atmospheric turbulence and gusts:

$$\Phi^n_{ww}(f) = \frac{L}{V}\,\frac{1+\frac{8}{3}\left(1.339*2\pi f\frac{L}{V}\right)^2}{\left[1+\left(1.339*2\pi f\frac{L}{V}\right)^2\right]^{\frac{11}{6}}} \tag{2}$$

where L = turbulence scale length = 2500 ft.
V = aircraft speed (TAS).
f = frequency (the above spectrum covers positive and negative frequencies, the so-called two-sided spectrum).

This power spectrum contains a scale parameter L that is set to 2500 ft here, in conjunction with reference 3.

The above Power Spectral Density describes the distribution of the total power of the turbulence over all frequencies. Integration of the normalized von Karman power spectrum yields a total

power of 1 for the turbulence signal. The energy per unit time (i.e. the power) of turbulence is defined by $\sigma_w{}^2$, where σ_w is the RMS value of the gust velocities. The total power spectrum of turbulence $\Phi_{ww}(f)$ is therefore represented by:
$\Phi_{ww}(f) = \sigma_w{}^2\Phi^n{}_{ww}(f)$.

2.2 PSD aircraft loads in Design Envelope Analysis

Note that in this report, the Mission Analysis approach will not be discussed.

2.2.1 The definition of PSD design loads

The response of output quantity y of a linear aircraft system represented by transfer function H to a Gaussian stochastic turbulence input with power spectrum Φ_{ww} will have a power spectrum Φ_{yy} of:

$$\Phi_{yy}(f) = |H_y(jf)|^2\Phi_{ww}(f). \tag{3}$$

If the aircraft system behaves linearly, aircraft responses to a Gaussian stochastic input will be Gaussian too. The variance of this Gaussian output signal y then is equal to the total power of y:

$$\sigma_y^2 = \int_{-\infty}^{\infty}\Phi_{yy}(f)df = \int_{-\infty}^{\infty}|H_y(jf)|^2\Phi_{ww}(f)df = \sigma_w^2\int_{-\infty}^{\infty}|H_y(jf)|^2\Phi^n_{ww}(f)df. \tag{4}$$

Taking into account only changes in output quantities (and not the stationary values), the mean of the output signal will be zero, because the mean of the turbulence input signal is zero.

The ratio of output standard deviation and turbulence standard deviation is called $\bar{A}$, and can be regarded as a weighted average response factor, because it is an integration of the multiplication of two functions that describe aircraft response sensitivity and turbulence contents, see equation 4:

$$\frac{\sigma_y}{\sigma_w} = \sqrt{\int_{-\infty}^{\infty}|H_y(jf)|^2\Phi^n_{ww}(f)df} = \bar{A}_y\ . \tag{5}$$

In the linear PSD method, this response factor is used to define design loads due to continuous turbulence, by prescribing a design value for a parameter U_σ (depending on altitude) and calculating the design level of load quantity y by multiplication of the response factor $\bar{A}$ by the design parameter U_σ:

$$y_d = \bar{A}_y U_\sigma . \tag{6}$$

In the PSD approach, a design load condition of an aircraft consists not only of design values for the load quantities, but one load quantity will have its design value, while all the other loads in the aircraft structure (or major component) are equal to their correlated or matched values. In linear PSD theory, the correlated value of a load z with design load y is defined as:

$$z_c = \rho_{zy} z_d = \rho_{zy} \bar{A}_z U_\sigma \tag{7}$$

where ρ_{zy} = correlation coefficient between load z and load y.

The correlation coefficient is defined by:

$$\rho_{zy} = \rho_{yz} = \frac{\int_{-\infty}^{\infty} H_z(jf) H_y^*(jf) \Phi_{ww}^n(f)\, df}{\bar{A}_z \bar{A}_y} . \tag{8}$$

2.2.2 PSD design loads definition that can be applied to nonlinear systems

For the application of the PSD requirement to nonlinear systems, both FAR 25 and JAR 25 state that: "*When a stability augmentation system is included in the analysis, the effect of system nonlinearities on loads at the limit load level must be realistically or conservatively accounted for*". In the line of reference 3, an interpretation of the Continuous Turbulence (CT) requirement will be given here, so that nonlinearities can be "realistically accounted for".

As follows from equations 5 and 6, the design load level y_d can also be regarded as the standard deviation of the Gaussian stochastic response of y to the Gaussian stochastic turbulence input with intensity U_σ. This definition of the design value as a standard deviation of the stochastic output process y indicates that the probability of y exceeding the design load level y_d is equal to a Gaussian process exceeding its standard deviation, if the standard deviation of the turbulence input process is U_σ. If the standard deviation of the input process is $k*U_\sigma$, the design level is defined by the probability:

$$P(y>y_d) = P(y>\frac{1}{k}\sigma_y) = P(w>U_\sigma). \tag{9}$$

For nonlinear systems, the transfer functions H(jf) are dependent of the input, so $\bar{A}$ and ρ cannot be calculated. However, the concept of a design load level being defined by a certain probability of exceedance can be applied to nonlinear systems by calculating the nonlinear system time responses to stochastic turbulence. Counting procedures can than be used to identify the load levels with the desired probability of exceedance. The intensity (standard deviation) of the stochastic turbulence signal to be applied in such a nonlinear time simulation has been studied thoroughly by Noback in reference 1. The results of that study will be discussed in subchapter 2.3.

When a linear aircraft model response to a stochastic patch of turbulence is calculated, the load z can have any value at the moment that load quantity y reaches its design value. As y and z are responses of the same aircraft to the same turbulence signal, there will be a certain correlation between y and z, defined by ρ_{zy}. Outputs y and z have a certain combined probability density function. The values of z at the moments that y is equal to y_d will have a Gaussian probability distribution in this case of a Gaussian stochastic turbulence input. It can easily be verified, see also reference 2 and chapter 2.5.2, that the load value of z as defined in equation 7 is the most probable (or, in this Gaussian case, the mean) value of z if y has its design value. In other words, a PSD design case is defined by one load value y_d that is exceeded a certain fraction of time, and all other loads z having their most probable values if $y=y_d$, when flying through a patch of Gaussian stochastic turbulence having the von Karman power spectral density:

$$P(y>y_d) = P(w>U_\sigma) \qquad p(z_c \mid y=y_d) = \max\left(p(z \mid y=y_d)\right). \tag{10}$$

These definitions of y_d and correlated load z_c can be applied to time responses of a nonlinear system.

2.3 Stochastic Simulation methodology

In the definition of linear PSD design and correlated loads, a value is prescribed for the continuous turbulence design parameter U_σ. It has been shown that the linear PSD design loads can also be regarded as load levels with a certain probability of exceedance if the turbulence is assumed to be a Gaussian stochastic process. Design parameter U_σ then becomes a measure for the turbulence standard deviation or intensity. If the turbulence intensity is taken equal to U_σ, the design load level coincides with the standard deviation of the load output. But if the turbulence intensity is taken equal to $k*U_\sigma$, the design load level coincides with 1/k times the standard deviation of the load output. This is a consequence of the linearity of the aircraft system: the response characteristics (H(jf)) are independent of the input intensity. In order to formulate a Stochastic Simulation procedure that is also applicable to nonlinear systems (where the response characteristics depend on the intensity of the input), a rational choice has to made with regard to the turbulence input standard deviation. In the following, a review will be given of Noback's study with respect to turbulence intensity in Stochastic Simulation (Ref. 1).

In the PSD philosophy, it is assumed that the intensity σ_w of atmospheric turbulence varies slowly; it is a stochastic variable. The probability density of σ_w is given by equation 1.

For a linear system, the probability density function p(y) of (incremental) load y in response to Gaussian stochastic turbulence ($\mu=0$, $\sigma=\sigma_w$) will be Gaussian:

$$p(y) = \frac{1}{\sqrt{2\pi}\,\sigma_y} e^{\left(-\frac{y^2}{2\sigma_y^2}\right)} = \frac{1}{\sqrt{2\pi}\;\overline{A}_y\sigma_w} e^{\left(-\frac{y^2}{2\overline{A}_y^2\sigma_w^2}\right)} \qquad (11)$$

where σ_w is not a constant value, but a stochastic variable with a probability density function as given in equation 1. The probability that y exceeds level $\overline{y}$ for a certain value of σ_w is:

$$P(y>\overline{y}\,|\,\sigma_w) = \int_{\overline{y}}^{\infty} \frac{1}{\sqrt{2\pi}\;\overline{A}_y\sigma_w} e^{\left(-\frac{y^2}{2\,\overline{A}_y^2\sigma_w^2}\right)} dy = 0.5\,\mathrm{erfc}\left(\frac{\overline{y}}{\sqrt{2}\;\overline{A}_y\sigma_w}\right) \qquad (12)$$

The probability of exceeding $\overline{y}$ when σ_w is a stochastic variable can be found by multiplying equation 12 by $p(\sigma_w)$ from equation 1 and integrating over all values of σ_w:

$$\begin{aligned} P(y>\overline{y}) \; = \; & \frac{P_1}{b_1}\sqrt{\frac{2}{\pi}} \int_0^{\infty} e^{-\frac{\sigma_w^2}{2b_1^2}}\, 0.5\,\mathrm{erfc}\left(\frac{\overline{y}}{\sqrt{2}\;\overline{A}_y\sigma_w}\right) d\sigma_w \\ + \; & \frac{P_2}{b_2}\sqrt{\frac{2}{\pi}} \int_0^{\infty} e^{-\frac{\sigma_w^2}{2b_2^2}}\, 0.5\,\mathrm{erfc}\left(\frac{\overline{y}}{\sqrt{2}\;\overline{A}_y\sigma_w}\right) d\sigma_w . \end{aligned} \qquad (13)$$

The non-storm contribution (defined by P_1 and b_1) in this equation is negligible (for altitudes up to 17000 m or 55000 ft), so only the second part of the expression remains. Noback found in reference 1, that the result of the integration in equation 13 is determined by a relatively narrow band of σ_w-values. A good approximation of the maximum of the integrand in equation 13 is found at a σ_w-value of:

$$\sigma_w = b_2 \sqrt{\frac{1 + \sqrt{1 + 4\left(\frac{\bar{y}}{\bar{A} b_2}\right)^2}}{2}} \tag{14}$$

The total probability of exceeding level $\bar{y}$ is thus determined by a narrow band of σ_w-values around the above value of σ_w. The design level of a linear aircraft load y is $y_d = \bar{A} U_\sigma$, so the band of turbulence intensities contributing the most to the probability of exceeding the design level $\bar{y}=y_d$ is located around the following "representative" σ_w-value:

$$\sigma_{wr} = b_2 \sqrt{\frac{1 + \sqrt{1 + 4\left(\frac{y_d}{\bar{A} b_2}\right)^2}}{2}} = b_2 \sqrt{\frac{1 + \sqrt{1 + 4\left(\frac{U_\sigma}{b_2}\right)^2}}{2}} . \tag{15}$$

It is suggested to use this property of the linear aircraft output distribution functions for the calculation of continuous turbulence design loads for nonlinear aircraft systems: The value of σ_{wr} according to 15 is to be used as the intensity of a Gaussian stochastic turbulence patch that serves as input to the nonlinear aircraft equations of motion. The output probability distribution will not be Gaussian.

The design value of a load quantity y can for a linear system be defined by its probability of exceedance:

$$P(y > y_d \,|\, \sigma_{wr}) = 0.5\,\mathrm{erfc}\left(\frac{y_d}{\sqrt{2}\ \bar{A}\sigma_{wr}}\right) = 0.5\,\mathrm{erfc}\left(\frac{U_\sigma}{\sqrt{2}\,\sigma_{wr}}\right) = P(w > U_\sigma \,|\, \sigma_{wr}) . \tag{16}$$

The probability of exceeding the design level is equal to the probability that w(t) exceeds level U_σ in the turbulence patch. This unambiguous definition of the design level in terms of a probability of exceedance can be used directly for nonlinear systems.

Thus, a "Stochastic Simulation" procedure has been defined to determine design levels of nonlinear aircraft loads, using a rational value of turbulence intensity, σ_{wr}.

Noback proposes in reference 1 to define the nonlinear correlated load z_c as the value of z having a probability of exceedance of 0.5 when the design load quantity y has its design level y_d:

$$P(z>z_c \mid y=y_d) = 0.5. \qquad (17)$$

In the case of a linear aircraft system, this means that z_c is the most probable value of z when y is equal to y_d; this need not be true for a nonlinear system. It may be more consistent to use this "most probable" criterion, as given in equation 10, for correlated loads in nonlinear cases too. It has been decided in this report to maintain Noback’s proposal of P=0.5.

2.4 Practical implementation of the Stochastic Simulation procedure

2.4.1 Generation of a Gaussian stochastic turbulence patch

Simulation of aircraft responses to a stochastic gust patch (Stochastic Simulation) is carried out by generating a gust patch of a certain length that serves as input to the aircraft equations of motion. The power spectral density of the gust speed is the von Karman spectrum, and the intensity equal to σ_w. This means that the amplitude of the frequency domain turbulence signal W(jf) is prescribed, but that the phase angle $\varphi(f)$ is arbitrary, because the power spectrum of a frequency domain signal W(jf) is:

$$\Phi_{ww}(f) = \frac{W(jf)W^*(jf)}{T_g} = \frac{|W(jf)|^2}{T_g} \qquad (18)$$

where T_g = length of gust patch.

The turbulence signal must have a Gaussian distribution. To generate a stochastic Gaussian signal, the phase angle of W(jf) should be a random value at every frequency. The gust speed input signal w(t) as function of time in a limited interval T_g is the inverse Fourier transform of W(jf):

$$w(t) = \mathcal{F}^{-1}\left[\sigma_w \sqrt{T_g}\ \sqrt{\Phi^n_{ww}(f)}\ e^{j\varphi(f)}\right] \qquad (19)$$

where j^2 = -1

φ = phase angle, in this case random between 0 and 2π.

In computational procedures, this Fourier transform is performed by application of a Fast Fourier Transform algorithm to a W(jf) signal that is known at a limited number of equidistant frequencies. The resulting discrete time domain turbulence signal w(t) thus contains contributions of frequencies according to the von Karman spectrum, $\sigma^2_w \Phi^n_{ww}$, and the gust speed exceedance curve has the shape of the normal distribution.

When generating a patch of stochastic turbulence, the following aspects have to be taken into account:

1. The lowest frequency present in the turbulence signal should not be less than $1/T_g$. If a lower frequency is present, the mean of the signal may not be zero, which means a shift of the probability distribution function (more positive than negative gust velocities, for instance).
2. Due to the description of the gust signal in the frequency domain W(jf) at a finite number of discrete frequencies, with $1/T_g$ the lowest frequency, the gust patch is in fact an infinite, but periodic, stochastic process with time period T_g. As the aircraft time response should be periodic with the same time period as the input signal (in order to make the time signal be the response to the periodic input signal), it is necessary that transient response behaviour has died out at the beginning of the turbulence patch at t=0. This can be achieved by starting the simulation "long enough before t=0". This means that the simulation is started with the last few seconds of turbulence from the "former" patch of length T_g in the infinite range of patches with this period. About 10 s of time to let the transient behaviour of the aircraft die out will generally be sufficient in aircraft simulation applications.
   The patches are continuous from one period T_g to another, because the patch contains contributions from frequencies that are multiples of $1/T_g$ only.
3. A patch of a certain length cannot be split up into several shorter patches. One reason for this is the effect discussed under point 1. Furthermore, the frequency content of a part of a longer patch is different from the frequency content of the total patch, because beginning and end of the shorter patch do not fit continuously: the shorter patch contains a step from $t=T_g$ to t=0. The frequency domain signal (and thus the power spectrum) of the shorter patch will contain contributions of this discontinuity (a step) over the entire frequency range.

It depends on the quality (the "randomness") of the random number generator generating the random phase angles in equation 19 whether the Gaussian distribution is approximated well or not, see also subchapter 2.5.1. An example of a distribution of turbulence velocities of a generated turbulence patch of 500 seconds is compared to a Gaussian distribution in figure 1 on probability paper. The generated turbulence approaches the straight line of the normal distribution reasonably well, but significant deviations occur above about 2.5*σ (probability parameter tp=2.5). The occurrence of higher levels of turbulence velocity ($|w(t)| > 2.5\ \sigma_w$) does not have a Gaussian distribution. This phenomenon caused by the limitations of the random number generator and the limited number of realizations of w(t) (a finite number of points) should be considered in the choice for the value of σ_w applied in the simulation.

It has been demonstrated in subchapter 2.3 (equation 16) that the probability of exceedance of a design load level is equal to the probability of exceedance of U_σ in the turbulence input. Therefore, the turbulence input probability distribution should be sufficiently Gaussian up to level U_σ. According to figure 1, σ_w should thus not be lower than $U_\sigma/2.5$.

In reference 1, the Stochastic Simulation with turbulence intensity $\sigma_w=\sigma_{wr}$ is presented as a procedure to obtain a good first estimate of the design load, and the estimate can be refined by a procedure that requires two more simulations with turbulence intensities $\sigma_{wr}/1.25$ and $\sigma_{wr}*1.25$. In the present investigation, a turbulence intensity of $\sigma_w=U_\sigma/2.5$ is selected for the Stochastic Simulation procedure, as this is a practical value not very different from σ_{wr}.

Figure 2 shows σ_w/U_σ as function of altitude for $\sigma_w=\sigma_{wr}$ and for $\sigma_w=U_\sigma/2.5$, and it can be seen that $\sigma_w=U_\sigma/2.5$ results in a turbulence intensity close to the "optimum" value σ_{wr} according to equation 15 in an altitude band of about 22,000 ft - 35,000 ft. At lower altitudes, the constant ratio of 2.5 leads to higher σ_w-values than σ_{wr}, which implies a certain conservatism when applying $\sigma_w=U_\sigma/2.5$ at these altitudes.

The response of a linear aircraft system to the Gaussian turbulence input will theoretically also have a normal distribution, see figure 3 where the distribution of the wing root bending moment response of the uncontrolled A310 aircraft model to the patch of 500 seconds is compared to the corresponding normal distribution. The normal distribution is approached even a bit better by this response than by the turbulence input signal. The higher frequencies of the turbulence signal are filtered out by the aircraft transfer functions; the approximation of sharp peaks with only a limited number of time points will therefore be better for the (low frequency) output signals than for the (high frequency) input signal. This observation means, that σ_w could in effect be chosen lower than $U_\sigma/2.5$ in a Stochastic Simulation, depending on the extent to which the output signals contain high frequencies. It is safest, however, to apply not lower than the recommended value of $\sigma_w=U_\sigma/2.5$.

2.4.2 Determination of design and correlated load levels

In the Stochastic Simulation procedure, the load level that has the same probability of exceedance as U_σ in the input signal is the design level of the load quantity under consideration. In order to determine this design load level, the (theoretical) probability of exceedance of U_σ in the turbulence input signal is multiplied by the number of realizations (time points) of the output. The resulting number N is the number of output realizations above the design level.

The array of output realizations is therefore sorted from high to low values, and the design level now is the N^{th} element of this sorted array. A linear interpolation is applied in the sorted array of the output signal if the calculated rank number N is not an integer. It should be noted, that the output signal must be given at a constant time step, because otherwise the levels where the simulation time step is small will have a relatively higher probability density than the levels where the simulation time step is large. The number of realizations above a level can only be related to a probability of exceedance if one realization (time point) represents one fixed amount of time (time step).

Having found the design level of a load y, the correlated level of another load z can be determined. The moments in time where the output signal of y has the value y_d are searched in the time response of y; linear interpolation between two values is applied if one value is lower than y_d and the next higher than y_d (or vice versa). At these moments in time (again applying linear interpolation if necessary) the values of load z are collected in a new array. A probability distribution can now be determined of this array, and the level with probability of exceedance P=0.5 can be found, which defines the correlated value of z. More practical is again to sort the correlated array of z-values from high to low, and find the correlated load z_c by the corresponding rank number, in the same manner as how the design level of y was found.

Appendix A.4 describes a computer procedure that comprises the selection of the turbulence patch length on the basis of a desired accuracy and Stochastic Simulation according to the procedure discussed above.

2.5 Accuracy of Stochastic Simulation results

2.5.1 Random number generator limitations

The random phase angle of the frequency domain turbulence signal in order to establish the Gaussian stochastic turbulence time signal is a first limitation to the accuracy of the Stochastic Simulation method. Random phase angles are generated using a random generator, that always has a limited "randomness" and a limited amount of "random" numbers. The random number generator used in this research is the standard (Workstation-)Matlab algorithm, based on a linear congruential method. The basic algorithm is:

$$\text{seed} = (7^7 \ \text{seed}) \ \text{mod} \ (2^{31} - 1). \qquad (20)$$

The quality of the random number generator can be evaluated by inspection of the probability distribution of the generated time signal. As could be seen in figure 1, the present random number generator performs satisfactorily up to values of about 2.5 times the standard deviation. At some point increasing the number of realizations (increasing T_g or decreasing the time step) will not lead to a better approximation of the Gaussian distribution. As stated before, it is recommended to choose σ_w not lower than $U_\sigma/2.5$, so that the design level of an output is not higher than 2.5 times the standard deviation of that output.

2.5.2 Number of realizations at or above design level

The accuracy of a statistical method like Stochastic Simulation will depend on the number of realizations of the different stochastic variables. A large number of realizations of a stochastic variable will lead to a better approximation of the theoretical probability distribution. For instance, let m be the mean of N realizations of the Gaussian stochastic variable x. The variable m will then also be a Gaussian stochastic variable with the following parameters:

$$\mu_m = \mu_x \quad , \quad \sigma_m = \frac{\sigma_x}{\sqrt{N}} . \tag{21}$$

In the same way, in the determination of design and correlated loads by Stochastic Simulation of a gust patch of a certain length, the loads found are Gaussian stochastic variables. The expected values are equal to the theoretical values y_d and z_c, and there will be a certain variation (standard deviation, σ) around these values.

The standard deviations of the stochastic variables 'design load' and 'correlated load' are measures for the attained accuracy of our results. The derivation of formulas for these standard deviations will now be described, so that the accuracy of our Stochastic Simulation results will be known. The formulas are verified by tests in Appendix A.

In the case of the 'design load found from Stochastic Simulation', it was felt that the standard deviation would be inversely proportional to the square root of the time that the load signal is above the considered level. This is confirmed by the tests in Appendix A.
In the case of a 'correlated load found from Stochastic Simulation', it can be derived theoretically that the standard deviation is inversely proportional to the square root of the number of realizations of the design load quantity at its design level.

The number of realizations of a stochastic output variable y above a level depends on:

- The turbulence patch length.
- The probability of exceedance of that level.

The number of realizations at a certain level y^* is determined by $N_y(y^*)$, where $N_y(y^*)$ is the number of positive crossings of level y^*, calculated from the equation of Rice for Gaussian processes: $N_y(y^*) = N(0) \exp\left(-(y^*)^2 / (2\sigma_y^2) \right)$.
N(0), the number of positive zero crossings, is a well-known PSD quantity that is a measure for the roughness of a Gaussian stochastic signal. A low N(0) indicates that the signal is smooth, which will mean that for instance the σ_y level is crossed only few times. Thus, with a low N(0), there are few moments in time where the signal is equal to the design value, so there will be a

small number of realizations of correlated load values.

In determining the design value of an output, the number of realizations above the design level $N(y>y_d)$ is proportional to the stochastic simulation length, and to the corresponding "design" probability of exceedance of equation 16:

$$N(y>y_d) = \frac{T_g}{\Delta t} * 0.5\,\mathrm{erfc}\left(\frac{U_\sigma}{\sqrt{2}\,\sigma_w}\right) \tag{22}$$

where Δt = the step width in the signal.

Choosing a smaller simulation time step will also increase the number of realizations above design level, but these additional realizations will not add any information with regard to the considered stochastic output probability distribution. If Δt has been chosen sufficiently small, an output value between the points t_1 and $t_1+\Delta t$ will be a linear interpolation between the output values at t_1 and $t_1+\Delta t$, and therefore does not improve the estimation of the output probability density function. The linear interpolation is implicitly assumed when determining the probability distribution from the points at time step Δt.

So diminishing the simulation time step will not increase design load accuracy if the smaller step does not imply a significant improvement of the discrete approximation of the continuous time signals.

If we normalize to the theoretical linear PSD design level, the standard deviation of the design value found for an output y will depend on T_g and σ_w according to equations 21 and 22:

$$\frac{\sigma_{y_d}}{\bar{A}_y U_\sigma} = \frac{\text{Constant}}{\sqrt{T_g * 0.5\,\mathrm{erfc}\left(\frac{U_\sigma}{\sqrt{2}\,\sigma_w}\right)}} = \frac{\text{Constant}}{\sqrt{T_g * P(w>U_\sigma)}}\,. \tag{23}$$

This standard deviation is a measure for the accuracy of the design load found by Stochastic Simulation, so the formula above can be used for the estimation of the design load accuracy.

Equation 23 implies that the accuracy of for instance the mean of the design values found from four Stochastic Simulations of four patches of $T_g/4$ is the same as the accuracy of the design value from one patch T_g. This is due to the fact that the number of realizations for the stochastic variable is linearly proportional to the patch length.

In the case of establishing a correlated value for a signal z, the number of times that y is equal to its design value determines the accuracy, or the standard deviation, of the stochastic variable "correlated load". Using the equation of Rice, this number of realizations (positive and negative level crossings) can be calculated:

$$N(y=y_d) = 2 * T_g N_y(0)\, e^{-\left(\frac{y_d^2}{2\sigma_y^2}\right)} = 2 * T_g N_y(0)\, e^{-\left(\frac{U_\sigma^2}{2\sigma_w^2}\right)} \tag{24}$$

where $N_y(0)$ = number of positive zero-crossings per second of output quantity y.

The number of positive zero-crossings $N_y(0)$ is calculated with:

$$N_y(0) = \left[\frac{\int_{-\infty}^{\infty} f^2 \Phi_{yy}(f)\,df}{\int_{-\infty}^{\infty} \Phi_{yy}(f)\,df}\right]^{\frac{1}{2}}. \tag{25}$$

In the linear aircraft response to a Gaussian turbulence patch, the probability distribution of the correlated load z at the moment that another load y has its design value, $p(z|y=y_d)$, is Gaussian with parameters:

$$\mu_{z_c} = \sigma_z \rho_{yz} \frac{y_d}{\sigma_y} \quad , \quad \sigma_{z_c} = \sigma_z \sqrt{1-\rho_{yz}^2} = \bar{A}_z \sigma_w \sqrt{1-\rho_{yz}^2}\,. \tag{26}$$

The value of μ_{zc} is equal to the correlated PSD value. The mean of the correlated load distribution (the values of z when $y=y_d$) is also equal to the correlated load according to the definition of Noback, $P(z>z_c)=0.5$, because $P(z>\mu_{zc})=0.5$ in this Gaussian correlated load distribution.

Combining 24 and 26, and normalizing to the theoretical correlated load value, the variation of the calculated correlated load with T_g, σ_w, and N(0) will be:

$$\begin{aligned}\frac{\sigma_{z_c}}{|z_{c,PSD}|} &= \frac{1}{\sqrt{2T_g * N_y(0)}}\, e^{+\left(\frac{U_\sigma}{2\sigma_w}\right)^2} * \bar{A}_z \sigma_w \sqrt{1-\rho_{yz}^2} * \frac{1}{|\rho_{yz}|\bar{A}_z U_\sigma} \\ &= \frac{1}{\sqrt{2T_g * N_y(0)}}\, e^{+\left(\frac{U_\sigma}{2\sigma_w}\right)^2} * \frac{\sigma_w}{U_\sigma}\, \frac{\sqrt{1-\rho_{yz}^2}}{|\rho_{yz}|}.\end{aligned} \tag{27}$$

In Stochastic Simulation, the value $y=y_d$ also has a certain standard deviation, so that the actual correlated load standard deviation will be somewhat higher than the above theoretical value.
The relations between the accuracy of Stochastic Simulation design and correlated loads and the parameters T_g, N(0), σ_w, and ρ that follow from equations 23 and 27 can be verified by performing a large number of Stochastic Simulation procedures. Design and correlated load values can than be treated as stochastic variables, and we can establish the distributions of the resulting design loads and correlated loads. The means of these distributions should be equal to the theoretical linear PSD load levels, and the standard deviations will show a certain dependence of T_g, N(0), σ_w, and ρ, which should comply with equations 23 and 27. These tests are discussed in appendix A. Practical correlated loads results usually show somewhat larger standard deviations than indicated by equation 27, but it is concluded that equations 23 and 27 can be used as estimations of Stochastic Simulation results deviations.

It will be seen in appendix A that the accuracy of correlated loads is considerably less than the accuracy of design loads. The desired accuracy of the correlated loads is therefore used to determine the necessary patch length in a Stochastic Simulation, on the basis of equation 27.

2.5.3 Numerical quality of response calculations, simulation time step

A first check on the output standard deviations of an A310 model stochastic turbulence response learned, that wing root bending moment standard deviation was slightly lower than according to linear (frequency domain) PSD theory, and the torsion moment standard deviation quite significantly lower. This effect will cause a systematic deviation of Stochastic Simulation design levels from the theoretical PSD levels.

In this first response calculation, the sample frequency was 32.8 Hz (time step of about 0.03 s), so that the highest frequency (Nyquist frequency) in the Fourier series of the turbulence signal was 16.4 Hz. This 16.4 Hz was the highest frequency in the linear PSD calculation (for $\bar{A}$) that had been performed, so this could not be a cause for the errors observed above.

Due to the Fourier series representation of the turbulence signal in the frequency domain, the time domain turbulence patch is the summation of a large number of sine-functions. A sine-signal of a certain frequency that is given at a limited number of time points will be approximated by straight lines between two time points. The frequency content of this approximation will always be somewhat lower than the original continuous signal, see figure 4. A 10-step approximation of a sine function has an amplitude in the frequency domain that is more than 3 % lower than the continuous signal.

The torsion transfer function of the A310 aircraft model has a pronounced peak at about 3.4 Hz (Fig. 5). The sample frequency of 32.8 Hz in this case means that there are about 10 steps-per-cycle in the time domain representation of the turbulence signal at the frequency of 3.4 Hz. According to the effect of figure 4, the torsion response to this discrete time domain signal will therefore be more than 6 % too low at this frequency (3 % reduction due to discrete input, 3 % reduction due to discrete output). The same effect will be present at the other frequencies around this peak. This is the reason why the torsion moment standard deviation in response to Gaussian stochastic turbulence will have a lower intensity (standard deviation) than expected according to linear frequency domain PSD theory.

For this reason, the sampling frequency has been doubled (time step of 0.015 s). The Nyquist frequency of this turbulence signal will be 32.8 Hz, but the aircraft models used here will hardly show any responses above 15 Hz. The discrete output signals of these simulations with twice the amount of time steps will therefore be "smoother" approximations of the continuous signals.

Figure 6 presents the standard deviations (or RMS) of stochastically simulated torsion and bending for the A310 model without the Load Alleviation controllers, as function of the time step in the signal w(t). These standard deviations are divided by their theoretical (from $\bar{A}$) standard deviations. It can be seen that the simulated bending moment standard deviation does not differ much from the theoretical value, but the torsion moment standard deviation gets a lot better for smaller time steps. The twice as small time step results in a σ for M_t of within 0.5 % of the theoretical value, so this time step can be accepted for this specific aircraft model and these outputs.

It can be seen in appendix A.3 that the time step has even more influence on the accuracy of correlated loads. A standard recipe for the selection of Δt can not be given; Δt should be determined by means of a preliminary test with a linearized model, or it should simply be set to a low value, such as 0.01 s, that will be sufficient for any aircraft response.

2.6 Review of the Stochastic Simulation procedure

The linear PSD method can be formulated as a procedure of finding output levels with a certain probability of exceedance for an aircraft flying through Gaussian stochastic turbulence with slowly varying intensity.

The probability of design level exceedance for a linear aircraft is determined mainly by a narrow band of turbulence intensity (standard deviation) values around σ_{wr}.

Assuming that σ_{wr} will also deliver the main contribution to the probability of design level exceedance of a nonlinear system, this turbulence intensity is to be used in a Stochastic Simulation procedure for nonlinear systems. The value of σ_{wr} is approximated by $U_{\sigma}/2.5$ in practical application of the procedure.

A Stochastic Simulation procedure for nonlinear systems has been described. A general formulation (equations 23 and 27) for the obtained accuracy of the results as function of turbulence patch length, N(0), and turbulence intensity level has been given for linear systems, where the results can be compared directly to the linear PSD design and correlated loads. The Stochastic Simulation results for linear systems comply reasonably with equations 23 and 27, as can be seen in appendix A.

For output quantities containing higher frequency contributions, the approximation of the stochastic inputs and outputs by discrete signals may cause errors in the design levels, and especially in the correlated levels. The time step should be chosen small enough, but a general formula describing the necessary sample frequency cannot be given.

It is the desired accuracy of the correlated loads that determines the necessary values of both patch length and step width in a Stochastic Simulation, because the correlated loads deviate more from their theoretical values than the design loads.

3 Comparison of Stochastic Simulation and Deterministic PSD methods

3.1 Introduction

Three Deterministic PSD methods that have also been used in previous research (Ref. 5 and 6) will be studied here:

- Matched Filter Based 1-Dimensional Search (MFB, Pototzky cs.).
- Indirect Deterministic Power Spectral Density method (IDPSD, Noback).
- Spectral Gust method (SG, Brink-Spalink).

A short description of these methods is given in appendix C.

For linear aircraft models, these Deterministic PSD methods and Stochastic Simulation result in design and correlated load values y_d and z_c that are equal to the "standard" PSD loads:

$$y_d = \overline{A}_y U_\sigma \qquad z_c = \rho_{yz} \overline{A}_z U_\sigma.$$

For nonlinear aircraft models, the standard PSD method cannot be applied, because the model transfer functions are then dependent on the input signal. The Stochastic Simulation method has been proposed for the definition of design and correlated loads in nonlinear cases. This method is based on the probability of exceedance of load levels. The Deterministic methods aim to comply with this Stochastic Simulation procedure in nonlinear calculations.

By showing results of calculations for three aircraft models it will be demonstrated that the Deterministic and the Stochastic Simulation procedures effectively lead to correct PSD loads in linear cases. The results for three nonlinear aircraft models are also presented, and the degree of compliance of the Deterministic methods with Stochastic Simulation will be investigated.

The three aircraft models used are the same as in reference 6, see also appendix B:

- Noback model: large transport with load alleviation through ailerons.
- F100 model: medium-sized transport with "Fokker-100-like" characteristics with load alleviation through ailerons.
- A310 model: an A310 model with load alleviation through ailerons and spoilers.

Nonlinearity is introduced in these models by limits on control surface deflections. The A310 model control surfaces can only deflect upward (max. 10 deg.) in the nonlinear version, so that a non-symmetrical nonlinearity is introduced.

3.2 Design and correlated loads calculation results

In Appendix C and in reference 5 it is explained that the considered Deterministic PSD methods follow a more or less similar scheme. An essential part in the procedures is the so-called gust filter.

The Power Spectral Density of the gust filter response to a pulse input should have the von Karman power spectrum shape. The impulse response power spectrum can be calculated directly from the frequency-domain representation of the gust filter G(jf):

$$\Phi(f) = \frac{G(jf)G^*(jf)}{T}$$

where T = length of impulse response.

Figure 7 shows the Power Spectra of the gust filter impulse response for the IDPSD filter (which gives by definition exactly the von Karman Spectrum), the original MFB gust filter ("NASA"), and a new MFB gust filter that has been taken from Hoblit, reference 7. The Hoblit filter clearly approaches the von Karman PSD better than the original NASA filter. The Hoblit gust filter has therefore been implemented in the present MFB procedure, which will be seen to result in correct PSD loads in linear cases, contrary to MFB with the original NASA gust filter, where slight deviations from $\bar{A}U_\sigma$ were found.

The parameters of the gust filter that is used in the original NASA MFB procedure and of the Hoblit gust filter used in this report are given in table 1.

The bar-charts in figures 8-13 show the results of the calculations for the three aircraft models and five calculation methods. The notation in the axis labels of these figures is as follows:

y,des	=	design load value of load quantity y.
y,cor z	=	correlated value of y if z has its design value.
nonlin	=	closed loop system, nonlinear (limited) load alleviation.
nolim	=	closed loop system, linear (unlimited) load alleviation.
nocon	=	open loop system (linear).
Stoch. Simul.	=	Stochastic Simulation result.
PSD	=	standard PSD result.
POS	=	"positive" design load case (A310 model only).
NEG	=	"negative" design load case (A310 model only).

Note that correlated load values in some cases are given with opposite sign, indicated by a minus sign in the legend. The results for the linear and nonlinear versions of the A310 model are given in separate figures, because there is a difference between "positive" and "negative" nonlinear design load cases, due to the fact that ailerons and spoilers can only deflect upward in the nonlinear version of this model.

These bar charts demonstrate that the three Deterministic PSD methods comply with the standard PSD results in linear cases, so it may be concluded that all Deterministic procedures lead to correct results for linear aircraft models. Figure 8 for the linear A310 model shows standard PSD results and Deterministic PSD results together with Stochastic Simulation results. It can be seen that the present Stochastic Simulation procedure gives design loads close to the standard PSD values, and correlated loads may deviate a few percent (of the design load value) from the theoretical value, see for instance the correlated bending for the uncontrolled A310 model.

In nonlinear conditions, where controller actions are limited, the stochastic and Deterministic methods lead to different results. MFB and IDPSD do not differ much, but especially correlated load values are different in some cases. It could be attempted to add a second optimization loop to MFB/IDPSD, calculating outputs at e.g. four more k/K_{eq} values around the optimum found, and find a higher maximum output with somewhat different correlated load values. However, an even more rigorous search routine, the "multi-dimensional search" (Ref. 8), has already been investigated by NASA. It was found that such a routine could change the design conditions by not more than one percent with respect to the one-dimensional search.

MFB and IDPSD both approach the Stochastic Simulation results reasonably; only the correlated value of Δn for the nonlinear F100 model is really very incorrect (wrong sign) for both methods, see figure 10. The corresponding MFB/IDPSD design levels of the bending moment in figure 11 differ more than 10 % from the Stochastic Simulation value. The SG procedure design loads and correlated loads can both deviate appreciably from Stochastic Simulation results.

The ailerons and spoilers of the A310 model can only deflect upward in the nonlinear version, so that different gust design loads will occur in positive and negative directions. In the IDPSD and MFB procedures, negative gust cases are created by reversing the sign of the gust inputs to the "first system". In the SG procedure the sign of a design load is determined, as suggested in reference 6, by calculating the sign of:

$$\int_0^\infty y\,|y|\,dt$$

where y = the load quantity response to an SG input.

It can be seen in figure 9, that the positive and negative design load cases of wing bending do not differ significantly, but the negative torsion design load is considerably lower than the positive design load in the results of Stochastic Simulation, MFB, and IDPSD. It is a good point for MFB and IDPSD that they appear to represent this effect in the same way as the Stochastic Simulation method.

As an indication of the relative computational effort required for each method, the amount of CPU-time used to calculate design and correlated loads for the nonlinear A310 model is given in table 2. The SG method is very fast, because only four time responses are calculated. The IDPSD method takes some more calculation time than MFB, because the "first system" response in IDPSD is twice as long as in MFB. Stochastic Simulation takes much more time than the other methods (14 times the MFB time!), mainly due to the counting procedures for finding design levels and correlated loads.

The following conclusions can be drawn from the comparison of Deterministic PSD methods with the Stochastic Simulation and "standard" PSD methods:

- With the Hoblit gust filter, MFB is equivalent to IDPSD and "standard" PSD in linear cases.
- The results of MFB and IDPSD are reasonably similar in nonlinear cases; correlated loads may deviate somewhat.
- MFB and IDPSD reasonably approach Stochastic Simulation results in nonlinear cases, but this is not enough for design load calculations.
- The SG method deviates significantly from the other methods in nonlinear cases.
- Stochastic Simulation takes much more calculation time than the Deterministic methods.

3.3 Theoretical justification of the methods discussed

Although an approximation, Stochastic Simulation results in design loads with a certain probability of exceedance ($P(y>y_d)$), assuming turbulence to be a stochastic process with a slowly varying statistical parameter (σ_w varies slowly). Noback has shown in reference 1 that $P(y>y_d)$ for linear systems is determined mainly by a narrow band of σ_w values. The approximation is, that only the most influential turbulence intensity is considered, and not the other intensities in the narrow band mentioned previously.

The assumption of the Stochastic Simulation method as applied to nonlinear systems is:

- The probability of exceeding the design level is also for nonlinear systems determined mainly by the turbulence intensity $\sigma_{wr} \approx U_\sigma/2.5$.

It could be useful to investigate this assumption for some different aircraft models with different nonlinearities.

The design level probability of exceedance $P(y>y_d)$ is equal for linear and nonlinear systems; it is equal to the proportion of time that the turbulence velocity is higher than U_σ in the stochastic turbulence patch with intensity $U_\sigma/2.5$. So the design load definition is based on a probability of exceedance, taking into account the statistical variations in the intensity of atmospheric turbulence. Thus, the Stochastic Simulation design loads are equivalent for linear and nonlinear systems, and they comply with $\bar{A}U_\sigma$ in linear cases.

The Deterministic PSD methods are expressions of the linear PSD method in the time domain, without considering the probabilistic aspects of the turbulence intensity. There is no indication at all that the probability of exceeding the design level will be equal for linear and nonlinear systems. There is no equivalence between linear and nonlinear results.

The Deterministic formulations of the PSD method can be seen as "worst case" concepts that search for the gust input shape giving the highest aircraft response. Some account is taken of the turbulence energy content at different wavelengths (the von Karman Power Spectral Density), but in MFB and IDPSD the gust shape itself (input to the "second system") does not have the von Karman PSD. In this formulation, MFB, IDPSD, and SG are alternatives for a Tuned Discrete Gust requirement, incorporating aspects of the Continuous Turbulence (PSD) requirement.

The different philosophies for the Stochastic Simulation method (based on realistic probabilistic aspects) and the Deterministic methods (based on imaginary worst case "design" conditions) can both be defended very well. Design conditions traditionally are somewhat stylistic representations of possible realistic events, so a worst case procedure would fit in well. The evolution of airworthiness requirements however shows a tendency to represent limit load conditions more and more realistically, so that design load cases become conditions that may occur with a certain probability during aircraft service. The Stochastic Simulation philosophy would be more appropriate in view of these latter developments. A Continuous Turbulence design level based on a probability of exceedance also seems to be more in the spirit of the development of the PSD approach in reference 3.

4 Conclusions

The PSD method for gust design and correlated loads calculation can be formulated as a procedure of finding output levels that are exceeded a certain proportion of time for an aircraft flying through Gaussian stochastic turbulence. In this formulation, the PSD method can be applied to nonlinear systems by means of Stochastic Simulation. The Stochastic Simulation results are equivalent for linear and nonlinear aircraft models.

A Stochastic Simulation procedure for nonlinear systems has been described. A general (but in practical applications approximate) formulation for the obtained accuracy of the results as function of turbulence patch length, N(0), and turbulence intensity level has been given for linear systems, where the results can be compared directly to the linear PSD design and correlated loads. These formulas are used for an estimation of the accuracy when Stochastic Simulation is applied.

The time step in the simulation should be chosen small enough to represent possible higher-frequency contributions in the loads responses, but a general prescription of the necessary sample frequency cannot be given.

MFB and IDPSD reasonably approach Stochastic Simulation results in nonlinear cases, but this is not enough for design load calculations. The SG method deviates significantly from the other methods in nonlinear cases. The present investigation was limited to three not very complex aircraft models with nonlinearities introduced by control surface deflection limits only. Other aircraft models with different types of nonlinearities may show larger differences between the Deterministic methods and Stochastic Simulation.

Stochastic Simulation takes considerably more calculation time than the Deterministic methods; for instance 14 times as much as MFB.

As the Stochastic Simulation definition of Continuous Turbulence design loads is based on realistic probabilistic considerations, it is believed that Stochastic Simulation is more appropriate than the worst case approach of the Deterministic PSD methods to comply with the Continuous Turbulence requirement.

5 References

1. Noback, R.; *Definition of P.S.D.-design loads for nonlinear aircraft*, NLR TP 89016 U, Jan. 1989.
2. Noback, R.; *The generation of equal probability design load conditions, using P.S.D.-techniques*, NLR TR 85014 L, Jan. 1985.
3. Hoblit, F.M.; e.a.; *Development of a power-spectral gust design procedure for civil aircraft*, FAA-ADS-53, Jan. 1966.
4. Noback, R.; *Statistical properties of atmospheric turbulence relevant to the calculation of airplane loads*, NLR TR 88013 L, 1988.
5. Jonge, J.B. de; Vink, W.J.; *Evaluation of deterministic spectral gust methods*, NLR TP 94368 U, 1994.
6. Vink, W.J.; *Evaluation of deterministic PSD methods with realistic nonlinear aircraft models*, NLR CR 96092 L, 1996.
7. Hoblit, F.M.; *Gust loads on aircraft: concepts and applications*, AIAA, Inc., 1988.
8. Scott, R.C.; Pototzky, A.S.; Perry III, B.; *Computation of maximized gust loads for nonlinear aircraft using matched-filter-based schemes*, Journal of Aircraft, Vol. 30, No. 5, 1993, pp. 763-768.
9. Anon., *Joint Aviation Requirements JAR-25, Amendment 93/1, JAR 25.341 (b) and ACJ 25.305 (d)*, 8 March 1993.
10. Anon., *Federal Aviation Regulations FAR-25, Amdt. 25-86, FAR 25.341 (b), and Appendix G*.

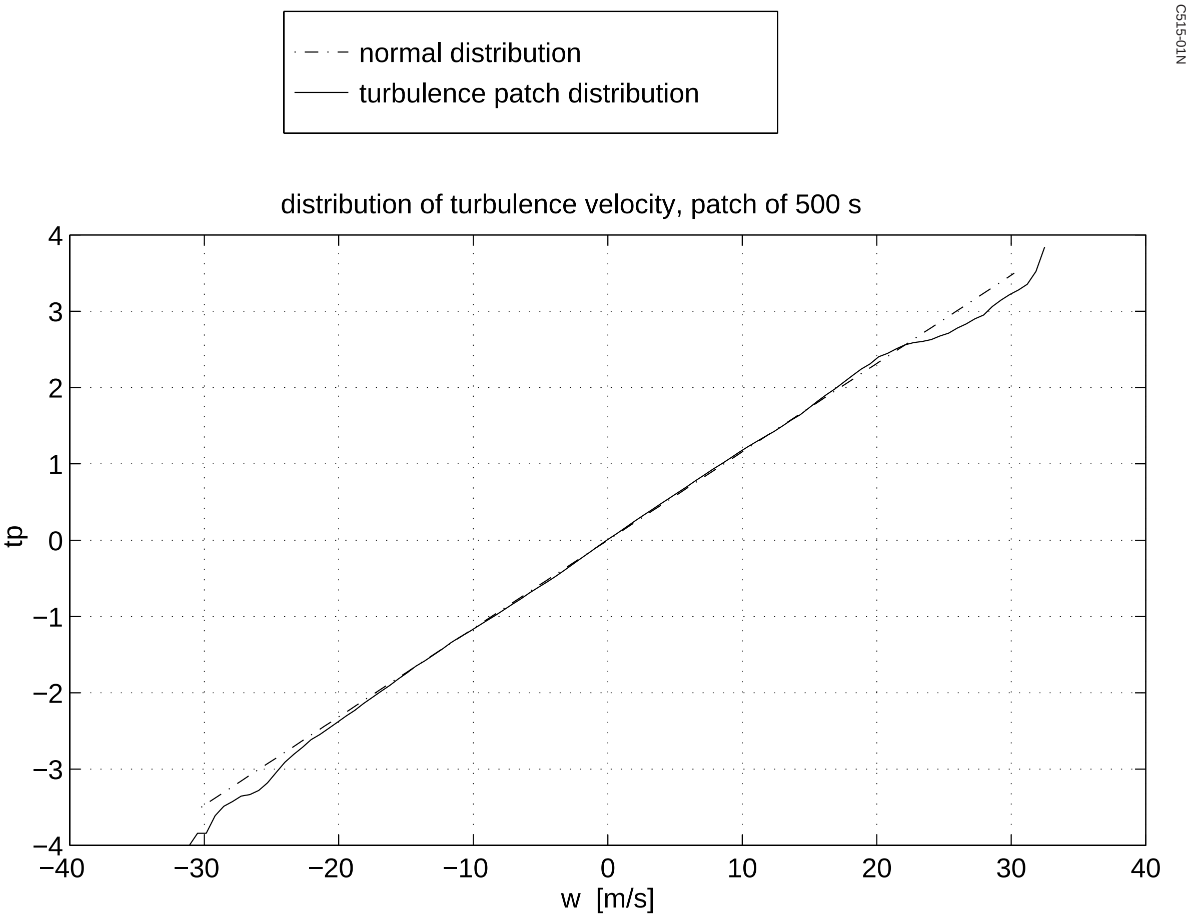
Table 1 NASA and Hoblit gust filters for the MFB method

$$G(s) = \sqrt{\frac{L}{\pi V}}\ \frac{(1+n_1\frac{L}{V}s)(1+n_2\frac{L}{V}s)(1+n_3\frac{L}{V}s)}{(1+p_1\frac{L}{V}s)(1+p_2\frac{L}{V}s)(1+p_3\frac{L}{V}s)(1+p_4\frac{L}{V}s)}\ ,\quad \text{with } s = j\omega .$$

	n_1	n_2	n_3	p_1	p_2	p_3	p_4
NASA	2.618	0.1298	0	2.083	0.823	0.0898	0
Hoblit	2.187	0.1833	0.021	1.339	1.118	0.1277	0.0146

Table 2 The necessary CPU-time for PSD methods applied to nonlinear A310 model

method	total length of responses (s)	CPU time (s)
Stochastic Simulation	1250	1067
IDPSD	540	86
MFB	450	76
SG	135	17



Note: The probability parameter tp is defined by $tp = \frac{x}{\sigma_x}$ with $x = N(0; \sigma_x)$

Fig. 1 Probability distribution of a generated patch of turbulence, 500 s

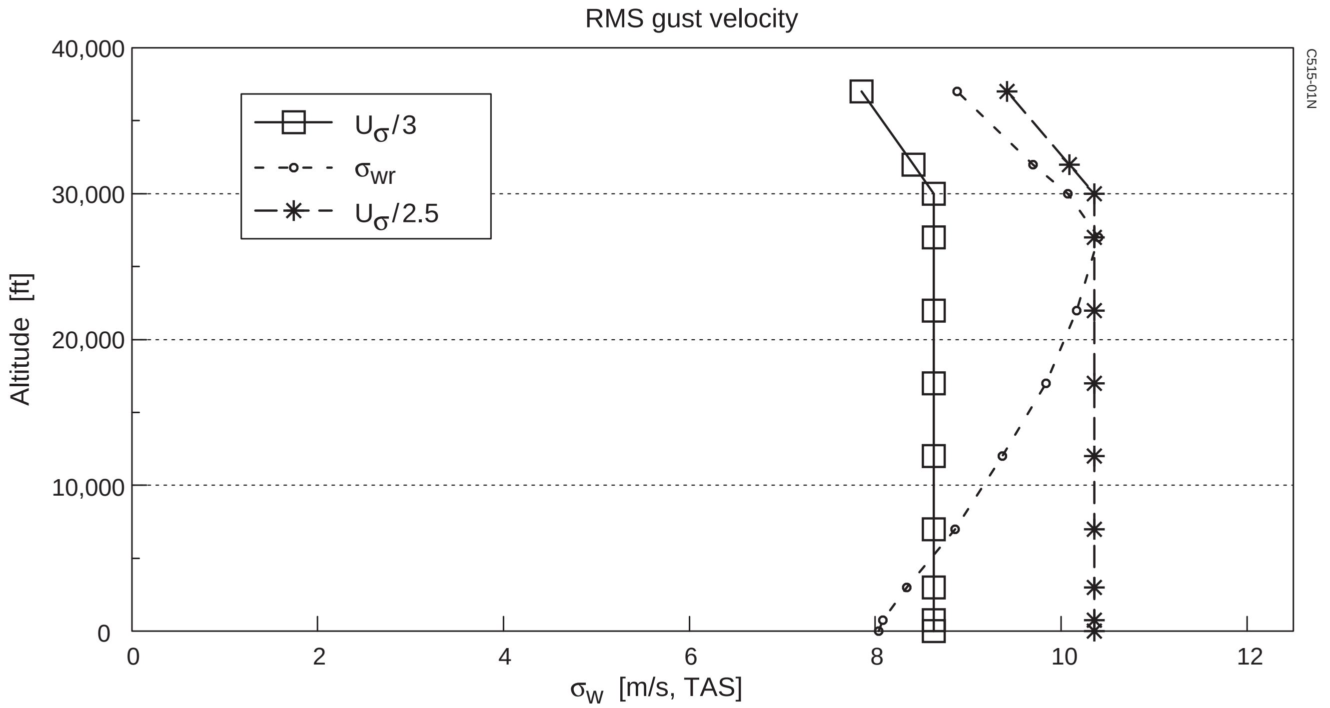
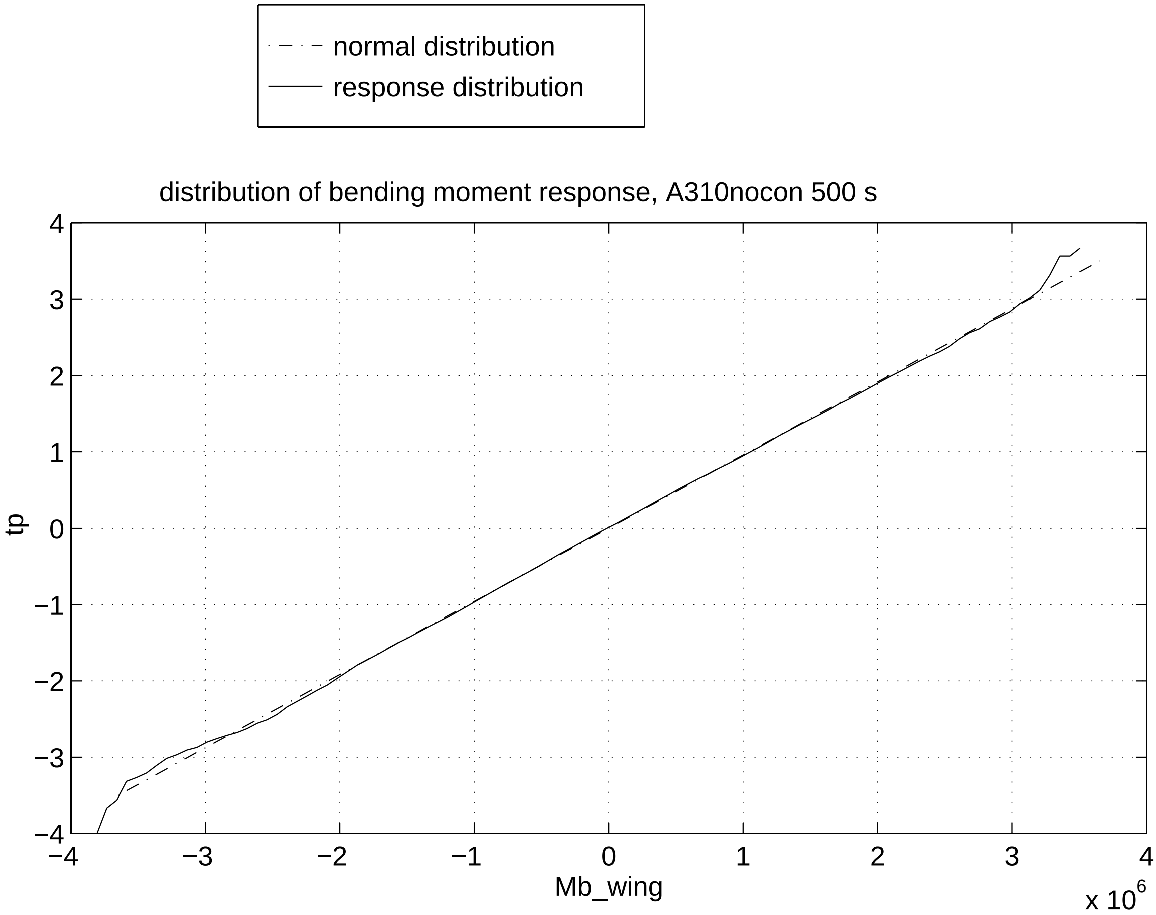


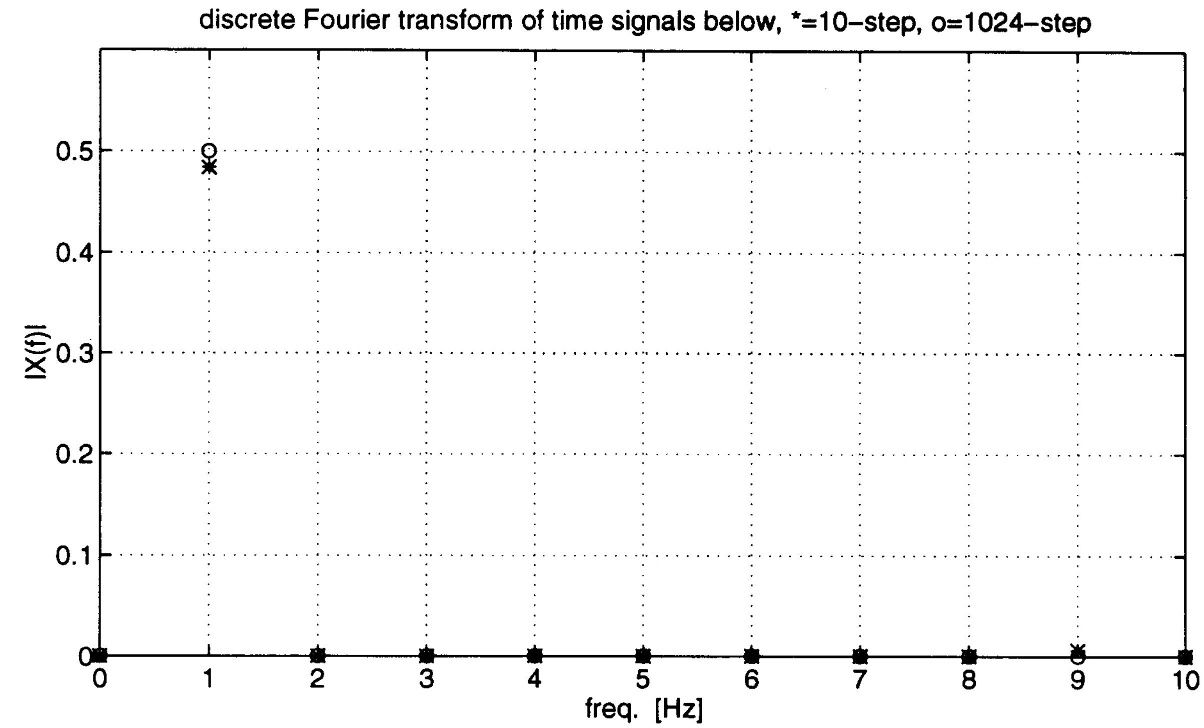
Fig. 2 Three functions of rms gust velocity versus altitude

C515-01N



Note: The probability parameter tp is defined by $tp = \frac{x}{\sigma_x}$ with $x = N(0; \sigma_x)$

Fig. 3 Probability distribution of a A310 bending moment response to a turbulence patch of 500 s



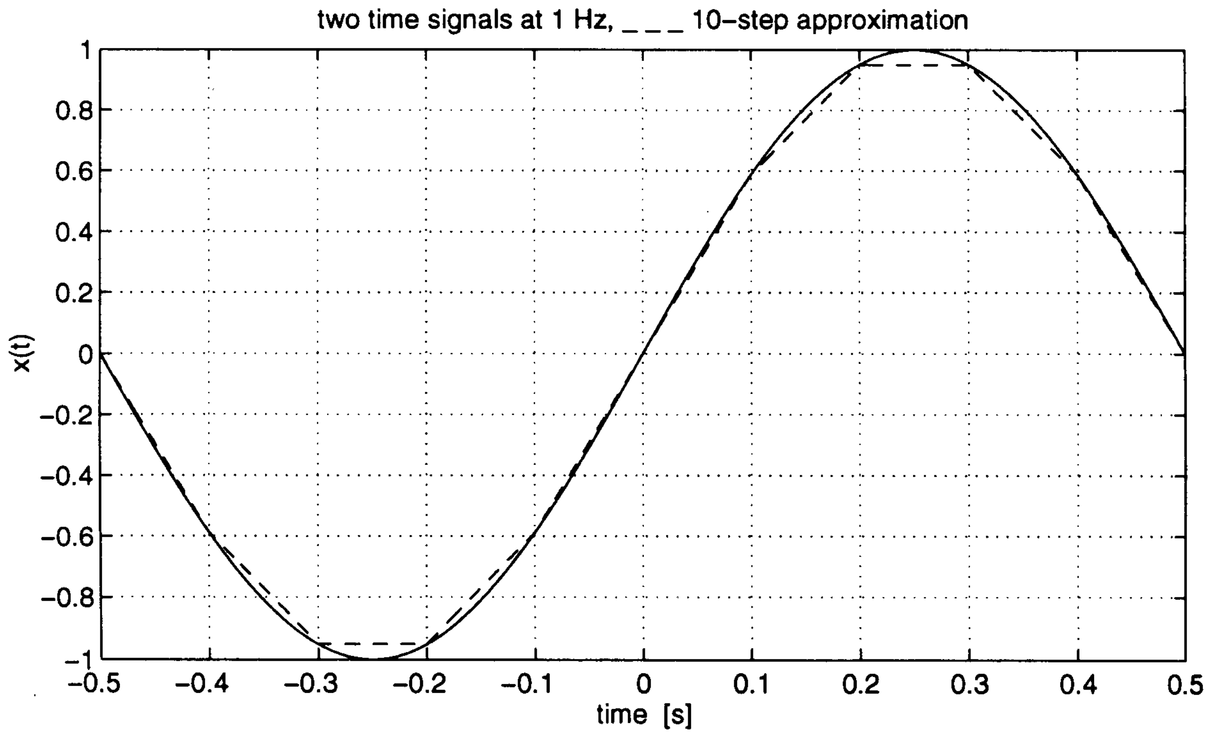


Fig. 4 Frequency analysis of a sine function at 1 Hz

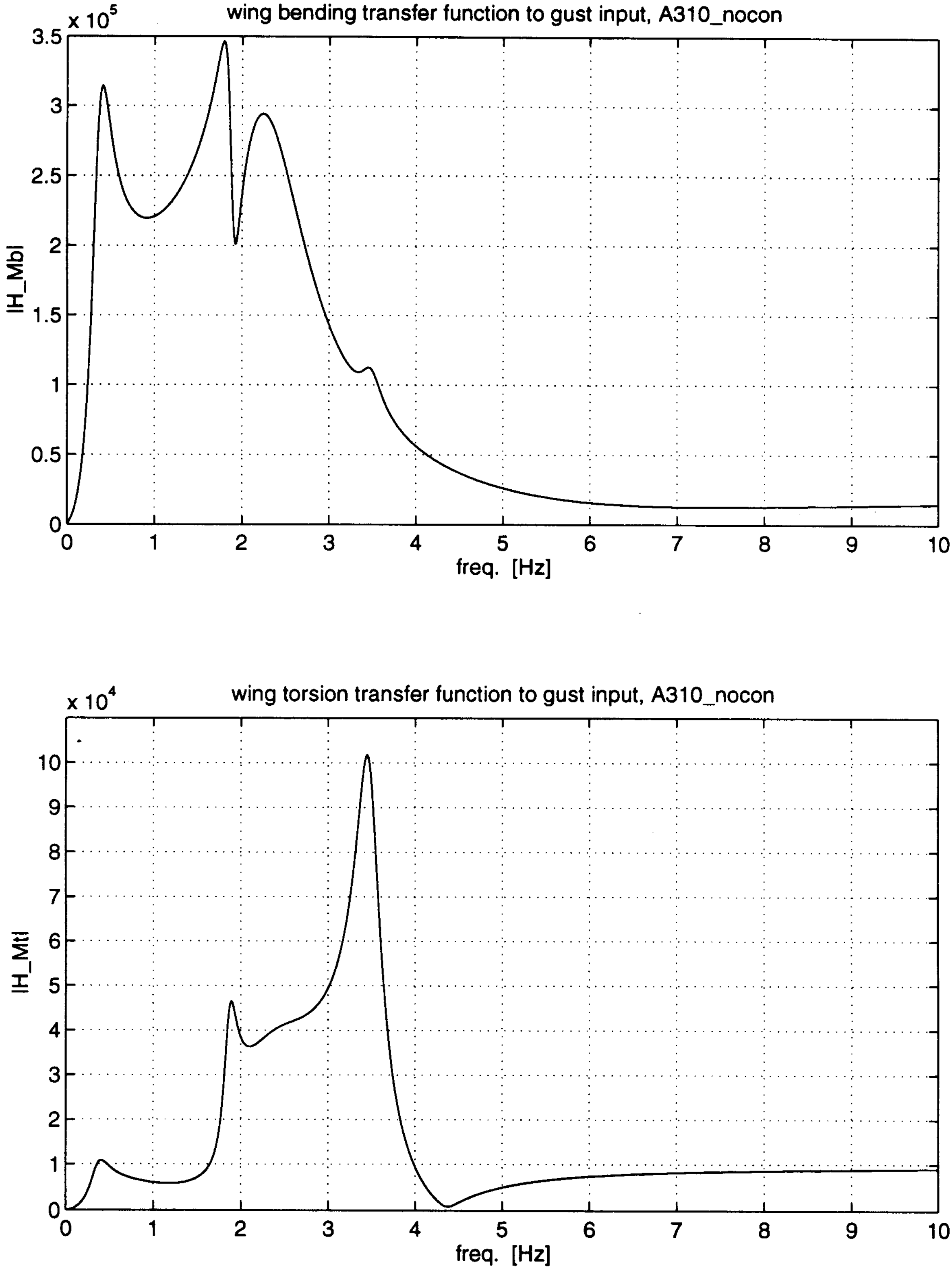


Fig. 5 A310 model transfer functions

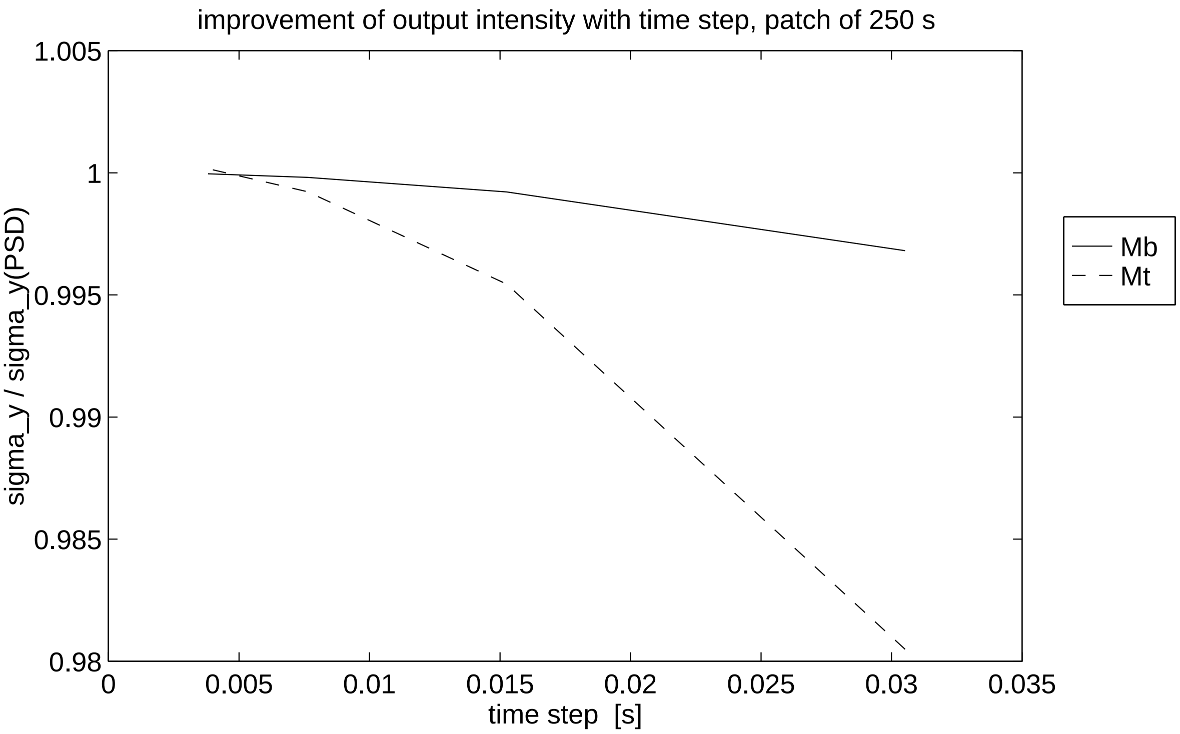


Fig. 6 Improvement of A310 output intensity with time step with respect to theoretical PSD value

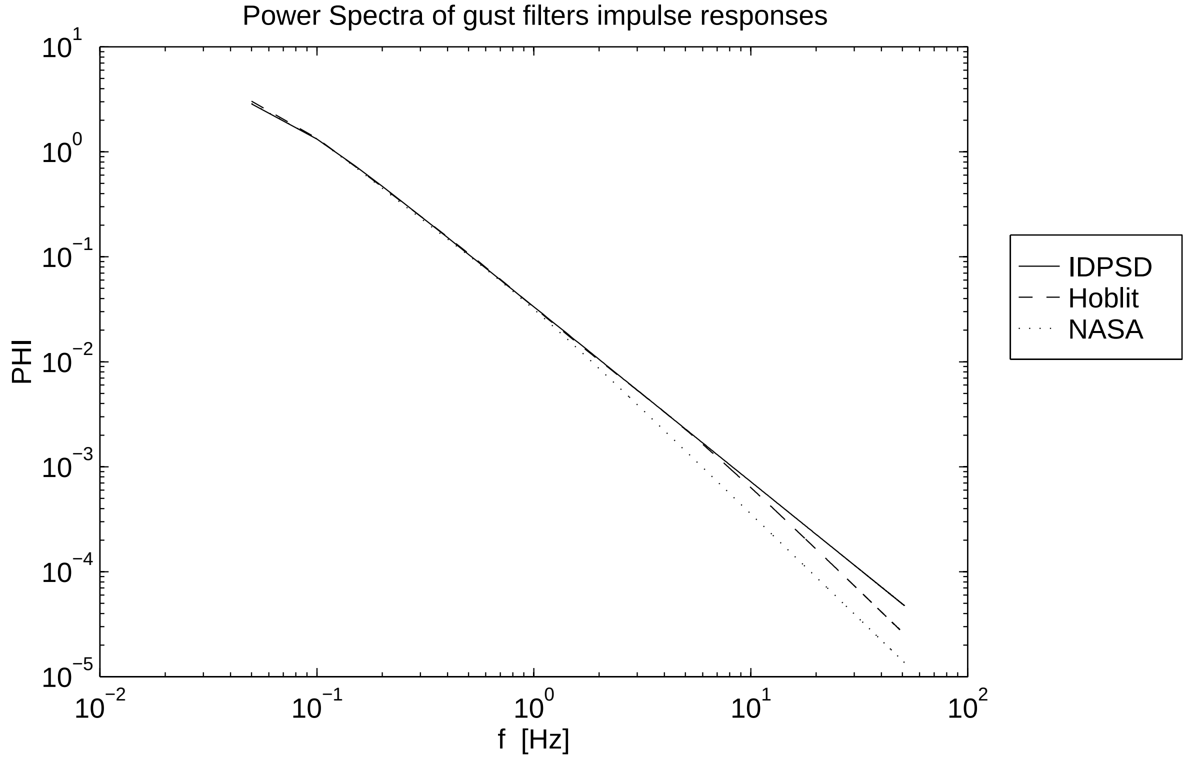
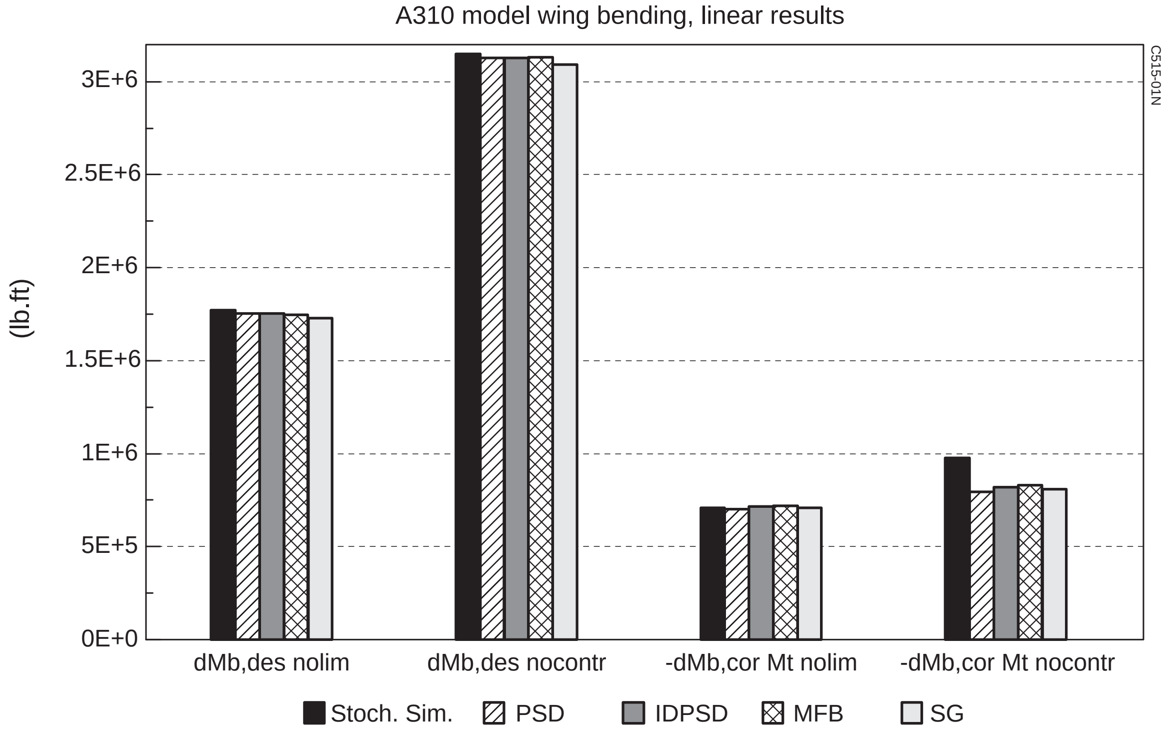


Fig. 7 Power Spectra of three gust filters impulse responses



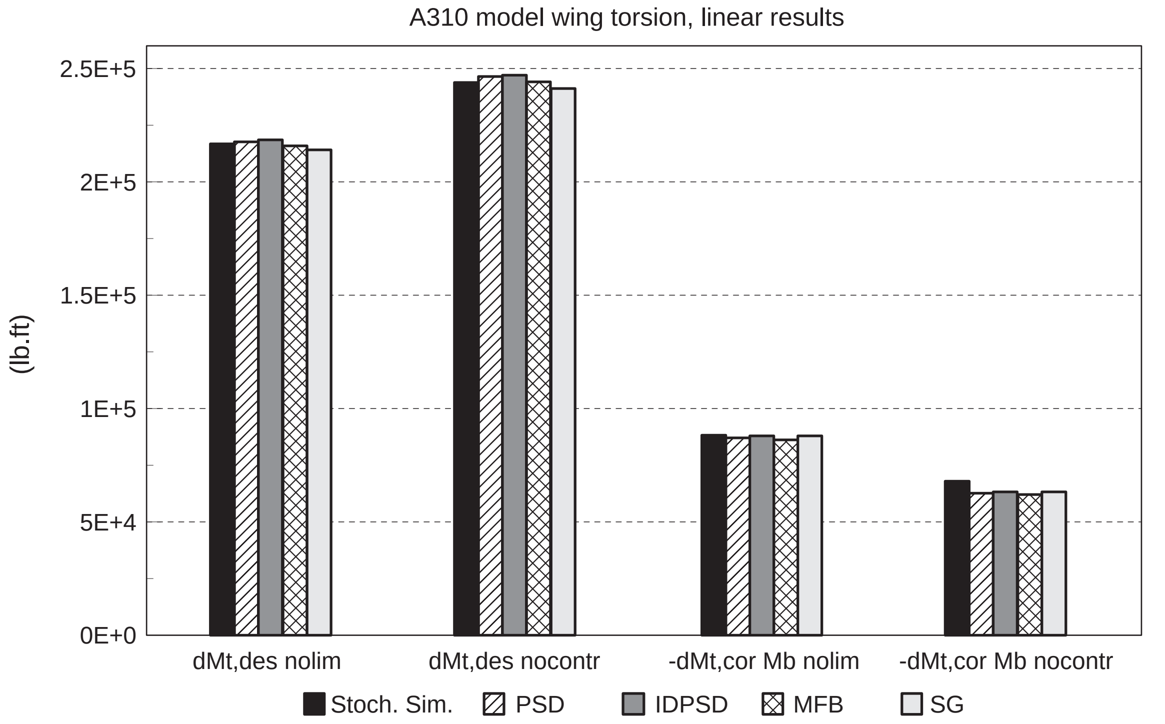
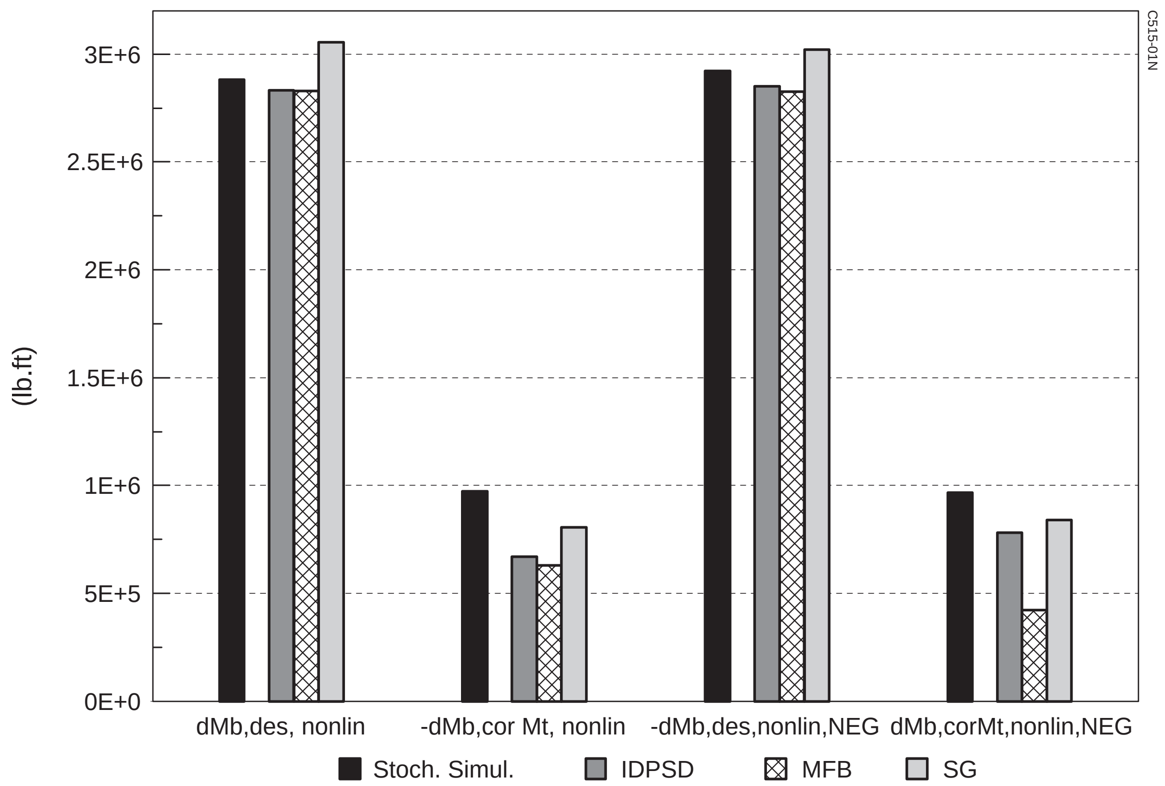


Fig. 8 Stochastic, Deterministic and "standard" PSD results for linear A310 models

A310 model wing bending, nonlinear results



A310 model wing torsion, nonlinear results

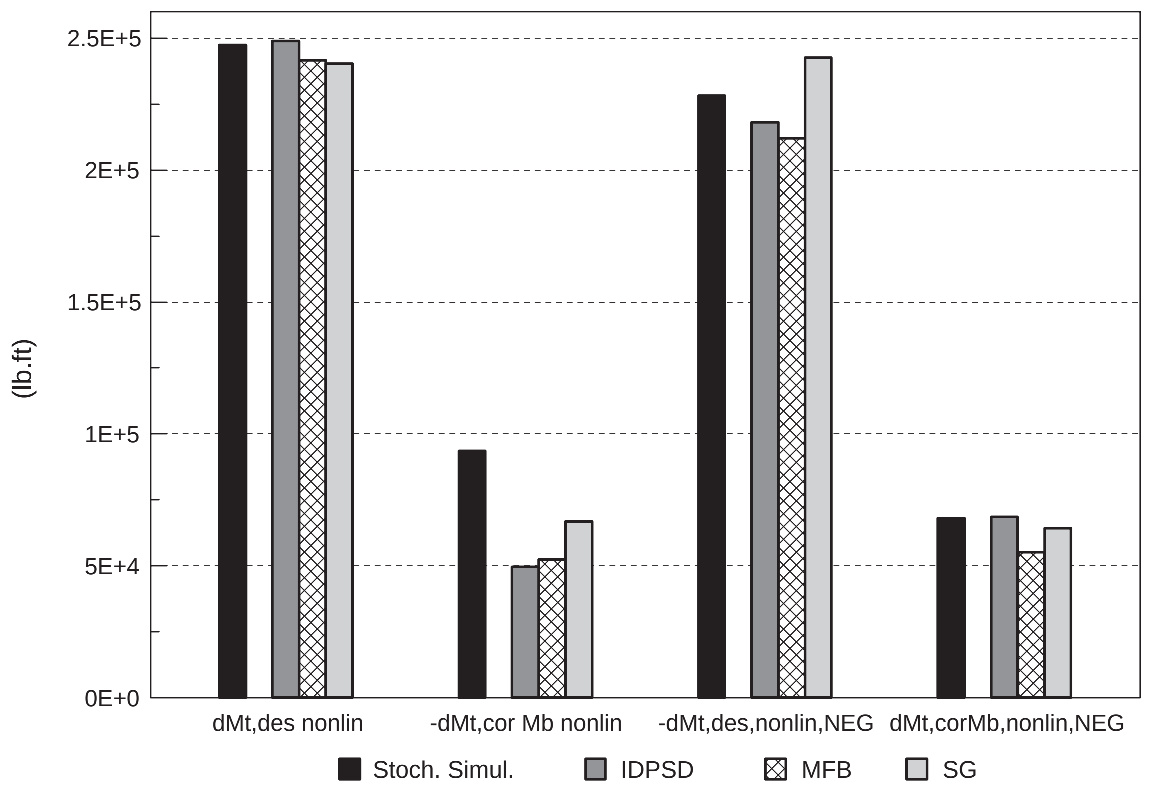


Fig. 9 Stochastic and Deterministic PSD results for nonlinear A310 model

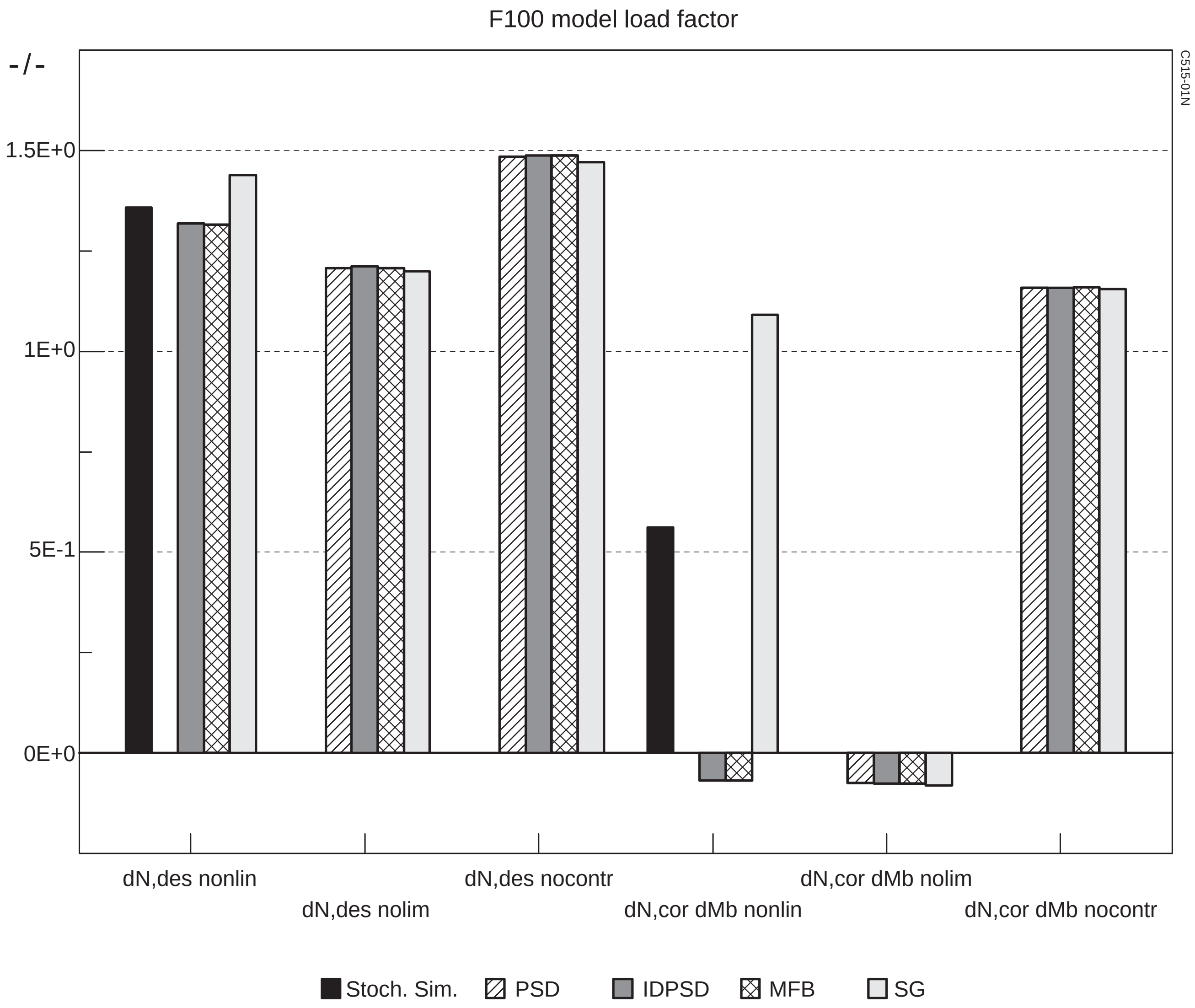


Fig. 10 Stochastic, Deterministic and "standard" PSD results for Fokker 100 models

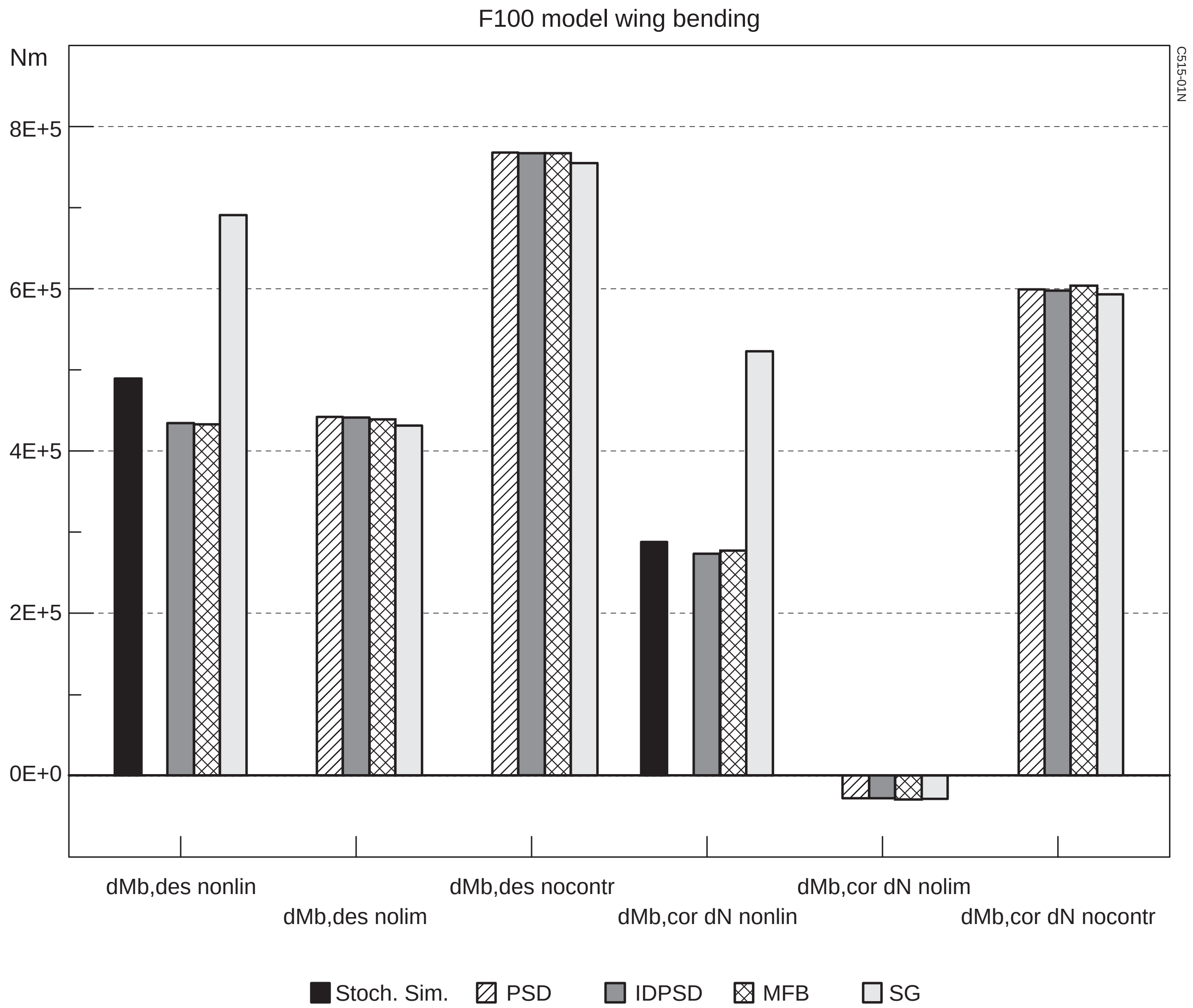


Fig. 11   Stochastic, Deterministic and "standard" PSD results for Fokker 100 models

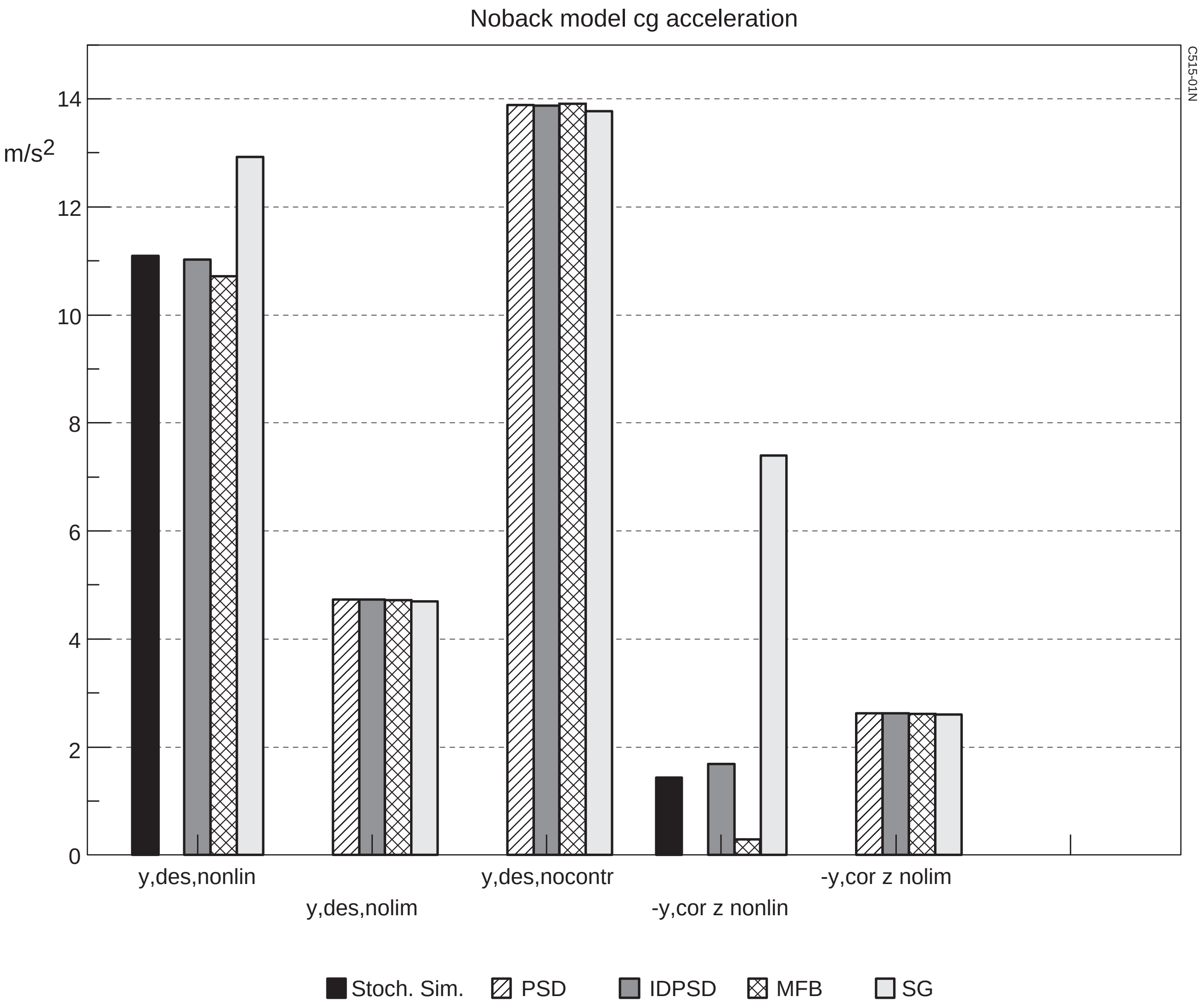


Fig. 12 Stochastic, Deterministic and "standard" PSD results for Noback models

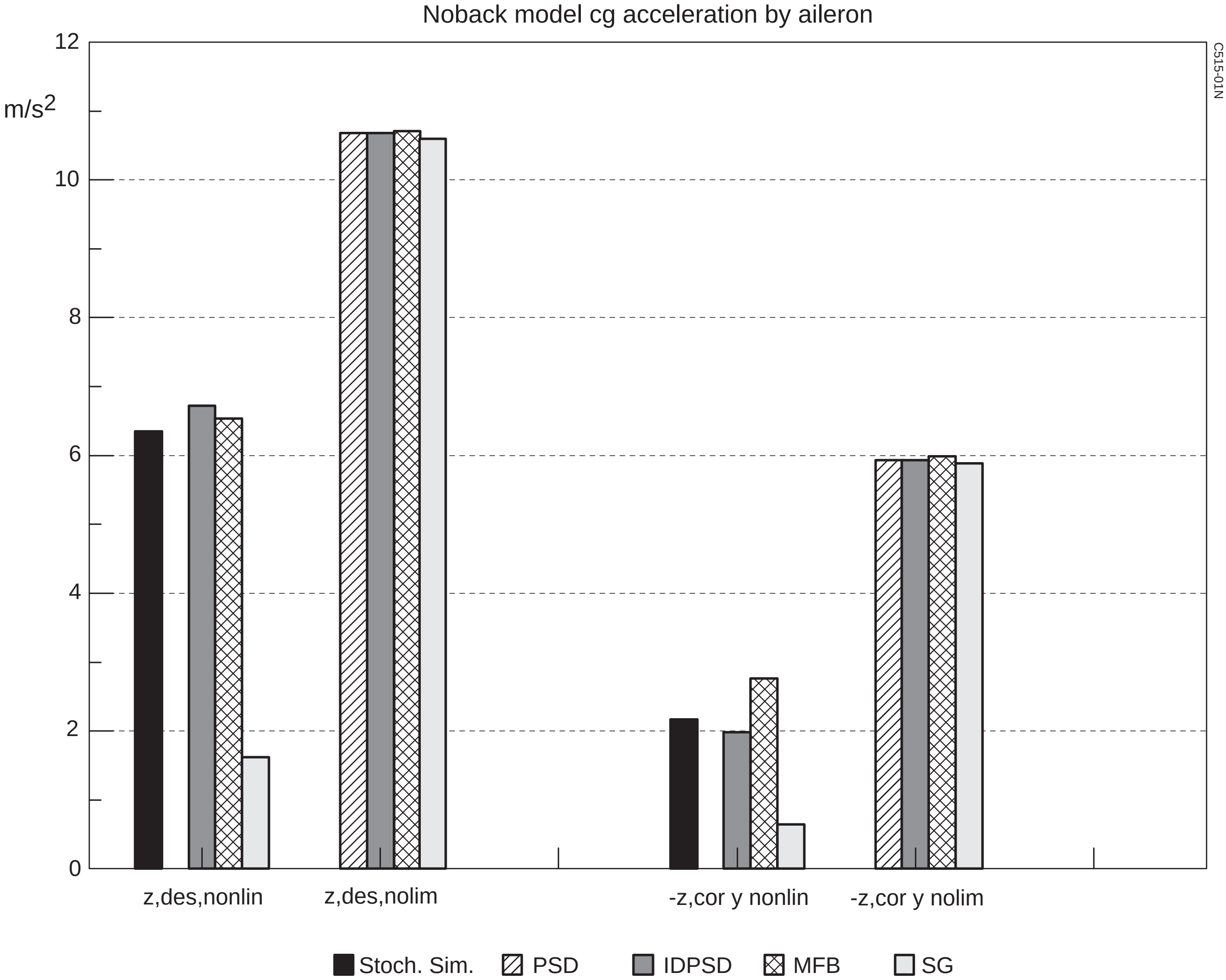


Fig. 13 Stochastic, Deterministic and "standard" PSD results for Noback models

Appendices

A Design and correlated loads as stochastic variables: the accuracy of the Stochastic Simulation procedure

In subchapter 2.5.2, equations 23 and 27 have been derived from stochastic process theory. These equations represent the relations between design and correlated loads scatter and the parameters gust patch length, gust intensity, and N(0) of the load quantity. In this appendix, equations 23 and 27 will be validated, and a practically applicable formula will be derived to estimate the attained accuracy of a Stochastic Simulation with certain length, gust intensity, and loads N(0)'s.

A.1 Accuracy of the design load level

The Stochastic Simulation procedure will now be evaluated by applying it to linear aircraft systems, for which design and correlated loads can also be calculated with the standard PSD method using $\bar{A}$ and ρ. So the results of Stochastic Simulation can be validated directly by comparison with the linear PSD results.

The "design load level found by Stochastic Simulation" and the "correlated load level found by Stochastic Simulation" are stochastic variables, as has been discussed in subchapter 2.5.2. They are Gaussian for a linear system with Gaussian input. The probability distribution parameters mean (μ) and standard deviation (σ) of these stochastic variables are investigated here. Mean and standard deviation are determined from 100 results of the Stochastic Simulation procedure (100 simulations), thus providing reasonably accurate estimations of these probability characteristics.

For the design load level, σ is caused by the numerical limitations of the Stochastic Simulation: limited patch length, limited number of points, and limited "randomness". For the correlated load, μ and σ result from theoretical probability distributions; numerical limitations may cause some deviations of μ and σ from the theoretical values.

The linear aircraft models used in these tests are discussed in appendix B:

- A310 model without Load Alleviation controllers.
- Large transport aircraft model ("Noback model") with unlimited controller.
- Model with characteristics similar to Fokker 100 without controller.

For these linear tests, the load alleviation control systems of the A310 and the Fokker 100 models have been switched off. In the linear Noback model, the control system is switched on (without limits), because one of the two possible outputs of this model is the centre of gravity acceleration caused by the controller.

The 100 different turbulence patches are generated according to the procedure in 2.4.1. Each patch is unique, because the phase relations of the frequency domain turbulence signals are all different random functions.

The mean and standard deviation of design and correlated loads as found from 100 Stochastic Simulations are divided by the corresponding linear PSD values of design load and correlated load. The results for the A310 model are presented in tables A1a-A1c for 3 different patch lengths T_g: 125 s, 250 s and 500 s. It can be seen in tables A1, that the "design level probability" has been varied between $P(y>1.75\sigma_y)$ and $P(y>3\sigma_y)$. This is equivalent to varying the turbulence input intensity σ_w between $U_\sigma/1.75$ and $U_\sigma/3$. We can thus verify the relationships between the probability characteristics μ and σ of the PSD loads and the quantities T_g, N(0), and σ_w from equations 23 and 27.

A first conclusion that can be drawn from the A310 results in table A1 is, that the means of the design values of bending and torsion are sufficiently close to the theoretical values. The means of the correlated loads, however, deviate systematically from the theoretical correlated loads. The correlated torsion (when bending has its design value) is consistently too low, and the correlated bending (when torsion has its design value) is consistently too high. These systematic errors in the correlated loads are influenced to some degree by the simulation time step. This will be studied in appendix A.2. Similar tests with the two other linear aircraft models did not show these systematic errors in mean correlated load values, see table A2a-A2c for the Noback model with unlimited controller and table A3 for the uncontrolled Fokker 100 model.

The accuracy of the results when simulating only one patch is determined by the standard deviations in tables A1-A3. Figure A1 shows the relation between the standard deviation of the positive and the negative design value of M_b and the length of the gust patch ($1/\sqrt{T_g}$) from the A310 model results, at some levels of turbulence intensity σ_w. Figure A2 shows the same for M_t. The linear relation between σ_{yd} and $1/\sqrt{T_g}$ from equation 23 is sustained reasonably by these graphs, so that this part of equation 23 has been validated.

Figures A3 and A4 show the relations between the design loads standard deviations and the function of turbulence intensity level as given in equation 23:

$$\frac{1}{\sqrt{\operatorname{erfc}\left(\dfrac{U_\sigma}{\sqrt{2}\,\sigma_w}\right)}}.$$

The validity of equation 23 is sustained again by these reasonably straight lines.

Tables A1-A3 show a difference between design load standard deviations for different outputs of an aircraft model. This difference may be due to the different smoothness of two output signals, characterized by N(0). In the case of the uncontrolled A310 model, bending has an N(0) of 1.0 s^{-1} and torsion of 3.0 s^{-1}. Torsion has a lower design load standard deviation than bending. Apparently, an output of a character with stronger variations (high N(0)) leads to more accuracy in the design value. This difference in accuracy may be caused by the fact, that design load exceedings of an output with low N(0) are more "clustered" than those of an output with high N(0).

Figure A5 shows two stochastic responses to turbulence, one with low N(0) and one with high N(0). The dashed line indicates the level with a certain prescribed probability of exceedance, comparable to a design level for each load output. This figure shows that realizations above a design level are more clustered (concentrated in only a few peaks) when N(0) is low than when N(0) is high, and this can very well be a cause for less accuracy, but this influence cannot be derived analytically using stochastic process theory.

In figures A6-A8, design load standard deviations for the three linear aircraft models have been plotted, together with the line:

$$\frac{\sigma_{z_d}}{z_{d,PSD}} = \sqrt{\frac{N_y(0)}{N_z(0)}} * \frac{\sigma_{y_d}}{y_{d,PSD}}.$$

It can be seen that the design load standard deviation is about proportional to $1/\sqrt{N(0)}$ in some cases, but the relation is not very clear. Especially the results of the Noback aircraft model deviate from this relation.

Based on stochastic process theory, a relation between Stochastic Simulation design load accuracy (or scatter) and N(0) is not expected; equation 23 therefore does not contain a contribution from N(0). Figures A6-A8 indicate roughly a linear relation with $1/\sqrt{N(0)}$, but this can be seen not to be valid in all cases. This rather unpredictable influence of N(0) thus makes equation 23 not very suitable for the estimation of Stochastic Simulation results accuracy. As will be seen in A.2, the accuracy of correlated loads is always considerably less than the design load accuracy, so that a general formula for accuracy estimation will be based on equation 27 for the correlated loads accuracy.

A.2 Accuracy of the correlated load level

The relation between the standard deviation of a Stochastic Simulation correlated load (normalized to the theoretical PSD correlated load) and $1/\sqrt{T_g}$ in equation 27 is confirmed by the reasonably straight lines of figures A9 and A10 for A310 bending and torsion respectively.

The relation between correlated value standard deviation and turbulence intensity σ_w is given in figures A11 and A12. The standard deviation is proportional to the function of σ_w as given in equation 27:

$$\frac{\sigma_w}{U_\sigma} e^{\left(\frac{U_\sigma}{2\sigma_w}\right)^2}.$$

The standard deviation of the Stochastic Simulation correlated loads also depends on N(0) of the design output according to equation 27 (see Figs. A13-A15), or, more specifically, on $N_y(y_d)$ of the design quantity. The number of positive and negative design level crossings of quantity y is the number of realizations for correlated quantity z. The linear relation of the standard deviation with 1/√N(0) from equation 27 is not confirmed very strongly by figures A13-A15. The relation will however be used as an estimation for correlated loads accuracy. When Stochastic Simulation is applied to nonlinear aircraft models, $N_y(y_d)$ cannot be calculated directly from N(0) by Rice’s equation (24), so equation 27 must be reformulated using

$$N_y(y_d) = N_y(0) e^{-\left(\frac{U_\sigma^2}{2\sigma_w^2}\right)}$$

into:

$$\frac{\sigma_{z_c}}{|z_{c,PSD}|} = \frac{1}{\sqrt{2T_g * N_y(y_d)}} * \frac{\sigma_w}{U_\sigma} \frac{\sqrt{1-\rho_{yz}^2}}{|\rho_{yz}|}. \qquad \text{(A1)}$$

For a nonlinear system response, $N_y(y_d)$ can be calculated by counting the crossings of y_d in a time response.

Thus far, experience with the linear models results in tables A1-A3 learned, that the Stochastic Simulation correlated load standard deviation will usually be 1.1 (for A310 model) to 2 (for Noback model) times larger than the standard deviation from the formula above, due to the fact that the practical dependence of N(0) (or $N_y(y_d)$) and ρ is not exactly according to theory.
Note that part of the correlated load standard deviation is caused by the error (σ) in the design load level, so that σ_{zc} will always be larger than the theoretical value. This latter effect can however not be the cause for a deviation of 100 % from the value calculated with equation A1.

A.3 The influence of simulation time step on correlated loads accuracy

It has been discussed in subchapter 2.5.3, that the approximation of continuous stochastic signals by discrete signals may lead to some deviations of the output RMS (σ_y) values.
The time step also has an influence on the correlated loads of table A1. As can be seen, the means of correlated M_b are consistently too high, and the means of correlated M_t are too low. Figure A16 shows the mean of the 100 correlated bending moments that were found with 100 Stochastic Simulations of a gust patch of 250 seconds as function of the time step. Two levels of turbulence input are depicted: $\sigma_w=U_\sigma/2$ and $\sigma_w=U_\sigma/2.5$. Figure A17 gives the mean of the correlated torsion. These two figures show, that the relation between time step and mean correlated load is not very well defined, but a tendency of improving results for smaller time steps can be recognized. The error in mean correlated bending is larger than the error in mean correlated torsion. A relative error of within about 10 % for the mean of the correlated (bending) loads can be achieved with the time step of 0.015 seconds, that has been chosen as the time step for the simulations in tables A1-A3.

It cannot be explained at this moment why there should be systematic errors in the A310 correlated loads, or why the bending moment error is larger than the torsion moment error.
The tests with the other two aircraft models give means of correlated load values that comply much better with linear theory, see tables A2 and A3.

A.4 A procedure for doing Stochastic Simulation with prescribed accuracy

To arrive at a general procedure for attaining a prescribed accuracy of the Stochastic Simulation method applied to any aircraft model, we can use the relations between design and correlated loads standard deviations and T_g, σ_w, and N(0). Systematic errors like discussed in the previous paragraph should not be present, so the time step should always be chosen small enough, preferably based on an evaluation of a large number of Stochastic Simulations of a linearized model. A time step of 0.01 s will probably be sufficient for any aircraft model.

For the turbulence signal intensity σ_w, the practical value of $U_\sigma/2.5$ has been chosen. This intensity is reasonably close to the "representative" σ_{wr} as given by Noback in reference 1, and it is a clear and simple definition of this design parameter, related in a simple way to the existing design parameter U_σ.

So after having verified that the step width in the output signals is sufficiently small, the only Stochastic Simulation parameter that has to be determined on the basis of a desired accuracy is the length of the simulation, T_g.

It has become clear from tables A1-A3 that the accuracy of the design load values is a lot better than the accuracy of the correlated load values. The choice for the length of a turbulence patch should therefore be based on the desired accuracy of the correlated loads. For a linear aircraft model, equation A1 gives an estimate of the standard deviation of the correlated load level, and thus of the accuracy of a correlated load that is calculated by one Stochastic Simulation patch.

If we for instance want to be 95 % confident that the calculated correlated load error is not more than ± 10 % of the "theoretical" correlated load, then the standard deviation from equation A1 should be 0.05. The 2σ value that corresponds to 95 % confidence is then 0.1. As the correlated load is sometimes low, it has been decided here to define the desired accuracy as 10 % of the theoretical design load. The standard deviation from equation A1 should then be $0.05/|\rho|$.

From the linear aircraft results it has been found that the design loads are a factor 1.5 to 5 more accurate than the correlated loads, with the accuracy expressed as a percentage of the design level. So if we express the accuracy in terms of the design level, the correlated load values still determine the necessary turbulence patch length. For practical applications it can be assumed that the design loads will be at least 1.5 times as accurate as the correlated loads.

Note that, as discussed previously, the "theoretical" equation A1 does not apply directly to Stochastic Simulation results. The correlated loads standard deviations are usually 1.1 to 2 times larger than what is calculated in equation A1. In the procedure proposed here, a multiplication factor of 1.3 is used for equation A1:

$$\frac{\sigma_{z_c}}{|z_{c,PSD}|} = \frac{1.3}{\sqrt{2T_g * N_y(y_d)}} * \frac{\sigma_w}{U_\sigma}\frac{\sqrt{1-\rho_{yz}^2}}{|\rho_{yz}|} \quad \wedge \quad \frac{\sigma_{z_d}}{z_{d,PSD}} \le \frac{1}{1.5}\frac{\sigma_{z_c}}{|z_{c,PSD}|}. \qquad \text{(A2)}$$

This equation A2 is a general formula for estimating the accuracy of Stochastic Simulation results.

The least accurate correlated load in a Stochastic Simulation is the correlated load z for which

$$\frac{\sqrt{1-\rho_{yz}^2}}{\sqrt{N_y(y_d)}\,|\rho_{yz}|}$$

is maximal. For this load z, the correlated load standard deviation $\sigma_{zc}/z_{c,PSD}$ can be calculated for a patch length T_1 according to equation A2. If the desired standard deviation is for instance

$$\frac{\sigma_{z_c}}{|z_{c,PSD}|} = \frac{0.05}{|\rho|}$$

the necessary patch length can be calculated with:

$$T_g = \frac{\left(\dfrac{\sigma_{z_c}}{z_{c,PSD}}\right)^2 T_1}{\left(\dfrac{0.05}{|\rho|}\right)^2} . \qquad \text{(A3)}$$

For a linear system, the necessary patch length can be calculated directly from A2 and A3, without having to perform the first simulation of T_1 seconds, because ρ and $N_y(y_d)$ can be calculated from the model's transfer functions.

For a nonlinear system, it is proposed to carry out a simulation of T_1 seconds. The values for $N_y(y_d)$ can then be determined by means of a counting procedure, and correlation coefficients can be calculated with:

$$\frac{\int_0^{T_1} y(t)\, z(t)\, dt}{\sqrt{\int_0^{T_1} y^2(t)\, dt \int_0^{T_1} z^2(t)\, dt}} . \qquad \text{(A4)}$$

The determination of the appropriate T_g then follows the procedure as described above in equations A2 and A3.

As the linear theory for stochastic processes is not valid for nonlinear systems, the correlated load distribution will not be Gaussian (Eq. 26) in nonlinear cases. Equation A2 can then only be an estimation of the correlated load accuracy. It is assumed that this estimation is at least reasonable.

A schematic overview of the proposed Stochastic Simulation procedure for nonlinear systems is given in figure A18. The procedure starts with an "identification patch" in order to find the necessary parameters for equation A2. The necessary total patch length is then calculated with A2 and A3. The total patch consists of separately generated patches of 250 s each. Design and correlated loads are calculated for each patch according to the procedure of subchapter 2.4. The final Stochastic Simulation results are the mean values of design and correlated loads from the series of patches.

Table A1 Results from A310 responses to 100 patches of stochastic gust

a Patches of 125 s

U_σ/σ_w	Prob.	Mb_{des}^+/PSD		Mb_{des}^-/PSD		Mt_{cor}^+/PSD		Mt_{cor}^-/PSD	
		μ	σ	μ	σ	μ	σ	μ	σ
1.75	0.04006	0.9960	0.0309	1.0007	0.0261	0.9467	0.3380	0.9486	0.2914
2.00	0.02275	0.9956	0.0347	1.0034	0.0304	0.9932	0.3594	0.9749	0.3893
2.25	0.01222	0.9925	0.0389	1.0016	0.0400	0.9740	0.4483	0.8746	0.4150
2.50	0.00621	0.9886	0.0500	1.0048	0.0549	0.9076	0.5262	0.9234	0.4914
2.75	0.00298	0.9899	0.0637	1.0036	0.0762	0.9257	0.5995	0.9248	0.6447
3.00	0.00135	0.9896	0.0856	0.9989	0.0879	0.8175	0.6778	0.9413	0.7416

U_σ/σ_w	Prob.	Mt_{des}^+/PSD		Mt_{des}^-/PSD		Mb_{cor}^+/PSD		Mb_{cor}^-/PSD	
		μ	σ	μ	σ	μ	σ	μ	σ
1.75	0.04006	0.9974	0.0155	0.9974	0.0144	1.0988	0.2154	1.0948	0.2277
2.00	0.02275	0.9961	0.0192	0.9958	0.0164	1.1250	0.2799	1.0965	0.2672
2.25	0.01222	0.9935	0.0243	0.9966	0.0236	1.1417	0.3172	1.0691	0.3408
2.50	0.00621	0.9918	0.0292	0.9960	0.0289	1.0805	0.3991	1.0399	0.4354
2.75	0.00298	0.9891	0.0419	0.9947	0.0386	1.0836	0.4968	1.0165	0.4968
3.00	0.00135	0.9862	0.0525	0.9968	0.0501	1.0320	0.6656	0.9913	0.6791

b Patches of 250 s

U_σ/σ_w	Prob.	Mb_{des}^+/PSD		Mb_{des}^-/PSD		Mt_{cor}^+/PSD		Mt_{cor}^-/PSD	
		μ	σ	μ	σ	μ	σ	μ	σ
1.75	0.04006	1.0026	0.0220	1.0007	0.0233	1.0223	0.2215	0.9491	0.2513
2.00	0.02275	1.0026	0.0249	1.0007	0.0272	0.9795	0.2914	0.9704	0.2569
2.25	0.01222	1.0040	0.0292	1.0013	0.0317	0.9674	0.2980	0.9678	0.3170
2.50	0.00621	1.0070	0.0356	1.0025	0.0383	0.9958	0.3966	0.9592	0.3682
2.75	0.00298	1.0035	0.0460	1.0001	0.0457	0.9631	0.4447	0.8087	0.4751
3.00	0.00135	1.0015	0.0569	1.0010	0.0582	0.9982	0.6366	0.9101	0.5894

U_σ/σ_w	Prob.	Mt_{des}^+/PSD		Mt_{des}^-/PSD		Mb_{cor}^+/PSD		Mb_{cor}^-/PSD	
		μ	σ	μ	σ	μ	σ	μ	σ
1.75	0.04006	0.9992	0.0100	0.9970	0.0111	1.1014	0.1716	1.0554	0.1593
2.00	0.02275	0.9985	0.0136	0.9989	0.0138	1.0961	0.1916	1.0747	0.1898
2.25	0.01222	0.9980	0.0158	0.9990	0.0172	1.0894	0.2437	1.0455	0.2611
2.50	0.00621	0.9982	0.0201	0.9970	0.0221	1.0797	0.2817	1.0663	0.3088
2.75	0.00298	0.9962	0.0244	0.9968	0.0294	0.9894	0.3942	1.1030	0.3407
3.00	0.00135	0.9991	0.0339	0.9985	0.0380	0.9816	0.5204	1.0455	0.4868

Table A1 (Continued)

c Patches of 500 s

U_σ/σ_w	Prob.	Mb_{des}^{+}/PSD		Mb_{des}^{-}/PSD		Mt_{cor}^{+}/PSD		Mt_{cor}^{-}/PSD	
		μ	σ	μ	σ	μ	σ	μ	σ
1.75	0.04006	1.0013	0.0151	0.9978	0.0142	0.9528	0.1745	0.9895	0.1687
2.00	0.02275	1.0012	0.0166	0.9980	0.0163	0.9509	0.1901	0.9631	0.1851
2.25	0.01222	1.0021	0.0215	0.9975	0.0206	0.9817	0.2247	0.9904	0.2172
2.50	0.00621	1.0048	0.0246	0.9976	0.0259	0.9501	0.2726	0.9567	0.2503
2.75	0.00298	1.0062	0.0364	0.9997	0.0314	0.9656	0.3085	0.9839	0.3039
3.00	0.00135	1.0030	0.0522	0.9980	0.0399	0.9909	0.4761	0.8846	0.5090

U_σ/σ_w	Prob.	Mt_{des}^{+}/PSD		Mt_{des}^{-}/PSD		Mb_{cor}^{+}/PSD		Mb_{cor}^{-}/PSD	
		μ	σ	μ	σ	μ	σ	μ	σ
1.75	0.04006	0.9955	0.0097	0.9951	0.0088	1.0875	0.1153	1.0780	0.1282
2.00	0.02275	0.9952	0.0107	0.9949	0.0104	1.0827	0.1413	1.0662	0.1601
2.25	0.01222	0.9950	0.0114	0.9959	0.0128	1.0791	0.1871	1.0767	0.1692
2.50	0.00621	0.9947	0.0161	0.9968	0.0154	1.1033	0.2324	1.0665	0.2175
2.75	0.00298	0.9947	0.0224	0.9967	0.0188	1.0498	0.2714	1.1060	0.2792
3.00	0.00135	0.9921	0.0298	1.0001	0.0267	1.0837	0.4065	1.1292	0.3637

Note: In these tables $[\ ]^{+}_{des}$ means: positive design level.
$[\ ]^{+}_{cor}$ means: correlated load with positive design load.

Table A2 Results from Noback model responses to 100 patches of stochastic gust

a Patches of 125 s

U_σ/σ_w	Prob.	y_{des}^+/PSD		y_{des}^-/PSD		z_{cor}^+/PSD		z_{cor}^-/PSD	
		μ	σ	μ	σ	μ	σ	μ	σ
1.75	0.04006	0.9930	0.0396	0.9956	0.0421	1.0257	0.1748	1.0187	0.1766
2.00	0.02275	0.9940	0.0513	0.9970	0.0559	1.0035	0.1729	1.0237	0.2237
2.25	0.01222	0.9959	0.0648	0.9966	0.0694	1.0076	0.2134	0.9988	0.2707
2.50	0.00621	0.9963	0.0873	0.9942	0.0852	1.0240	0.2382	0.9919	0.2595
2.75	0.00298	0.9901	0.0969	0.9881	0.0968	1.0438	0.2958	1.0012	0.3095
3.00	0.00135	0.9756	0.1020	0.9793	0.1025	1.0157	0.3119	1.0339	0.3399

U_σ/σ_w	Prob.	z_{des}^+/PSD		z_{des}^-/PSD		y_{cor}^+/PSD		y_{cor}^-/PSD	
		μ	σ	μ	σ	μ	σ	μ	σ
1.75	0.04006	1.0046	0.0375	0.9964	0.0389	0.9922	0.2073	0.9972	0.2028
2.00	0.02275	1.0001	0.0479	0.9977	0.0489	0.9934	0.2219	1.0089	0.2044
2.25	0.01222	0.9965	0.0557	0.9999	0.0634	0.9738	0.2828	1.0320	0.2472
2.50	0.00621	0.9988	0.0707	0.9978	0.0775	0.9927	0.3697	1.0188	0.3040
2.75	0.00298	1.0072	0.1048	0.9970	0.0978	1.0018	0.4026	0.9856	0.3511
3.00	0.00135	0.9975	0.1154	0.9896	0.1236	0.9686	0.3928	0.9526	0.4177

b Patches of 250 s

U_σ/σ_w	Prob.	y_{des}^+/PSD		y_{des}^-/PSD		z_{cor}^+/PSD		z_{cor}^-/PSD	
		μ	σ	μ	σ	μ	σ	μ	σ
1.75	0.04006	0.9966	0.0251	0.9967	0.0262	1.0252	0.1232	1.0280	0.1189
2.00	0.02275	0.9963	0.0300	0.9979	0.0335	1.0308	0.1307	1.0148	0.1368
2.25	0.01222	0.9982	0.0380	0.9973	0.0437	1.0366	0.1662	1.0119	0.1780
2.50	0.00621	1.0019	0.0492	0.9941	0.0547	1.0315	0.1979	1.0196	0.2082
2.75	0.00298	1.0015	0.0585	0.9902	0.0708	1.0655	0.2402	0.9883	0.2562
3.00	0.00135	0.9963	0.0741	0.9893	0.0824	1.1017	0.3260	0.9908	0.2783

U_σ/σ_w	Prob.	z_{des}^+/PSD		z_{des}^-/PSD		y_{cor}^+/PSD		y_{cor}^-/PSD	
		μ	σ	μ	σ	μ	σ	μ	σ
1.75	0.04006	1.0024	0.0292	1.0017	0.0272	0.9686	0.1560	0.9996	0.1434
2.00	0.02275	1.0046	0.0369	1.0048	0.0299	0.9861	0.1806	1.0268	0.1727
2.25	0.01222	1.0017	0.0444	1.0032	0.0409	1.0176	0.2234	1.0282	0.1939
2.50	0.00621	1.0008	0.0528	1.0024	0.0613	1.0072	0.2209	1.0410	0.2027
2.75	0.00298	0.9969	0.0627	0.9999	0.0715	0.9808	0.2943	1.0310	0.2587
3.00	0.00135	0.9871	0.0738	0.9980	0.0774	1.0007	0.3269	1.0185	0.3564

Table A2 (Continued)

c Patches of 500 s

U_σ/σ_w	Prob.	y_{des}^+/PSD μ	σ	y_{des}^-/PSD μ	σ	z_{cor}^+/PSD μ	σ	z_{cor}^-/PSD μ	σ
1.75	0.04006	0.9943	0.0189	0.9967	0.0189	1.0323	0.0874	1.0246	0.0792
2.00	0.02275	0.9956	0.0257	0.9975	0.0226	1.0269	0.1009	1.0289	0.0966
2.25	0.01222	0.9962	0.0315	0.9984	0.0304	1.0298	0.1142	1.0215	0.1254
2.50	0.00621	0.9936	0.0374	0.9971	0.0420	1.0326	0.1417	1.0119	0.1481
2.75	0.00298	0.9923	0.0479	0.9958	0.0541	1.0381	0.1834	1.0070	0.1690
3.00	0.00135	0.9917	0.0659	0.9933	0.0646	1.0613	0.2673	1.0164	0.2347

U_σ/σ_w	Prob.	z_{des}^+/PSD μ	σ	z_{des}^-/PSD μ	σ	y_{cor}^+/PSD μ	σ	y_{cor}^-/PSD μ	σ
1.75	0.04006	1.0010	0.0217	1.0037	0.0207	1.0107	0.1008	0.9863	0.1014
2.00	0.02275	0.9992	0.0261	1.0042	0.0218	0.9947	0.1270	0.9879	0.1180
2.25	0.01222	0.9989	0.0308	1.0044	0.0290	0.9986	0.1356	1.0011	0.1342
2.50	0.00621	1.0000	0.0401	1.0061	0.0379	0.9842	0.1822	0.9979	0.1833
2.75	0.00298	0.9965	0.0463	1.0070	0.0506	0.9956	0.1893	0.9806	0.2332
3.00	0.00135	0.9963	0.0612	1.0071	0.0697	0.9833	0.2660	1.0002	0.2942

Table A3 Results from F100 model responses to 100 patches of stochastic gust

Patches of 125 s

U_σ/σ_w	Prob.	Δn_{des}^{+}/PSD		Δn_{des}^{-}/PSD		Mb_{cor}^{+}/PSD		Mb_{cor}^{-}/PSD	
		μ	σ	μ	σ	μ	σ	μ	σ
1.75	0.04006	0.9979	0.0274	1.0001	0.0270	1.0129	0.0864	1.0099	0.0676
2.00	0.02275	0.9946	0.0340	1.0012	0.0332	0.9930	0.0892	1.0077	0.0798
2.25	0.01222	0.9945	0.0444	1.0038	0.0416	1.0037	0.1055	1.0163	0.0853
2.50	0.00621	0.9921	0.0530	1.0035	0.0539	1.0222	0.1114	1.0108	0.1146
2.75	0.00298	0.9853	0.0656	1.0007	0.0693	1.0102	0.1361	0.9749	0.1398
3.00	0.00135	0.9762	0.0847	0.9954	0.0945	1.0041	0.1737	1.0108	0.1626

U_σ/σ_w	Prob.	Mb_{des}^{+}/PSD		Mb_{des}^{-}/PSD		Δn_{cor}^{+}/PSD		Δn_{cor}^{-}/PSD	
		μ	σ	μ	σ	μ	σ	μ	σ
1.75	0.04006	0.9988	0.0217	0.9977	0.0213	0.9910	0.0606	0.9966	0.0647
2.00	0.02275	0.9952	0.0249	0.9978	0.0268	0.9908	0.0735	0.9908	0.0722
2.25	0.01222	0.9948	0.0266	0.9948	0.0295	0.9943	0.0931	0.9907	0.0931
2.50	0.00621	0.9945	0.0336	0.9949	0.0377	0.9664	0.1143	0.9944	0.1148
2.75	0.00298	0.9896	0.0438	0.9957	0.0492	0.9643	0.1320	1.0020	0.1305
3.00	0.00135	0.9826	0.0565	0.9933	0.0626	0.9701	0.1683	0.9896	0.1853

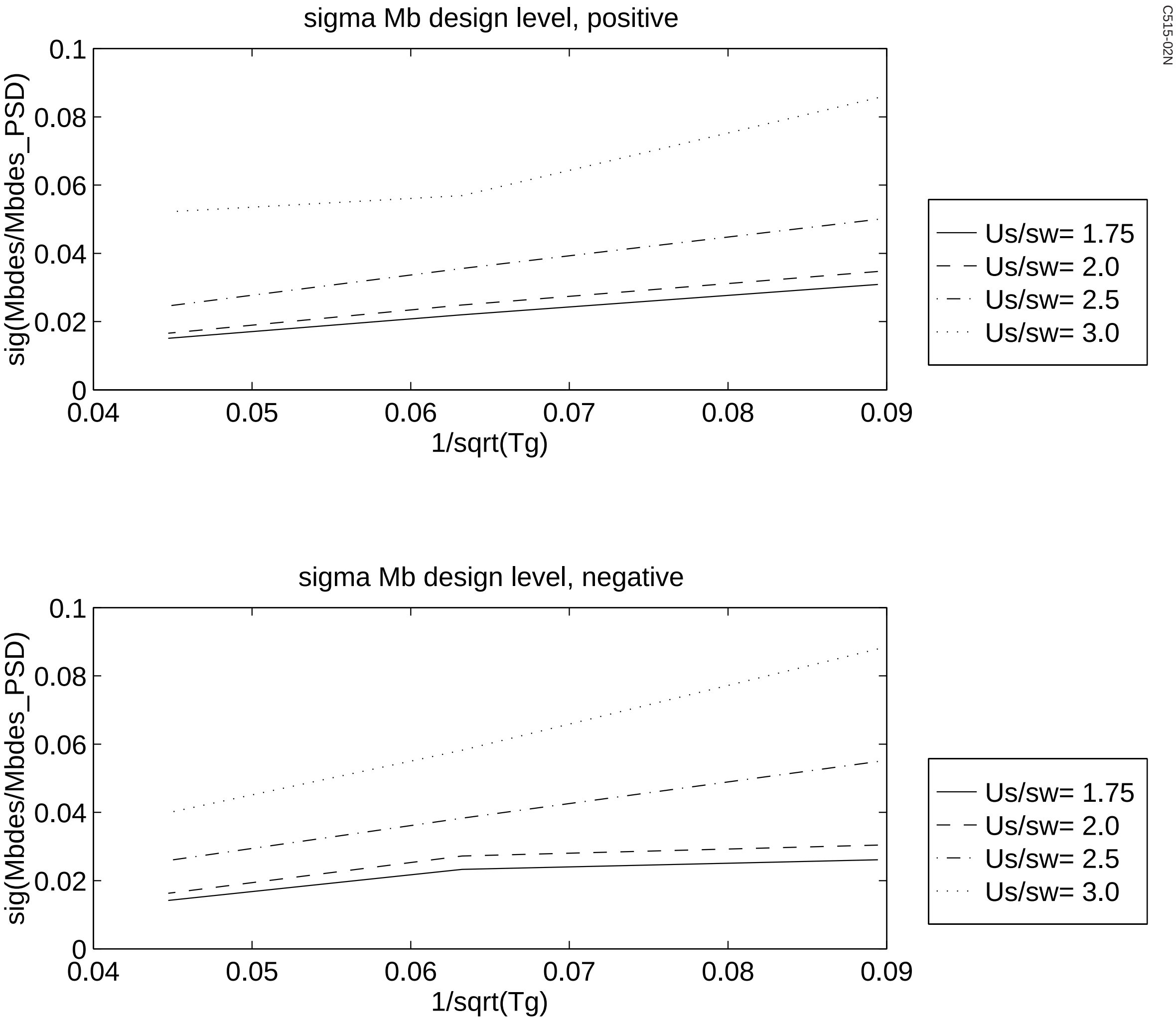
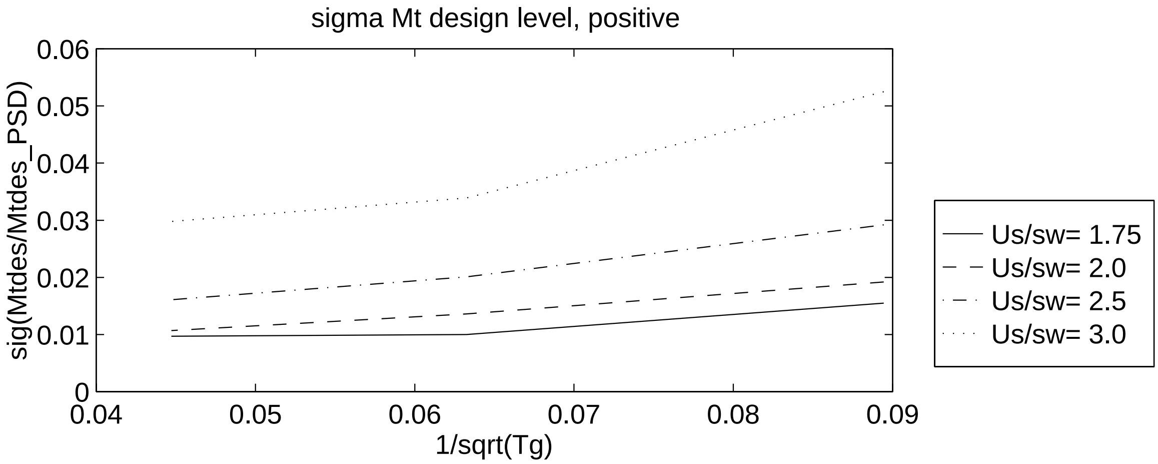


Fig. A.1 Linear A310 bending moment design level standard deviation as function of turbulence patch length



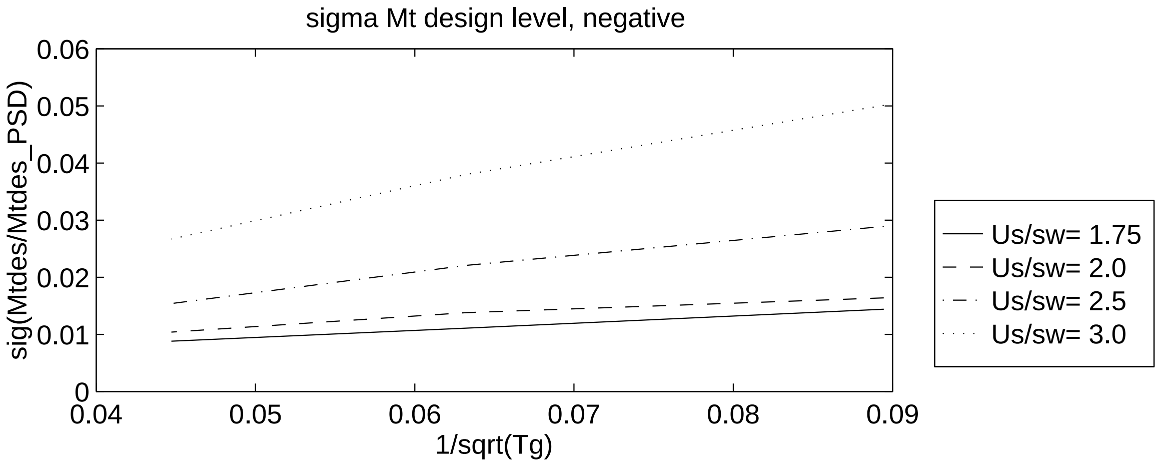


Fig. A.2 Linear A310 torsion moment design level standard deviation as function of turbulence patch length

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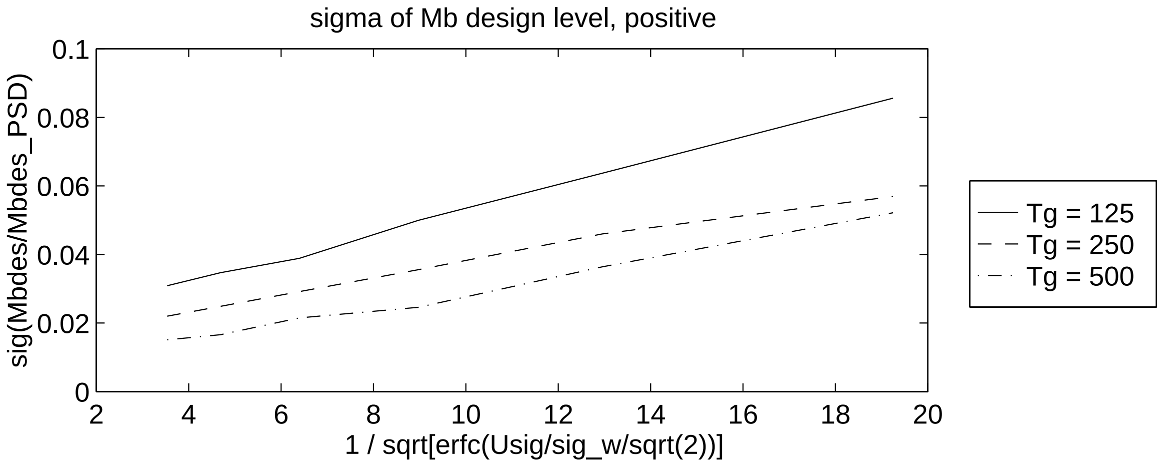
C515-02N



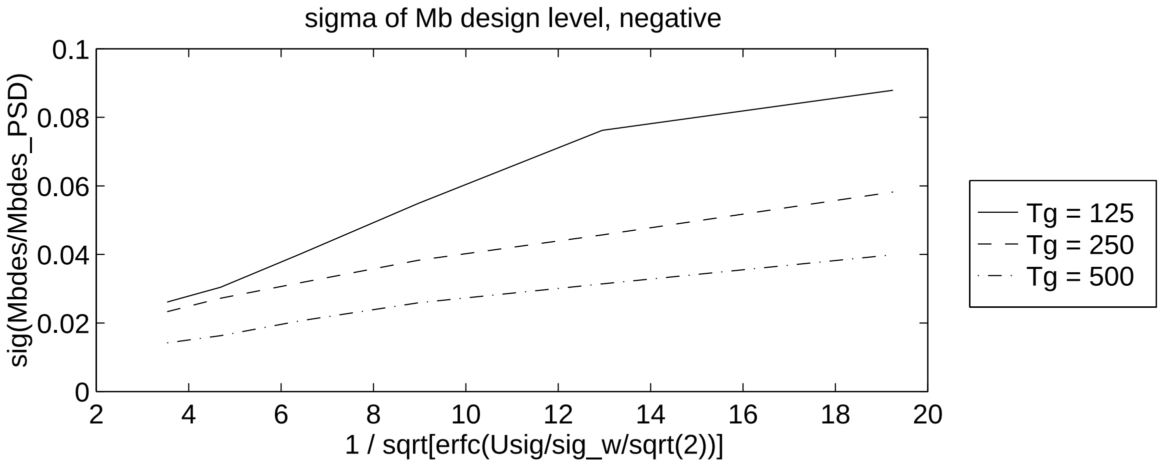


Fig. A.3 Linear A310 bending moment design level standard deviation as function of turbulence intensity σ_W

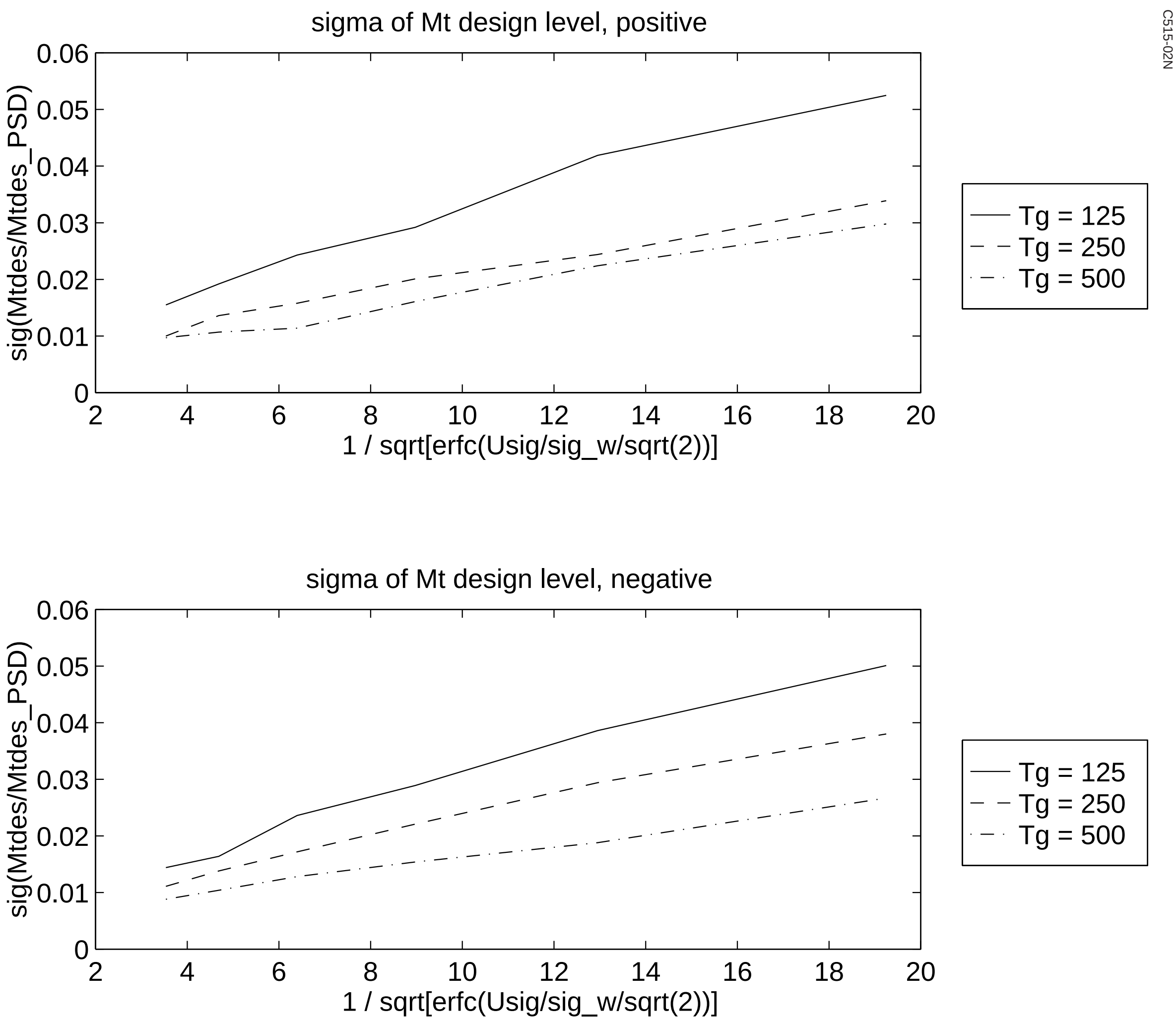


Fig. A.4 Linear A310 torsion moment design level standard deviation as function of turbulence intensity σ_w

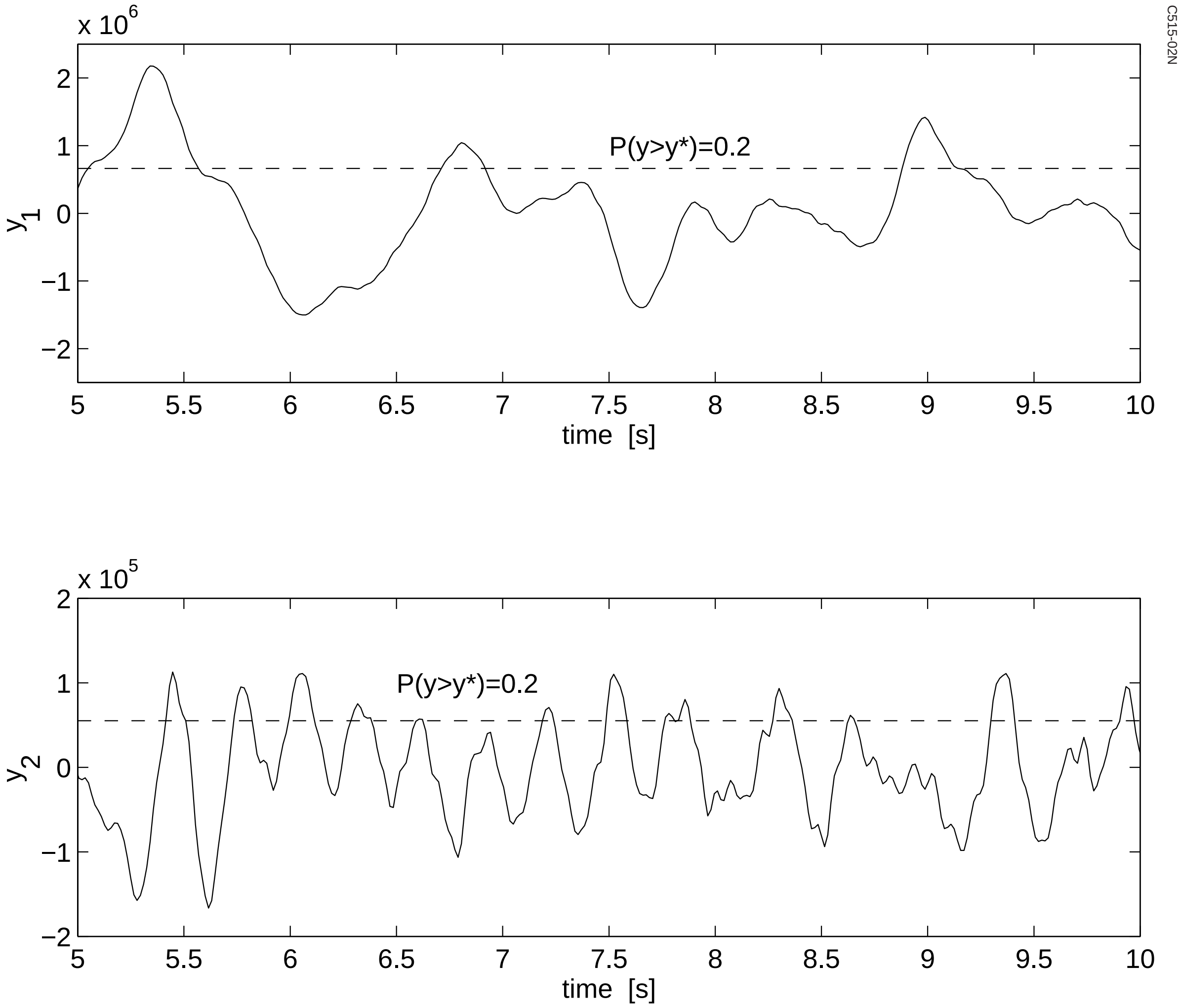


Fig. A.5 Time responses to stochastic gust of two load quantities of an aircraft

C515-02N

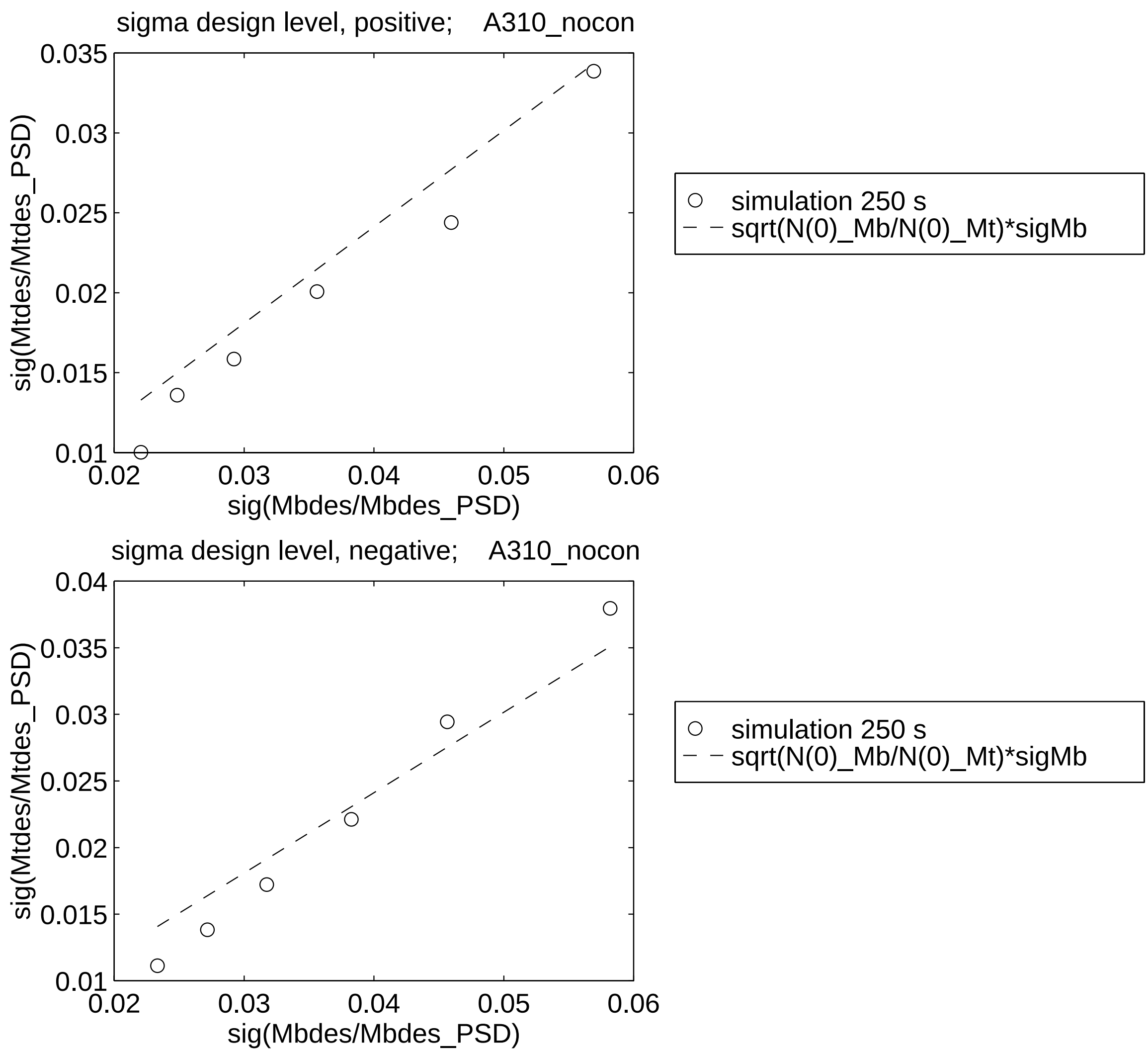


Fig. A.6 A310 torsion design level standard deviation as function of bending design level standard deviation

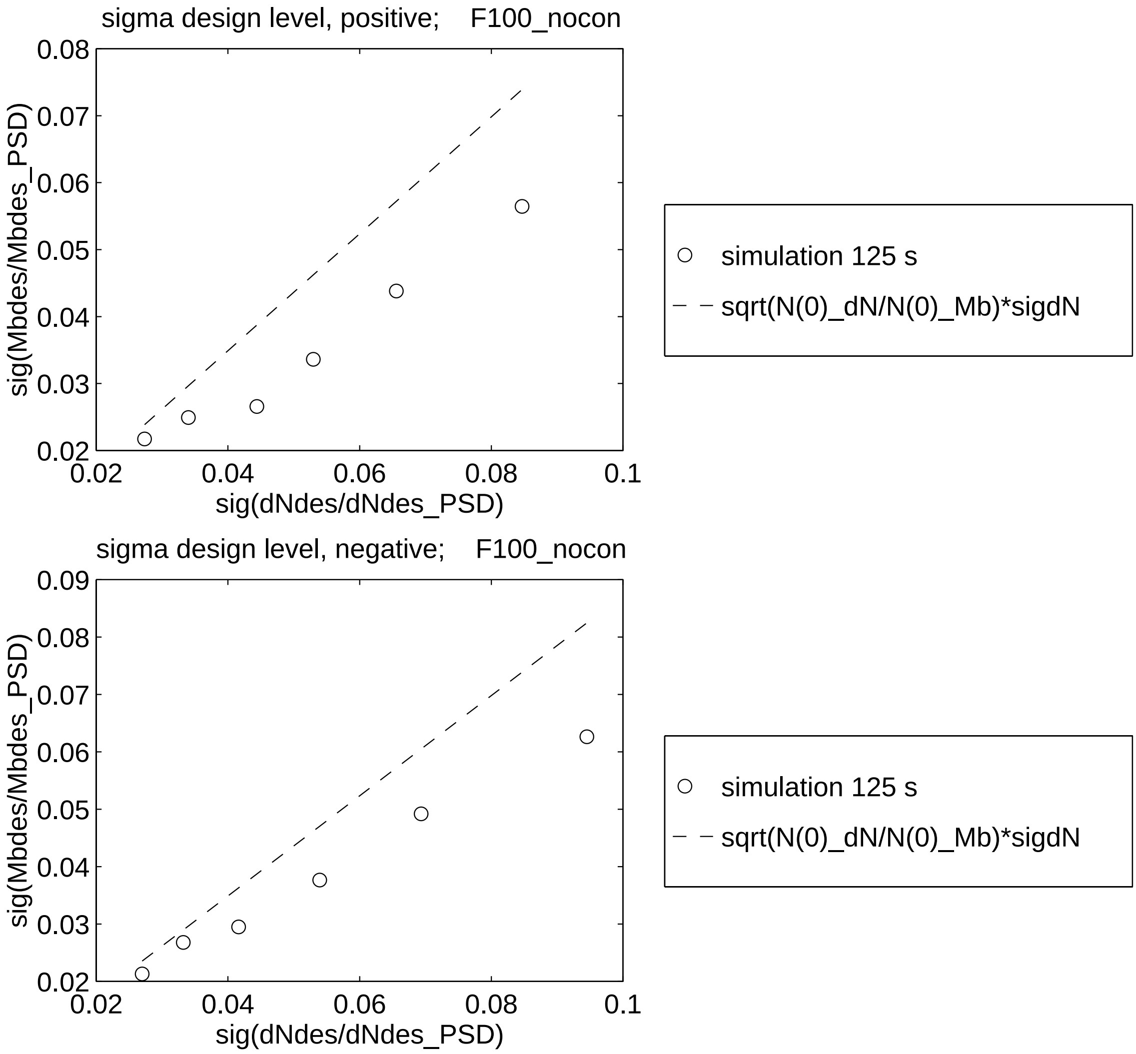


Fig. A.7   F100 bending design level standard deviation as function of load factor design level standard deviation

C515-02N

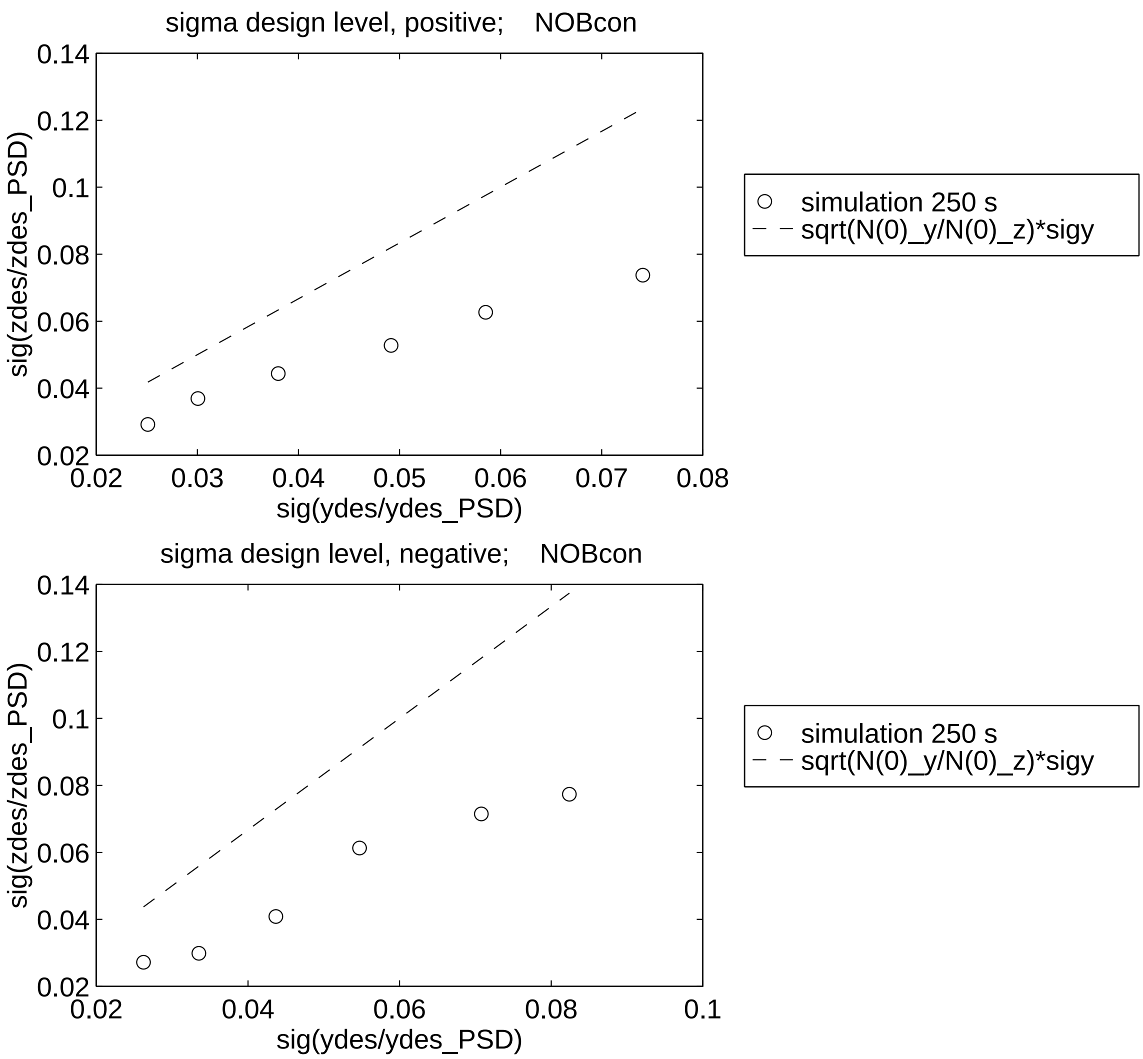


Fig. A.8 Noback model output z design level standard deviation as function of output y design level standard deviation

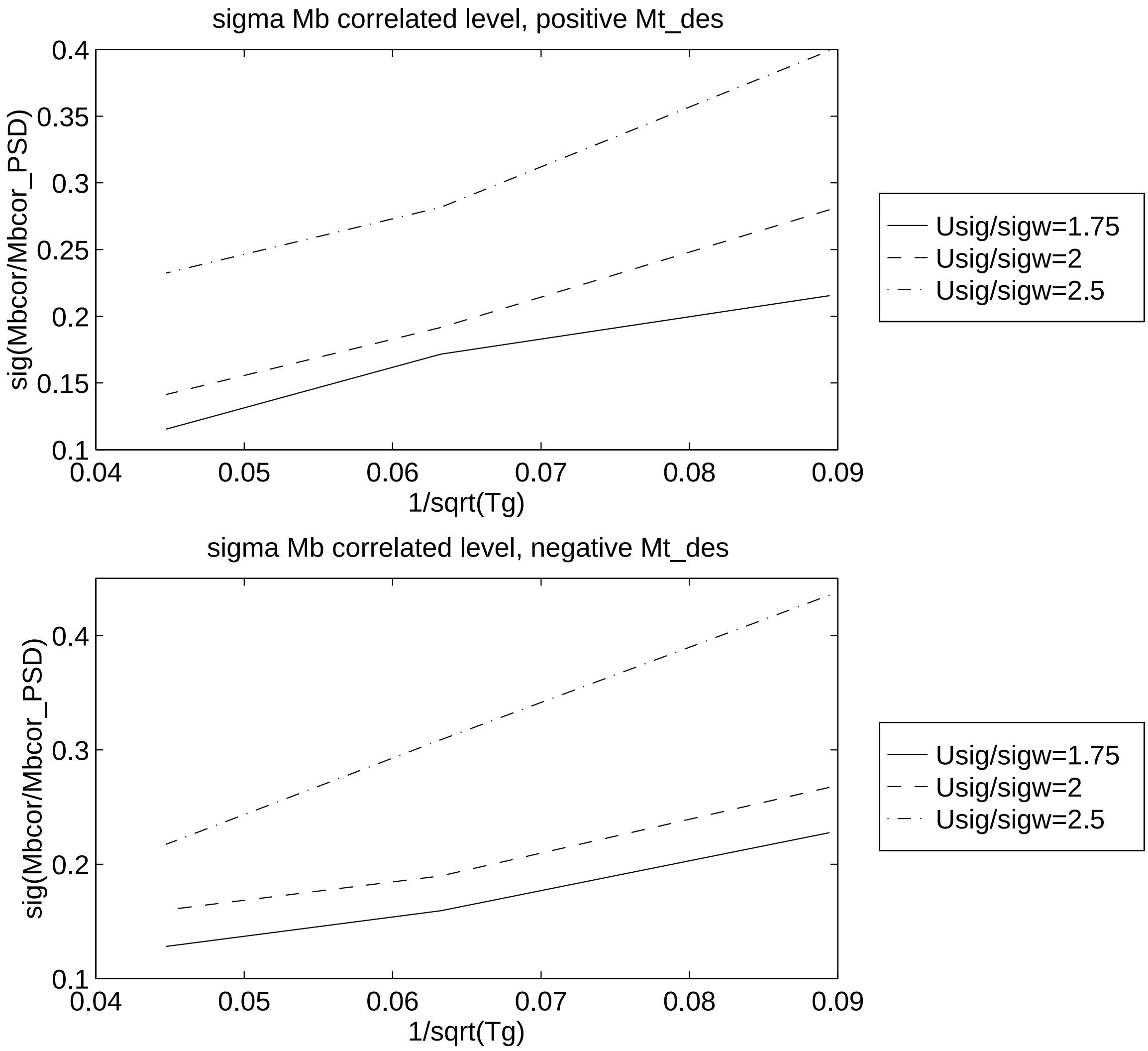


Fig. A.9 Linear A310 bending moment correlated level standard deviation as function of turbulence patch length

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C515-02N

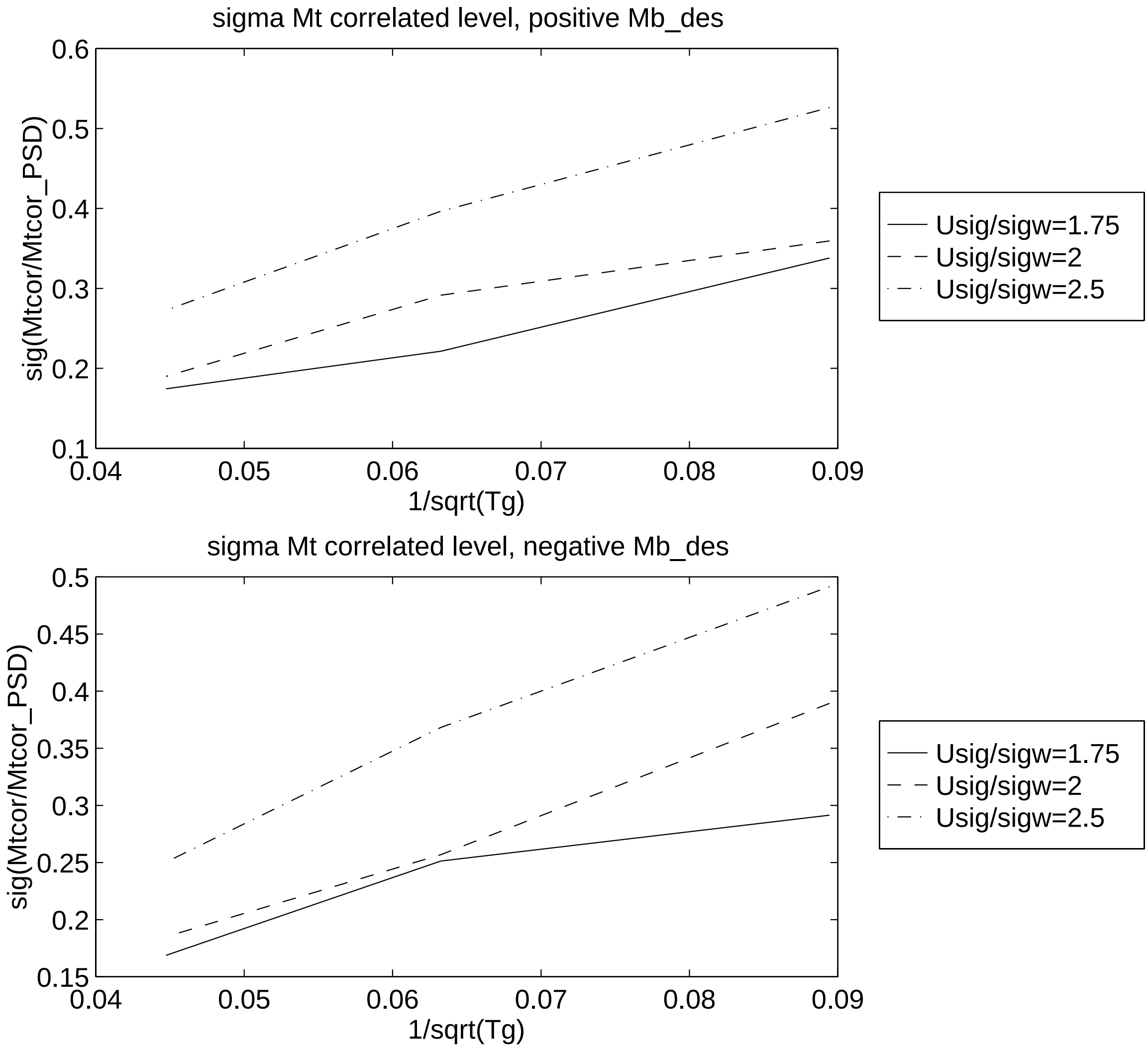
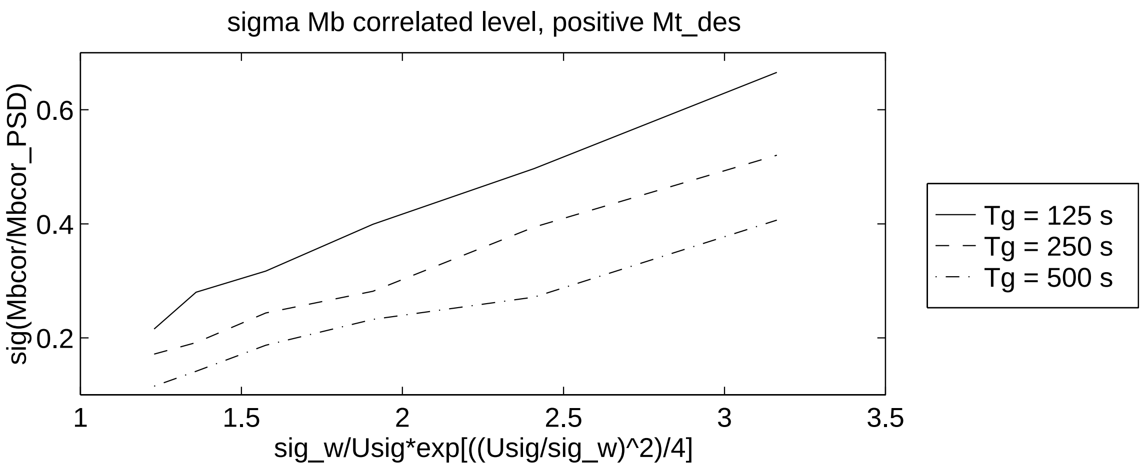


Fig. A.10 Linear A310 torsion moment correlated level standard deviation as function of turbulence patch length



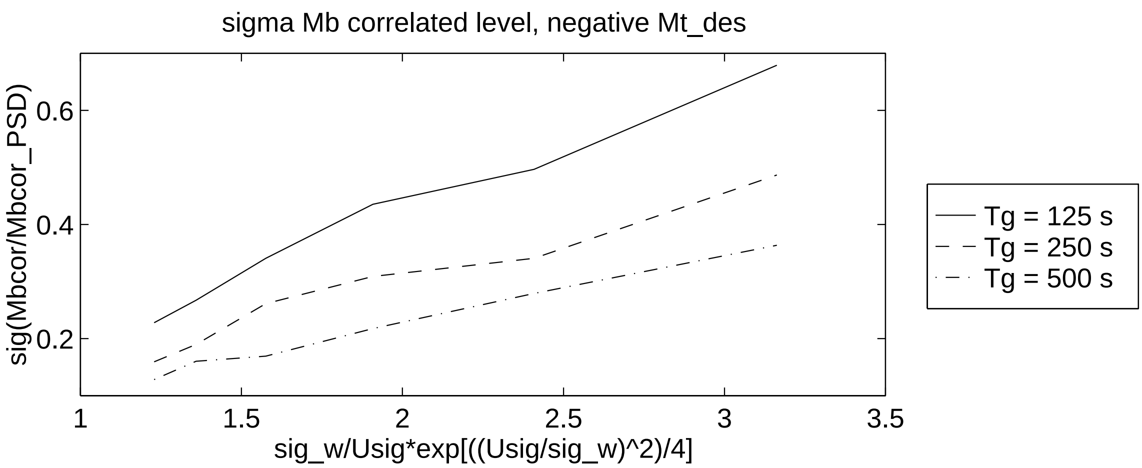


Fig. A.11 Linear A310 bending moment correlated level standard deviation as function of turbulence intensity σ_w

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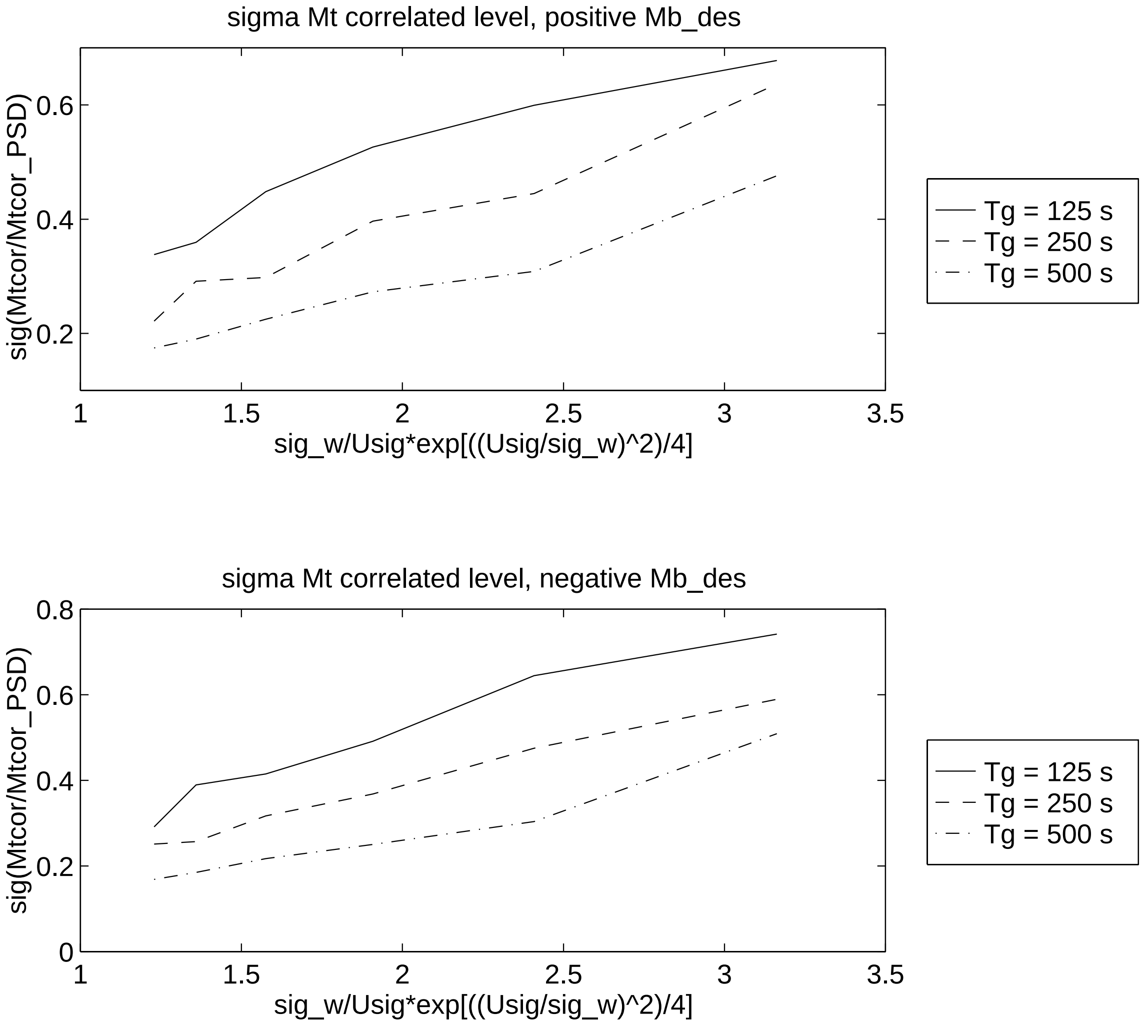
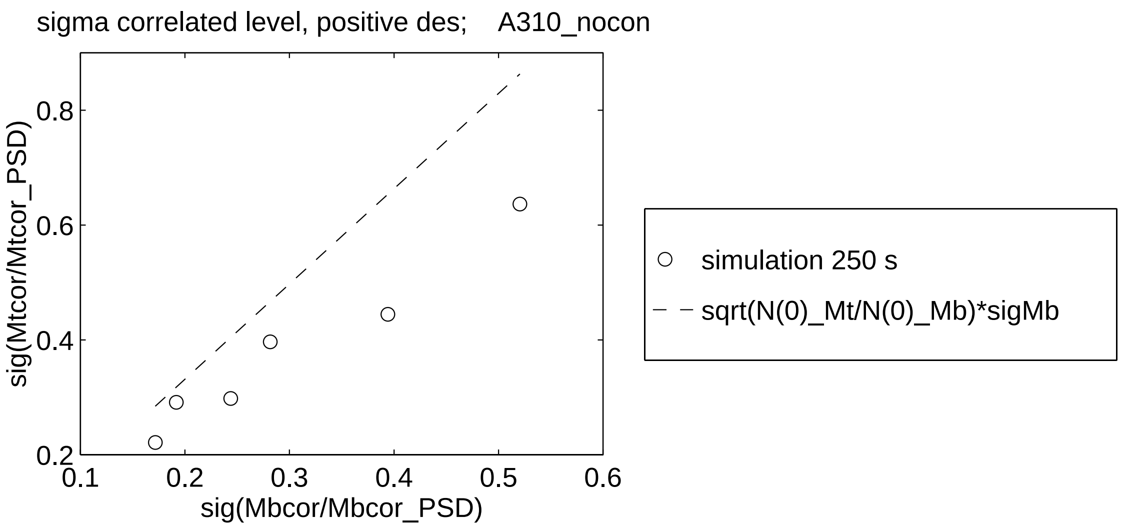


Fig. A.12 Linear A310 torsion moment correlated level standard deviation as function of turbulence intensity σ_w



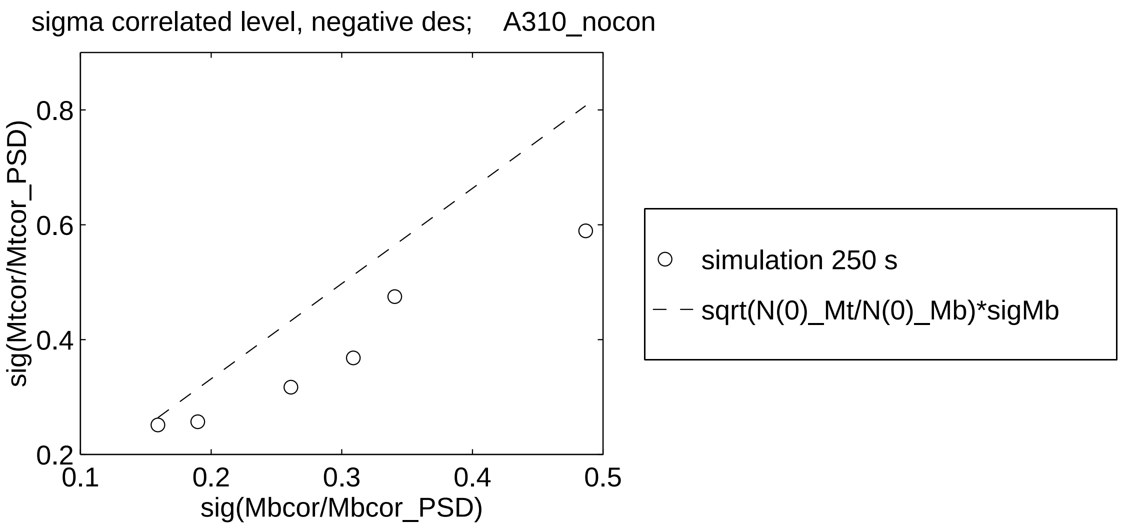


Fig. A.13   A310 torsion correlated level standard deviation as function of bending correlated level standard deviation

C515-02N

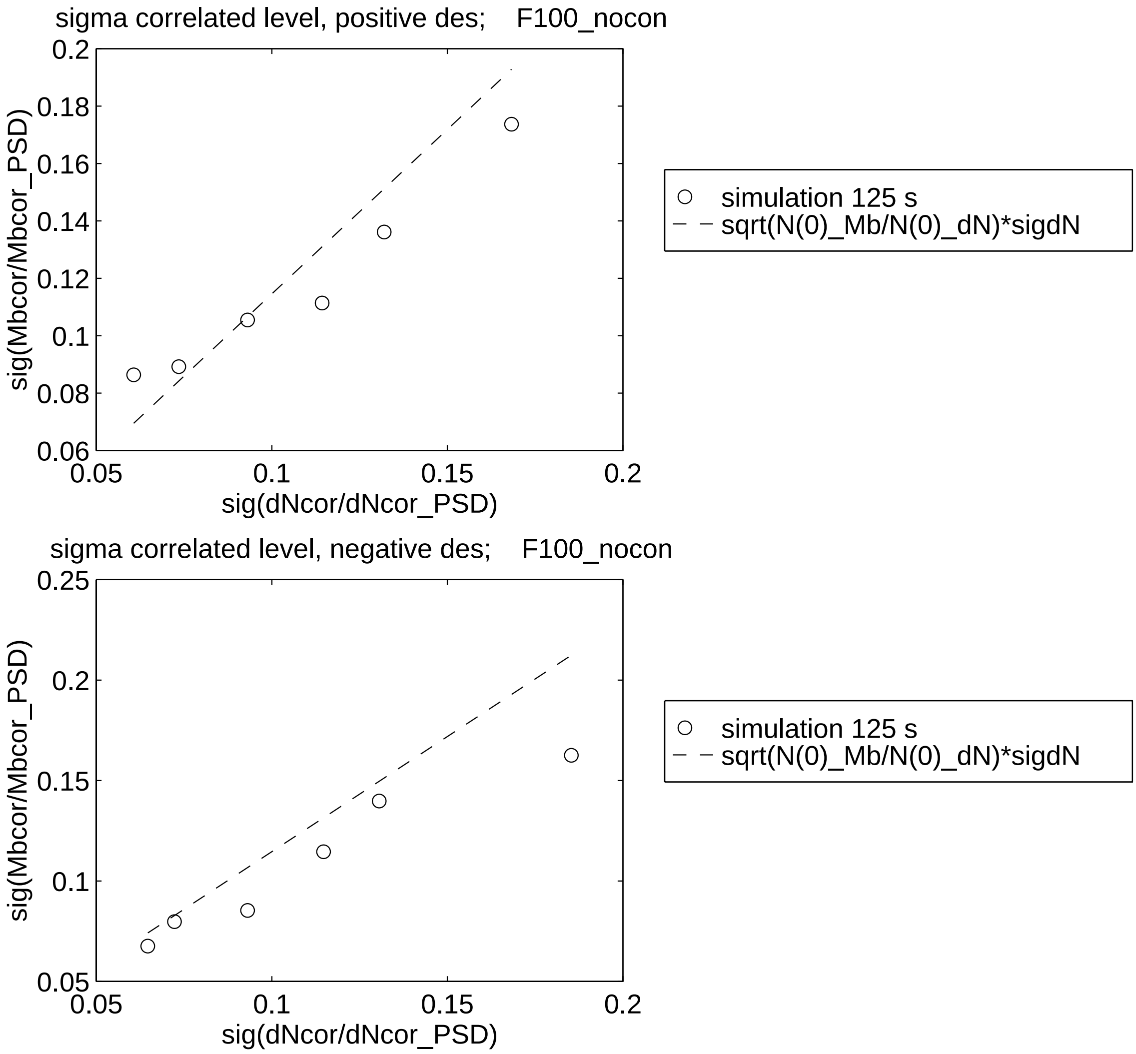
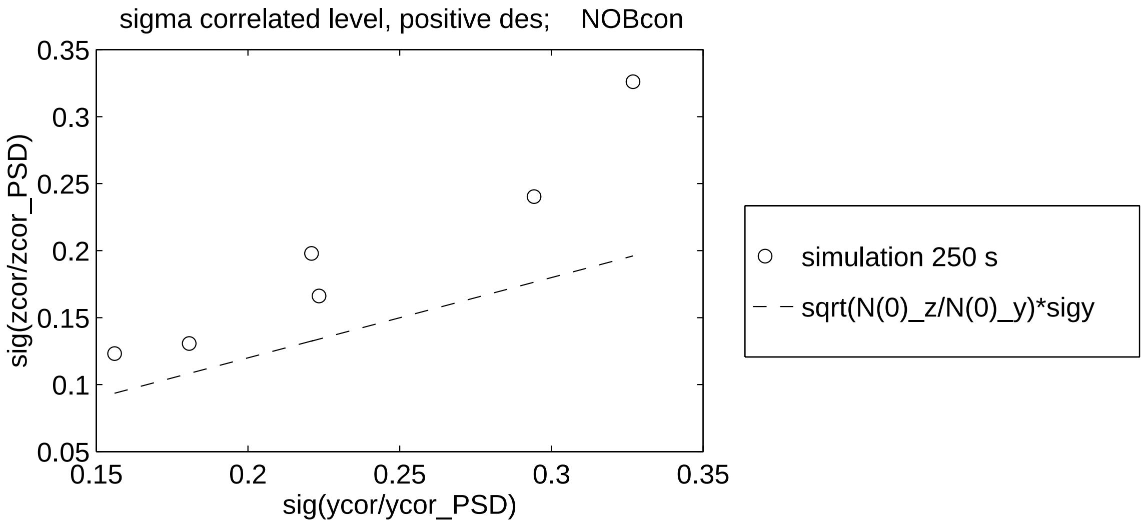


Fig. A.14 F100 bending correlated level standard deviation as function of load factor correlated level standard deviation



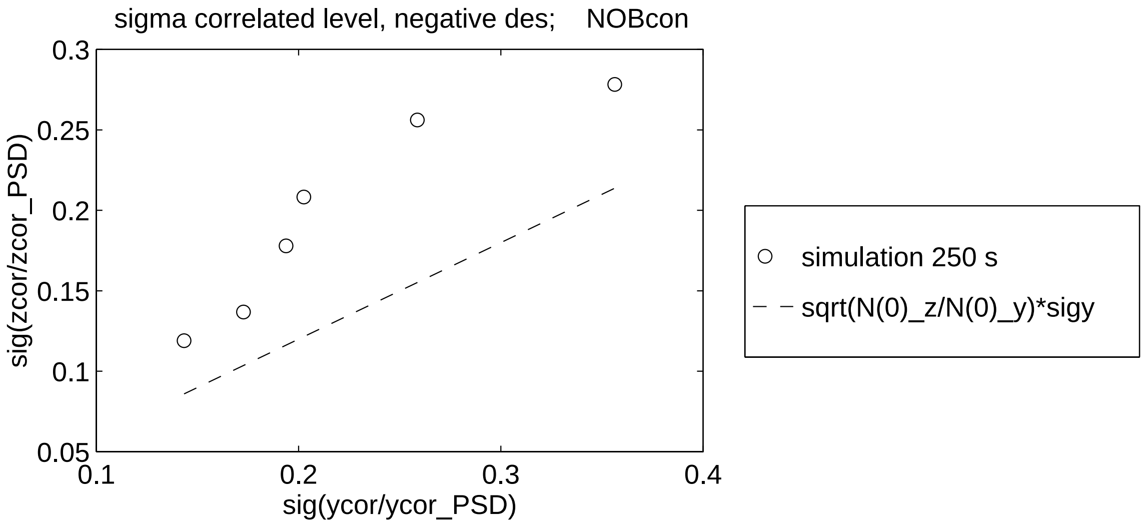
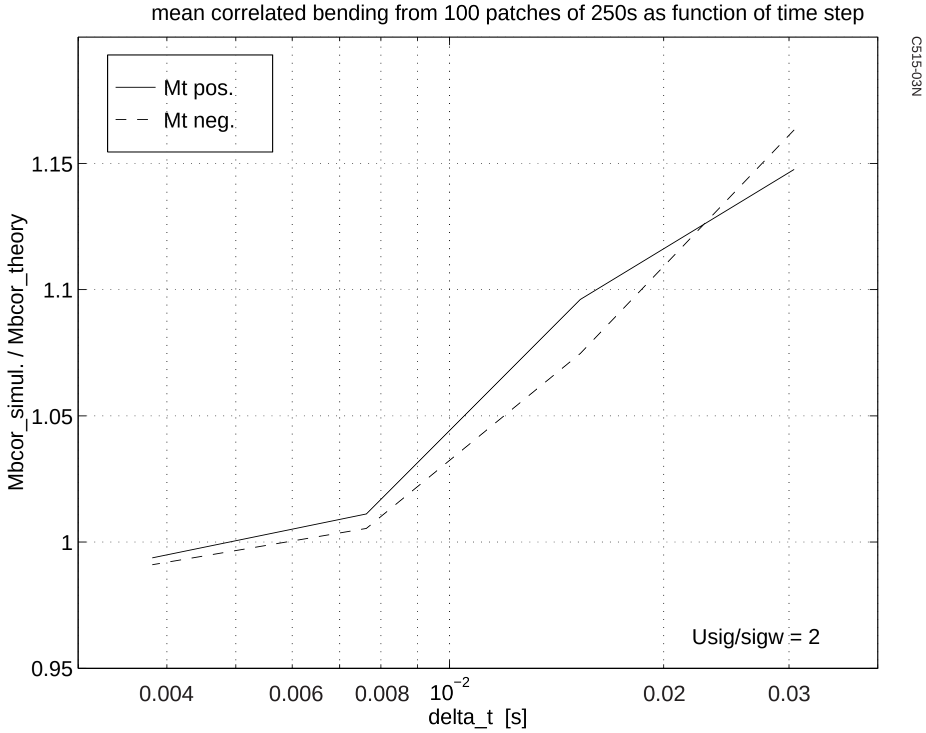


Fig. A.15 Noback model output z correlated level standard deviation as function of output y correlated level standard deviation



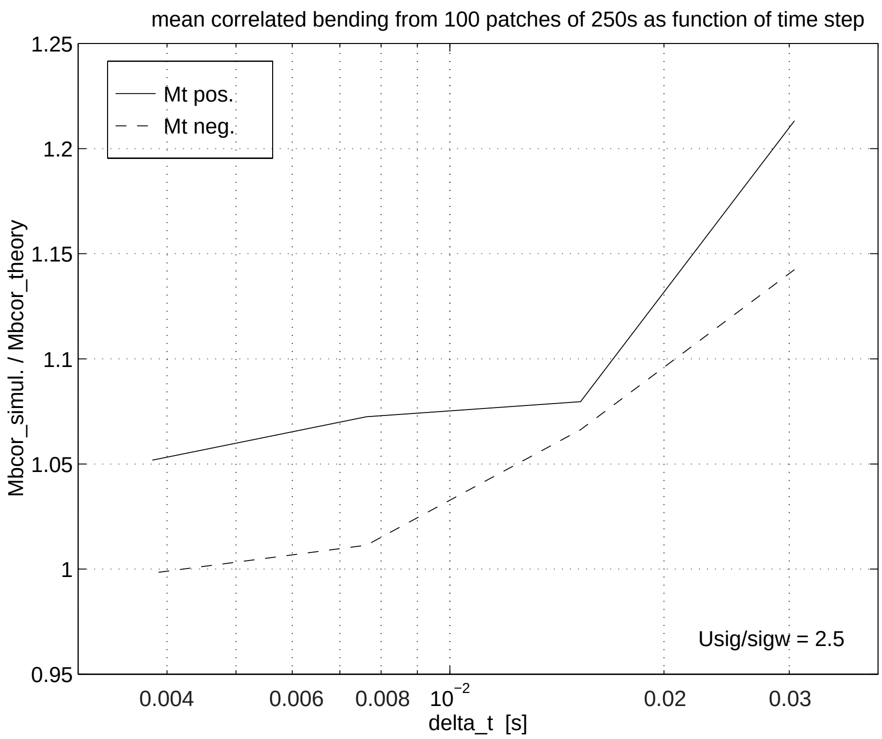
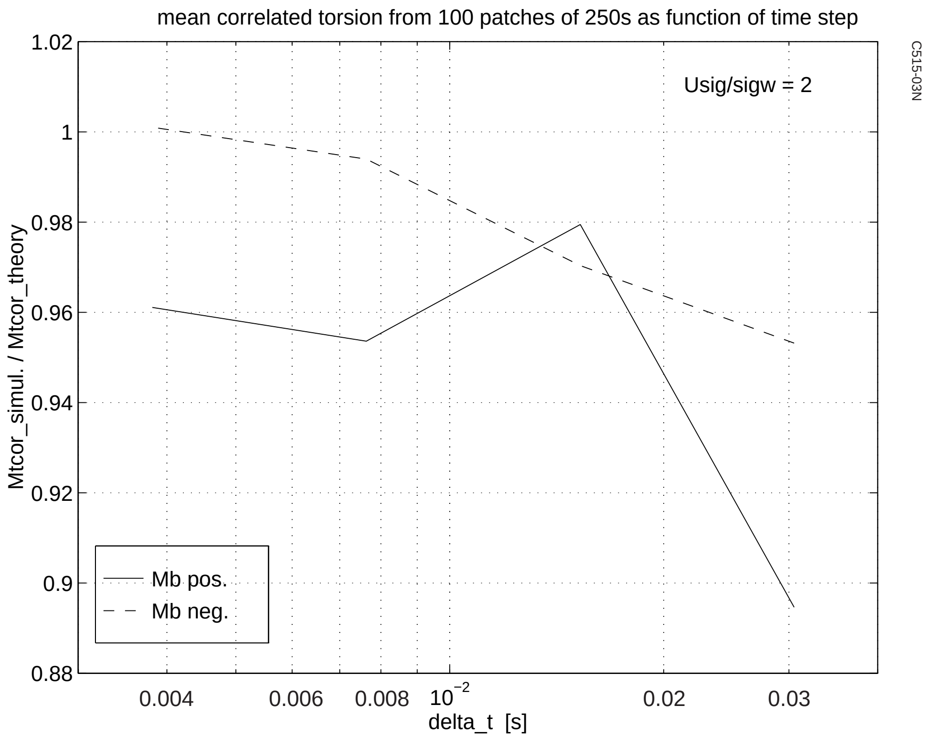


Fig. A.16 A310 mean correlated bending from 100 patches of 250 s as function of time step, for two turbulence intensity levels



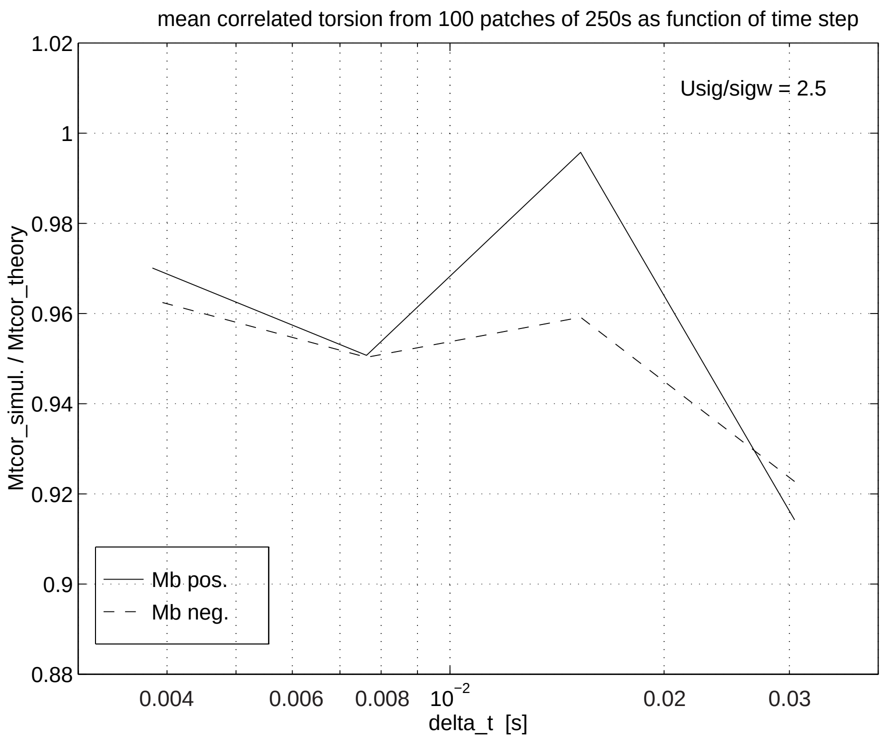


Fig. A.17 A310 mean correlated torsion from 100 patches of 250 s as function of time step, for two turbulence intensity levels

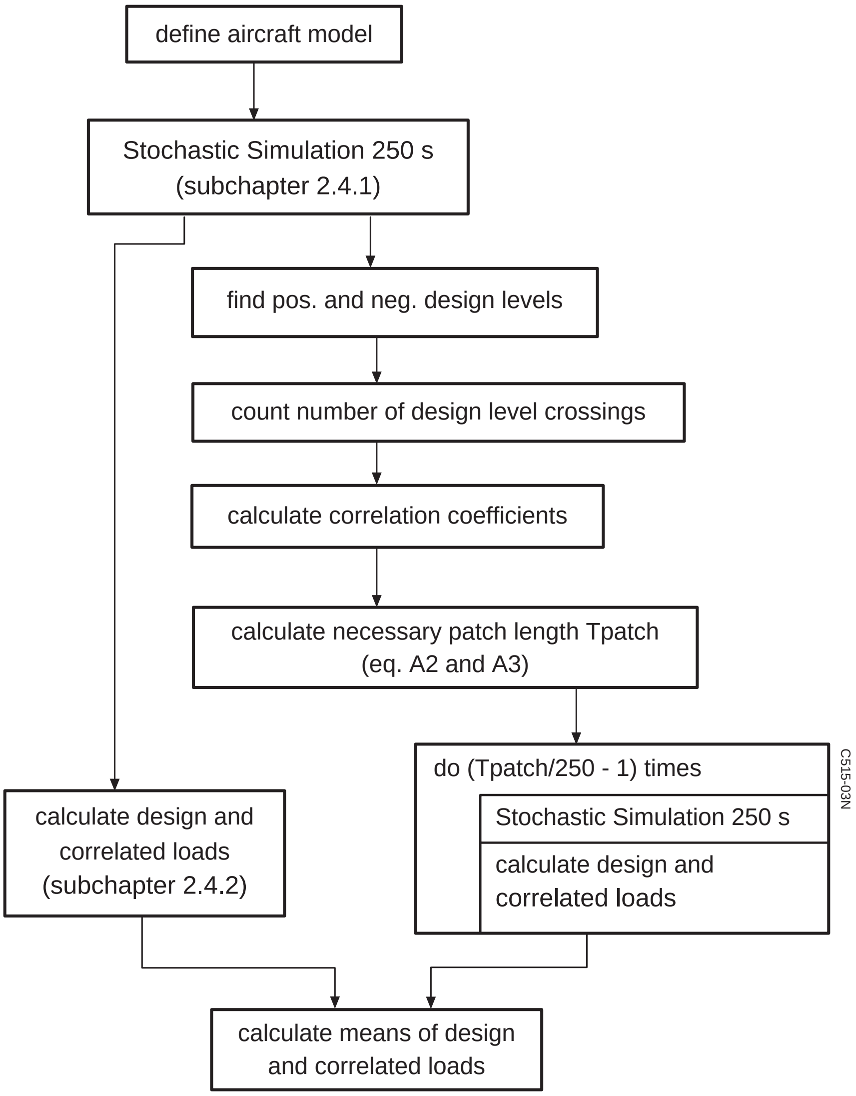


Fig. A.18 The proposed Stochastic Simulation procedure

B The aircraft models used

Three symmetrical aircraft models have been used in this research. These are the same models as in reference 6. The first one is a simple model of a large transport aircraft with two degrees of freedom, pitch and plunge, and a load alleviation system that feeds back the centre of gravity acceleration to aileron deflection. The model is shown in figure B1. The functions C(s) and D(s) are the transformed Wagner - and Küssner functions representing unsteady aerodynamic loads. Output y in the figure is the centre of gravity acceleration, and output z is the centre of gravity acceleration caused by aileron action only. This model is called the Noback-model in this report.

The second model represents an aircraft with "Fokker-100-like" characteristics. This model has the two rigid degrees of freedom pitch and plunge, and ten symmetric flexible degrees of freedom. This flexibility is represented by the first ten natural modes of the aircraft structure. Aerodynamic forces are calculated with strip theory, and unsteady aerodynamics is accounted for by Wagner - and Küssner functions. The wing has 27 strips and the tail 13; the fuselage is considered as one lifting surface. The Wagner - and Küssner functions are calculated at 3 locations on the wing and at 1 location on the horizontal tail.

The gust penetration effect and the time delay of the downwash angle at the tail with respect to the wing are included. Taking these two effects into account, makes it necessary to apply time delays to the gust input, and to the state variables (because the angle of incidence at the reference point on the wing is a function of all states) respectively. Especially the latter considerably increases the total number of system states.

A Load Alleviation System is implemented in the model that feeds back the load factor to a (symmetrical) aileron deflection. Figure B2 shows the aircraft system with the feedback loop to the aileron input. The configuration of the Fokker 100 model used in this report is:

$m_{a/c} = 40{,}000$ kg $I_y = 1.782\ 10^6$ kgm^2

V = 220 m/s   altitude = 7000 m

centre of gravity location at 25 % mean-aerodynamic-chord.

The third model has been distributed at the Gust Specialists Meeting of March 1995. It represents an A310 aircraft, containing plunge, pitch, and 3 symmetric flexible degrees of freedom. Unsteady response is assumed instantaneous, and gust penetration is not represented. The aircraft with control system is depicted in figure B3. The centre of gravity acceleration is fed back to both the ailerons and the spoilers through a feedback gain of 30 degrees per g load factor. Ailerons and spoilers have the same authority: deflections between 0 and 10 degrees. This means that the nonlinearity in this control system is "non-symmetric"; the control surfaces can only deflect upward. The load quantity outputs of this system are the increments of:

- Engine lateral acceleration [g].
- Wing bending moment [lb.ft].
- Wing torque [lb.ft].
- Load factor [g].

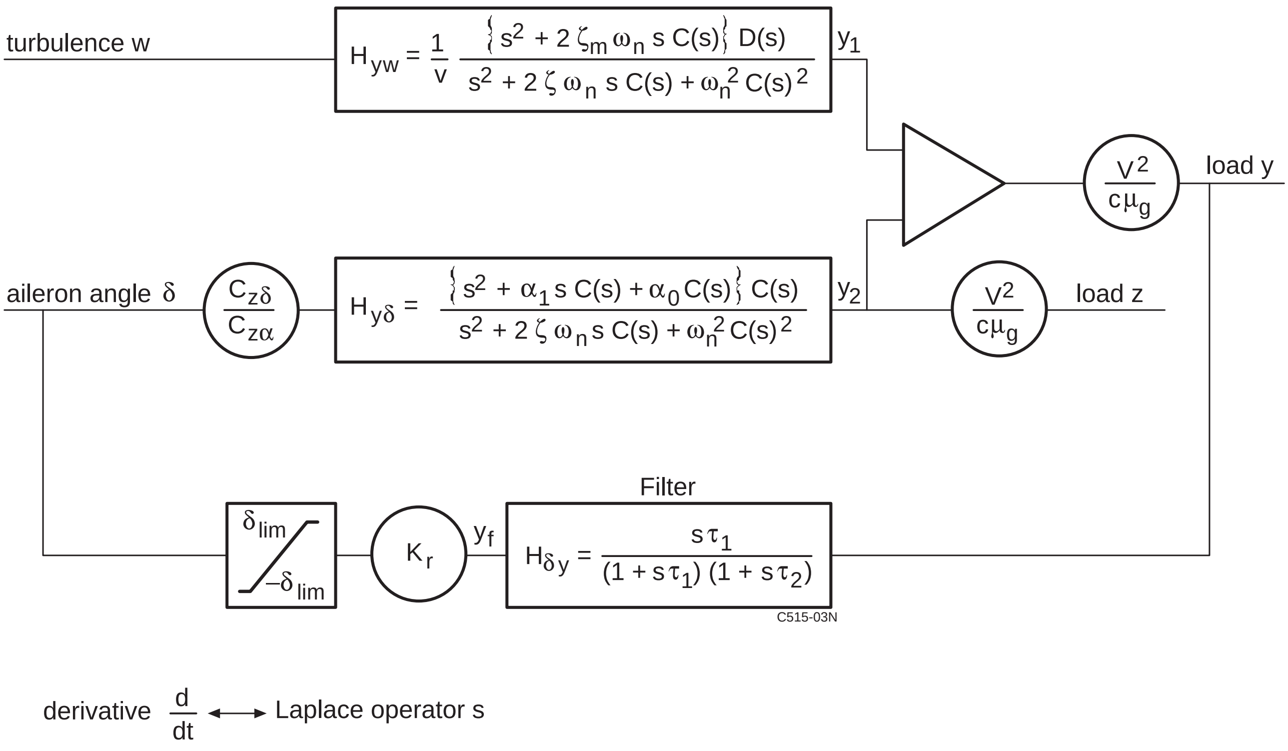


Fig. B.1 Noback model with load alleviation system (from Ref. 1)

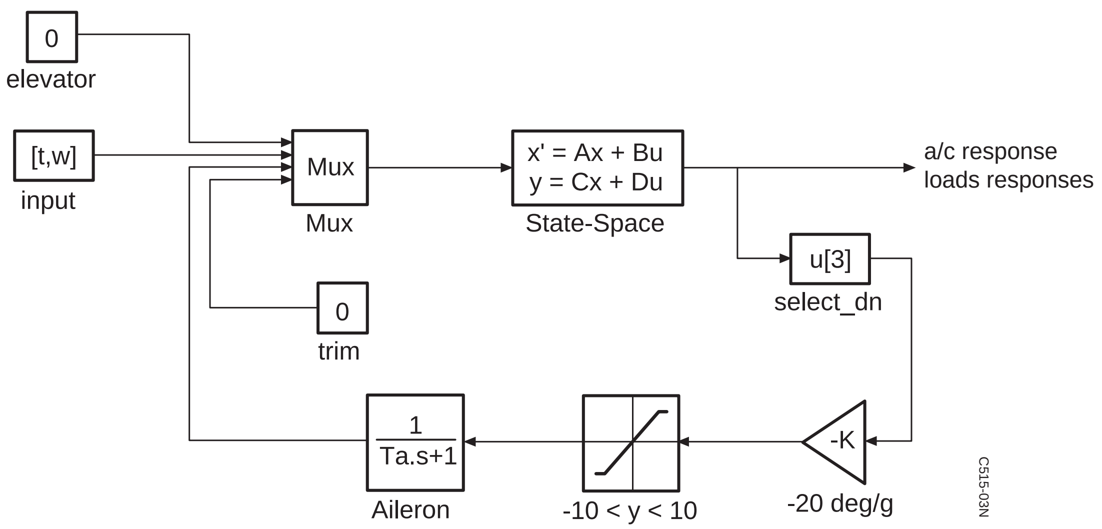


Fig. B.2 Fokker 100 aircraft model with load alleviation using ailerons (Ref. 6)

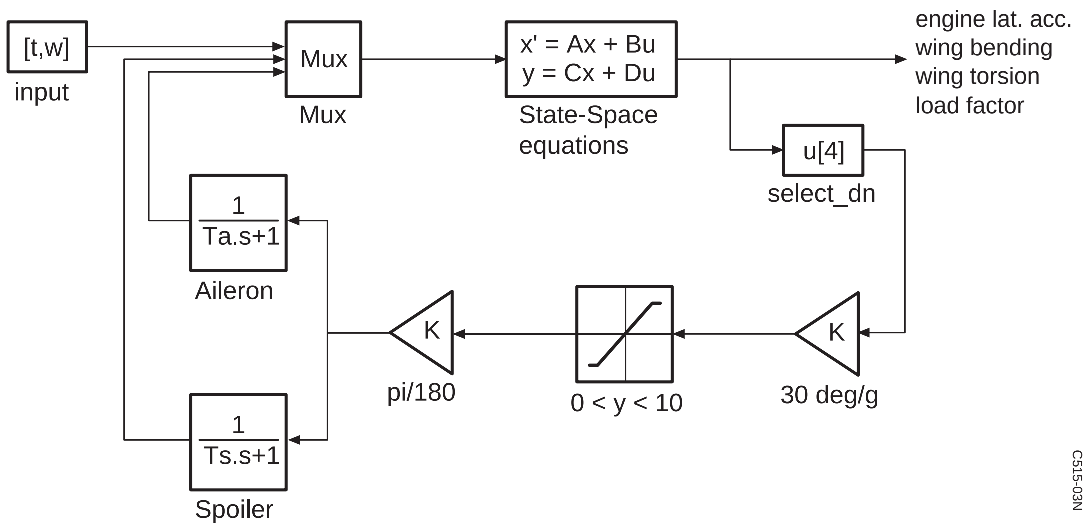


Fig. B.3 A310 model with load alleviation system (from Ref. 6)

C Deterministic PSD methods

The three Deterministic PSD design load calculation methods studied here are:

- Matched Filter Based 1-Dimensional Search (MFB).
- Indirect Deterministic Power Spectral Density method (IDPSD).
- Brink-Spalink's Spectral Gust procedure (SG).

Figure C1 and table C1 from reference 5 summarize the procedures. An input signal to the "first aircraft system", H_1, is generated by feeding a pulse through a (von Karman) gust filter G, with $|G(jf)|=[\Phi^n_{ww}(f)]^{1/2}$. The power spectrum of the input to the first system will thus have the shape of the von Karman spectrum. The pulse strength k is variable in the MFB method, and constant in the IDPSD ($k=U_\sigma$) and SG ($k=U_\sigma\sqrt{T}$, where T = length of gust input) methods. It should be noted, that the gust filter in the MFB method is only an <u>approximation</u> of the von Karman spectrum, and in the version used in this report it is the Hoblit approximation of reference 7.

The first aircraft system, H_1, represents the nonlinear aircraft equations of motion in MFB and SG. In IDPSD, H_1 is a linearized version of the nonlinear aircraft, by replacing the nonlinearity by a linear element with an "equivalent gain", K_{eq}. K_{eq} is a multiplication factor to the original gain in the feedback loop, with $0 \le K_{eq} \le 1$.

For the output y for which the design level is to be determined (output s of the first system), a norm is calculated as follows:

$$y_{norm} = \sqrt{\int_0^\infty s^2(t)\,dt}\ .$$

It can easily be verified in table C1, that $y_{norm}=\bar{A}U_\sigma\sqrt{T}$ in the SG method, if H_1 is a linear transfer function. The SG method now concludes by dividing y_{norm} by $\sqrt{T}$, which is then equal to the linear PSD design load $y_{des}=\bar{A}U_\sigma$ if the system is linear.

In MFB and IDPSD, the first system response is now inverted in time (conjugation of the frequency domain representation of the signal), normalized by y_{norm}, and multiplied by U_σ. The resulting signal is fed through the gust filter again, and then serves as turbulence input to the "second aircraft system", H_2, which represents the nonlinear aircraft system in both methods. For linear systems, H_1 is equal to H_2, and in table 1 it is shown that y_{norm} is equal to $\bar{A}U_\sigma$ (the linear PSD design load) if $k=U_\sigma$.

The output of the second system in MFB and IDPSD can be described in the frequency domain as:

$$Y(jf) = \frac{U_\sigma}{y_{norm}} \sqrt{\Phi^n_{ww}(f)}\, H_2(jf)\, k \sqrt{\Phi^n_{ww}(f)}\, H_1^*(jf).$$

For linear systems, H_1 and H_2 exist and are identical transfer functions of the linear system, H. The value of y_{norm} is equal to $k\overline{A}$, see table C1. The time domain signal y(t) can be found from the frequency domain representation Y(jf) by inverse Fourier transformation:

$$y(t) = \int_{-\infty}^{\infty} \frac{U_\sigma k}{k\overline{A}} |H(jf)|^2 \Phi^n_{ww}(f) e^{j2\pi ft} df.$$

The design load in MFB and IDPSD is now defined as the maximum of this output signal. The maximum in this signal will be reached if the exponent j2πft is equal to zero for all frequencies. This occurs at t=0, and the maximum output is:

$$y_{max} = y(0) = \frac{U_\sigma}{\overline{A}} \int_{-\infty}^{\infty} |H(jf)|^2 \Phi^n_{ww}(f) df = U_\sigma \overline{A} = y_{des,PSD}.$$

This shows that MFB and IDPSD also result in the PSD design load in linear cases.

A correlated load z_{cor} in MFB and IDPSD is the value of output z at the moment that y reaches its maximum (design) value. In the linear case this results in the correlated value that would also follow from the linear PSD method:

$$z_{cor} = z(0) = \frac{U_\sigma}{\overline{A}_y} \int_{-\infty}^{\infty} H_z(jf) H_y^*(jf) \Phi^n_{ww}(f) df = \rho_{yz} \overline{A}_z U_\sigma.$$

With the correlation coefficient ρ_{yz} as defined in equation 8 of subchapter 2.2.1.

The correlated SG loads are calculated with:

$$z_{cor} = \frac{\rho_{yz} z_{norm}}{\sqrt{T}}$$

where ρ_{yz} is calculated from the deterministic time responses by:

$$\rho_{yz} = \frac{\int_0^T s_y(t)s_z(t)dt}{y_{norm} z_{norm}}.$$

The SG correlated load is equal to $\rho_{yz}\bar{A}_z U_\sigma$, which is the correct correlated load again, according to linear PSD theory.

For nonlinear systems, the three Deterministic methods apply different procedures:

- MFB varies the strength k of the input pulse to the first gust filter.
- IDPSD varies the value of the equivalent gain that represents the nonlinearity in the first system.
- SG varies the phase relation of the gust filter, which is limited to only four different phase relations.

The variation in each of these methods is used to find a maximum response of the aircraft system. In the SG method, the maximum y_{norm} of the different simulations of the first system is defined as the design load. In MFB and IDPSD, the maximum of the maxima (depending on k and K_{eq} respectively) in the responses of the second system is considered to be the design load.

Table C1 Elements of deterministic PSD-Procedures (Ref. 5)

<table>
<tr><td>Element</td><td>Matched filter
(Scott e.a.)</td><td>IDPSD
(Noback)</td><td>Spectral Gust
(Brink-Spalink e.a.)</td></tr>
<tr><td>Impulse
strength k</td><td>k variable</td><td>$k = U_\sigma$</td><td>$k = U_\sigma * \sqrt{T}$</td></tr>
<tr><td>Gust
Prefilter G(jf)</td><td>$|G(jf)| \approx \sqrt{\Phi^n(f)}$
One set $\varphi(f)$</td><td>$|G(jf)| = \sqrt{\Phi^n(f)}$
One set $\varphi(f)=0$
for all f</td><td>$|G(jf)| = \sqrt{\Phi^n(f)}$
four sets $\varphi(f)$</td></tr>
<tr><td>Aircraft
system $H_1(y)$</td><td>(Nonlinear)
set of equations
for output y</td><td>Linearized
equations;
variable
"equivalent gain"</td><td>Nonlinear
set of equations
for output y</td></tr>
<tr><td>Calculation
y-norm:</td><td colspan="3">$$y_{norm} = \left[\int_{-\infty}^{\infty} s^2(t)\,dt \right]^{1/2}$$
$$= \left[\int_{-\infty}^{\infty} s(jf)\ s^*(jf)\,df \right]^{1/2}$$

For linear system:
$$y_{norm} = \left[k^2 \int_{-\infty}^{+\infty} H_1.H_1^*\ G.G^*\,df \right]^{1/2} = k\ \bar{A}_y$$
if $k = U_\sigma \rightarrow y_{norm} = y_{des}$</td></tr>
<tr><td>"Critical
gust
profile" w(t)</td><td colspan="2">For linear systems
same profile for
matched filter and IDPSD</td><td rowspan="3">SG stops
here:
Four values
for y_{norm},

$$y_{des} = \frac{y_{norm}(max)}{\sqrt{T}}$$</td></tr>
<tr><td>Aircraft
system $H_2(y)$</td><td colspan="2">Nonlinear set
of equations</td></tr>
<tr><td>Y_{des}</td><td>Variable k
$y_{des} = [y_t]_{max}$</td><td>Variable gain
of $H_1(y)$
$y_{des} = [y_t]_{max}$</td></tr>
</table>

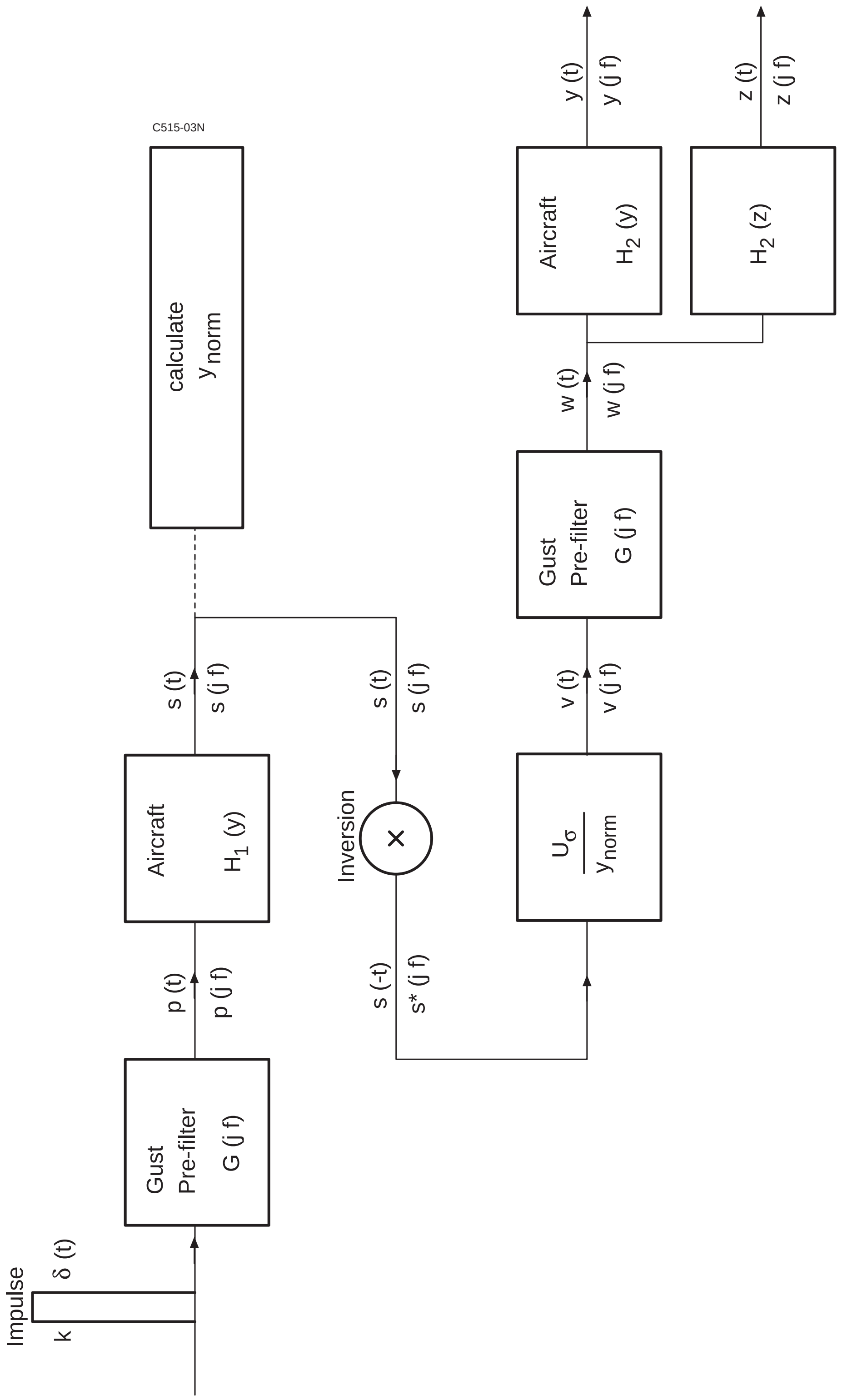


Fig. C.1 Schematic presentation of Deterministic PSD procedures (Ref. 5)